FINANCING OF INVESTMENT IN A MULTIPLIER-ACCELERATOR FRAMEWORK

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CERTIFICATE

This is to certify that the dissertation entitled **"FINANCING OF INVESTMENT IN A MULTIPLIER-ACCELERATOR FRAMEWORK"** submitted in partial fulfillment for the degree of **Master of Philosophy** of this university has not been previously submitted for any other university and is my original work.

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We recommend that the dissertation may be placed before the examiners for evaluation.

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Needless to add, the errors and mistakes, that still might remain, are all mine.

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CHAPTER 1:
INTRODUCTION

The Great Depression of the 1930s is widely regarded as a turning point in the evolution of economic theory. Many of the conventional economic principles, based on the assumption that market always clears at the full-capacity level of output, failed to explain an event like the Great Depression with evidence of gross underutilization of both capital and labor all over the economy. This led to faulty and misguided policies, probably best exemplified by then US President Herbert Hoover's attempt at balancing budgets and the Federal Reserve's attempt at defending the Gold Standards in the face of an economic slump. The failure of conventional economics and policy premises created a space for emergence of new economic principles seeking to explain these events from the demand side. John Maynard Keynes' classic book, *"The General Theory of Employment, Interest and Money"* (Keynes, 1936) probably stands out as the most famous representative of this school of thought. However, other writers like Richard Kahn (1972) and Michal Kalecki (1939 and 1971) also contributed to the development of this school of thought. The subsequent success of 'Keynesian' policy prescriptions increased its popularity and led to its inclusion in most of the basic macroeconomics textbooks of the Post World-War-II period.[1]

Two related concepts were made popular by this new emerging literature – Multiplier and the Accelerator. *Multiplier* is the amount by which the equilibrium output changes when autonomous aggregate demand increases by one unit (Dornbusch & Fischer, 1994, Chapter 3). *Accelerator* is the amount by which investment responds to change in income (or profits) by one unit. Integration of these two concepts led to the emergence of 'Multiplier-Accelerator models'.

A study of Keynesian economics and the simple interaction of multiplier and accelerator would, however, tell us that, though the Keynesian economics independently explains both a boom and depression, it does not offer a story of transition from one phase to another. In other words, by predicting a monotonic behavior of investment, income and employment, it fails to explain the phenomenon of business cycles. This was

[1] There are, however, many interpretations of Keynes. Unless otherwise specified, we use the word 'Keynesian' in this paper to imply the Hicks-Samuelson interpretation, often referred to as the 'Neoclassical Synthesis'. This interpretation is found in some of the standard macroeconomics textbooks like Dornbusch and Fischer (Dornbusch and Fischer, 1994)

probably not as relevant in the 1950s and 1960s, when the developed countries enjoyed a prolonged boom, helped to a great extent by the Marshall Plan in the 1950s and major wars in various parts of the world in the 1960s. The Great Depression was then beginning to be regarded increasingly as an outcome of a policy mistake, never to be repeated again.[2] Many developing countries, no doubt, were going through much slower rates of growth. These were, however, regarded as results of structural imbalances and not cyclical factors. Once corrected, these countries could also repeat the developed country experience of sustained high growth of investment, income and employment. The experience of the last three decades since 1970s, however, brought the idea of business cycle back into academic discussions. The cyclical behavior of investment, income and employment has become quite evident since mid-seventies, with booms interspersed with recessions, and even depressions. It has also been observed that transition from boom to recession is often either accompanied with or preceded by problems in the financial sector – much like the Great Depression – an aspect not dealt with in the simple multiplier-accelerator models.

The simple multiplier-accelerator models, if viewed as a reflection of the real world, thus, would have two major limitations:

1. They would predict a monotonic movement of income, employment and profits, with no explanation for a turnaround. In other words, these models would offer an explanation for that part of the business cycle where the income and employment either increase or decrease monotonically. Without addition of anything else, this would make these models unbounded.
2. Financing of investment (which is an important aspect in any modern economy) would find little mention in most models of this genre.

We know from our experience in the last three decades that both the above limitations are too serious to be ignored. The purpose of the following chapters is to close this gap in the multiplier-accelerator models, in order to make it more relevant in light of the recent experience.

[2] There were, in fact, instances of conferences being held by economists with topics like, "Is Business Cycle Obsolete?" (Krugman, 1999)

The next chapter consists of a survey of existing theories of both standard multiplier-accelerator models and the literature attempting to explain cyclical behavior of financial sector. Chapter 3 then tries to integrate these aspects of financial sector in the multiplier-accelerator models. The conclusions from the entire study are summarized in the final chapter.

CHAPTER 2:

A SURVEY OF EXISTING LITERATURE

The survey is divided into three sections: Section I looks at some of the early multiplier-accelerator models and their limitations, Section II looks at some of the literature trying to use finance as an explanation for turnaround, Section III contains an assessment of the entire survey of literature. The survey is not meant to be exhaustive but only illustrative of some of the major works in the above-mentioned areas.

Section I:
Multiplier-Accelerator Models

The models of this genre are based on the simple interaction between multiplier and accelerator. The investment (or any autonomous expenditure) positively affects income and profits via multiplier. The income and profits, on the other hand positively affect investment through the accelerator, typically with some time lag.

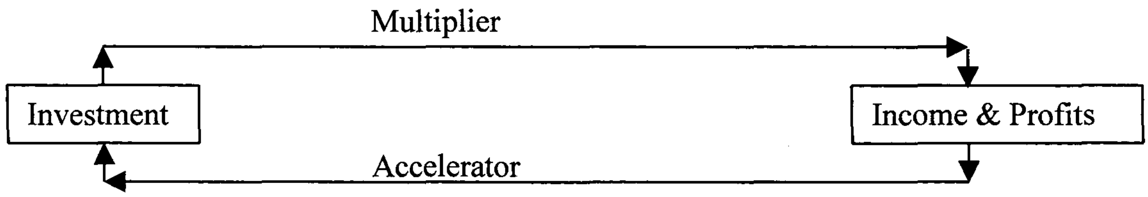

Figure 2.1

Even such an elementary exposition brings out one of the limitations of these models: absence of any mechanism for a turnaround. As we will see, this indeed is a problem with some of the simple multiplier-accelerator models. Some of the models try to bypass this problem by bringing in additional restrictive assumptions, like interaction of time lags.

To better analyze this problem, let us consider a simple multiplier-accelerator model based on Kaleckian framework.

A Simple Multiplier-Accelerator Model

Let us consider a simple Kaleckian two-sector framework (Kalecki, 1971), consisting of the investment and the consumption good sector, and two social classes, wage-earners and profit-earners. Following classical savings assumption, we assume that all wages are consumed and savings consist of profits only. This gives us the following saving-investment identity:

$$I_t^{\text{realized}} \equiv S_t^{\text{realized}}$$

$$\Rightarrow I_t^{\text{realized}} \equiv sP_t^{\text{realized}} \qquad \text{...(2.1)}$$

where *I, S, P* and *s* stand for investment, savings, profits and propensity to save out of income (which is assumed constant) respectively. Equation 2.1 is an *ex poste* identity in terms of realized variables. This becomes an *ex ante* equilibrium condition as follows:

$$\Rightarrow P_t = \frac{1}{s} I_t$$

$$P_t = \frac{1}{s} I_t \qquad ...(2.2)$$

Equation 2.2 forms the basis for a multiplier relationship. Investment in a period generates profits by an amount equal to investment times the reciprocal of saving propensity, $\frac{1}{s}$. $\frac{1}{s}$ is known as the multiplier.

Let the investment in period *t* to be a function of expected profits in period *t*. This would be similar to the Kaleckian investment function (Kalecki, 1971). This gives us:

$I_t = a(P_t^e)$, which with an assumption of linearity becomes

$$I_t = aP_t^e \qquad ...(2.3),$$

which with further assumption of static profit expectation, $P_t^e = P_{t-1}$, becomes:

$$I_t = aP_{t-1}$$

$$i.e.\ I_{t+1} = aP_t \qquad ...(2.4)$$

From equations 2.2 & 2.4, we get

$$I_{t+1} = \frac{a}{s} I_t$$

$$I_n = \left(\frac{a}{s}\right)^n I_0 \qquad ...(2.5)$$

Equation 2.5 gives us a few unrealistic results:

- If $a \succ s$, the system escapes to ∞.
- If $a = s$, the system is static at the same level of investment forever.
- If $a \prec s$, investment drops to 0.

The simple Kaleckian framework, thus, highlights both the problems we discussed earlier – it predicts an unbounded system if $a \succ s$, with no mechanism for turnaround, and there is no consideration of how the investments are to be financed.

Kalecki's 1935 model (in Kalecki, 1971) actually explains a turnaround using fixed capital and interaction of time lags. This model, however, is more complicated and involves additional concepts like fixed capital stock, inventory decisions and gestation lags. Since we are mainly concerned with simple interaction between multiplier and the accelerator, we do not discuss it here. We instead discuss another well-known model, using interaction of lags to explain turnaround in multiplier-accelerator models.

Hicks

One of the better-known models of the multiplier-accelerator genre, Hicks' model (J.R. Hicks, 1950) is similar to the model developed earlier by Samuelson and Hansen (Samuelson, 1939). However, Hicks' model better emphasizes the limitations of multiplier-accelerator models. All these models attempt to solve the problem of unboundedness of the multiplier-accelerator models by lagging both the multiplier and the accelerator. Following is a simplified version of the Hicks' model (Gandolfo, 1971), which retains most of the characteristics of the original model.

The basic equations of the model are as follows:

Equilibrium Condition : $Y_t = C_t + I_t$ ...(2.6)

Consumption function : $C_t = bY_{t-1}$ ...(2.7)

where *Y, C* and *I* have their usual meanings of income, consumption and investment respectively; *b* represents the marginal propensity to consume.

The investment function consists of two parts, the induced and the autonomous investment. The induced part depends on the change in income lagged by one period. The autonomous investment, consisting mainly of the government expenditure, increases exponentially over time. (This indicates that the role of government increases with time)

Induced investment: $I_{\text{induced}} = k(Y_{t-1} - Y_{t-2})$, where *k* is the accelerator.

Autonomous investment: $I_{\text{autonomous}} = A_0(1+g)^t$

Total investment = Induced Investment + Autonomous Investment

$\therefore I_t = k(Y_{t-1} - Y_{t-2}) + A_0(1+g)^t$ ...(2.8)

By standard substitutions, we obtain:

$$Y_t - (b+k)Y_{t-1} + kY_{t-2} = A_0(1+g)^t \qquad ...(2.9)$$

Trying the solution $\bar{Y}_t = Y_0(1+g)^t$, and substituting it in equation 2.9, we get:

$$(1+g)^{t-2}\left[Y_0(1+g)^2 - (b+k)Y_0(1+g) + kY_0 - A_0(1+g)^2\right] = 0 \quad ...(2.10)$$

If $g \neq -1$, equation 2.10 holds true iff,

$$Y_0(1+g)^2 - (b+k)Y_0(1+g) + kY_0 - A_0(1+g)^2 = 0 \qquad ...(2.11)$$

$$\Rightarrow Y_0 = \frac{A_0(1+g)^2}{(1+g)^2 - (b+k)(1+g) + k} \qquad ...(2.12)$$

Therefore, given Y_0, the equilibrium or the trend level of income at time t is $\overline{Y}_t$, which itself moves over time.

The homogenous equation corresponding to equation 2.10 yields the following characteristic equation:

$$\lambda^2 - (b+k)\lambda + k = 0 \qquad ...(2.13)$$

The stability conditions of the above equation are as follows (Gandolfo, 1971, p-78):

$$\begin{aligned} &1 - (b+k) + k = 1 - b \succ 0 \\ &1 - k \succ 0 \qquad ...(2.14) \\ &1 + (b+k) + k \succ 0 \end{aligned}$$

The first inequality is satisfied since the marginal propensity to consume is always less than 1.The third inequality is satisfied since L.H.S. is a sum of positive variables. Therefore, the crucial inequality is the second one:

$$k \prec 1 \qquad ...(2.15)$$

Discriminant of equation 2.13 is

$$D = (b+k)^2 - 4k = k^2 - (4-2b)k + b^2 \qquad ...(2.16)$$

We have real roots when $D \succ 0$

Solving equation 2.16 for $D = 0$, we get,

$k_1, k_2 = \left(1 \pm \sqrt{s}\right)^2$, where s = Marginal propensity to save $= 1 - b$

Roots are real and distinct when $D \succ 0$, i.e. when $k \prec (1-\sqrt{s})^2$ or $k \succ (1+\sqrt{s})^2$

Roots are complex conjugates when $D \prec 0$, i.e. when $(1-\sqrt{s})^2 \prec k \prec (1+\sqrt{s})^2$ ...(2.17)

Roots are real and equal when $D = 0$, i.e. when $k = (1 \pm \sqrt{s})^2$

Equation 2.15 and 2.17 gives us the following behavior of Y_t around the equilibrium value:

Table 2.1

Values of Parameters	Behavior of Deviation around Trend as $t \to \infty$
$k \prec (1-\sqrt{s})^2$	Monotonic and convergent
$k = (1-\sqrt{s})^2$	Monotonic and convergent
$(1-\sqrt{s})^2 \prec k \prec 1$	Oscillatory and damped
$k = 1$	Oscillatory with constant amplitude
$1 \prec k \prec (1+\sqrt{s})^2$	Oscillatory and divergent
$k = (1+\sqrt{s})^2$	Monotonic and divergent
$k \succ (1+\sqrt{s})^2$	Monotonic and divergent

However, according to Hicks, k, the acceleration coefficient, is always greater than 1. Empirical studies would support this claim[3]. This would put the models in one of the unstable regions, making the trajectory of income an explosive one.

Hicks, however, noted that there is an intrinsic ceiling – the full employment level of output, which cannot be crossed. Similarly, there exists a floor, the output corresponding to zero level of gross induced investment, below which the economy cannot go. The economy fluctuates between these two bounds, hitting the ceiling, rebounding, hitting the floor and again turning upward and so on.[4]

The full employment level of output grows at the rate of growth of population and productivity as follows:

$$B_t = B_0(1+g)^t \qquad \ldots(2.18)$$

[3] For instance, the capital output ratio of the OECD countries in 1996 varied within the range of 2 and 4, except USA, which had a capital output ratio of 1.91. The ratio was much higher in some of the European countries, like 3.87 in Denmark and 3.57 in Finland.

[4] This makes the analysis of discriminant as in equation 16 irrelevant, since the trajectory would always be oscillating, irrespective of the value of discriminant.

where B is the value of the ceiling, and B_0 is assumed to be greater than Y_0.

Similarly at the floor, where the gross induced investment is equal to zero (i.e. the net induced investment is equal to negative of the rate of depreciation), the total gross investment would be equal to the gross autonomous investment. Thus the floor grows at the rate of growth of autonomous investment.

Let us assume that the economy is at the trend rate of income, when an exogenous shock gives an upward shock to the income. Since $k \succ 1$, this would trigger an explosive movement upwards till it hits the ceiling. Once it hits the ceiling, it can either move along the ceiling or rebound back. However, for it to stay at the ceiling, the rate of growth of income and the rate of growth of full employment must be equal. If this condition is not satisfied, then the income must turn around. Once it turns around, it will keep decreasing till it reaches the floor. Once again, it can either stay growing along the floor, for which the rate of growth of income and the floor must be equal, or it must turn upward. This process continues indefinitely.

Thus, the concept of ceilings and floors enable Hicks to make the system bounded, within which the economy keeps fluctuating. In the real world, however, we often find that the economy fluctuates within much narrower boundaries and rarely touches the particular ceilings and floors referred to by Hicks. In other words, the upward movement ends much before it actually reaches the full employment level of output. Similarly, the downward movement does not always continue till it reaches the level of zero gross induced investment. This compels us to look for other explanations of turnaround. We next look at the models involving financial sector to investigate whether such models may be forthcoming in these contexts.

Section II

Role of Finance

The literature reviewed so far does not discuss how the investments are financed. However, we know from real world experience that finance forms an important aspect of investment decisions in any modern economy. The financial sector can both facilitate and constrain investment.[5] The most relevant case in the current context, perhaps, is when finance acts as a constraint on investment.

Finance can act as a constraint on investment in a number of ways. The issue most commonly discussed in the literature is the problem of asymmetric information, leading to adverse selection.[6] The 'market for lemons' problem discussed by Akerlof (Akerlof, 1970), for instance, is an offshoot of this. These arguments led Stiglitz and Weiss to suggest that, credit is often rationed in the market, so that the demand does not equal supply at the equilibrium rate of interest (Stiglitz and Weiss, 1981). We, however, do not discuss these issues here. We are merely concerned with how problems in the financial sector emerge while the economy is in the phase of upward monotonic movement of income and investment and how these problems act as an effective brake to the upward monotonicity. The literature variously refers to these problems as 'financial crisis', 'financial instability', 'financial fragility' etc.

Seen in this light, financial crisis / instability / fragility can be observed in terms of either of the following factors (or any combination thereof):

- Widespread difficulties faced by agents all over the economy (or at least, in many of the sectors) in honoring their payment commitments.
- An abnormal fall in the price of financial assets.
- A general unwillingness in the economy to hold illiquid assets and a tendency to shift to more liquid assets, like money.

However, it is important to note that none of the above factors leads to an actual crisis or a problem of any sort (except affecting a handful of financial investors) unless it starts

[5] A survey of various theories of the relationship between the financial sector and investment may be found in Gordon (1994).
[6] For a detailed discussion on the concept of asymmetric information and adverse selection in the financial markets, refer to Hillier (1997).

affecting the real economy by constraining investment. Hence the whole idea of a 'financial crisis' must be looked upon in terms of its impact on real sector. As would be evident, this impact will rise with an increase in exposure of various economic agents to the financial markets – financial crisis is more likely to affect an economy where both the firms and households are heavily exposed to the financial market then an economy with a lesser degree of separation between the savers and investors. In other words, possibilities of financial crisis increase with an increase in the degree of financial intermediation.

In the real world, a 'financial crisis' is often known to have caused a dramatic turnaround in the upward movement of income. Similarly, though not in as dramatic fashion, an easing of cost of financing investment has often led to a turnaround in the downward movement of income, putting an end to recession. Writers like Minsky (1982) and Kindleberger (1978) have tried to provide a theoretical basis for this. In this section, we look at a selection of writings from this school of thought.

Minsky's 'Financial Instability Hypothesis'

Minsky, one of the most influential Post-Keynesians in the recent times, proposed his *'Financial Instability Hypothesis'* in a number of writings over a period of time. These writings have a consistent microeconomic flavor. The microeconomic flavor, while providing certain new insights, is also a source of a number of problems in both understanding and formalizing of his ideas. We here discuss certain basic features of Minsky's analysis relevant to our study.

The object of Minsky's analysis is a microeconomic firm at the decision making stage of investment. As we know, investment at the microeconomic level, unlike macroeconomic investment[7], does not lead to immediate profits – there is a gestation lag involved. The length of this lag after which the profit streams are realized depends on the

[7] Writers like Kalecki (Kalecki, 1971) have shown us that investment at the macroeconomic level must lead to macroeconomic profits within the same period, either by expansion of output (if there is excess capacity) or by increase in share of profits (if the economy is close to the full-employment level), though the investment and profits might not occur in the same unit or the same sector.

nature of the investment. Investment in modern industries, according to Minsky, is more capital intensive compared to earlier industries and might have a longer gestation lag.[8]

Let the desired investment depend on present value of expected future profits, expressed as follows:

$$I_{\text{desired}} = f(V), \text{ where } V = \frac{Q_1}{1+r_1} + \frac{Q_2}{(1+r_2)^2} + \frac{Q_3}{(1+r_3)^3} + \ldots$$

V is the present value and Q's form the expected future stream of profits.

Expectations regarding future are based on past experiences. This would imply that, the values of Q_1, Q_2, Q_3 ... would be a function of profits in the past.[9]

$Q_1, Q_2, Q_3 = g(\text{Profits in the recent past})$

During a boom, with high profits fresh in the recent memory of investors, the expected future profits are likely to be high, pushing up V as well. This would result in a desire for higher investment.

An investment can be financed either from internal (from retained profits) or external (by selling debt and equity instruments) sources. When the desired investment is high, then internal financing is unlikely to be enough to finance it, making the firm go for external finance. However, this is not a problem during the boom time, as the lenders (namely, the financial intermediaries) are also willing to lend. Typically, there are two factors determining the access to loans for the firms:

1. The liability structure of the borrower, i.e. some measure of debt-equity ratio. Usually, there is some tolerable upper limit to the debt-equity ratio.
2. Future perceptions about the economy by the lender, which would determine the ability of the borrower to pay back.

However, none of this is a problem during the boom time. The tolerable upper limit to the debt-equity ratio as well as the future perceptions about the economy by the lender is high during boom time. Thus, increasingly, investment starts being financed from external sources, especially debt.

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[8] It might, however, be argued that the gestation lags have in fact decreased in the recent decades with the introduction of new technology in some sectors. This might specially hold true for the information technology sector.

[9] This idea might have originated from Keynes, who postulated that the recent past could act as a guide towards future, especially when the future is uncertain. (Keynes, 1936)

Minsky distinguishes between three different kinds of financing options by the firms:

1. Hedge Financing: The expected cash flows of a firm from income producing activity exceed payment commitments in every period.
2. Speculative Financing: The expected cash flows from income production fall short of payment commitments in the next few periods in the immediate future.
3. Ponzi Financing: The expected cash flows from income producing activities in the next few periods not only fall short of the payment commitments but the interest commitments as well – i.e. the firms have to borrow every period to even pay back the interests. In this case, the debt must grow over time. Hence, this is the riskiest of all the financial postures. However, all Ponzi firms need not necessarily default and actually get into a financial crisis.

Firms belonging to Speculative and Ponzi categories survive if and only if the lenders believe that their condition will improve within some finite time horizon, i.e. these firms will turn into hedge firms.

During boom time, there are two tendencies that push the firms from Hedge financing towards the direction of Speculative and Ponzi Financing:

1. During boom, profit expectations are high, making it difficult for the expectations to be realized. Once the desired profits fall short of the expected profits in any particular period, the firms might face a sudden credit squeeze from the lenders, adversely affecting their financial postures.
2. During boom time, external financing covers a greater part of the investment than normal times. Therefore, there is a steady decline in liquidity of the portfolio of the asset holders. This tends to push up the interest rates, thus pushing up the debt commitments of the firms. This again adversely affects the financial postures of the firms.

Both the above factors might prompt the lenders to squeeze credit, pushing the Hedge firms to Speculative and Speculative to Ponzi. Some of the Ponzi or non-Ponzi speculative firms might start defaulting as a result of a credit squeeze. The credit squeeze adversely affects investment, which in turn adversely affects profitability. Thus, once the crisis sets in, the turnaround occurs due to a downward revision of expectations.

Many of Minsky's writings were published in the 1950's and the 1960's, when the American economy was undergoing a prolonged boom. The general view prevalent at that time believed that events like the Great Depression were things of the past. The Post World-War-II reforms in the world financial architecture would be able to prevent any such crisis. Minsky contested this view. According to him, there had been no fundamental change in the working of the economy since the 1930's to rule out another Great Depression. He believed that the US government had merely postponed a financial crisis (which would have ended the upward phase) through two sets of policies:

1. Counter-cyclical fiscal policies: This basically referred to the expansionary fiscal policies in the wake of Marshall Plan in the 1950's and some major wars, like the Vietnam War in the 1960's.
2. Monetary policy playing the Lender-of-Last-Resort Role: The Federal Reserve played the Lender-of-Last-Resort role efficiently in the Post World-War-II period, providing liquidity at times of crisis.

However, Minsky believed that the above policies would not be able to prevent occurrence of another Great Depression for too long. Hence, he prophesied financial crises in the near future putting an end to the Post World-War-II boom.

Kindleberger

Kindleberger's book, "Manias, Panics and Crashes" (Kindleberger, 1978) provides an alternative approach to explain cycles arising out of financial processes. Using plenty of examples from history, Kindleberger explained the occurrence of booms and financial crises in an international context. The framework of his analysis is largely borrowed from Minsky. Though the analysis mostly refers to an economy in international context, some of the insights are relevant in the closed economy settings as well. The basic features of his work are briefly described below.

According to Kindleberger, every international economy with a financial system goes through the following stages:

1. Boom
2. Euphoria

3. Bubble or Speculative Mania
4. Financial Distress
5. Revulsion and Discredit
6. Panic
7. Crash

The various stages of the cycle are explained below.

Consider an exogenous shock[10] creating optimistic expectations in an economy. This optimistic expectation could either be widespread in the entire economy or limited in certain sectors.[11] This leads to an increase in demand for both goods and financial assets as more and more people want to take advantage of the situation. Increase in demand for goods increases profits, whereas increase in demand for financial assets increase asset prices reducing cost of investment. This represents the *boom* stage.

Positive feedback from increase in profits and a reduction in costs is carried forward to the next stage of *euphoria*. Increase in prices, especially of financial assets creates expectations about possibilities of further increases. Such expectations are self-fulfilling and result in actual increase in prices and an expansion in trading of financial assets. Some writers like Adam Smith have referred to this condition as 'overtrading'. In this stage, people tend to shift from more liquid to less liquid assets – for instance from cash to short-term bonds and from short-term bonds to shares. For those investing in the real sector, larger and larger part of the investment is financed by selling financial assets at higher prices.

Such overtrading of financial assets often creates condition for transition to the next stage – development of a *bubble* or *speculative mania*. A consistent increase in prices in the previous two stages attracts new entries to the financial market. Many of the new entrants are not as well informed as professional investors and are attracted by possible capital gains from financial transactions, often based on an overestimation of future price of financial assets. Kindleberger refers to them as the 'outsiders'. Many of

[10] The shock could be anything from new invention, wars, a change in policy to change in fortune in some important sector or companies.
[11] The recent boom in the IT sector could be taken as an example of optimism being limited to a few sectors.

the firms, on the other hand, use this opportunity to increase their gearing ratio[12] and raise funds from the market at low costs. Much of these funds are pumped back by these firms into the financial market in speculative transactions, in an effort to make capital gains. All these events create excess liquidity in the market, driving up prices to unrealistic levels and a bubble is said to have developed.

Since the 'insiders' are aware that the expectations are unrealistically optimistic, at some point they decide to book their profits (i.e. sell while the prices are high). Such profit-booking activities exert a downward pressure on the prices. However, the outsiders are not aware of this and continue to enter the market. Thus, the downward pressure due to profit booking is matched by the continued upward pressure due to new entries. The prices, thus tend to stabilize, (i.e. there is a reduction in the rate of increase in prices of assets) creating payment problems for some of the units expecting higher capital gains. Kindleberger calls this stage *financial distress*. The insiders manage to book profits and move out of the market at this stage.

As the stage of financial distress continues, rest of the market, either gradually or suddenly, realizes that the prices cannot go any higher and it is time to withdraw. This starts the beginning of the stage of *revulsion* or *discredit*. The selling pressure starts dominating and no one wants to enter the financial market anymore. The prices start falling sharply as everyone, especially the outsiders start running out of the market. Many of them sell only after the prices have fallen. The outsiders, thus, often end up buying high and selling low. Those who had a high gearing ratio in the previous stages and could not get out in time, start facing liquidity crisis, leading to bankruptcies.

The bankruptcies often create chain effects as units that had earlier entered into payment problems with the bankrupt firms also start facing problems. Anticipating such a situation, there is a mad rush for everyone to end all contracts and get out of the market. Kindleberger calls this stage *Panic*, or, using a common German term, *Torschlusspanik* (i.e. door-shut panic, with people crowding to get out of the door before it gets shut). This panic feeds on itself, driving the prices lower, finally resulting in a *crash*.

[12] Gearing ratio, according to the Palgrave Dictionary of Economics, refers to the relative importance of loans in the capital structure of a firm. It is also known as 'debt ratio' and, in the USA, 'leverage'. There are different ways of measuring the gearing ratio.

The turnaround in prices can occur when one of the following happens:

- Prices fall so low that the people are tempted to move back into less liquid assets, thinking that it cannot fall any more.
- Trade in financial assets is cut off by setting limits on price declines.
- A lender of last resort manages to convince the market that money will be made available in sufficient quantities to meet the demand for cash, enabling the various units to honor their commitments.

Thus, both Kindleberger and Minsky come to a similar conclusion – of financial processes ending the upward movement of income and profits.

Attempts to Model Minsky

There have been a number of attempts to model a financial crisis of the kind described by Minsky (often referred to in the literature as 'Minsky Crisis'), using different interpretations of financial instability hypothesis. Most of them considered only the macroeconomic aspects of Minsky, ignoring the microeconomic aspects. For instance, Taylor and O'Connell (1985) attempted modeling Minsky using a framework similar to the IS-LM framework. This was later extended by Semmler (1987). Steve Keen (1995, 1996) modeled Minsky by adding a financial sector to Goodwin's growth cycle, with chaotic implications. One of the exceptions, however, is Vercelli's (Vercelli, 2000) model. This captures the essence of Minsky Crisis and includes some of the microeconomic and institutional aspects as well. We consider these contributions next.

Taylor & O'Connell (1985)

Taylor and O'Connell ignored the microeconomic and institutional aspects of Minsky and tried to model the macroeconomic aspects of Minsky. In particular, they emphasized two main features stressed by Minsky: "

1. The total nominal wealth in the system is macro determined, dependent on the confidence and the state of the cycle.

2. There is high substitution among assets in household portfolios under certain circumstances. For instance, there could be a flight to money when the conditions are ripe." (Taylor and O'Connell, 1985)

Symbols:

r = Rate of profit

K = Capital stock

w = Nominal Wealth

b = Labor-output ratio

X = Output

P = Price of investment goods

τ = Rate of mark-up

ρ = Difference between anticipated return to holding capital and current rate of profits

g_0 = Constant autonomous capital stock growth

Following Kalecki, these authors assume mark-up pricing for goods. By mark-up pricing,

$P = (1+\tau)wb$ ...(2.19)

Rate of profit, $r = \dfrac{PX - wbX}{PK} = \dfrac{\tau}{1+\tau}\dfrac{X}{K}$ ...(2.20)

The firm uses "a rule of thumb for investment that depends on anticipated profits and a discount factor" (Taylor and O'Connell, 1985, p-873). The authors assume that the capitalized value of expected earnings per unit of investment is an appropriate shadow price, P_k for investment decision. This is expressed as follows:

$$P_k = (r+\rho)\frac{P}{i}$$

The authors, following Minsky, make investment demand depend on price differential $P_k - P_i$, where P_i is the supply price of new investment goods. The authors replace P_i with P, i.e. the actual price of investment goods is taken as the supply price.

$$P_k - P = (r+\rho)\frac{P}{i} - P = \frac{\rho}{i}(r+\rho-i)$$

Investment demand in nominal terms,

$$PI = [g_0 + h(r+\rho-i)]PK$$

With the classical savings assumption of all wages being consumed, the supply of savings

$= srPK = s\tau wbX$

From demand-supply equality of nominal savings and investment,

$$[g_0 + h(r + \rho - i)]PK = s\tau wbX$$

Dividing both sides by *PK*,

$$g_0 + h(r + \rho - i) = \frac{s\tau wbX}{PK}$$

Substituting the value of *r* from equation 2.20,

$$g_0 + h(r + \rho - i) - sr = 0 \qquad \text{...(2.21)}$$

Equation 2.21 represents the equilibrium condition for the commodity market.

Similarly, the equilibrium condition for the asset market is derived. The authors assume that there is an outside primary asset, Fiscal Debt or *F*, consisting of money, *M* and short-term bonds, *B*.

Capitalized value of plant and equipments held by firms:

$$P_k K = (r + \rho)\frac{PK}{i}$$

Total wealth of the rentiers would consist of the sum of outside assets and the capitalized value of equity held by them.

$$W = P_E E + M + B = P_E E + F \qquad \text{...(2.22)}$$

At each point in time, the rentiers allocate their wealth according to the following equations for market balance[13]:

$$\mu(i, r + \rho)W - M = 0 \qquad \text{...(2.23)}$$

$$\frac{\varepsilon(i, r + \rho)}{P_E}W - E = 0 \qquad \text{...(2.24)}$$

$$-\beta(i, r + \rho)W + B = 0 \qquad \text{...(2.25)}$$

where μ, ε and β are the shares of wealth allocated by rentiers to holding money, equities and short-term bonds respectively. $\mu + \varepsilon + \beta = 1$.

From equations 2.23, 2.24 and 2.25, we get

[13] These equations are based on the assumption that all the financial markets clear at the equilibrium rate of interest and profits. As many writers have argued, this need not be true, especially in a market characterized by asymmetric information. Stiglitz and Weiss (1981), for instance, argued that the equilibrium rate of interest will be lower than the rate at which the demand equals supply, the excess demand for credit being rationed out.

$$W = \frac{F}{1-\varepsilon(i, r+\rho)} \quad \ldots(2.26)$$

$$\mu(i, r+\rho) = \frac{M(1-\varepsilon(i, r+\rho))}{F} \quad \ldots(2.27)$$

If $\alpha = \frac{M}{F}$, then equation 2.27 reduces to:

$$\mu(i, r+\rho) - \alpha[1-\varepsilon(i, r+\rho)] = 0 \quad \ldots(2.28)$$

Equation 2.28 gives us the equilibrium condition for the asset market.

Putting equations 2.21 and 2.28 together gives us, like IS-LM, two loci of equilibrium points in the commodity and the asset market respectively[14]:

$$g_0 + h(r+\rho-i) - sr = 0 \quad \ldots(2.21)$$

$$\mu(i, r+\rho) - \alpha[1-\varepsilon(i, r+\rho)] = 0 \quad \ldots(2.28)$$

This is represented in the following diagram:

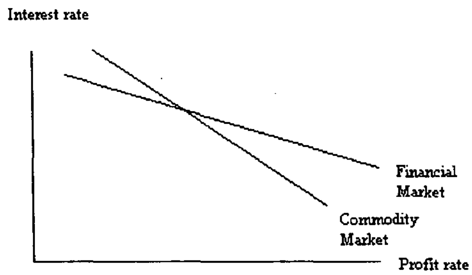

Figure 2.2

Any point above and to the right of the locus of equilibrium of the commodity market represents a situation where the rate of interests is higher than the equilibrium level for a given rate of profits. This implies an excess of savings over investment. Equilibrium would be restored in the commodity market by a reduction in the rate of profits. A point below and to the left of the locus of commodity market equilibrium, on the other hand,

[14] It might, however, be noted that this system, like IS-LM, is a combination of a flow equilibrium, namely equation 2.21, and a stock equilibrium, namely equation 2.28. Point on a flow equilibrium is determined over a period of time, whereas that on a stock equilibrium is determined at a point in time. This creates a problem, especially at the point of intersection of the two curves, unless the time period used to calculate the flow equilibrium is infinitesimally small. (See, for instance, Chapt. 8 of Hicks, 1965)

represents excess of investments over savings, leading to an increase in profits and restoration of equilibrium in the commodity market.

Similarly, any point above and to the right of the locus of equilibrium in the financial market represents a situation where the rate of profits is higher than the equilibrium level for a given rate of interests. This implies an excess demand for equities and an excess supply of money. This would lead to a shift in the portfolio of the financial investors from money to equities. A shift away from money, however, would cause the rate of interests to fall, leading to restoration of equilibrium in the financial market. A point below and to the left of the locus of financial market equilibrium, on the other hand, would represent excess demand for money and excess supply of equities, leading to a rise in the rate of interests and restoration of equilibrium.

Dynamics of the stable case can be represented by the following diagram:

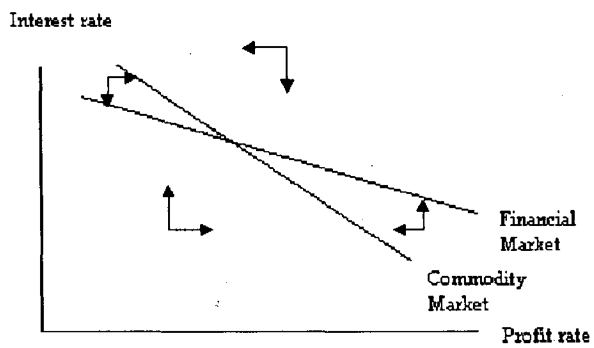

Figure 2.3

The local stability (around equilibrium values) of the above system can be determined from the following Jacobian:

$$J = \begin{bmatrix} -h & -s \\ \mu_i + \alpha\varepsilon_i & \mu_r + \alpha\varepsilon_r \end{bmatrix}$$

where the subscripts i and r refer to partial derivates with respect to i and $r + \rho$ respectively.

For local stability, trace < 0, determinant > 0.

We know that, $s > 0,\ h > 0,\ \mu_i < 0,\ \varepsilon_i < 0,\ \mu_r < 0,\ \varepsilon_r > 0$.

From Gross Substitution Assumption: $\varepsilon_r > |\mu_r|$.

However, the authors propose that, since money and equities are close substitutes of each other in the asset demand, the magnitudes of the two partial derivatives will be close to each other. Further, if α is a small enough fraction, then $\mu_r + \alpha\varepsilon_r < 0$. This will make the trace < 0. However, an open market operation by the Central Bank would increase α. This might make the trace positive.

Sign of the determinant, as would be evident, is ambiguous. One can see that $\mu_r + \alpha\varepsilon_r < 0$ is a necessary but not sufficient condition for the determinant to be positive.

By assuming that the ratio of the financial debt to capital stock to be fixed, the authors derive a second dynamical system as follows:

$$\dot{\rho} = -\beta\left(i - \bar{i}\right) \qquad \text{...(2.29)}$$

$$\hat{\alpha} = \hat{M} - g \qquad \text{...(2.30)}$$

where $\dot{\rho}$ is the time rate of change of ρ, $\bar{i}$ the normal long-run rate of interest, β a reaction coefficient and $\hat{M}$ a fixed growth rate of money supply. The stability conditions of this system can be derived from the following Jacobian:

$$J = \begin{bmatrix} -\beta i_\rho & -\beta i_\alpha \\ -\left(g_i i_\rho + g_\rho\right) & -g_i i_\alpha \end{bmatrix}$$

Signs of the various elements in the above Jacobian are as follows:

$i_\rho \prec 0,\ i_\alpha \prec 0,\ g_i \prec 0,\ g_\rho \succ 0.$

Possibilities of local instability around equilibrium point occur when i_ρ is a large enough negative value to make trace of the Jacobian is positive, i.e. $\text{Tr}\,J = J_{11} + J_{22} \succ 0$. This means that the fall of the variable ρ causes the interest rate to rise sufficiently due to shift of equity capital into money by rentiers, so that the negative feedback effect from the interest rate on g will drive down expected profits. Due to high asset substitution, an increase in interest rate and fall in profits drive rentiers into money, pushing up the interest rate further. This process goes on, accompanied with fall in capitalized quasi rents and equity prices and 'general disintermediation' – in other words, the economy moves into the trap of a 'Minsky Crisis'.

The above point would be clearer if we look at the following diagram representing equations 2.29 and 2.30:

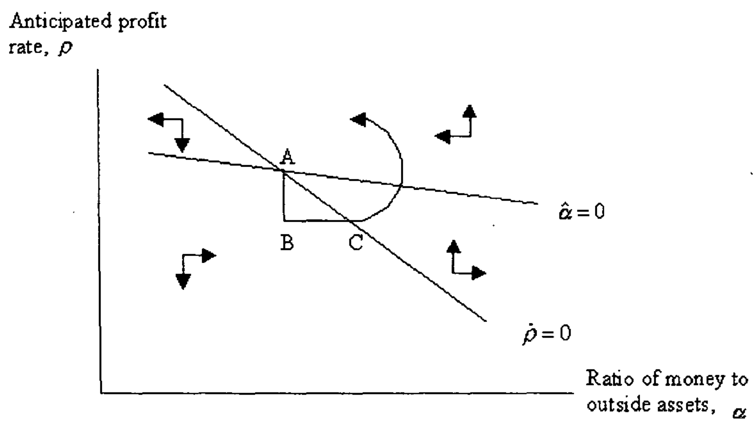

Figure 2.4

Suppose the economy is at equilibrium at the point A, from where, say, a loss of confidence occurs, reducing ρ and pushing the economy to B. This will slow down the rate of growth of capital stock. If the authorities maintain a constant rate of growth of supply of money, then there would be an increase in money-debt ratio, *M*. This would reduce the rate of interest, increasing ρ. If the feedback to ρ is strong enough, then the economy would converge back to the equilibrium point through *C*. According to Taylor and O'Connell, this represents a minor crisis. On the other hand, if the feedback to ρ is weaker, then the economy would move away from the equilibrium and the 'crisis' would continue. According to Taylor and O'Connell, this case is similar to a 'liquidity trap', though unlike the traditional liquidity trap, the interest rate would continue to increase in this case.

Steve Keen (1995, 1996)

Keen tried modeling a specific aspect of Minsky's work, which, in his words, is the following: "In a capitalist economy with finance, an endemic tendency towards euphoric expectation will generate both cycles and secular trend of rising debt, ultimately leading to a debt-induced crash." (Keen, 1995, p-614)

Keen modeled this by adding a finance sector to Goodwin's 1967 model of trade cycle (Goodwin, 1967). This converts the two-dimensional phase space of Goodwin to a three-dimensional one. Recalling the assumptions of Goodwin's 1967 model:

1. There are only two factors of production, labor and capital, both homogenous and non-specific. Corresponding to this, there are two social classes, laborer and capitalist, earning wages and profits respectively. (Keen adds a third banking sector, earning profits separately)
2. All quantities are real and net.
3. All wages are consumed and all profits saved and automatically invested.
4. The capital-output ratio, k, is constant.
5. The real wage rate, w, rises in the neighborhood of full-employment.

Keen inherits all the assumptions and equations of Goodwin's 1967 model and adds the equations for the financial sector to it. Following are the main equations of the resultant model:

$$a = a_0 e^{\alpha t} \qquad \ldots(2.31)$$

where a = Productivity of labor.

The equation shows steady exponential growth of embodied labor productivity.

$$N = N_0 e^{\beta t} \qquad \ldots(2.32)$$

where N = Labor force.

The equation shows steady exponential growth of the labor force.

$$Y = aL \qquad \ldots(2.33)$$

where Y = output, L = Employment.

$$K = vY \qquad \ldots(2.34)$$

where K = Stock of capital, v = Accelerator.

i.e. a fixed accelerator relation.

$$\lambda = \frac{L}{N} \qquad \ldots(2.35)$$

where λ = The employed fraction of the labor force.

$$\frac{dw}{dt} = w(\lambda).w \qquad \ldots(2.36)$$

i.e. rate of change of real wages is a non-linear function of the rate of employment.

The specific functional form that Keen considers for $w(\lambda)$ is as follows[15]:

$$w(\lambda) = \frac{0.000641}{(1-\lambda)^2} - 0.0400641$$

$$I = \frac{dK}{dt} = k\left[\frac{\Pi}{K}\right]Y - \gamma K \qquad \ldots(2.37)$$

where Π = Profits, γ = Rate of depreciation.

Equation 2.37 implies that net investment is a function of profit times the level of output minus depreciation.

The functional form for $k\left[\frac{\Pi}{K}\right]$ is similar to $w(\lambda)$:

$$k\left[\frac{\Pi}{K}\right] = \frac{0.0175}{\left(0.53 - 6\frac{\Pi}{vY}\right)} - 0.065$$

$$\frac{\Pi}{K} = \frac{\Pi}{vY} = \frac{\pi}{v} \qquad \ldots(2.38)$$

where π = Share of profits in the national income.

Equation 2.38 implies that the rate of profit equals the ratio of profit share and the accelerator.

$$\pi = 1 - \omega - b \qquad \ldots(2.39)$$

where ω = Share of wages in the national income, b = Share of bankers' profits in the national income.

i.e. profit share is the residual after workers' and bankers' income.

$$\omega = \frac{W}{Y} = \frac{wL}{aL} = \frac{w}{a} \qquad \ldots(2.40)$$

$$b = \frac{B}{Y} = \frac{rD}{Y} \qquad \ldots(2.41)$$

where D = Outstanding debt.

This means that banker's income is the interest rate times the outstanding debt.

$$\frac{dD}{dt} = rD + I - \Pi \qquad \ldots(2.42)$$

---

[15] Keen considers a specific numerical example to run a simulation program. The values of the parameters given above is from the numerical example.

$$r = \varsigma + \phi \frac{D}{Y} \qquad \text{...(2.43)}$$

where ς denotes the base rate of interest and ϕ the level of bank sensitivity to the debt-output ratio.

i.e. interest rate is a linear function of the debt to output ratio.

Equations 2.31 to 2.43 give us the following six equations:

1. Rate of change of output $= \frac{dY}{dt} = \frac{d}{dt}\frac{K}{v} = \left[\frac{k\frac{\pi}{v}}{v} - \gamma\right]Y$ ...(2.44)

2. Rate of change of employment $= \frac{dL}{dt} = \frac{d}{dt}\frac{Y}{a} = \frac{1}{a}\left[\frac{dY}{dt} - \alpha Y\right]$ ...(2.45)

3. Rate of change of employment rate $= \frac{d\lambda}{dt} = \frac{d}{dt}\frac{L}{N} = \lambda\left[\frac{k\frac{\pi}{v}}{v} - \alpha - \beta - \gamma\right]$ ...(2.46)

4. Rate of change of worker's share of output $= \frac{d\omega}{dt} = \frac{d}{dt}\frac{w}{a} = \omega[w(\lambda) - \alpha]$ ...(2.47)

5. Rate of change of debt ratio $= \frac{dd}{dt} = \frac{d}{dt}\frac{D}{Y} = b - \pi - (v - d)\left[\frac{k\frac{\pi}{v}}{v} - \gamma\right]$ ...(2.48)

6. Rate of change of banker's share $= (\varphi d + r)\left[b - \pi - (v - d)\left\{\frac{k\frac{\pi}{v}}{v} - \gamma\right\}\right]$ ...(2.49)

Based on the above equations, the author performs a simulation exercise for the following values of the parameters:

$a = 1,\ N = 100,\ \alpha = 0.015,\ \beta = 0.035,\ \gamma = 0.02,\ v = 3,\ \lambda = 0.9,\ \omega = 0.96$

Keen considers interest rate as the basic parameter. Simulations are done for two kinds of variations on the interest rate:

1. Changing the base rate, with zero bank sensitivity to debt-output ratio, i.e. changing ς when $\varphi = 0$. According to Keen, this corresponds to the "conventional

governmental action to control an overheated economy using monetary policy" (Keen, 1995, p-618).

2. Changing the level of bank sensitivity to debt-output ratio, i.e. changing φ keeping ς constant.

These simulations are presented in the articles by Keen (Keen, 1995, Fig. 4-10). The main results of these simulation exercises are presented below.

Results of simulations from base rate (ς) variations are as follows:

- When the base rate is less than 4.6%, the system approaches a stable equilibrium over time. During booms, investments are higher than normal, leading to greater than normal accumulation of debt by the capitalists. This leads to an increase in interest commitments. This leads to a gradual decline in investments, till the debt-output ratio reaches a stable level. Thereafter the system grows at a steady pace, with constant share of output for the three sectors.
- When the base rate is more than 4.6%, the system gets explosive. Initially, there is a boom, where both the workers and the capitalists try to maintain their share of outputs with the total output increasing. Thus, there is an increase in the share of wages and profits in the alternative periods, with an increase in one of these feeding the other. This leads to excessive accumulation of debt by the capitalists. Finally, excessive debt leads to a turnaround in profits, where the profits start declining, leading to a decline in investment, employment and wages. Finally, the system moves towards zero employment, profits and wages, with the share of output for the financial sector spiraling upward. Once the system gets into this stage, there is no way it can get out on its own without a change in the rules of the game. Thus, the booms are unproblematic in the beginning, but then gets out of hand due to attainment of unsustainable debt-output ratio – a conclusion similar to Minsky.

The results of simulations from variations in sensitivity of the banks to change in debt-output ratio (φ), when the base rate (ς) is within the stable range (i.e. less than 4.6% in the present case) are as follows:

At low rates of debt sensitivity, with variable interest rates, the system "stabilizes rapidly" (Keen, 1995, p-623). As the debt sensitivity increases, instability of a kind different from the earlier case is observed. The investment boom, in this case, is muted due to an increase in the share of the financial sector to output from an increase in debt-output ratio. Thus, there is only a 'minor boom' in the investment, which causes a 'minor boom' in the employment. However, the rate of increase in banker's share exceeds the rate of decline in worker's share of output, leading to a steady fall in the rate of profits, which finally falls to below zero, and the model breaks down.

Thus, according to Keen, the model might, depending on the values of the parameters, display any of the following two behaviors:

1. A system that tends to stability.
2. A system that breaks by attaining unsustainable debt-output ratio.

Keen performs further simulation exercises for the role of government. Government, through appropriate fiscal policies, plays two kinds of counter-cyclical role in this setting:

1. Dampen the increase in debt-output ratio during boom time by muting the investment boom itself.
2. Provide capitalists with cash flows during slump to enable them to honor the debt commitments.

The results of simulations with the government sector shows that "government intervention greatly diminishes the possibility of complete breakdown, but does not eliminate cycles. Instead, the system displays apparently random, irregular cycles." (Keen, 1995)

The basic conclusions that Keen derived from these simulations are as follows:

1. Financial instability of the kind noticed in Great Depression is one of the possibilities that an economy might get into. Once it gets into this stage, it might not be able to get out without the help of something outside the system.
2. Government can play an effective counter-cyclical role to prevent occurrence of such crises.
3. Since high debt-output ratio is source of the problem, institutional reforms to prevent an increase in this ratio during boom might make the economy more stable.

Alessandro Vercelli (2000)

Vercelli's paper does not explicitly model Minsky. However, he tries to model financial instability using some of the insights of Minsky. He makes one useful point throughout the article – dynamic instability in the mathematical sense (i.e. the tendency of a system not to come back to the equilibrium) is not a necessary condition for existence of financial instability. He defines the additional concept of 'structural instability', a condition where a small exogenous shock alters the dynamic behavior of the system by changing the functional or parametric structure.

Vercelli also tries to model the financial behavior of the microeconomic units. Each unit *i* in each period *t* is characterized by the following financial ratio:

$$k_{it} = \frac{e_{it}}{y_{it}} \qquad \text{...(2.50)}$$

where e_{it} = Sum of cash outflows corresponding to purchases of goods and services

and y_{it} = Sum of cash inflows corresponding to sales of goods and services.

k_{it} is allowed to exceed 1, provided the unit *i* is able to finance it either by a reduction in the stock of cash balances or by increasing the stock of debt.

Each unit is also characterized by the following intertemporal financial ratio calculated over a time horizon *m*:

$$k_{it}^{*} = \frac{\dfrac{\sum_{n=0}^{m} e_{it+n}^{*}}{(1+r)^n}}{\dfrac{\sum_{n=0}^{m} y_{it+n}^{*}}{(1+r)^n}} \qquad \text{...(2.51)}$$

where r = Rate of discount, e^{*} = The sum of discounted expected outflows, y^{*} = The sum of discounted expected inflows. $k_{it}^{*} \leq 1$ is the condition for financial sustainability of the unit, beyond which the unit is pushed towards bankruptcy. Therefore, Vercelli considers k_{it}^{*} as an indicator of financial fragility of the firm.

Let each unit define a threshold of financial fragility $1-\mu_i$ beyond which it does not want to go. When $k^*_{it} \geq 1-\mu_i$, the unit tries to reduce it by reducing k_{it}, leading to a reduction in k^*_{it}. When $k^*_{it} < 1-\mu_i$, the pressure of competition forces the unit to increase k_{it}, leading to an increase in k^*_{it}. The feedback between the two variables is shown in the following system of equations:

$$\frac{\Delta k_{it}}{k_{it}} = \alpha\left(k^*_{it} - 1 - \mu_i\right) + \varepsilon_i, \quad \alpha \prec 1 \qquad \ldots(2.53)$$

$$\frac{\Delta k^*_{it}}{k^*_{it}} = \beta\left(k_{it} - 1\right) + \eta_i, \quad \beta \succ 1 \qquad \ldots(2.54)$$

where ε_i and η_i are shocks around the rest of the equation, which might change the behavior of the system. The system represented by equations 2.53 and 2.54 is shown in the following diagram:

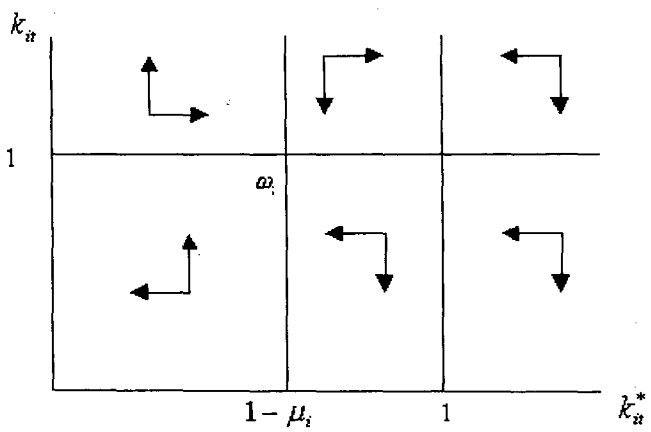

Figure 2.5

According to Vercelli, this system has a tendency to be cyclical, though the final outcome might not be regular due to the effects of the shocks, ε_i and η_i, and also of the other control decisions of the units and policy variables outside the purview of the above equations.

Aggregating the inflows and the outflows at the macroeconomic level, we arrive at a macroeconomic system similar to that represented by equations 2.53 and 2.54, though the macroeconomic system is likely to be more regular due to the correlation between the individual behaviors. This system is shown below:

$$\frac{\Delta k_t}{k_t} = \alpha\left(k_t^* - 1 - \mu\right) + \varepsilon, \quad \alpha \prec 1 \qquad \ldots(2.55)$$

$$\frac{\Delta k_t^*}{k_t^*} = \beta\left(k_t - 1\right) + \eta, \quad \beta \succ 1 \qquad \ldots(2.56)$$

This is shown in the following diagram:

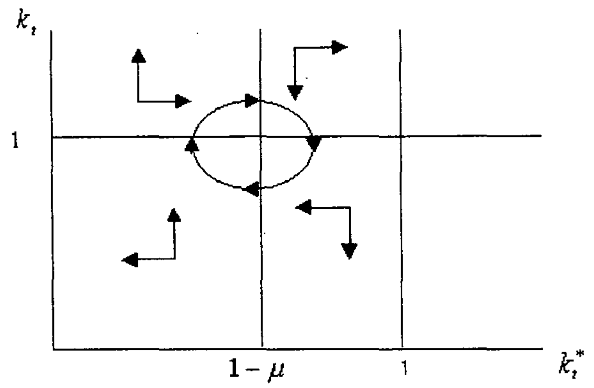

Figure 2.6

The author assumes that the aggregate outflows, E_t, consists of the private aggregate expenditure, $E_t^{'}$ and the public expenditure, ΔM_t (representing a change in the stock of money). With the assumption of a lag of one period between realized inflows and realized outflows, the author derives the following relation:

$$k_t = \frac{E_t^{'}}{Y_{t-1}} \qquad \ldots(2.57)$$

The public expenditure, ΔM_t consists of the deficit of the government (i.e. purchases of the government of goods and services from private sector) and the bonds purchased by Central Bank from private sector.

$$\Delta M = D + \Delta B \qquad \ldots(2.58)$$

From equations 2.55-2.57, the author derives the following dynamic system:

$$Y_t = E_t \qquad \ldots(2.59)$$

$$E_t = k_t Y_{t-1} + \Delta M_t \qquad \ldots(2.60)$$

The dynamic behavior of the above system of equations would depend on the cyclical fluctuations of k_t as described by equations 2.55 and 2.56. With diagrams for these two equations in *Y-E* phase space separately for the following two cases (Vercelli, 2000, p-151):

1. $\Delta M \succ 0$: The author calls this 'inflationary regime'.
2. $\Delta M \prec 0$: The author calls this 'deflationary regime'.

The author derives the following conclusions from the diagrams:

1. Cyclical fluctuations are observed in both cases. In both the cases, these are characterized by upward dynamic instability (i.e. income and expenditure tends to explode to infinity) in boom phase (boom is defined as the phase where income exceeds certain critical value, i.e. $Y \succ \hat{Y}$) and downward dynamic instability (i.e. income and expenditure tends to collapse to zero) in the depression phase (depression is defined as the phase where income is less than the critical value, i.e. $Y \prec \hat{Y}$).
2. However, a change in k_t might shift the curve and bring the economy from one phase to another, i.e. from depression to boom and vice versa. Thus, a change in parameters over time causes a change in the behavior of the system.
3. In the inflationary regime, "there may be an expansion characterized by upward dynamic stability which endogenously transforms itself in an expansion characterized by upward dynamic instability in consequence of the progressive increase in k_t, triggered by the expansion itself on the basis of equations" 2.55 and 2.56 (Vercelli, 2000, p-151).
4. In case of deflationary regime, the economic system may be trapped into a downward path of dynamic instability even if $k_t \succ 1$. According to Vercelli, the crisis of the Great Depression was characterized by a situation similar to this.

Section III

An Assessment of the Literature

An assessment of the literature reviewed so far leads us to the following conclusions:

1. Pure multiplier-accelerator models are likely to lead to unbounded behavior of income and profits. Considerations of full-employment ceilings and zero gross investment floors might give us boundaries. However, in the real world income and profits often fluctuate within much narrower boundaries. These models do not offer much explanation for this.
2. Integrating the financial sector with multiplier-accelerator analysis might give us a system bounded within narrower boundaries. Much of the literature emanating from Minsky (1982) and Kindleberger (1978) does precisely this.

Historical Evidence

Historical evidence would suggest that the theoretical framework discussed in Section II (especially the one developed by Minsky) captures at least one crucial aspect of the modern economy – that of a tendency of firms to increase their indebtedness during boom, leading to financial crisis putting an end to the boom. Kindleberger (Kindleberger, 1978) deals with plenty of examples of this kind. A more recent example could be the South Korean experience, which we briefly look at in the following paragraphs.

A Brief Account of the Experience in South Korea[16]

East and South-East Asia was one of the fastest growing regions in the post World-War-II period, leading to a dramatic transformation in standards of living. Initially, in the 1960s and 1970s, this transformation was confined mainly in the East Asian region, in countries like South Korea and Taiwan.[17] Much of the literature attributed this to an aggressive state-directed export-oriented growth strategy (For instance, Jomo, 1998). In

[16] A more detailed description of the events in South Korea and other economies in the region may be found, among others, in Islam and Chowdhury (1997), Jomo and others (1997), Jomo (1998), Jackson (1999) and Radelet and Sachs (2000).

[17] We do not include Japan here, since it was already substantially developed in the period prior to World War-II. However, following the destructions of war, it had to reconstruct the economy in the post World-War-II period. In doing so, it had a growth experience similar to the countries referred here.

the 1980s and 1990s, they were joined by countries from the South-East Asian region, like Thailand, Malaysia, Indonesia, and at a later stage, Philippines. By mid-nineties, the region was enjoying export growth rates in excess of 10%, with some countries like South Korea, Thailand and Malaysia enjoying export growth rates between 25 to 30% (World Trade Organization, various years). Due to various reasons, however, the exports slowed down in the period after 1995.[18] The resulting gap in the Balance of Payments was closed by inflows of capital into these countries, which, with the liberalization in the capital accounts played an increasingly larger role in these countries in the nineties.[19] These took the form of foreign direct investment, portfolio investment and short-term borrowing, with the last factor playing an important role, especially in countries like South Korea, Thailand and Indonesia. In Korea and Thailand, the borrowings were mainly undertaken by the banks, which subsequently utilized this by expanding domestic credit. The cost of finance, thus, was kept low by an expansion of both domestic and external credit. For instance, the domestic credit in South Korea expanded from around 85% of GDP in 1987 to around 165% of GDP in 1997, just before the crisis. This was in addition to even more dramatic increase in external debt by private sector in this period (IMF International Financial Statistics, various years). This enabled these economies to sustain the high rates of growth in spite of falling exports. For instance, in South Korea, the real growth rate of GNP was 8.4%, 8.7% and 6.9% in 1994, 1995 and 1996 respectively.

There were certain signals of the impending trouble in most of these economies right from the mid-nineties, though the markets (and, to a great extent, the analysts) largely ignored these signals. In South Korea, this was in the form of rather high debt-equity ratio in the indigenous corporate conglomerates, known as *chaebols*, with major portion of debt from international financial markets. For instance, in 1996, the debt-equity ratio of top 30 *chaebols* (who together produced over half of Korea's GNP) ranged from 250% to 8500%, with the average debt-equity ratio at 386.5%. The average debt-equity ratio of top 30 *chaebols* increased to 518.9% in 1997. (Pyo, 1999) Most of the debt was used in making strategic investment, aimed more at capturing a share of the market

[18] These factors are discussed in detail in Jomo, 1998.
[19] The capital inflows into this region is discussed in more detail by Rangarajan (2000).

than immediate profits. A large portion of the investment was made in horizontal diversification and vertical integration. For instance, Kia Motors, one of the important *chaebols* (and one of the first to get into trouble), is alleged to have invested in drastically increasing its automobile production to 'unsustainable' (Pyo, 1999) levels, and also invested heavily for vertical integration (like Kia Metals Co.) and horizontal diversification (like construction and financing business). In fact, most of the firms in automobile and semiconductor business were known to have increased their investment substantially in an attempt to put themselves in a strategic position in the global market.

The literature typically identifies beginning of the 'crisis' in the region with a major devaluation of Thai Baht (which till then was pegged to the U.S. dollar) on July 2, 1997, following a speculative attack against it. The magnitude of devaluation was about 20% within a single day. This was soon followed by similar speculative attacks and devaluations of Philippines Peso, Singapore Dollar, Indonesian Rupiah, Malaysian Ringgit and, finally South Korean Won.

However, it might be more appropriate to identify beginning of the crisis, at least in South Korea, with the *chaebols* beginning to default on their payment commitments. Hanbo Steel was the first major *chaebol* to go bankrupt in January 1997. This was soon followed by two major *chaebols,* Sammi Steel and Kia Motors, also defaulting on their payment commitments. By October 1997, seven of the nation's top 30 *chaebols*, including the Kia Motors Group had already started defaulting and had filed for court-mediated protection. Since a large proportion of the debts of *chaebols* were from international financial markets, this had severe implications in the balance of payments for South Korea as well. The foreign loans were channeled through merchant banks, pushing these banks to default as well. The foreign lenders started pulling out of South Korea, leading to pressures of depreciation on the South Korean won. This was also followed with a sharp fall in investment and employment, leading to a recession. The growth rates in fact, was negative – minus 6.7% in 1998 (World Bank, 2001).[20]

[20] The slow rates in the aftermath of the crisis have often been attributed in the literature to lack of capital inflows. It could also, however, be an outcome of widespread defaults by *chaebols* and their takeovers, which made the surviving *chaebols* extra careful while making investment decisions.

Like many other countries in the region, South Korea entered an agreement with IMF, which resulted in one of the largest rescue packages in the recent history of IMF (Pyo, 1999). Like all other IMF packages, this also consisted of certain policy commitments on the part of the South Korean governments (Government of Korea, 1997 and 1998). These can be summarized as follows:

- Fiscal contraction
- Closure of banks which are in financial trouble
- Enforcement of capital adequacy standards in banks
- Tightening of domestic credit
- Repayment of international debt
- Non-financial structural changes, like reduction in tariffs, opening sectors to foreign investment etc.

In other words, the policy package basically amounted to a contractionary fiscal and monetary policy.[21]

Since then, many of the corporate groups in South Korea have been restructured. There have also been takeovers of some of the major South Korean firms by multinational enterprises from USA and Europe – a trend observed all over East and South-East Asia. The growth rates, however, have failed to reach anywhere near the pre-crisis levels till now, except for a short boom during 1999 and first quarter of 2000 (World Bank, 2001).

Minsky's Contribution: Some Problem Areas

In spite of overwhelming historical evidence in support of Minsky's writings, there seems to be certain problem areas in his analysis, which are not resolved by those trying to model it. Let us briefly look at some of these:

1. The microeconomic analysis of Minsky, when translated to the macroeconomic level, seems to get into a typical problem of aggregation (or what is more popularly known as the fallacy of composition). Let us, for instance, recall one of the problems

[21] Many of these prescriptions were heavily criticized, not just by the usual critiques of IMF, but even by international organizations like World Bank, for intensifying both the recession and the bankruptcies (Krugman, 1999).

discussed by Minsky – that of the high profit expectations during a boom resulting in high investment financed by debt. The problem occurs only when the unrealistically high profit expectations are not actually realized. This, however, is a problem only for some firms at the microeconomic level. As long as higher expected profits actually translate to higher investment, one should not expect a problem at the macroeconomic level. This is because of an important difference in the way profits are realized at the microeconomic and the macroeconomic level. For a firm at the microeconomic level, investment in a particular time period might lead to realization of profits in some future time period, with a gestation lag. The length of this lag will depend on the nature of the investment. Within this lag, there could be quite a few changes in the economy. For instance the demand might decrease, or the demand for the specific commodity the firm was producing might decrease. At a macroeconomic level, however, investment leads to an increase in profits at the end of the same time period due to the operation of the multiplier. This might occur either by an expansion of output (when there is excess capacity in the economy), or by increasing the share of profits at the same level of output by squeezing real wages (the case of profit inflation).[22] Thus, individual firms could get into trouble, but there is no reason to believe that this will create any problems at the macroeconomic level.

2. Lower liquidity during a prolonged boom is the other source of problems discussed by Minsky. This, however, completely depends on the monetary policy of the Central Bank. Higher level of economic activities during a boom will imply higher transaction demand for money. Interest rates will increase if and only if the monetary authorities decide not to provide additional liquidity to meet this demand.

The above two arguments does not take away the relevance of Minsky's analysis, especially given the overwhelming support of historical evidence. It, however, questions one of the central propositions of Minsky – the *inevitability* of a financial crisis resulting

[22] For instance, a car manufacturer, say General Motors, invests in India by setting up new factories. The profits will be materialized only when the factory is completed, the cars manufactured and actually sold in the market. Lot of changes could have happened within this time, creating problems for the realization of profits for General Motors. However, at a macroeconomic level, the investment generates additional demands the moment the investment orders are placed through the multiplier effect. This could be from spending out of wages paid to the workers employed by the General Motors, or from investment orders placed by General Motors for new machines, creating new employment and profits elsewhere etc.

in an end to economic boom. Nothing in Minsky's work gives us a theoretical basis for this. This gives us a reason to suspect if such crises are indeed inevitable, or whether appropriate policies can postpone such problems indefinitely without any drastic change in institutional structure (as suggested by Minsky). The next chapter looks into these problems.

CHAPTER 3:

FINANCING INVESTMENT – MACROECONOMIC IMPLICATIONS OF MICROECONOMIC DECISION-MAKING

The main purpose of this section is to specify the relationship between various microeconomic and macroeconomic variables while incorporating the financing decisions in the multiplier-accelerator models.

The economy consists of households and firms, with a Central Bank to conduct the monetary policy of the economy. Following classical savings assumption, we assume all wages to be consumed. Savings are out of profits only. Income of the households consists of both wages and profits. The income of firms consists of profits only.

All agents follow *static profit expectation* throughout, i.e. $P_t^e = P_{t-1}$, P being the profits.

Investment costs in this model consist of both fixed and variable costs. However, the fixed capital has a life of only one period. In other words, all machines and other fixed capital are completely depreciated within a period.[23] This converts the formal model into a circulating capital model.

The rate of interest in our model is affected by two factors: the transaction demand for money (which is a function of income and investment) and monetary policy of the Central Bank. (Speculative demand for money is not considered in our model) This is shown in the following formulation of the rate of interest in period t:

$$r_t = \bar{r} + lI_t \qquad \ldots(3.1)$$

where $\bar{r}$ is the historically given 'normal rate of interest', and l an indicator of the degree of the accommodation of the monetary policy. Higher the l, lower is the degree of financial accommodation by Central Bank. (Financial Accommodation implies the degree to which the Central Bank expands or contracts the money supply in response to an increase or decrease in transaction demand for money) $l = 0$ would imply that the Central Bank completely accommodates every change in transaction demand for money with a corresponding change in supply, making the rate of interest completely unresponsive to transaction demand for money. l is allowed to be negative, as well.

[23] This can be justified since the length of period t is not specified in the model.

Though changing money supply seems to be the most obvious route for Central Bank to control l, it might be noted that there exist other means for changing l as well. In many economies, for instance, the interest rates are directly administered by the Central Bank. In this case, change in interest rates by the Central Bank in response to change in investment would determine l. Similarly, the way the monetary authorities play the regulatory role in the financial sector might also affect l. For instance, a relaxation of controls, by allowing financial innovation, might make the broad money supply[24] more responsive to transaction demand, reducing l. On the other hand, a tightening the regulatory framework might reduce the flexibility of broad money supply, increasing l.

There are three ways an investment can be financed:

1. Internal finance out of retained earnings of the firm
2. Debt
3. Equity

Decisions regarding specific financing mode are typically made by the firms at the micro-level. There already exists a considerable body of literature in this area. The concern of this section, however, is to look at the macroeconomic consequences of these decisions. A simple aggregation of microeconomic terms often leads to fallacy of composition, giving a misleading picture of macroeconomic events. This is because many variables behave differently at the micro and the macro level – and macroeconomic variables are not always sums of various microeconomic terms. The behavior of each variable at micro and macro level must be distinguished explicitly. Hence, we begin our analysis in this section by examining the relevant variables separately at the microeconomic and macroeconomic level.

[24] The term 'broad money' is used to imply a broader measure of money supply then the more conventional M1. It can be argued that, in the contemporary world, the relevant measure of money supply must include many of the modern financial instruments of transaction and credit, which the conventional M1 fails to consider.

Variables at the Microeconomic Level

Internal Finance

Firms may typically finance a part of their investment out of retained profits from last period (i.e. profits after paying out dividends to the shareholders). The management of the firm decides on the fraction of profits to be paid out as dividends to the shareholders and the fraction to be retained for financing investment.

Let the fraction of profits retained for financing investment be σ. Therefore, the fraction paid out to the shareholders is $1-\sigma$.

Dividend policies might differ from firm to firm, and might also change for the same firm over time. However, certain kinds of shares (like the preferred shares) require a legal commitment to prioritize paying dividends. Even for other shares having no formal legal commitments, certain fraction of profits is normally expected to be paid out as dividends. If the dividends are less than the expected fraction, the owners of shares might be unhappy with the management and replace it. On the other hand, paying out extra dividends would be bad for the long-term growth of the firm. So, there is relatively little flexibility of σ both across firms and for the same firm overtime. We take σ as constant for the sake of simplicity. Thus:

Internal finance of the i^{th} firm = σP_{t-1}^{i}

Debt Financing

A firm *i*, in the period *t*, approaches the debt market to borrow an amount B_t^i at a rate of interest r_t. If the loan application is granted (i.e. if firm *i* is creditworthy), this leads to a debt commitment D_{t+1}^i in the next period *t+1*, where $D_{t+1}^i = B_t^i(1+r_t)$. This commitment is fixed, irrespective of the profits of the firm. This amount is either paid back in the period *t+1*, if necessary, by further borrowing (rolling over of debt), or the firm defaults. In other words, default occurs when a firm cannot meet its loan requirements to repay past debt, and the debt can no longer be rolled over.

Equity Financing

In our study, we look at a simple model of equity financing, where a firm finances a part of investment by diluting the ownership of the firm and inviting new owners to have a stake in the firm.

At the beginning of a period t, let the i^{th} firm sell equities worth ξ_t^i to the general public. The public buys these shares for a stake in the profits of the firm and trades it freely among themselves within period t at the face value of the shares.[25] Without capital gains, trading between public within a period takes place purely due to liquidity reasons. At the beginning of the next period, the firm buys back all its outstanding shares and sells fresh shares of an amount ξ_{t+1}^i in the market.[26]

$\therefore$ In period t, the i^{th} firm sells ξ_t^i and buys back ξ_{t-1}^i. This defines the amount of equity financing over period t, viz.:

$$E_t^i = \xi_t^i - \xi_{t-1}^i \qquad \ldots(3.2)$$

As would be clear, equity financing involves two kinds of costs for the firm:

1. Cost of profit-sharing: Equities involve sharing a fraction of profits with new shareholders, unlike debt, which leads to fixed commitments irrespective of profits. Sharing of profits could be especially costly when the profits are high. For instance, if new owners own a fraction θ of shares, then $\theta(1-\sigma)P_t^i$ must be paid to them. This is directly proportional to P_t^i.
2. Risk of take-over: Dilution of equities might change the structure of ownership, leading to increased risks of takeover by the new dominant shareholders. We do not, however, consider this in the formal model. In our model, takeover occurs only when a firm defaults.

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[25] This rules out possibilities of capital gains. Capital gains, no doubt, play an important role in equity markets. This has to be modeled separately. This, however, is a more complex issue and beyond the scope of the current paper.

[26] The firm might buy back all outstanding shares due to legal commitments or due to certain conventions. Since the firm can always buy and sell the same shares back into the market, the transaction of buying back and reselling can occur in a purely notional sense in the accounts of various entities without actual changing of hands. The assumption of buying back, thus does not affect the real world validity of the model. This is conceptually similar to the notion of rolling over of debt, and is done for the sake of consistency with the rest of the circulating capital model.

A firm might also face two kinds of supply constraints on equity financing:

1. Lack of buyers: Many new public offerings fail to find adequate buyers in the markets. (This is especially true for Initial Public Offerings, and also for some of the Seasoned New Offerings) This might force the underwriter to buy the entire offering at a previously negotiated price. So, the underwriters will be especially wary to enter a contract with lesser-known firms. These firms, therefore, might not find an underwriter to market the offerings in the first place. Similar problems might arise if these firms attempt to sell shares privately.
2. Value of the firm: Value of the firm might prove to be an upper limit to the amount of equity financing the firm might generate. However, a good firm might enhance the value of the firm by undertaking investments out of the fresh equities. Thus, this constraint, like the first one, would vary from firm to firm.

Microeconomic Balance-sheet

We have the following balance-sheet at the level of the firm:

Table 3.1

Source of Funds	Use of Funds
Retained Profits from last period.	Investment in the current period.
Borrowing in the current period.	Paying back debt commitments from last period.
Equity financing in the current period.	

The items in the above balance-sheet are realized or *ex poste* quantities, and hence must be distinguished from the planned variables discussed so far. This makes the equality in the above balance-sheet an *ex poste* identity, which always holds true. The *ex* *ante* counterparts of the above variables, however, would show the above equality under equilibrium condition as follows[27]:

$$B_t^i + E_t^i = I_t^i + B_{t-1}^i(1+r_{t-1}) - \sigma P_{t-1}^i \qquad ...(3.3)$$

A Measure of Financial Fragility

Literature provides various ways of measuring financial fragility at the level of the firm. Some of these are: the debt-equity ratio, ratio of inside to outside finance (due to Kalecki) and Tobin's q-ratio $\left(= \dfrac{\text{Market value of equity and debt}}{\text{Replacement cost of assets}}\right)$. The first and the last measure are most widely used in the corporate world. However, Tobin's q-ratio depends on market valuation. Since our model provides no scope for capital gains through market valuation, this measure would be inappropriate for the present context.

The basic difference between the first and the second measure lies in the treatment of equity financing. Equity of all kinds is considered safe in the measurement of debt-equity ratio. Hence, increase in equity, *ceteris paribus* reduces financial fragility when measured in terms of debt-equity ratio. It ignores the fear of takeover or mergers. This might be more appropriate where the shareholding is so disperse that the controlling group is sure of not losing control. On the other hand, Kelecki had a different conception of the firm, where the shareholding is concentrated in the hands of controlling shareholders. Since shareholding is the basis for control in this case, takeovers or mergers and acquisitions through dilution of shareholding could be a real threat. So Kalecki considers equity financing by outside shareholders risky and groups it with debt as a risky asset. Higher issue of outstanding equities, when this measure is used, increases the financial fragility of the firm.[28] However, most of the contemporary firms are somewhat in between these two extremes, where the dominant shareholder does control the

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[27] This distinction made here between planned and actual quantities is analogous to the distinction between planned and actual investment and savings, made by Keynes (1936, Chapters 6 & 7), and also discussed in most of the standard macroeconomics textbooks, for instance, Dornbusch and Fischer (1994), Chapter 3. It is also similar to the debate over Quantity Theory of Money between Keynes and his followers and the monetarists like those belonging to the Chicago School.

[28] We can, for instance, contrast the traditional American style of corporates having disperse shareholding with the traditional Japanese conglomerates or modern IT firms like Microsoft, where the shareholding is concentrated with the controlling group. Takeovers through mergers and acquisitions present a real threat to the latter, and not to the former.

management, but only in the major decisions through threats of takeovers or mergers. This puts equities in an ambiguous category. To bypass this problem, we do not include equities in our measure of financial fragility. In other words, our measure of financial fragility is neutral to equity financing and is a function of only debt and internal financing.

It must be noted that, it is not borrowing itself but the debt commitments arising out of it that makes a firm vulnerable.

Therefore, we propose the following formulation of financial fragility or leverage of a firm:

$$\lambda_{t+1}^{i} = \frac{D_{t+1}^{i}}{\sigma P_{t}^{i}} = \frac{B_{t}^{i}(1+r_{t})}{\sigma P_{t}^{i}} \qquad \ldots(3.4)$$

(The denominator takes into account only the retained profits, and not the total profits, in spite of equity holders being residual claimants. This is because, our model does not allow flexibility to σ, due to reasons discussed earlier)

Typically, in an economy, we should witness firms with a whole spectrum of λ^{i}s, ranging from very high to very low levels. The ones with the highest λ^{i}s are more likely to default, though whether a specific firm will default will depend on its creditworthiness, which in turn would depend on a whole range of factors within and outside the control of the firm.

This can help us quantify three financial postures by firms corresponding to Minsky's classification (with the assumption of static expectation):

1. Hedge Finance: A firm i is said to be following hedge financing option if $\lambda_{t}^{i} < 1$, i.e. expected retained earnings are enough to pay back the debt commitments. This is the safest of the three options.
2. Speculative Finance: A firm i is said to be following speculative financing option if $\lambda_{t}^{i} > 1$, i.e. expected retained earnings fall short of the total debt commitments.
3. Ponzi Finance: This is the riskiest of the three options. A speculative firm i is said to be following Ponzi option if $r_{t}B_{t}^{i} > \sigma P_{t}^{i}$, i.e. expected retained earnings are not

enough to cover the interest payments. Ponzi finance, thus, is a special case of speculative finance.[29]

All speculative units, including the Ponzi units exist because their long-run expected $\lambda^i < 1$. This, in fact, is a necessary condition for the firm to exist in the first place.[30]

An economy consists of firms with a spectrum of λ^is, ranging from very high to low levels. We further assume that, not all firms in the economy ever turn Ponzi in the same period – i.e. there exists at least one hedge or speculative firm at any given period of time. The firms with higher λ^is are the most likely to default, though whether a specific firm will default or not would depend on its creditworthiness, which in turn would depend on a whole range of other factors within and outside the control of the firm.

Macroeconomic Counterparts of Micro Variables

Internal Finance

At macroeconomic level, retained profits are assumed to be a fixed proportion of macroeconomic profits in the last period.

Internal finance in period $t = \sigma P_{t-1}$

Debt Finance

Let the credit policy of lenders be such that, *firm i has access to credit if and only if it is not Ponzi. Ponzi financing, thus, is not allowed in our model.* A firm defaults due to lack

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29 $\lambda^i_{t+1} = \frac{B^i_t(1+r_t)}{\sigma P^i_t} = \frac{B^i_t}{\sigma P^i_t} + \frac{r_t B^i_t}{\sigma P^i_t}$, where, in case of Ponzi finance, the second term on the right hand side by itself is greater than 1, making λ far in excess of 1. Although all speculative firms must borrow to pay their debt commitments, what makes Ponzi units special is the fact that, they have to borrow to pay back both the principal and the interest. Non-Ponzi speculative firms, on the other hand, can at least pay back interest commitments, $r_t B^i_t$ out of their retained profits, and need credit to only pay back the principal, B^i_t. The outstanding debts of Ponzi units must increase over time.

30 This is an aspect highlighted by Minsky as well, when he consistently defines the three financial postures in terms of short-run cash flows only. Within some finite long-run time horizon, even the Ponzi unit hopes to turn hedge, provided it survives the length of time required for such a transformation to take place.

of access to credit as soon as it enters the Ponzi stage. *If the takeover market is sufficiently active and there exist sufficient buyers for defaulting firms at book value, the defaulting firms would immediately be taken over by firms that are financially sounder, i.e. the hedge or speculative firms*[31]. The new owners would assume the debt commitments of these units, though they will not face any problems since they will have access to credit as long as they are themselves non-Ponzi. In other words, owners change but good investment projects never fall short of funds. The financial market, through takeovers, only ensures that the most creditworthy owner and management undertake the investment.

This means that, there is an important distinction in the way debt must be treated at the micro and the macro level. At micro level, individual firms might face supply constraint for credit. But at the macroeconomic level, credit is completely demand-determined. In other words, we are in an endogenous credit regime.

Equity Financing

Let E_t be the equity financing at the macroeconomic level, corresponding to E_t^i at the microeconomic level. However, unlike micro level, *there does not exist any supply constraints on equity financing at the macroeconomic level.* This, in fact is just a reiteration of the earlier assumption that a good investment project never falls short of funds – the financial markets only picks the most worthy firm or management to undertake a project. If it is unwilling to fund investment undertaken by one particular firm, it will find another firm more worthy to undertake the investment. But at the macroeconomic level, the investment will always be financed.

Macroeconomic Balance-sheet

We have the following balance-sheet at the macroeconomic level:

[31] This means, those acquiring defaulting firms must satisfy two conditions: they must be attracted by the long-run profitability of the Ponzi units, and must be non-Ponzi themselves in order to have access to funds to undertake investments as well as payment commitments of defaulting firms.

Table 3.2

Source of Funds	Use of Funds
Macroeconomic Retained Profits from last period.	Macroeconomic Investment in the current period.
Borrowing at the macroeconomic level in the current period.	Paying back debt commitments at the macroeconomic level from last period.
Equity financing at the macroeconomic level in the current period.	

Like the microeconomic balance-sheet, the macroeconomic balance-sheet also consists of actual quantities, and hence must be distinguished from the planned variables discussed so far in this section. The balance-sheet identity must always hold true. The corresponding *ex ante* equilibrium condition is as follows:

$$B_t + E_t = I_t + B_{t-1}(1 + r_{t-1}) - \sigma P_{t-1} \qquad \ldots(3.6)$$

Due to the assumptions made about the debt and the equity market, the direction of causation of equation 3.6 must be from right to left. This would mean that we are in an endogenous finance regime, where both debt and equity are completely demand determined. This, in fact, is the critical difference between behavior of the micro and macroeconomic variables – individual firms at the microeconomic level often faces a supply constraint of both debt and equity, but at the macroeconomic level these variables only face a demand constraint.

Financial Fragility at the Macroeconomic level

The macroeconomic λ would be a weighted average of various λ^is across the economy. When the λ^is increase for all firms, the macroeconomic λ will increase as well. Of course, some of the firms with high λ^i (Ponzi firms) are going to default. But, as we have already argued, these will be taken over by better (i.e. non-Ponzi) firms. These firms will also inherit higher debt commitments reflected by high λ^i of the defaulting firms, increasing their λ^i as well. Therefore the λ at the macroeconomic level will increase.

$$\lambda_t = \frac{D_t}{\sigma P_{t-1}} = \frac{B_{t-1}(1 + r_{t-1})}{\sigma P_{t-1}} \qquad \ldots(3.7)$$

This allows us to make the following proposition:

If in any time period an exogenous event moves all the λ^is in any particular direction, then the macroeconomic λ will also move along in the same direction.

Financial Structure

The specific financial structure at the macroeconomic level is decided from the demand side. Internal finance is given from the past. The firms at the microeconomic level make decisions between debt and equity as alternative modes of finance, leading to *B/E* at the macroeconomic level. Since capital gains are ruled out by assumption in our model, firms do not worry about valuation while deciding on this ratio.[32]

Let h_t be the fraction of investment financed by borrowing in period *t*.

$$B_t = h_t I_t \qquad ...(3.8)$$

Equity financing and retained profits from last period finance the rest.

$$\sigma P_{t-1} + E_t = B_{t-1}(1 + r_{t-1}) + (1 - h_t)I_t \qquad ...(3.9)$$

There is usually a tendency to finance a greater fraction of investment from debt when the investment is at a high level. This could be explained as follows:

1. Retained profit is given from the past and is beyond the control of the firms. Hence, it is likely to fall short when the desired investment is at higher level, forcing them to increasingly seek outside sources of funds.
2. Debt leads to fixed commitments irrespective of profits. Cost of equities (due to profit sharing), on the other hand, is asymmetric and increases with increase in profits. Higher desired investments, however, are usually associated with higher profit expectations as well. Both are affected by same set of exogenous market conditions. Therefore, within debt and equity, debt will be preferred when investment is at high level.
3. Higher proportion of debt magnifies both returns and losses from an investment. This can be shown as follows:

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[32] If our model included capital gains, then either valuation would have explicitly entered as one of the factors determining *B/E* or, following Modigliani and Miller, we would have had to assume the value of a firm to be independent of capital structure.

Let a firm invest an amount x in a project having a rate of return π. A fraction α of this is financed by debt at a rate of interest r and the rest financed out of internal finance.

Rate of profit for the firm $= \Omega = \dfrac{x(1+\pi) - \alpha x(1+r) - (1-\alpha)x}{(1-\alpha)x} = \dfrac{\pi - \alpha r}{1-\alpha}$

$$\frac{\partial \Omega}{\partial \alpha} = \frac{\pi - r}{(1-\alpha)^2}$$

When $\pi \succ r$, $\dfrac{\partial \Omega}{\partial \alpha} \succ 0$, i.e. when the rate of return exceeds rate of interest, financing a larger fraction of investment out of debt increases profits.

When $\pi \prec r$, $\dfrac{\partial \Omega}{\partial \alpha} \prec 0$, i.e. when the rate of return is less than the rate of interest, financing a smaller fraction of investment out of debt increases profits.

It might be noted that the functional form for Ω has no maxima or minima, i.e. to maximize profits, the firm would like to have α as either 0 or 1. However, there are other factors, like risk of default etc. which affect this decision in real world but have not been included in the model. Thus, in real world, we may expect the desired fraction of debt, α to lie somewhere between 0 and 1 and be a smooth function of the difference between rate of profits and interests.

$\alpha = f(\pi - r)$

Thus, at a given rate of interest, higher profit expectations would imply a move towards debt financing, whereas a lower expected profits would imply a move away from debt financing. But profit expectations and desired investments move together. Thus, debt financing is likely to go up with an increase in investment and vice versa.

Based on this, we propose the following functional form for h:

$h_t = kI_t$ ...(3.10)

$\Rightarrow B_t = kI_t^2$ ...(3.11)

Equation 3.22 is represented as follows:

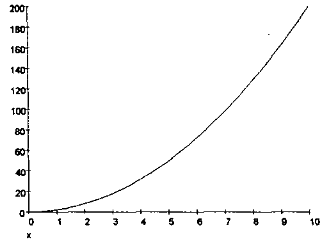

Figure 3.1: Borrowing vs. Investment

Effect of Increasing Investment on λ

Substituting the values of various variables, we get,

$$\lambda_t = \frac{sk}{\sigma} I_t \left(1 + \bar{r} + lI_t\right) \qquad \ldots(3.12)$$

Thus,

$$\lambda_{t+1} = \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)I_t + lI_t^2\right]$$

$$\lambda_t = \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)I_{t-1} + lI_{t-1}^2\right]$$

$$\therefore \lambda_{t+1} - \lambda_t = \frac{sk}{\lambda}\left[\left(1+\bar{r}\right)\left(I_t - I_{t-1}\right) + l\left(I_t^2 - I_{t-1}^2\right)\right]$$

$$= \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)\left(I_t - I_{t-1}\right) + l\left\{\left(I_t + I_{t-1}\right)\left(I_t - I_{t-1}\right)\right\}\right]$$

$$= \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)\left(I_t - I_{t-1}\right) + l\left\{I_t\left(I_t - I_{t-1}\right) + \left(I_t - \left(I_t - I_{t-1}\right)\right)\left(I_t - I_{t-1}\right)\right\}\right.$$

$$= \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)\left(I_t - I_{t-1}\right) + l\left\{I_t\left(I_t - I_{t-1}\right) + I_t\left(I_t - I_{t-1}\right) - \left(I_t - I_{t-1}\right)^2\right\}\right]$$

$$\therefore \Delta\lambda_{t+1} = \frac{sk}{\sigma}\left[\left(1+\bar{r}\right)\Delta I_t + l\left\{2I_t\Delta I_t - \left(\Delta I_t\right)^2\right\}\right]$$

$$\Rightarrow \frac{\Delta\lambda_{t+1}}{\Delta I_t} = \frac{sk}{\sigma}\left[\left(1+\bar{r}\right) + l\left(2I_t - \Delta I_t\right)\right] \qquad \ldots(3.13)$$

$\frac{\Delta\lambda_{t+1}}{\Delta I_t}$ denotes the sensitivity of macroeconomic financial fragility to change in investment. Let us call this expression θ.

Considering the expression $2I_t - \Delta I_t$:

For $2I_t - \Delta I_t \leq 0,\ I_t + I_t - (I_t - I_{t-1}) \leq 0,\ \Rightarrow I_t + I_{t-1} \leq 0,$

which cannot be true since both I_t and I_{t-1} are positive.

So we can unequivocally claim that: $2I_t - \Delta I_t \succ 0$.

We next consider the expression inside the square brackets, $(1+\bar{r}) + l(2I_t - \Delta I_t)$:

$$(1+\bar{r}) + l(2I_t - \Delta I_t) \leq 0 \text{ only if } l \leq -\frac{(1+\bar{r})}{2I_t - \Delta I_t} = \hat{l}, \text{ where } \hat{l} \prec 0$$

$$\succ 0 \text{ otherwise.}$$

Any value of l lower than $\hat{l}$ would represent an extremely expansionist monetary policy during boom and a contractionary monetary policy during a recession.

We can now examine the impact of various parameters on θ:

1. $$\frac{\partial \theta}{\partial l} = 2I_t - \Delta I_t \succ 0 \qquad \text{...(3.13a)}$$

   Thus, l, the indicator of monetary policy of the Central Bank, has an important role to play in determination of sensitivity of λ to change in investment. A situation where $l \geq 0$ would ensure that macroeconomic financial fragility and investment moves in the same direction. A high l, implying a less accommodating monetary policy regime, would increase the sensitivity of financial fragility to change in investment. Central Banks, therefore, through appropriate monetary policy can control change in financial fragility in response to change in investment.

2. $$\frac{\partial \theta}{\partial k} = \frac{s}{\sigma}\left[(1+\bar{r}) + l(2I_t - \Delta I_t)\right] \succ 0 \text{ provided } l \succ \hat{l} \qquad \text{...(3.13b)}$$

   $$\frac{\partial \theta}{\partial s} = \frac{k}{\sigma}\left[(1+\bar{r}) + l(2I_t - \Delta I_t)\right] \succ 0 \text{ provided } l \succ \hat{l} \qquad \text{...(3.13c)}$$

   Thus k, the sensitivity of the share of borrowing in investment to investment levels, and s, the savings propensity both have important roles to play. Higher k or s would make $\Delta\lambda$ more sensitive to change in investment. The monetary authorities, however, have the power to reverse this relationship by following a heavily expansionist monetary policy such that $l \prec \hat{l}$.

3. $\frac{\partial\theta}{\partial\sigma} = -\frac{sk}{\sigma^2}\left[\left(1+\bar{r}\right)+l\left(2I_t - \Delta I_t\right)\right] \prec 0$ provided $l \succ \hat{l}$ ...(3.13d)

Thus σ, the proportion of profits retained, has the opposite effect on financial fragility. Higher σ would make $\Delta\lambda$ less sensitive to change in investment. However, once again monetary authorities have the power to reverse this relationship.

4. $\frac{\partial\theta}{\partial\bar{r}} = \frac{sk}{\sigma} \succ 0$ ...(3.13e)

Higher normal rate of interest, $\bar{r}$, would make $\Delta\lambda$ more sensitive to changes in investment.

We note that, most of the time monetary authorities have the power to control the direction and degree of movement of macroeconomic financial fragility as a response to change in investment. This would imply that, once the monetary authorities make some estimate of the change in investment in the current period, if they have sufficient information regarding the parameters of the system, they can guide the economy towards the desirable level of financial fragility.

Role of λ in the Determination of Investment

Typically, in any economy, there exists a spectrum of λs across firms. At any point of time, there would be firms with very low λ^i as well as those with high λ^i. There will be firms with a whole range of $\lambda^i s$ in between. This is because different firms would have financed their investment in different ways, both in the current period and in the history.

Let us consider a situation where, due to some exogenous factor, $\lambda^i s$ increase systematically across firms. Let this growth rate be g per period. Thus, for i^{th} firm, $\lambda^i_{t+1} = (1+g)\lambda^i_t \Rightarrow \lambda^i_{t+n} = (1+g)^n \lambda^i_t$

As g and n cross some critical value, some of the Hedge firms move to being Speculative, and some of the Speculative firms start being Ponzi. The Ponzi firms, due to arguments presented above, default and are taken over by financially sounder (non-Ponzi) firms. The new owners taking over such firms would also inherit debt commitments of the defaulting firms, so $\lambda^i s$ of new firms would also increase. If the macroeconomic λ is a weighted average of the $\lambda^i s$ at the microeconomic level, then one can postulate that the $\lambda^i s$ and macroeconomic λ move together.[33]

High $\lambda^i s$, however, worry the managers. They do not like a change in ownership, since, typically they are the first ones to lose jobs in a hostile takeover. Since a high λ^i makes the firms vulnerable to takeover, there would be an attempt from the managers of the firms to keep the λ^i within some reasonable limit. In other words, if λ^i is high in period *t*, then they would like it to be lower in the period *t+1*. This would require a reduction in B_t^i

However, equation 3.3 would suggest that they have very little options. All the variables except I_t, B_t & E_t come from past period and beyond the control of the management. This leaves the management with only two options to reduce B_t:

1. Increase E_t^i
2. Reduce I_t^i

Resorting to the first option will imply that, the existing shareholders will have to share a greater fraction of the profits with the new shareholders. This might make them unhappy with the management threatening the jobs of management. So, this cannot be a viable option for the management beyond a certain limit. So, the management maybe expected to prefer the second option, i.e. reduce current investment.

Similarly, the management of a firm which has already defaulted and witnessed a takeover will be especially cautious in making new investments.

Thus, the exogenous factor which resulted in increasing $\lambda^i s$ systematically across firms (resulting in increase in λ as well) has forced many firms to cut investment, decreasing investment at the macroeconomic level, I_t. This makes λ_t one of the important determinants of I_t - a high λ_t would necessarily have a depressing effect on

[33] However, the earlier assumption of the defaulting firms being taken over by non-Ponzi firms becomes critical here. If the defaulting firms are allowed to shut down without honoring their payment commitments, then $\lambda^i s$ of these firms do not enter in the calculation of λ in the next period. Consequently, even if all the $\lambda^i s$ move in the same direction at the microeconomic level, the λ at the macroeconomic level might as well move in the opposite direction.

investment. However, this will be effected through an active takeover market; i.e. before the investment is actually depressed, the market would witness hectic takeovers and change in corporate controls. This captures, though in a limited sense, certain aspects of a 'financial crisis' in a modern economy. This interpretation of financial crisis also resembles that of Minsky – where a widespread failure to honor debt commitments by firms lead to a reduction in investment and income at the macroeconomic level. Real world examples of financial crises are, no doubt, much more complex, and the current model might not be a good reflection of these. Nevertheless, the pattern of events narrated above does resemble real life events in at least some of the crises, like the South Korean crisis discussed in section III of chapter 2.

This enables us to modify the Kaleckian investment function and explicitly include our indicator of financial fragility as one of the arguments:

$$I_{t+1} = aP_t + b(1 - \lambda_{t+1}) \qquad \ldots (3.14)$$

Equation 3.14 implies that, $\lambda = 1$ is the benchmark at which financial fragility ceases to have an effect on investment. A higher λ would have a depressing effect on investment, whereas a lower λ would positively affect investment.[34]

Two Explanations of 'Minsky Crisis'

The 'Minsky crisis' discussed in the previous chapter can have two possible explanations through alternative assumptions of exogenous and endogenous credit. This is explained below for clarity:

Exogenous Credit Regime: An ex ante cut in credit leads to defaults and reduction in investment and achieves ex poste equality of demand and supply of credit.

The lender stops lending, leading to drying up of funds for borrower. This means that, the borrower can no longer finance investment or pay its debt commitments, and starts defaulting on its past debt commitments. This leads to widespread default in the

[34] Since the λ^is are distributed around the mean of macroeconomic λ, $\lambda = 1$ would imply that for some of the firms $\lambda^i \succ 1$. These are the firms that could be either in the Ponzi state or rather close to Ponzi state, and thus have started worrying about their financial state, leading to a reduction in investment.

economy. This seems to be the simplest and a popular interpretation of Minsky's Financial Instability Hypothesis.

Endogenous Credit Regime: An ex ante cut in investment leads to fall in need for credit and achieves the ex poste equality of the demand and supply for credit.

As has been maintained throughout this paper, a good investment project does not fail due to lack of funds, though a particular management might be credit-rationed. Thus, access to borrowing cannot be a constraint for investment. The borrowers, however, are themselves worried with high financial fragility (λ^i), and responds by cutting down their investment. This reduces the demand for borrowing as well. However, before this happens, high λ^i turns some of the firms to Ponzi and forces them to default. These firms, due to the presence of active takeover market, are taken over by others. A widespread occurrence of this will have some aspects of a 'Minsky Crisis'.

Thus, it might be argued that the Minsky Crisis (or at least some aspects of it) can be explained with alternative assumptions of exogenous and endogenous credit.[35] However, since the purpose of this section is to analyze how financing of investment affects the results of multiplier-accelerator models and not to interpret Minsky, we do not explore this topic anymore in the rest of our study.

Formal Model

The investment function from equation 3.14

$$I_{t+1} = aP_t + b(1-\lambda_{t+1})$$

$$\Rightarrow I_{t+1} = \frac{a}{s}I_t + b\left[1 - \frac{sk}{\sigma}I_t(1+\bar{r}+lI_t)\right]$$

[35] The current debate between exogenous and endogenous credit, however, must be distinguished from the traditional Keynesian debate of exogenous and endogenous money. The Keynesian debate was concerned with money, a stock variable. Followers of endogenous money typically used substitution between various money-like assets as a justification in favor of their argument. For an overview of this debate, refer Sikorski (1996). The assumption made in this paper, however, regards credit, a flow, as endogenous, and is therefore a stronger assumption then that of endogenous money.

$$\Rightarrow I_{t+1} = -\frac{bskl}{\sigma} I_t^2 + \left[\frac{a}{s} - \frac{bsk}{\sigma}(1+\bar{r}) \right] I_t + b$$

$$\Rightarrow I_{t+1} = -AI_t^2 + GI_t + C \qquad \text{...(3.15)}$$

where $A = \frac{bskl}{\sigma}$, $G = \frac{a}{s} - \frac{bsk}{\sigma}\left(1+\bar{r}\right)$, $C = b$

$A, C \succ 0$

$G \succ 0$ if $\frac{a}{s} \succ \frac{bsk}{\sigma}(1+\bar{r}) \Rightarrow s \prec \sqrt{\frac{a\sigma}{bk(1+\bar{r})}}$.

Thus, $G \succ 0$ holds true if s does not cross a threshold limit. Let us call this threshold s^*. This, however, may not be a constraint since we know that $0 \leq s \prec 1$. This would give us the following two possible situations:

a) $s^* \succ 1$

b) $s^* \prec 1$

These two scenarios are shown in the following diagram:

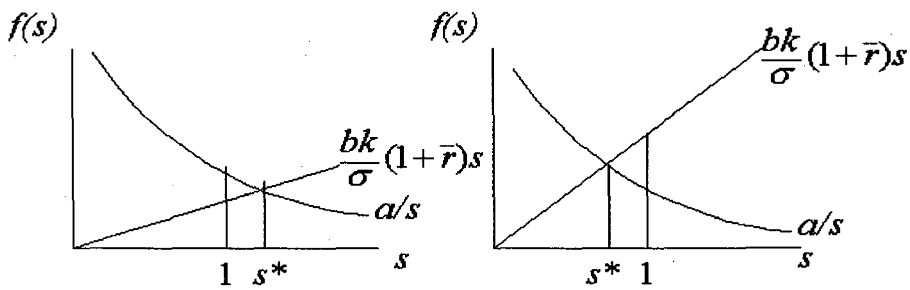

Figure 3.2(a) **Figure 3.2(b)**

In case (a) represented by figure 2(a), the condition $G \succ 0$ holds true unequivocally since $s \prec 1 \prec s^*$.

In case (b) represented by figure 2(b), however, the condition $G \succ 0$ holds true only if we make additional assumptions regarding the threshold limit for s: $s \prec s^* \prec 1$, which we assume to be true.

Equation 3.15 corresponds to a parabola with a unique maxima at $\frac{G}{2A}$ and first-order fixed points at $I^* = \frac{(G-1) \pm \sqrt{(1-G)^2 + 4AC}}{2A}$ as follows:

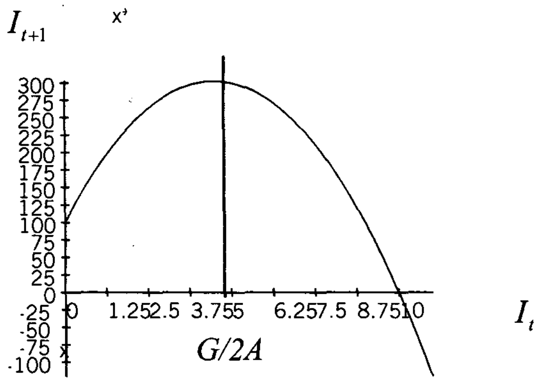

Figure 3.3: Dynamics of Investment

We also note that:

$$I_{\max} = f\left(\frac{G}{2A}\right) = \frac{G^2}{4A} + C \succ 0$$

Thus, solution to equation 3.15 always remains positive.

Investment in this system would always be bounded between 0 and $f\left(\frac{G}{2A}\right)$.

It would be convenient to transform equation 3.15 into a more well-known and well-analyzed format.

Let γ be the lower fixed point, i.e. $\gamma = \frac{(G-1) - \sqrt{(1-G)^2 + 4AC}}{2A}$

Let $J_t = I_t - \gamma$

Substituting the value of J_t in equation 3.15, we get

$$J_{t+1} = -AJ_t^2 + (G - 2A\gamma)J_t + [-A\gamma^2 + (G-1)\gamma + C] \qquad \text{...(3.16)}$$

The terms inside the square bracket reduces to *0* when γ is the lower root of equation 3.15.

$$J_{t+1} = -AJ_t^2 + (G-2A\gamma)J_t \qquad ...(3.17)$$

Let $\eta_t = \dfrac{A}{G-2A\gamma} J_t$ and $\mu = G - 2A\gamma$

Substituting these values in equation 3.10, we get

$$\boldsymbol{\eta_{t+1} = \mu\eta_t(1-\eta_t)} \qquad \textbf{...(3.18)}$$

Equation 3.18 corresponds to the well-known example of logistic map.[36]

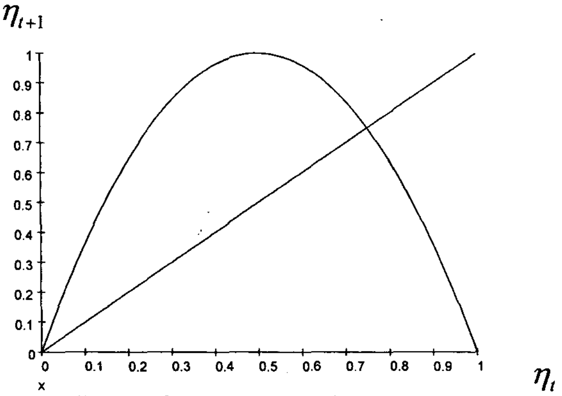

Figure 3.4: The Logistic Map

The logistic map has 2 fixed points: *0* & $1-\dfrac{1}{\mu}$.

$\eta = 0$ is the stable fixed point in the range $0 \prec \mu \prec 1$.

However, $\mu = G - 2A\gamma$

$$\mu = G - 2A \bullet \frac{(G-1) - \sqrt{(1-G)^2 + 4AC}}{2A}$$

$$\Rightarrow \mu = 1 + \sqrt{(1-G)^2 + 4AC} \qquad ...(3.19)$$

Since the expression inside the root is positive, $\mu > 1$. Thus, $\eta = 0$ or the area around it is of no interest to us.

The above expression can be expanded to:

$$\mu = 1 + \sqrt{\frac{(s-a)^2}{s^2} + \frac{b^2s^2k^2}{\sigma^2}\left(1+\bar{r}\right)^2 + 2\frac{(s-a)bsk}{s\sigma}\left(1+\bar{r}\right) - 4\frac{b^2skl}{\sigma}} \qquad ...(3.20)$$

[36] A more detailed study of logistic maps may be found, for instance, in Lauwerier (1986) and Kumar (1996).

When $1 \prec \mu \prec 3$, $\eta = 1 - \frac{1}{\mu}$ is the stable fixed point.[37]

When $\mu \succ 3$, $\eta = 1 - \frac{1}{\mu}$ becomes unstable. However, new cycles of two periods emerge, which exchange stability with the first order fixed point $\eta = 1 - \frac{1}{\mu}$. This process is known in the literature as pitchfork or period-doubling bifurcation.[38] Very soon, further period-doubling bifurcations takes place, with the 2-period cycles exchanging stability with 4-period cycles, followed by the 4-period cycles exchanging stability with 8-period cycles etc. In general, at every point of pitchfork bifurcation, a 2^n cycles becomes unstable and is replaced by stable 2^{n+1} cycles, forming an infinite series. Following table shows the range of μ where first few of these cycles are stable:

Table 3.3

Range of μ	Period of stable cycle
3 to 3.449499	2-period cycles
3.449499 to 3.544090	4-period cycles
3.544090 to 3.564407	8-period cycles
3.564407 to 3.568759	16-period cycles
3.568759 to 3.569692	32-period cycles
3.569692 to 3.569891	64-period cycles
3.569891 to 3.569934	128-period cycles

These values of μ converge to 3.569946. Thus, the range (3, 3.569946) has stable 2^n-period cycles as well as many unstable cycles with periods of all the powers of 2. In the

---

[37] An equilibrium $\bar{x}$ of a system $x_{t+1} = f(x_t)$ is stable if $\left|f'(\bar{x})\right| \prec 1$. In the present situation, the absolute value of derivative evaluated at $1 - \frac{1}{\mu}$ is $2 - \mu$.

[38] The concept of bifurcation is dealt with in more detail in Kumar (1996), Lauwerier (1986) and Lorenz (1993)

interval (3.569946, 4), there is an infinite number of small windows of μ-values for which there exists stable *m*-cycles. These are first of even periods, and then of odd periods in descending order. The period-3 cycle first appears at $\mu = 3.828427$ and stays stable up to $\mu = 3.841499$, beyond which there are further period-doubling bifurcation.

Outside these windows of stable cycles, there are no stable periodic orbits, though there are an infinite number of unstable cycles of all periods.

Thus, the behavior of the system becomes extremely complicated beyond $\mu = 3$, especially as we move closer to 4.[39]

μ is the crucial parameter in the above equation. Any value of μ close to 4, (specially values between 3.6 and 4) will give us wide range of unpredictable values of our iterates of η. On the other hand, for lower values of μ the dynamics of the system would be much simpler, with much more predictability regarding the values of η.

Since we not know exact values of some of the parameters, we cannot make any reasonable estimate about the value of μ. This would mean that, η can show any of the following behaviors:

1. It might have a stable one-period fixed point.
2. It might have stable cycles with periods which are powers of 2.
3. It might have stable cycles of odd and even periods.
4. The trajectory might be aperiodic.
5. There might not be any stable periodic orbit.

However, all the above behaviors will occur within the boundary: $\eta \in (0,1)$. It escapes this boundary only if $\mu \succ 4$, but these points do not return within the boundaries, and hence must be rejected.

Since we do not have the exact value of μ, we examine the influence of some of the parameters on the behavior of the system. We start by looking at the impact of monetary policy by partially differentiating μ w.r.t. *l*.

---

[39] For a formal proof of the chaotic behavior with the onset of period-3 orbits, refer to Li and Yorke, 1975.

$$\frac{\partial\mu}{\partial l}=\frac{1}{2\sqrt{\frac{(s-a)^2}{s^2}+\frac{b^2s^2k^2}{\sigma^2}\left(1+\bar{r}\right)^2+2\frac{(s-a)bsk}{s\sigma}\left(1+\bar{r}\right)-4\frac{b^2skl}{\sigma}}}\left(-4\frac{b^2sk}{\sigma}\right)\prec 0 \quad ...(3.21)$$

This would mean that, higher the l (i.e. the less accommodating the monetary policy), *ceteris paribus* the less is the value of μ and less are the chances of complex dynamics.

We had, however, seen earlier that increasing l (i.e. less accommodating monetary policy) increases θ, the responsiveness of macroeconomic financial fragility to change in investment, increasing possibilities of a financial crisis. A less accommodating monetary policy, thus, performs the dual role of increasing the chances of financial fragility but making the system more predictable.[40]

Keeping l constant, let us now examine the effect of changing the other parameters. For convenience, let us keep l constant at $l=0$ (i.e. perfectly accommodating monetary policy). Plugging this value in equation 3.20 gives us:

$$\mu=1+\sqrt{\left\{\frac{s-a}{s}+\frac{bsk}{\sigma}\left(1+\bar{r}\right)\right\}^2}$$

$$\Rightarrow \mu=1+\left|\frac{s-a}{s}+\frac{bsk}{\sigma}\left(1+\bar{r}\right)\right|$$

$$\Rightarrow \mu=1+\left|1-G\right|$$

$$\Rightarrow \mu=\begin{Bmatrix}2-G & \text{if } G\prec 1\\ G & \text{if } G\succ 1\end{Bmatrix}$$

Since G by earlier assumption is positive, μ cannot exceed 3 in the first case (i.e. when $G\prec 1$). The only way $\mu\succ 3$ is when $G\succ 3$. Thus, higher the G, more are the chances of obtaining complex dynamics.

We recall that: $G=\frac{a}{s}-\frac{bsk}{\sigma}\left(1+\bar{r}\right)$

[40] Distinction must be made here between the concept of 'predictability' of a system in the sense of dynamic stability and 'financial stability' as used in day-to-day parlance – one might not necessarily lead to other. Even a very stable system might display signs of what is known as 'financial instability', for instance our model might show financial instability, in the sense discussed before even when $\mu\prec 3$. Similarly, we can have dynamic instability even without involving financial sector.

Partially differentiating: $\frac{\partial G}{\partial a} \succ 0,\ \frac{\partial G}{\partial s} \prec 0,\ \frac{\partial G}{\partial b} \prec 0,\ \frac{\partial G}{\partial k} \prec 0,\ \frac{\partial G}{\partial \sigma} \succ 0,\ \frac{\partial G}{\partial \bar{r}} \prec 0$

From the analysis so far, we can make the following conclusions:

1. Higher the sensitivity of investment to expected profits, *a*, more are the chances of complex dynamics.
2. Higher the savings propensity, s, lower are the chances of complex dynamics.
3. Higher the sensitivity of changing the share of borrowing in total finance as a response to investment, *k*, lower the chances of complex dynamics.
4. Higher the fraction of profits retained, σ, higher are chances of complex dynamics.
5. Higher the normal rate of interest, $\bar{r}$, lower are the chances of complex dynamics.

The above results, along with the earlier results on financial fragility are summarized in the following table:

Table 3.4

The variable whose values are increased	Impact on θ, the responsiveness of financial fragility to investment	Impact on possibilities of complex dynamics
l	Increases	Decreases
k	Increases	Decreases*
s	Increases	Decreases*
σ	Decreases	Increases*
$\bar{r}$	Increases	Decreases*

****Note: This analysis is done holding*** $l = 0$.

$\eta_t = \frac{A}{G - 2A\gamma}(I_t - \gamma)$, investment, I will show a behavior similar to η – it will show a bounded behavior, but the exact behavior of the system would depend on the values of parameters whose values are uncertain.

The above table seems to suggest that under most conditions there exists an inverse relationship between θ and μ. It might, therefore, be suggested that economies that show relatively simpler dynamics might as well be financially more fragile during the upward phase of the business cycle. On the other hand, economies where financial crises (in the limited sense discussed in our model) are of rare occurrence could be the ones where the investment dynamics shows highly complex and unpredictable behavior. In other words, financial crises could be playing a role in keeping the dynamics simple in an economy. This might, as we will see soon, have important consequences in terms of policy formulation.

Policy Implications

The monetary authority, in our model, has one control variable, l (i.e. the degree of financial accommodation). However, there are usually multiple objectives of monetary policy. We discuss some of them here.

Controlling Investment and Inflation

Monetary authorities are frequently expected to control investment and inflation. We see here how this may be carried out.

Equation 3.13a establishes a link between l, the degree of financial accommodation of monetary policy, and λ, the indicator of macroeconomic financial fragility. Equation 3.14 establishes the link between λ and the rate of investment at the macroeconomic level. Together, these two equations link the monetary policy of the Central Bank to the investment at the macroeconomic level via changes in macroeconomic financial fragility. During the upswing (i.e. when $\Delta I_t \succ 0$), an increase in l (i.e. reduction in degree of financial accommodation), subject to fulfillment of certain conditions, leads to a cut in investment. Financial crisis (in the limited sense of an increase in incidence of defaults and takeovers) might arise as a byproduct of the process, provided the increase in financial fragility is enough to force some of the firms to enter Ponzi stage.

Consider, for instance, a situation where the investment is increasing, and the Central Bank, to prevent overheating of the economy, wants to cut the rate of investment.[41] So it restricts the degree of credit accommodation, increasing *l*. This leads to increase in macroeconomic financial fragility and reduction in investment by the process described above, with financial crisis as a possible byproduct. This makes financial crisis (or a fear of firms to get into a crisis) one of the tools for the monetary authorities to affect real variables.

The above account, in fact, gives us an alternative to the Keynesian route for monetary policy and interest rates to affect investment. Instead of the usual 'marginal efficiency of capital' (or the more popular IS-LM) route, a tight monetary policy here depresses investment by increasing financial fragility and inducing fear of defaults among firms.

Preventing Financial Crisis

In the recent times, quite often the monetary authorities have also been called upon to control financial fragility and prevent a financial crisis from occurring. There could be a number of reasons for this:

- As we have seen, financial crisis has a depressing impact on investment. Therefore, especially when an economy is going through recession, prevention of financial crisis is likely to be one of the priorities of the Central Bank.
- Even if there is no fear of immediate recession in the economy, financial fragility by itself is usually not considered desirable. This could be either due to the impact a widespread occurrence of defaults and takeovers have on the healthy functioning of the financial system (something we have not modeled so far), or due to pressures from firms that are close to defaults.

Equation 3.13a shows that a more accommodative monetary policy might help in controlling the financial fragility in the economy during boom time (i.e. when the economy is in the upswing). In the real world, this might resemble the 'lender-of-last-resort' role that a monetary authority is often expected to play.

[41] A number of factors might make the Central Bank act this way. Typically the Central Banks are concerned with possibilities of high inflation, especially if it feels that the economy is close to the full-employment level. A deficit in the current account might also prompt it to act this way.

The above discussion would suggest that, during a boom, reducing l, implying a more expansionary monetary policy would both boost investment and control financial fragility. Thus, during a boom the only reason why a Central Bank should consider increasing l, i.e. reducing the degree of accommodation of the monetary policy is when reducing investment is quite high on its priority.

However, situation is different during the downswing. Now, the to control financial fragility, the Central Bank would like the responsiveness of λ to ΔI as high as possible, so that the financial fragility decreases at a faster rate and the financial crisis ends sooner as a response to the declining investment. In other words, it would like θ, and therefore l to be as high as possible.

There could, however, be another implicit policy objective of the Central Bank.

Keeping the Dynamics Simple

This is unlikely to be an explicit policy objective of the monetary authority. However, as we discussed earlier, the dynamics of investment, under certain values of the parameters get complicated and the model loses much of its predictability. Under these circumstances, the role of policy formulation gets weakened. The Central Bank, through its monetary policy, might attempt to move the real variables like investment in a specific direction, but the exact outcome of such a move could be completely unexpected when the dynamics is complex. The final outcome could well be quite different from the desired outcome, undermining the role of monetary policy. To prevent this, the policymakers would like to keep the dynamics simple.

However, we know from Table 3 and equation 3.21 that lower the l during boom, higher are the chances of complex dynamics and unpredictable behavior. However, lower l is also associated with lower θ, and hence less chances of financial crises during a boom. Therefore, a policy excessively geared towards controlling financial fragility during an upswing beyond a certain point might make the system more and more unpredictable, weakening the role of policy formulation itself. Seen in this light, there seems to be a trade-off between controlling financial fragility (which would also boost investment) and keeping the investment dynamics simple.

The situation, however, is different when the economy is in a downswing. In this case, both the objectives of controlling financial fragility and keeping the dynamics simple are achieved by increasing l. This would correspond to an easy money policy during downturn.

In other words, to keep the dynamics simple, the Central Bank must follow tight money during an upswing and easy money during a downturn. Hence, if this is indeed one of the priorities of the Central Bank, it is likely to step in and play its role in prevention of financial crisis only when the investment has started decreasing. During a boom time, it is also more likely to concentrate on playing a regulatory role (which, as we have discussed earlier, has the effect of tightening money at a given rate of supply of M1) along with a steady and low rate of growth of money supply.

Thus, the policy implications of our model seem to unequivocally prescribe an easy money policy during a recession induced by a financial crisis (in the limited sense discussed in this chapter).[42]

However, before using our model as a reflection of the real world, certain limitations of the model must be kept in mind. These limitations are discussed in the concluding chapter.

[42] One might contrast the policy implications of our model with those followed in South Korea, as discussed in Section III of Chapter 2. South Korea enjoyed a prolonged boom, during which the firms increased their indebtedness, leading to an increase in financial fragility, as suggested by equations 3.13 and 3.13a. This led to some *chaebols* defaulting on their payment commitments. Our model would predict a decrease in investment under such circumstances. Our model also would have prescribed an easy money policy (corresponding to high l when $\Delta I_t \prec 0$) to both revive the economy and prevent complex dynamics. The IMF prescriptions, however, amounted to a tight money (i.e. low l and θ), both by increasing interest rates and tightening prudential norms for banks. Equation 3.13 and 3.13a would predict that this would worsen financial fragility (leading to increase in bankruptcies). On the other hand, equation 3.21 would predict the economy to show more complex behavior leading to loss in effectiveness of policy formulation. The actual events in the aftermath of IMF policies, as we know, were not too far from this.

CHAPTER 4:
CONCLUSIONS

We had begun our study by looking at how the simple multiplier-accelerator models might predict an unbounded behavior of income, employment and investment. We also saw that, incorporation of the financial sector might provide us with endogenous boundaries for the system; but since the financing decisions are typically made at the microeconomic level, there is a need for specifying the relationship between various variables at the microeconomic and macroeconomic level. Subsequently, in the last chapter, we outlined a model, aimed at specifying this relationship.

However, the elementary model outlined in the previous chapter, as would be evident from a study of real world experience (like the South Korean case discussed in section III of chapter 2), suffers from certain limitations.

Limitations of the Model

1. Possibilities of capital gains and losses have been ruled out by assumption in the model, though we know that in the real world this is one of the important factors affecting financial decisions, especially in the equity market. This, in fact, also ignores an important aspect of a financial crisis – a rapid fall in the value of assets is often one of the principal characteristics of a financial crisis.[43]
2. Ignoring possibilities of capital gains and losses has also meant that we are ruling out speculative demand for money, an important insight of Keynes, throughout the model.
3. All expectations are static in our model. However, it might be noted that expectations in the real life are often much more complicated. This is one of the sources of some of the financial crisis. A framework based on static expectations ignores such possibilities.
4. Another concept related to the above three factors is that of investor confidence. If there exist possibilities of capital gains and losses, and if we do not know how expectations are formed, then typically there is a tendency to fall back on

[43] This aspect is discussed in detail by Pratten (1993).

conventions, in an attempt to guess other player's expectations.[44] Thus, when investor confidence is an important factor, the policies in addition to being correct must also *be seen* to be conventionally correct.[45]

5. Considerations of stock variables have been avoided throughout the model by converting it into a circulating capital model. This has enabled us to avoid some of the complications and contradictions of standard stock-flow models (for instance the problems discussed in the earlier chapter with IS-LM). However, this has been at the cost of ignoring one of the important aspects of a financial market – the interaction between the stock and the flow variables.[46]
6. The closed economy setting ignores the fact that most of the contemporary economies are integrated with the international commodity and the financial markets. This integration with the international commodity and financial markets has often acted as one of the causes of many of the financial crises in the recent times.
7. The role of government in the model is limited to monetary policy only. Fiscal policy, which has played an important role since World War II, has been ignored.
8. The strict rule by which the takeover market operates in the model ignores one important real life factor – inter-linkages between liabilities and assets of different economic units. This is especially relevant during a financial crisis when the defaulting unit has payment obligations to other units. When the firm fails to pay, even other good and solvent firm might start facing liquidity problems, pushing these firms to bankruptcy as well. This chain effect of defaults has been ignored.
9. Defaults often leads to the units being shut down instead of being taken over by other firms – an aspect ignored.

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[44] The aspect of falling back on conventions is discussed in detail by Keynes (chapter 12 of Keynes, 1936). He has, in fact, compared this with a newspaper competition where the readers are awarded for choosing the prettiest face among a given number of photographs. Each person tries not to actually choose the prettiest face, and not even whom the average opinion considers prettiest – everyone would in fact try to guess what the average opinion considers to be the average opinion of a pretty face. In other words, we have already reached at least third degree of estimation. Convention would, in this case, play a greater role in formation of opinion than a rational assessment of the situation.

[45] The IMF prescription of a contractionary monetary policy (along with contractionary fiscal policy) as a response to financial crisis could also be explained in terms of this. It could have been, for instance, simply aimed at boosting investor confidence.

[46] The interaction between the stocks and the flows is dealt with in more detail in Scitovsky (1994).

10. Many of the functional forms used in the model are arbitrary and have not been tested econometrically.
11. We have assumed a single interest rate throughout the economy, when in the real world, due to asymmetric information, there could be multiple interest rates depending on risk.

In spite of the above problems, the model seems to capture at least one important aspect of the financial crisis in the recent times, namely, increased tendency of various firms to access debt during boom time finally resulting in a payment crisis and recession. This is a pattern observed, perhaps most explicitly, in South Korea, but also to some extent in a number of Asian and Latin American countries. The model also helps us better assess some of the standard policy prescriptions of the international organizations, especially the International Monetary Fund.

Finally, the exercise carried out in the previous chapter leads us to conclude the following:

1. The Central Bank, through monetary policy, can influence the rate of investment. Given certain conditions, an expansionary monetary policy (or low rate of interests) will boost investment, whereas a contractionary monetary policy (or high rate of interests) will restrict investment. This result is similar to the Keynesian result (using marginal efficiency of capital schedule) as well as that of the Hicksian IS-LM analysis.
2. Monetary policy can also prevent, or at least postpone a financial crisis from occurring during boom. This might be done by following a heavily expansionary monetary policy, not letting the financial fragility of the firms increase over time, in spite of the firms financing a larger and larger fraction of investment from debt. Thus, financial crisis *need not inevitably* put an end to the boom phase, as Minsky seemed to be claiming. However, as we have seen, such extreme policies might make the system unpredictable, especially in the boom time, undermining the role of the policymakers. So, in the real world, such extreme policies might not be followed. In

other words, the monetary authorities might allow a financial crisis to end the boom phase, even if it had the power to prevent this.

3. The behavior of the system is also critically dependent on the values of the parameters. Even a slight change in the values of the parameters might completely change the behavior of the system. Hence, the empirical studies need to be accurate for us to make any reasonable estimate about the future behavior of the economy – even a small approximation or error while collection of data or econometric analysis can cause considerable difference in the outcome.

As we have indicated above, the model does not claim to reflect all the complexities of a real world financial crisis. A more realistic model, as we just saw, must consider factors like capital gains / losses, speculative hoarding of money and other stocks, international capital flows and exchange rate variations, etc. Such a model must also clearly spell out the nature of transmission of microeconomic effects on the macroeconomic variables and the consequent feedback on the agents at the microeconomic level. The main purpose of the current study was to make a beginning precisely in this direction.

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