Stability in a Volatile Market:
The Role of Convention and Strategic Behaviour

*Dissertation submitted to Jawaharlal Nehru University
in partial fulfilment of the requirement for
the award of the degree of*

Master of Philosophy

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1999

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Certificate

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Acknowledgement

First of all, I would like to thank my supervisor for extremely insightful and thought-provoking discussions and suggestions, and that too, despite my almost unforgivable unpunctuality. His enthusiasm towards the new approaches of enquiry has been both amazing and encouraging for me.

Alok and Deepankar went through my arguments tediously and provided some valuable suggestions. Alok had a tough time typing the equations with the software which was extremely moody and fun-loving. Lima typed the rest of the text despite his time-constraints. Arup was helpful in bringing everything together.

The mistakes that remain are undoubtedly mine.

Avinash

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CHAPTER I

INTRODUCTION

Keynes, in his General Theory was the first to talk of the role of expectations in the determination of market outcomes. Taking expectations formation as exogenous mostly in the case of "long-term normal rate of interest", he did not elaborate any method or process by which the market participants may revise it. What was clearly stated was the fact that expectations work as a destabilizing factor in the markets where they matter. Some Keynesians have tried to categorize the markets where expectations would play a crucial role and the markets where they do not, concentrating mostly on stock versus flow equilibrium in certain commodities (Ackley,1983). But the issues raised here are different in nature. They revolve around the notion of self-fulfilling equilibrium and its stability in the markets where expectations play a crucial role. What is proposed to be analysed here are various conditions which characterise the stability or instability; and also the meaning and significance of such results.

The notion of self-fulfilling equilibrium was articulated by Hicks(1939) and later it was used by Muth (1961) to argue the importance of such a concept. What was not questioned in these works was the role of expectations as a destabilizing factor. The works of Lucas brought the notion of self-fulfilling equilibrium to the centre-stage of mainstream Macroeconomic thinking. This time, it was in the form of "Rational expectations" literature.[1] What happened in this journey from Keynes to Lucas is that the role of expectations in market stability is reversed.

To understand this reversal, one has to look at Friedman's work. Friedman (1968) criticised Phillip for his assumption that everyone anticipated the normal prices to remain stable whatever happened to actual prices and wages. He then made a case for what is known as "expected-augmented" Phillips Curve where the rise in nominal wages depends on anticipated rather than the actual prices in addition to unemployment rate.

In this analysis instead of taking expectations as given, he argued for changing expectations depending upon the past observation(s) of the individuals. Adaptive expectation that was used is one such example of changing expectations. This was first step towards endogenising individual expectations. But did not argue that individuals would always form correct expectations about the future; and thus there existed a scope of effective exogenous policy measures. His defense of non-intervention in the market and minimal and stable use of monetary policy derives strength from (i) inability of such policies especially in terms of accelerating rate of inflation to sustain their usefulness in achieving the goal, (ii) the limitations of the government about the information to act at the right time, and (iii) 'inefficiencies' associated with government interventions.[2] Expectations, though endogenised in this line of argument, do not ensure stability in the market, at least in the 'short run'. What came to be known as 'shocks' in the rational expectation literature was a matter of routine for Friedman, even

[1] The use of "Rational expectation" here is for the Lucas type of analyses and not for Muth who used the same term.
[2] The latter arguments are elucidated in Friedman (1953).

though undesirable.

In contrast, in the Rational expectation literature which also came to be known as "Monetarist Version 2" as distinguished from Friedman's "Monetarist Version 1", individuals' expectations about the outcome coincide with it; hence the notion of self-fulfilling equilibrium. This means that individuals have full information about all the markets including all the markets of the future extended to infinitely many periods. Added to it is the assumptions that individuals are homogenous who as "signal processors"- a favourite term of Lucas, process all this information in the same manner that is through the same model. Thus, the same outcome is also the one anticipated by everyone and hence acted upon accordingly. Whereas the assumption of homogenous individuals using the same model to process information has also never been adequately justified, the complete informational requirements bestowed upon individuals is totally unrealistic and unacceptable. The assumption of homogenous individuals is never relaxed, and the informational requirements are defended as only the first approximation for the story. The justification of this first approximation is given by bringing in the notion of learning in a sense very different from Bayesian learning. This is done through endogenous expectation functions. As mentioned above, learning was used by Friedman also, but in the extreme Rational Expectation version, learning is such that it anticipates the exact outcome. Expectations, since they are always correct exert a stabilizing, rather than destabilizing influence in the markets. This is very different from the views expressed by Keynes. This reversal of role of expectations in markets seems to defy our day-to-day observations. Heuristically, one may ask: if expectations are stabilizing how come we typically observe more instability in the markets where expectations have a larger role to play, than in those markets in which their role is not that important? This is the argument on which Grandmont (1998)[3] relies to attack Rational expectation theory and the associated learning processes, as he contrasts his own framework of Temporary equilibrium to that of Rational expectation.

According to Grandmont, the stability of any stationary state outcome in a market where expectations play a crucial role depends upon the range which individuals (traders, for Grandmont) are willing to extrapolate. If "large" deviations have occurred in past and traders do take those into account while forming expectations about tomorrow, then the stationary state is highly likely to be unstable. Further, if one is talking about a market where role of expectations is not very important and/or traders do not take into account the large derivations from stationary state while forming their expectations then market is likely to be stable.[4] Grandmont argues that in effect this is precisely what is done by various exponents of Rational expectations theory using assumptions about "Projection facility", which deliberately and quite arbitrarily restrict the range within which traders extrapolate. According to Grandmont, such restrictions on the range have no basis and showing convergence on the basis of it is devoid of any meaning.

Grandmont's results on stability and instability state equilibrium with learning and their relevance is discussed in chapter 2. His framework allows

[3] Henceforth any reference of Grandmont refers to his 1998, unless other works are mentioned.

[4] This conclusion is based on theorem proved in Grandmont (1998). This has been proved earlier in Grnadmont (1985) and Grandmont and Loroque (1986, 1990).

for heterogeneous individuals; individuals being characterized by their weightage in determining the aggregate (or average) expectation function and their respective expectation functions. The average expectation function alongwith the structural parameters of the market determine the outcome and the stability of stationary state.

Further, in chapter 2, a result is derived to argue that it is possible to have local instability in the market even if we ignore the large deviations considering them as 'shocks'. *Once we allow for ignoring 'shocks' there is no justification to the limits or bound on the values of estimators of learning parameters, through any "projection facility."* The case that illustrates this point specifies expectation functions of individuals (logically, of at least one individual) taking into account their respective notion of average expectation in the market and their notion of others' notions of average expectation in the market, to higher and higher degree. Such individual behaviour was first indicated by Keynes (chapter 12, The General Theory) in his famous parable of the "beauty contest."[5] Indeed such assumption about individuals' analysing behaviour is central to the game-theoretic literature. With these specifications of individual expectation function(s), certain restrictions are imposed upon a functional parameter (or estimator) of at least one individual. The analytical

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[5] It will be wrong to conclude that Keynes took expectations to be endogenous. In chapter 12, Keynes talks of many types of individual behaviours to show that how instability in the market is created due to these. Essentially he wanted to bring out the point that how tendency of speculative behaviours in the investment market makes the economy unstable. Expectations, for Keynes, in his final analysis, was exogenous as in his emphasis on the importance of "animal spirit" in the market. This is even much clearly emphasised upon in Keynes(1937). Further, definitely Keynes talked of importance of convention in analysing expectations, he did not have any "stationary state" or self-fulfilling outcome in mind.

result follows that stationary state would be unstable (and hence globally unstable) even when the "shocks" are ignored.

In chapter 3 we discuss the restriction and the assumptions made to get this result, from the point of view of giving a critique of Rational expectations theory. Whether the assumptions made can be justified without allowing any individual to be crazy or totally abnormal is discussed. To what extent this result coupled with Grandmont's results can provide a critique of Rational expectation theory is discussed in chapter 4,which ends with concluding observations and conjectures.

CHAPTER 2

ON THE QUESTION OF STABILITY OF RATIONAL EXPECTATION BASED STATIONARY STATE

In this chapter we first restate Grandmont's results of stability and instability of stationary state and discuss their implications. In the markets where expectations are important, the requirement for stability is that traders do not extrapolate the large deviations. If they do extrapolate then the market would most likely be locally unstable, unless of course, the influence of expectation about tomorrow on today's outcome is very weak[1]. Grandmont argues that learning in Rational expectation theory is ensured by doing precisely this; that is, deliberately ignoring the large derivations from stationary state. The manner in which it is done is explained in this chapter. Further, Grandmont gives a critique of basic intuition of Rational expectation theory. According to him in this theory expectations are always correct or people learn to form expectations which ensure stationary state outcome so that expectations work as stabilizing rather than destabilizing factor in any market. This is to say that markets where expectations play a crucial role would be more stable than the markets where they do not. This is in total contradiction to our day-to-day observations, whereas, Grandmont's own formulation (temporary equilibrium framework), and his results explain these observations.

[1] This influence of "tomorrow" on "today" was first analysed by Hicks(1939) through his notion of the role of the elasticity of expectation on the stability of temporary equilibrium. In particular, the elasticity of expectation has to be less than unity for the stationary state to be stable.

Towards the end of this chapter we explore further the question of stability, starting with Grandmont's framework. We point out a case of instability even when individuals ignore the large deviations is given. The moving factor of this result is the fact that individuals are strategic. Strategic individuals are ruled out by Grandmont even though he allows for heterogeneous individuals. The discussion on this result is given in next chapter.

For the sake of completeness of argument Grandmont's results of stability and instability and their implications are briefly discussed here on the analytical background.

Assumptions made in Grandmont's analysis are:

- No strategic considerations among different agents in the market.
- Time is discrete.
- State of the system is completely described at every date t by a single real number x_t.
- ${}_t x^e_{t+1}$ or x^e denotes average forecast about period t+1 at time t, each individual's forecast being weighted by it's relative local contribution to the dynamics of the system.
- The current equilibrium state x_t depends on the x^e and x_{t-1} through the temporary equilibrium relation

(1) $T(x_{t-1}, x_t, x^e) = 0$

- The analysis will be local i.e. near a stationary state $\bar{x}$, defined by $T(\bar{x},\bar{x},\bar{x})=0$. T is supposed to be well defined and continuously differentiable when its arguments are near $\bar{x}$.

  b_1 ,b_0 and a are the partial derivative of T with respect to x_{t-1} , x_t and x^e ,evaluated at stationary state.

  Assume, a $\neq 0$ (expectations matter).

- Equation (1) describes the structural characteristics of the system. Average (or common, if x^e is common forecast) expectation function is denoted by:

(2) $x^e = \psi(x_{t-1},\ldots,x_{t-L})$

- Assume that either traders know $\bar{x}$, or if not, they are prepared to extrapolate constant sequence x near $\bar{x}$.We assume that for all x in the immediate vicinity of $\bar{x}$, we have ψ (x, ...,x) = x. In any case $\psi(\bar{x},\bar{x},\bar{x})=\bar{x}$.

Let c_j stand for the partial derivative of ψ with respect to x_{t-j} at $\bar{x}$, for j=0,1,2, ..., L. Replacing the forecast in(1) by expression (2), we get the actual temporary equilibrium dynamics:

(3) $T(x_{t-1},x_t,\psi(x_t,\ldots,x_{t-L}))=0$.

Assuming certain regularity conditions, we write (3) as local difference equation of the form:

(4) $x_t = W_{loc}(x_{t-1},\ldots\ldots,x_{t-L})$, for all x_t near $\bar{x}$.

Linearizing (3) and (4) near the stationary state, we get

(5) $Q_w(z) = b_1 z^{L-1} + b_0 z^L + a\sum_0^L c_j z^{L-j} = 0$.

If all the roots of (5) have modulus less than 1, then the system is stable. Otherwise, except for a very "thin" set (for Lebesgue measure 0), the trajectories generated by dynamics (3) and (4) are pulled away from the stationary state.

Linearizing (1) and (2), we get (6) and (7) respectively:

(6) $Q_F(z) = b_1 + b_0 z \ + az^2 = 0$.

(7) $Q_\psi(z) = z^{L+1} - \sum_0^L c_j z^{L-j} = 0$.

The roots of $Q_F(z)$, namely $\lambda_1$and λ_2 characterize the local behaviour of perfect foresight dynamics near stationary state.The L+1 roots $\mu_1,\ldots..,\mu_{L+1}$ of the polynomial $Q_\psi(z)$ characterize the local regularities that traders are, on the average, ready to extrapolate out of small past deviations from the equilibrium.

We can write,

(8) $Q_W(z) = z^{L-1}Q_F(z) - aQ_\psi(z)$.

(9) $Q_W(z) = a[z^{L-1}(z-\lambda_1)(z-\lambda_2) - \Pi_k(z-\mu_k)]$.

To interpret this algebraic procedure economically, local stability of actual dynamics with learning is seen to depend on how the local eigenvalues of traders' average forecasting rule interact with local characteristic roots of perfect foresight dynamics. The manner in which these roots affect the stability of the system and its meaning and significance is explained by propositions 1 and 2 given below.

General Instability Result[2]

Proposition 1

Assume $a \neq 0$ and $b_0 + ac_0 \neq 0$ (the actual dynamics with learning is well defined). Let ψ have two local real eigenvalues $\mu_1 < 0 < \mu_2$ that differ from perfect foresight roots λ_1 and λ_2.

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[2] Grandmont and Laroque(1991), and Grandmont(1998).

1. Let n_F and n_W be the number of real roots of the polynomial $Q_F(z)$ and $Q_W(z)$ respectively that lie outside the interval $[\mu_1, \mu_2]$.Then n_W is odd iff n_F is even(i.e. 0 or 2).
2. If $\mu_1 \leq -1$ and $\mu_2 \geq 1$ and the interval $[\mu_1, \mu_2]$ contains in its interior all perfect foresight characteristic roots that are real, then $Q_W(z)$ has a real root r that satisfies r< $\mu_2 \leq -1$ or r> $\mu_2 \geq 1$(i.e. system is unstable).

Interpretation:

Among the L+1 roots $\mu_1, \ldots, \mu_{L+1}$, there would be two roots (say μ_1 and μ_2) such that $\mu_1 \leq -1$ and $\mu_2 \geq 1$, if traders are willing to extrapolate wide range of local regularities out of small past deviations from the equilibrium. Now, if $|a|$ is not too small relative to $|b_0|$ and $|b_1|$, i.e. market is such that expectations play an important role, then $\lambda_1 + \lambda_2 (= -b_0 a^{-1})$ and $\lambda_1\, \lambda_2 (= -b_1 a^{-1})$ would be small in magnitude. This implies that λ_1 and λ_2 would be small in magnitude and will fall in the interval $[\mu_1, \mu_2]$. This to say that n_F=0 which implies that n_W is odd and hence the system is unstable.

So, if the range upto which traders extrapolate is sufficiently "wide enough" and expectations are "important enough" in determining market outcomes, then the market is likely to be unstable. Grandmont (1998) discusses a few examples of learning common in the literature to show that "width" and "importance" required for instability are not stringent.

General Stability Result[3]

Proposition 2.1

Assume $b_0 + ac_0 \neq 0$ and let α be the maximum of $\left|\sum_0^L c_j z^{L-j}\right|$ when $|z| = 1$.

1. Then the stationary state is locally stable in the actual dynamics with learning if $|b_0| > |b_1| + \alpha|a|$.
2. In particular, assuming that traders extrapolate constant sequence for all x near $\bar{x}$, and that $c_j \geq 0$ for all j = 0,1,....,L , we get $\alpha = 1$ and the sufficient condition for stability becomes $|b_0| > |b_1| + |a|$.

Interpreting the above result:

For all z s.t. $|z| = 1$,we have:

$$\sum_1^L |c_j| \geq \alpha \geq \left|\sum_0^L c_j z^{L-j}\right| = \left|Q_\psi(z) - z^{L+1}\right| \geq \left|Q_\psi(z)\right| - 1$$

If μ_k s are small, then c_j s would be small, and thus α would be small. Hence, α in a sense is a measure of the width of the distribution of μ_k s (here k =

[3] Grandmont 1998

1,....,L+1 and j = 0,.....,L). Now consider the condition $|b_0| > |b_1| + \alpha|a|$. If $b_0 > b_1$ and $|a|\alpha$ is small enough then the condition is satisfied. $|a|\alpha$ is small iff $|a|$ is small and/or local eigenvalues of the expectation functions are close enough to zero. $b_0 > b_1$ is rather a very mild condition. So, one can produce stability in this framework, but that would require a system in which the influence of the system is weak or there exist strong restrictions on the range of regularities that traders stand ready to extrapolate.

Conceptually, the sufficient condition for stability is merely negation of sufficient condition for instability, even though mathematically this is not so. Strictly speaking, the theorems do not constitute a necessary and sufficient condition as both involve probability and likelihood.

Nevertheless, the point is that it is the "range of regularities" which determines the stability in the markets where expectations matter. Grandmont argues that that the defenders of rational expectation theory ensure the stability of the system by precisely doing this that is manipulating the "range of regularities". He gives some examples to show how the restrictions are imposed upon individual's forecast estimators (estimate depends upon the value of past observations) with the help of the some "projection facility". This "projection facility" specifies some lower and upper limit beyond which if estimate falls, it takes the corresponding limit values. The estimator would take values beyond these limits only if large deviations from the past are taken into account. Specifying such "projection facility" from outside the model then means that traders are assumed to ignore such large deviations. The permissible range of deviations (or estimate to be precise) depends on the learning process (i.e. expectation functions) and the structural parameters of the system. Any deviation out of this range (so that true value of estimator falls outside the limits given by projection facility) is termed as "shock" in the rational expectation literature.

According to Grandmont this method of ensuring stability of actual dynamics with learning is quite arbitrary and cannot be justified in any meaningful way. The strength and limitation of this criticism is discussed in the next chapter. Here, we discuss the impact on the stability of stationary state when the expectations of the individual are such that they depend upon both past observations and the notion about the other individual's expectations (through average expectation in the market).

A case of instability in the absence of "shocks", when individuals are strategic:

All the notations used by Grandmont are retained here. We specify the expectation function of any individual i as follows:

(10)

$$x^{e_i} = \psi_i \left[x_t, \ldots x_{t-j}; f_i \left(\sum_{l_1} \psi_{il_1} (x_t, \ldots, x_{t-j}; f_{il_1} (\sum_{l_2} \psi_{il_1 l_2} (x_t, \ldots, x_{t-j}; f_{il_1 l} (\sum_{l_3} \psi_{il_1 l_2 l_3} (\ldots\ldots)) \ldots \right) \right]$$

where j=0,1,2,...,L

l_1 , l_2, l_3,......=1,2,3,....,n

n is the number of individuals

x^{e_i} is i's expectation about period t+1 at period t

ψ_i is i's expectation function

f denotes the market aggregation of individual expectation function, i.e., $x^e = f(\psi_1, \psi_2, \ldots\ldots \psi_n)$.

f_i is individual i's notion of f.

$\psi_{il_1}, \psi_{il_1l_2}, \ldots$denote i's notion of l_1's expectation function, i's notion of l_1's notion of l_2's expectation function and so on. Similarly $f_{il_1}, f_{il_1l_2}, \ldots\ldots$ are interpreted.

Stationary state: Let $\bar{x}$ is a stationary state. If $x_t = x_{t-1} = \ldots\ldots = x_{t-L} = \bar{x}$ then $x^{e_i} = \bar{x}$.

All the partial derivative below are evaluated at the stationary state.

$$c_{ij} = \frac{\partial \psi_i}{\partial x_{t-j}}, \quad c_{il_i j} = \frac{\partial \psi_{il_1}}{\partial x_{t-j}} \text{ and so on.}$$

$$d_i = \frac{\partial \psi_i}{\partial f_i}, d_{il_1} = \frac{\partial \psi_{il_1}}{\partial f_{il_1}} \text{ and so on.}$$

$$\alpha_{il_1} = \frac{\partial f_i}{\partial \psi_{il_1}}, \alpha_{il_1l_2} = \frac{\partial f_{il_1}}{\partial \psi_{il_1l_2}} \text{ and so on.}$$

We make the following assumptions[4] :

(A1) $\alpha_{il_1} = \alpha_{il_1l_2} = \alpha_{il_1l_2l_2} = ... = \alpha = \frac{1}{n}$.

(A2) $d_{il_1}, d_{il_1l_1}$,........are greater than equal to 1.

(A3) $\sum_{l_T} \alpha \bullet c_{il_1.....l_T j} dx_{t-j} = c_{ij} dx_{t-j}$ where T=1,2,3,.....

Now, linearizing equation (10) around the stationary state we get:

(11)

$$dx^{e_i} = \sum_j c_{ij} dx_{t-j} + d_i \Big[\alpha \sum_{l_1} \sum_j c_{il_1j} dx_{t-j} + \alpha^2 \sum_{l_1} \sum_{l_2} \sum_j d_{il_1} \bullet c_{il_1l_2j} dx_{t-j}$$

$$+\alpha^3 \sum_{l_1} \sum_{l_2} \sum_{l_3} \sum_j d_{il_1} \bullet d_{il_1l_2} \bullet c_{il_1l_2l_3} dx_{t-j} + ...\Big]$$

We fix j = J (J is any of the numbers 1,2, ...,L) and analyse the term in the bracket in the above equation. We have,

(12) $\alpha \sum_{l_1} c_{il_1j} dx_{t-J} + \alpha^2 \sum_{l_1} \sum_{l_2} d_{il_1} \bullet c_{il_1l_2J} dx_{t-J} + ...$

[4] Interpretations of these are given in chapter 3.

The Tth term in (12) is

$$\alpha^{T}\sum_{l_1}\cdots\sum_{l_T} d_{il_1}\bullet d_{il_1l_2}\bullet\cdots\cdots\bullet d_{il_1\ldots l_{T-1}}\bullet c_{il_1\ldots\ldots l_TJ}\,dx_{t-J}$$

Similarly the (T+1)th term in (12) is

$$\alpha^{T+1}\sum_{l_1}\cdots\sum_{l_{T+1}} d_{il_1}\bullet d_{il_1l_2}\bullet\cdots\cdots\bullet d_{il_1\ldots lT}\bullet c_{il_1\ldots\ldots l_{T+1}J}\,dx_{t-J}$$

For $T \geq n$,

$$\sum_{l_T}\alpha\bullet c_{il_1\ldots\ldots l_TJ}\bullet dx_{t-J} = \sum_{l_T}\alpha\bullet c_{il_1\ldots l_pl_p\ldots l_TJ}\bullet dx_{t-J}$$

where p= 1,2,….,n.

$$\Rightarrow n\sum_{l_T}\alpha\bullet c_{il_1\ldots\ldots l_TJ}\bullet dx_{t-J} = \sum_{l_T}\sum_{p}\ \alpha\bullet c_{il_1\ldots l_pl_p\ldots l_TJ}\bullet dx_{t-J}$$

$$\Rightarrow \sum_{l_T}\alpha\bullet c_{il_1\ldots\ldots l_TJ}\bullet dx_{t-J} = \alpha\sum_{l_T}\sum_{p}\ \alpha\bullet c_{il_1\ldots l_pl_p\ldots l_TJ}\bullet dx_{t-J}$$

$$\Rightarrow \qquad d_{il_1}\bullet d_{il_1l_2}\bullet\cdots\cdots\bullet d_{il_1\ldots\ldots l_{T-1}}\sum_{l_T}\alpha\bullet c_{il_1\ldots\ldots l_TJ}\bullet dx_{t-J}$$

$$\leq$$

$$\alpha\bullet d_{il_1}\bullet d_{il_1l_2}\bullet\cdots\cdots\bullet d_{il_1\ldots\ldots l_T}\sum_{l_T}\sum_{p}\ \alpha\bullet c_{il_1\ldots l_pl_p\ldots l_TJ}\bullet dx_{t-J}$$

[due to (A2)]

$$\Rightarrow \sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_{T-1}} \sum_{l_T} \alpha \bullet c_{il_1\cdots\cdots l_T J} \bullet dx_{t-J}$$

$$\leq \alpha \sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_T} \sum_{l_T} \sum_{p} \alpha \bullet c_{il_1\cdots l_p l_p \cdots l_T J} \bullet dx_{t-J}$$

$$\Rightarrow (13) \quad \alpha^T \sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_{T-1}} \sum_{l_T} \alpha \bullet c_{il_1\cdots\cdots l_T J} \bullet dx_{t-J}$$

$$\leq \alpha^{T+1} \sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_T} \sum_{l_T} \sum_{p} \alpha \bullet c_{il_1\cdots l_p l_p \cdots l_T J} \bullet dx_{t-J}$$

Consider the right-hand side of equation (13). This is equal to

$$\sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_T} \sum_{p} \sum_{l_T} \alpha \bullet c_{il_1\cdots l_p l_p \cdots l_T J} \bullet dx_{t-J}$$

$$= \sum_{l_1} \cdots\cdots \sum_{l_T} d_{il_1} \bullet d_{il_1l_2} \bullet \cdots\cdots\cdots \bullet d_{il_1\cdots\cdots l_T} \sum_{l_{T+1}} \alpha \bullet c_{il_1\cdots\cdots l_{T+1} J} \bullet dx_{t-J} \quad \text{[due to (A3)]}$$

But this is nothing but the (T+1) th term. The left-hand side of equation (13) is the T th term. So (T+1) th term is at least as large as T th term.

Therefore, equation (12) which is the sum of an infinite series, after (n-1)th term, is non-declining. Hence, the sum i.e. i's expectation tends to infinity.

Now, $x^e = f(\psi_1, \ldots\ldots, \psi_n)$

$$\Rightarrow dx^e = \sum_k \alpha_k \bullet d\psi_k \,, \quad k=1,2,\ldots\ldots,n.$$

Since, $\alpha_i = \alpha$ is positive and finite and ψ_i tends to infinity, dx^e would also tend to infinity. *So, the system becomes locally unstable (and hence globally unstable also) regardless of the actual learning process of other individuals.*

Therefore *with only one individual's expectation function* of the above mentioned form and satisfying assumptions (A1), (A2) and (A3), the market is unstable.

This result, its meaning and implications are discussed in the following chapters.

CHAPTER 3

AN EXAMINATION OF THE UNDERLYING ASSUMPTIONS

In this chapter we discuss the meanings and implications of the way we have specified the individual expectation function and the assumptions we have made about the learning parameters of individuals i in the "case of instability", in the last chapter. The point of reference of this discussion is an attempt to provide a critique of arguments given in Rational expectation theory in justification of stability of stationary state outcome. The specification about the expectation functions, assumptions made thereupon and the result is not an attempt to characterize what might actually happen in the market. Our justification of the specification and assumptions is more of an analytical nature, to counterpose it against Rational expectation analysis.

We have taken individual i's expectation function to depend upon his notion of average expectation in the market where i thinks that everyone's expectation function depends upon their notions of average expectation function, and further i thinks that every one thinks that everyone's expectations functions depend upon their respective average expectation and so on. Such behaviour is not unexpected in the markets where expectations play a very crucial role in determining the outcome(s). The role of expectation is particularly important in the market for assets as their valuation today would be determined by their expected (average) price about tomorrow. So while forming one's expectation about tomorrow, individuals act

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strategically[1] taking into account the average expectation. Such behaviour has been indicated by Keynes by drawing a parallel with the beauty contest. In chapter 12 of The General theory Keynes indicates the agents' behaviour when all they are interested in is the short term variations in the asset prices. In this venture they want to match their expectation (forecast) with the average expectation. They react as in, "... a game of snap, of Old Maid, of Musical Chair - a past time in which he is victor who says Snap neither too soon nor too late, who passes the Old Maid to his neighbour before the game is over, who secures the chair for himself when the music stops...... Or, to change the metaphor slightly, professional investment may be linked to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached a third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

As it is clear from the above quotation, Keynes expected his readers to be surprised from the fact that some people are known to do this exercise upto

[1] **Grndmont has not taken into account any strategic behaviour in his analysis.**

fourth or fifth level. But it is assumed here that individuals do this exercise recursively infinitely many times. This enables one to endogenise the expectation function as ultimately the expectation would depend upon the past observations. There can be two justifications for this. Firstly, Keynes' own emphasis upon the importance of convention in the stability analysis as is clear from this statement of Keynes, "... We are assuming, in effect, that the existing market valuation, however, arrived at, is uniquely correct in relation to our existing knowledge of the facts which will influence the yield of the investment, and that it will only change in proportion to changes in this knowledge; though philosophically speaking, it cannot be uniquely correct, ... Nevertheless, the above conventional method of calculation will be compatible with a considerable measure of continuity and stability in our affairs, *so long as we can rely on the maintenance of the convention.*"[2]

Of course, Keynes did not have in mind local stability analysis of the stationary state. And as such, this justifications has limited validity. But as argued earlier, the whole exercise is done here to provide a critique of Rational expectation theory in an analytical framework.

Interpretation of Assumption 2:

Now we take up the assumption 2 that we've made. This assumption states that there is an individual i such that all the d_i ... are as large as one. This mean that if there is a small change in i's notional average expectation

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[2] Chapter 12, The General Theory. Such argument is stated even more forcefully in Keynes(1937).

then his expectation changes by more than the former. Also, i thinks that all other individuals react similarly to any small change in their respective notional average expectations. Further, i thinks that everyone thinks similarly about everyone else.

To clarify the point further we assume a hypothetical share market where infinitesimal changes occur. The assumption says that if i thinks that market is bullish about any share (i.e. average expectation about the share is that it's Price would go up), then because of this, her expectation about that share's price would be even higher than the average expectation. That is, she is more bullish than the market. Further, i thinks that everyone wants to be more bullish than the market, i.e., everyone increases one's expected increase in price by more than the increase in average expected increase in price. Also, i thinks that this tendency to be more bullish than the market for everyone is a "common belief" in the market. "Common belief" is used here to mean the similar thing as "common knowledge", except that it's about notion or belief of individuals rather than any actual fact.

The question remains that why is that the individual i more bullish than the market about that particular share. Why is that she wants to outperform others in getting the share which she thinks that everyone is more bullish about than the market and is trying to outperform others.[3] This is possible only if i thinks that due to any increase of average expectations over the stationary state, the actual outcome would increase more than average

[3] In chapter 12, The General Theory, Keynes talks of such behaviour when people have poor belief about some asset. He says, "The actual, private object of the most skilled investment to-day is "to beat the gun", as Americans so well express it, to outwit the crowd, and to pass the bad, or depreciating, half-crown to other fellows."

expectation. Also, i is thinking that everyone is thinking in the same manner and this is a common belief. This means that the graph between i's notion of average expectation and the actual outcome near the stationary state would look like the following diagrams

Diagram I

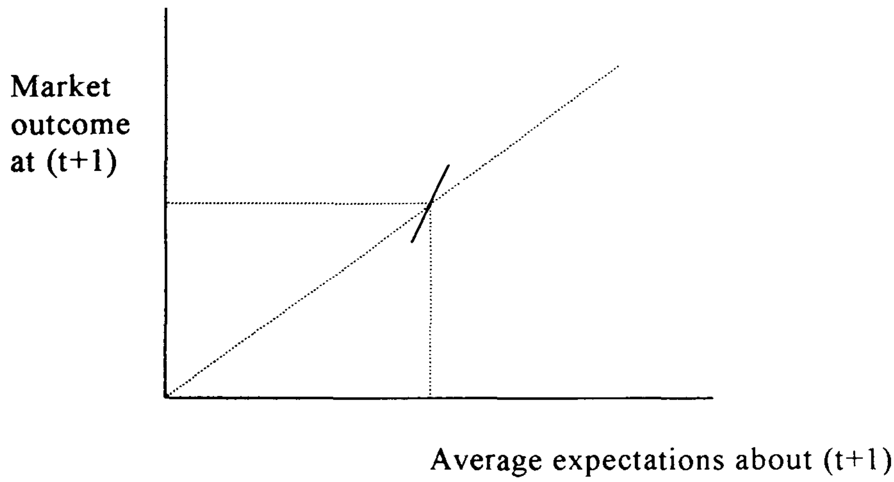
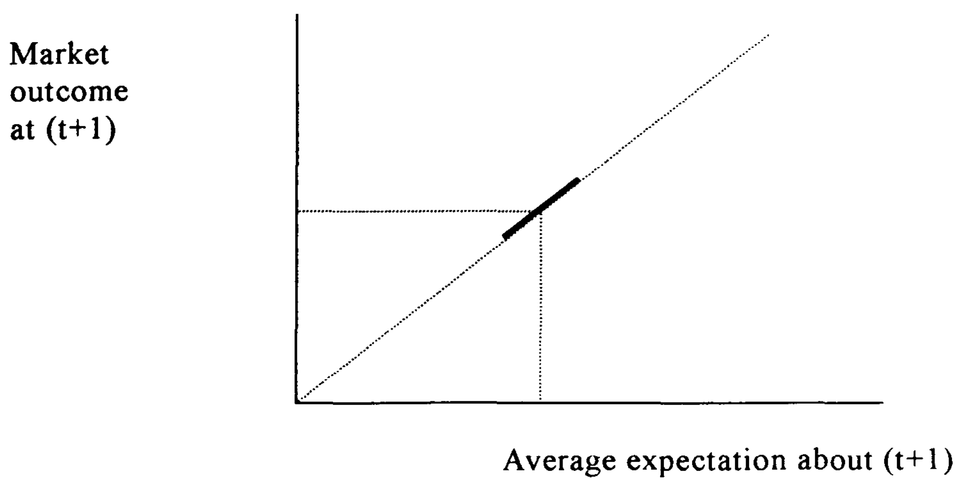


Diagram II



The idea is that at the stationary state the line showing market outcome at period (t+1), for any average expectation about (t+1) at period t, either cuts the 45 degree line from below or overlaps it, in the neighbourhood of the stationary state. Either of these graph is there in i's mind and she thinks that everybody thinks that either of these graphs is there in others' minds. As it is, this simply means that i satisfies the assumption 2.

But why should any normal individual think in the manner described above? Is such thinking totally absurd? Is i then a crazy and abnormal individual whose behaviour cannot be rationalized on the basis of conventions? An example is given below to show that why the answer to these questions is no.

Suppose we have a convention of stationary state outcome $\bar{x}$, and suppose there is a perturbation of ε in some stationary period where $\varepsilon \rightarrow 0^+$. With the convention of stationary state one can argue that this perturbation is due to the external factors, and also due to the convention, traders would form such expectations that market comes back to $\bar{x}$. But now suppose, in the next period, the outcome is $\bar{x}+\varepsilon+\eta$ where $\eta \rightarrow 0^+$; again due to external factors. In this circumstance, an individual i who thinks that these deviations in the consequent periods is due to the learning process of individuals, then the assumption 2 is justified. The deviations are not large enough to be considered as 'shocks' and individuals do not know that whether these have occurred due to some external reasons or change in learning parameters of others. In such a situation it's not weird to assume that one individual is thinking that such deviations have occurred because people have become more bullish about that particular outcome. The deviations are so small that they cannot be called "shocks", and hence there is no meaning in putting any restriction on the value of i's estimators and hence "Projections facility." The strength of the result being discussed here is that we require only one individual to think in such a manner; allowing others to follow any learning process.

We have discussed one situation where the i's notion about market and other individuals has justification in the past observations. One can expect many more situations like this. The general idea is that once we allow for such strategic behaviour on the part of individuals (which indeed is totally justified), we can have stable convention turning into unstable convention in many circumstances, even if the 'shocks' are ignored, and all the individuals behaving quite normally. *So we have a case where 'shocks' and 'psychopaths' are ignored and still the stationary state outcome might be unstable.*

Interpretation of Assumption 3:

To get the result, assumption 3 is used. Assumptions 3 implies that i thinks that an average people react to $(t-j)^{th}$ period derivation as much as she reacts. Further, i thinks that everyone thinks on an average everyone react to $(t-j)^{th}$ period deviation by the same magnitude and so on .

To put it differently, i reacts to $(t-j)^{th}$ period deviation as much as she on an average people react. The latter is same as what she thinks that on an average people would react and so on. This means that i has certain notion of people on an average react to $(t-j)^{th}$ deviation and this she takes to be the "Common notion", and then, she reacts only that much.

This assumption could be justified as i observes only the outcome and then forms some notion of average reaction to period's deviation, which is in accordance to her notion of other learning parameters of everyone, the structural parameters of the market and the past outcomes.

Indeed, if one makes a vector of everyone's learning parameters and one's notion of structural parameters of market, then there can be(possibly infinite) values of vector which explains the past outcomes.

The requirement is that i chooses that one which satisfies (A3) and uses it to make her forecast. The crucial distinction with rational expectation theory and the analysis done here is that, whereas the former requires any particular value of the above mentioned vector to be adopted by everyone, here we allow everyone to make personal choice of any value which explains the past outcomes. One can say that individuals have rational beliefs rather than Rational expectation.[4] These beliefs are allowed to be different from each other.

So Assumption 3 is justified if all that i observe is the past outcomes and learns on the basis of it. Indeed, the justification of Assumption 2 is given precisely on this ground.

Interpreting Assumption 1:

Assumption 1 says that all individuals have equal weightage in the market. This is very unrealistic assumption. The defence of this assumption

[4] If there is only such value of vector which explains the past outcome then, the two coincides, we rule out such case here.

can be given by taking any individual whose weight is M times the weight of the smallest individual, as M individuals all having the same expectation function as the large (parent) individual. In that case N would represent in the population such constructed, rather than the number of individuals in the actual population. Individuals then are characterized by their learning schemes only and their weightage in the market. This solves the problem and thus assumption is relaxable in some form. Nevertheless, it would be desirable to show the result without this assumption, which would require a different proof.

It has been argued above that how we can show instability (local, and hence global) without extrapolating large deviations and without any individuals behaving totally absurd. The result is due to the specifications of the expectation function (for at least one individual) taking into account the strategic nature of human beings and the Assumptions 1, 2 and 3 made in the last chapter.

CHAPTER 4

CONCLUDING OBSERVATIONS AND CONJECTURES

We began our analytical exercise in chapter 2 by focusing on Grandmont's interesting interpretation of how stability is achieved in 'Rational expectation' theory. He pointed out that stability of stationary state outcome is ensured in this theory by essentially restricting the 'domain' of individual behaviour or what we may call the relatively narrow basin, borrowing a term from dynamical systems. Individuals learn, according to this theory, only from those outcomes which lie within this narrow basin.

This limits imposed by this basin, according to Grandmont is meaningless in economics. Nevertheless, the fact remains that ignoring the 'shocks' is justified to some extent if one has a market where most of the past outcomes have been closed to the stationary state. The general idea is that if 'convention' is stable as Keynes (1937) had argued then there is good reason to believe that future outcome would also be close to that convention, as all individuals simply extrapolate the future from the past conventions shaped by past experience. In such a case ignoring the large deviations might be meaningful procedure.

In chapter 2 and 3, it has been argued that one can have instability even when individuals ignore the large deviations in making their forecasts. The argument basically hinges on two points; *First, individuals are strategic; Secondly, individuals observe only the past market outcomes and these are the only inputs in their expectation function.*

Given these two basic points, we could construct a story formalized in our model which leads to the instability results. This was discussed in previous chapters through justifying the assumptions made in chapter 2. Here we present heuristically the main argument in outline.

In our story, individuals are strategic in the sense that they take into account others' expectation functions while framing their own expectation, realizing that everyone is involved in the same exercise.[1] Obviously, individuals do not know about others' expectation functions, and hence have certain 'beliefs' about these. Also, individuals have their personal beliefs about others' beliefs about everyone's expectation functions, and so on. Based on this, individuals do a recursive exercise, realizing that the expectation is endogenous as it depends ultimately on the past observation.

The question that emerges is that how do individuals term their respective beliefs about others' expectation functions (to be more particular, the parameters involved, since functional form has been specified)? We consider a particular individual i. All that i observes are the past outcomes, and hence there is no reason that her belief matches with the actual ones. To explain i's attitude, we allow for an infinitesimally small deviation from the conventional stationary state outcome observed in all the previous periods. Since there is no history of deviation, i would recognize it as occurring due to some external factor (even though it falls into the prescribed 'basin') and ignore it while making forecast about tomorrow. Now, suppose in the next period also the outcome deviates more in the same direction, still the

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[1] Logically, we require only one such individual.

deviation from previous period being infinitesimally small. Now, i has good reasons to believe that deviations in two consecutive periods are forming a trend and this is due to changing beliefs of others. It is not necessary that i believes that convention has changed and the stationary-state outcome is not highly likely in the "long-run". All that we require her to think is that people have become more speculative or less confident about the outcome in the short-run. Knowing that previous two periods' outcome are not the stationary-state outcome, i realizes that average expectations in these periods are not necessarily self-fulfilling. In that case, what could be the relationship between average expectation about tomorrow and tomorrow's outcome? We have assumed that i thinks that everyone believes outcome to deviate more or equal to average expectation and further that this belief is a "common-belief." Naturally, i also expects the same relationship between the market outcome and average expectation of tomorrow.

Two related questions emerge here. Firstly, why should i have similar belief about others' notion of relationship between average expectation about tomorrow and tomorrow's market outcome? Also, if the first question is answered why should i have any particular belief (i.e., the above mentioned relationship is such that market outcome is at least as volatile as the average expectation about it) and not other (it's less volatile)?

One answer that could be given to the first question is that in absence of any information about other individuals, i has no way of distinguishing between them and hence takes them to have similar beliefs. *All that i observes is the market outcomes or 'sentiments'. Consequently, any belief about others' parameters is as good as other beliefs, in so far as the former* *explains the past outcomes.* And given the informational limitations, any individual is likely to choose the belief we are posing, i.e., one which treats everyone similarly in the sense discussed above.

The second question that why individual i chooses any particular belief about others i.e., the belief that everyone believes that market outcome tomorrow would deviate more than the average expectations about it, is more difficult to answer. Nevertheless, we could argue that once the first question is answered, the two particular beliefs amongst which i has to choose one are equally likely, and hence it's not observed to assume that atleast one among many in the market chooses that particular one between the two alternative beliefs.

Individual i also faces the problem of choosing the parameters associated with deviations in the past. Facing the same informational limitations we argue that i uses 'introspection' as the only source of information. She thinks that on an average participant in the market react to any deviation in any period as much as she reacts to it. In the story, we are discussing here, i is ready to react only to the last period's deviation(s), and she reacts only that much as she thinks people on an average are reacting to it which she takes to be same as what everyone thinks that on an average others are reacting, and so on. The justification of this belief of i, thus came from the same arguments which justify the belief previously discussed.

In the story, described above is plausible then the stationary-state outcome is likely to be unstable, even in a presence of only one individual like i. This puts a question mark on the robustness of stability results of

Rational expectation theory, as it is unable to sustain the presence of even one such individual like i.

If the idea that strategic behaviour causes instability, could be generalized further than one can make following conjectures on the basis of our analysis:-

a. Grandmont's emphasis on the range of deviation or what we call the basin considered while making forecasts by individuals may not be critical factor in the consideration of stability. Instead, strategic behaviour on the part of individuals in expectation formation may play a more crucial role. Indeed, despite realizing and emphasizing the importance of convention justifying the beliefs, Keynes talked of instability in the investment market, and he tried to locate the root of such instability in the strategic behaviour of the individual. The beauty contest parable discussed here is one type of such strategic behaviour. Definitely, the result of chapter 2 suggests a different line of enquiry in the stability analysis of any market.

b. The counter-example that we've offered against Rational expectations theory against basis of strategic nature of individuals is analytically too complex and involved to describe reality. If one argues that such behaviour is observed in reality the analysis has to be much simpler. If one could base the problem as an algorithm of smaller steps, then the exercise would be simpler and then any empirical testing on the basis of it would be meaningful. Indeed, such exercise have been carried out in economics, particularly for the complex models related to the area of Bounded Rationality. It can be worthwhile to take up this exercise as a future research project.

c. Strategic nature of individuals and it's implications are rigorously discussed in the Game-theoretic literature. Further, a vast literature on the stability of some of equilibrium concept (and thus learning behaviour of individuals) is available in Game-theoretic studies, and one might gain certain insights from these in the case we've discussed. The general problem that one may face is the following. Discussion on learning and stability is mainly done for the "long-run", ignoring the "short-run" analysis and it's possible impact on "long-run" behaviour. This is because of the fetishness of the literature to show stable equilibrium of some sort or other, and often it's difficult to prove it for short run. These has been same work which take the notion of different "histories" available to individuals who choose one of them depending upon the outcomes in recent past. Further, it's impact on Long-run convergence is discussed. Unfortunately there are very few games like this at the present stage. Nevertheless, one can expect some useful hints from such works in a discussion like our's.

With the above conjectures, we can conclude that there is ample scope for meaningful enquiry into the question of market stability characterized by individuals acting strategically. A vast literature is available in different areas to support such exercise. One could use insights developed in different branches of economic theory to discuss the perhaps one of the most difficult and challenging question in economic theory: how strategic behaviour generate 'market sentiments' that lead to fluctuations in the stock market?

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