Teaching of Algebraic Concepts in Middle Schools in Kerala: A Study of Metaphor and Metonymy of Algebra

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DOCTOR OF PHILOSOPHY

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DECLARATION

The thesis titled, **'Teaching of Algebraic Concepts in Middle Schools in Kerala: A Study of Metaphor and Metonymy of Algebra'**,is a presentation of my original research work. Wherever contributions of others are involved, every effort has been made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions. The work was done under the guidance of Prof.Dr.Minati Panda, ZHCES, School of Social Sciences, Jawaharlal Nehru University,New Delhi.

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CERTIFICATE

In my capacity as supervisor of the candidate's thesis titled, **'Teaching of Algebraic Concepts in Middle Schools in Kerala: A Study of Metaphor and Metonymy of Algebra'**, I certify that this of a bona fide work of the candidate and that the above statements are true to the best of my knowledge.

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ABSTRACT

Even though mathematics education has acquired new insights from cognitive revolution in psychology after late 1950s, the prevalently celebrated teaching methods of algebra still cannot properly solve the algebraic difficulties which the students face. One of the profound reasons behind this is that only a few studies have been conducted on both the teaching and learning of algebra with most following the popular constructivist framework of Piaget. The difficulty of algebra is is seen as one of that failure to transit from arithmetic to algebra. Therefore, the reform in the conventional teaching methods focused more on solving these blocks alone leaving a bulk of the problem. Access to instruction and visualizing the abstract representational system of algebra for every student is a significant issue in most schools in India. Therefore the present research focuses on the teaching and learning of algebra with emphasis on the technique of visualization of algebraic concepts to overcome difficulty in comprehension among students.

The cultural historical theory of Vygotsky has provided the theoretical and methodological paradigm for this study that attempts to examine if the students can learn algebra better if metaphors from everyday lives of children are used and the specific metonymy of algebra is introduced using Davydovian and Lackoff approach. It aims to analyse if the students benefit from a reverse movement in the discourse i.e. from algebra to arithmetic. The study has five research objectives on the teaching and learning process of both conventional and experimental classrooms in comparison to each other as well as on the effectiveness of teaching-learning materials and the classroom pedagogy. The study is conducted in Kerala in the Government Model HSS, Calicut University. VI standard students from the school are selected using random sample technique, and are divided into two groups: control group and experimental group. The 'between group design' of experimental method is used where the controlled group was exposed to the regular Algebra teaching activties and the experiemental activities are exposed to various activities specially designed by the researcher based on the theoretical principles of Davydov and 'Lackoff. The discourse

techniques were partially derived from Panda and Cole's (2007) study. The data ares interpreted using simple content analysis.

In regular class, the teacher taught algebra through the method of substitution of numbers with the initial letters of the names of the objects or geometrical figures in translation i.e. from Malayalam to English, and that the method of replacement of numbers by letters was the vehicle to the introduction of the concept of the unknown. The teacher made use of the examples and problems from the student's text book and their solutions in the teacher's text book to teach the concepts of algebra. As a result, greater emphasis was laid on helping the students arrive at the correct answers and not on making they understand the concept of unknown, variability and the relations between quantities as contained in an equation. Each new concept was introduced without ensuring that the students had understood the former concepts. Stress on the use of English alphabets made students internalize the notion that algebra is a kind of mathematics where mathematical operations use English letters in an operation and not numbers. The teacher introduced algebra as a new area of knowledge and not as an abstract system in correspondence with the already acquired arithmetic system. The teacher did not resort to the use of knowledge or tools from the culture of the students in the introduction of algebraic symbols and concepts. She served as a mere agent who transmitted the information and procedures in the text books to students. The students remained as recipients who recorded the information they received.

In the experimental group, students were aided to think beyond numbers with the help of two games; *kothan kallu* and *nootam kolu*. They were introduced to algebraic symbols, concepts and operations using their own everyday ideas and concepts in a mathematically oriented axiomatic discourse. In order to teach the concept of variability and equations, the researcher made use of the examples suggested by the students themselves rather than imposing textual examples on them. The method adopted was not limited to the substitution of numbers by letters. It involved comparison of quantities to find out which has a greater/lesser quantity using objects like red lucky seeds, palm tree seeds, and green gram whose exact count was difficult to estimate owing to their small size and big quantities. With the employment of a common balance, the researcher demonstrated how classroom talk about quantities and their relationships in abstract terms facilitate learning of algebraic concepts. Additionally, the children were encouraged to use symbols to talk about quantities and appreciate arbitrariness of abstract symbols. The classes were carried forward using the everyday knowledge of the students. The children initially struggled to think about quantities without employing numbers. But the use of common balance helped them to talk about quantities using abbreviations like the first letter '*che*' derived from the word '*cherupayar'* (green gram) to represent the given quantities in different coconut shells. The children compared these quantities using their already acquired algorithmic knowledge and talked about the quantities using abstract non-number symbols.

The erroneous comparison by some students provided opportunities to the teacher to introduce pedagogically the concept of 'unknown' and 'variability'. It also helped students to naturally reflect on the errors and engage meta-mathematically with the process that led to erroneous comparisons or equations. The concept of 'variability' that an abstract symbol can represent different quantities depending on the mathematical context was not fully derived by the students from their everyday observations. The teacher handled the problem of translating the mathematical concepts as visualized by the students while playing the games '*kothan kallu'* and '*noottan kolu'* into mathematical operation. The students showed how children moved from concreate thinking (i.e. arithmetical operation) into abstract and theoretical thinking (i.e. algebraic operation) using symbols that are not counting numbers.

Use of first letter of the object names in Malayalam language by the students to represent a quantity and subsequent replacement of the these by textbook symbols as an "agreed upon" symbols by a community of formal mathematicians help children understand the social as well as the intellectual history of evolution of metonymy of algebra. Most of the classroom dialogues mediated by the researcher involved metaphorical extension of already acquired mathematical imaginations of the children. For example most of the arithmetic operations and quantity ideas like more, less, equal to etc. worked as metaphorical expressions that hold the new algebraic imagination.

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LIST OF ABBREVIATIONS

APA	American Psychological Association
B.Ed.	Batchelor in Education
CCE	Continuous and Comprehensive Evaluation
DCT	Dual Coding Theory
HSS	Higher Secondary School
SCERT	State Council Educational Research and Training
Std. VI	Standard Sixth
TTC	Teachers Training Course
XII	Plus Two
ZPD	Zone of Proximal Development

INTRODUCTION

Algebra is considered as a gateway to significant ideas of mathematics and also to higher mathematical studies (MacGrigor & Prince, 1999). But it is often experienced as a stumbling block to students rather than as a gateway to higher studies or mathematically significant ideas (Stacey, Chick & Kendal, 2004). In other words, on the one hand, mathematicians and mathematics educators tend to remain satisfied with their knowledge of mathematics and general theories of mathematics education through classroom algorithmic practices, demonstration of mathematics models and working with heuristics of Eurocentric mathematics (Vergnaud, 1982), while on the other hand, mathematicians like Vergnaud (1982), Sfard (2002) and Cobb (2004) have emphasized the need of Mathematics education to be widely open to different ideas coming from other fields as well as communities other than middle class European and American communities. Frudenthal, a scintillating figure in the contemporary mathematics, said that mathematical concepts, structures and ideas have been invented as tools to organize the phenomena of physical, social and mental world (Treffers, 1993). He advocates for incorporating everyday lives, experiences into academic mathematics. The everyday world can be a source for mathematics learning (Panda, 2004; 2006: Panda & Cole, 2007; Panda & Mohanty, 2009; Greer, Mukhopadhyay, Powell & Nelson-Barber, 2009; Rampal & Marker; 2011; Rampal & Subramanian, 2011). Later mathematicians like Tall (1996) and Kaput (2005) emphasized the importance of visualizing mathematical concepts, and bringing in the inactive experiences into the school mathematics classrooms.

In the recent times, mathematics education discourses emphasize innovative mathematical activities that support the process of visualization of mathematical concepts in mathematics classrooms (Brating, 2008). Based on their Indian research, Panda (Panda, 2004; 2006) and Panda & Cole (2007) show how cogently initiated classroom discourse can create visual reifications for mathematical concepts. Everyday metaphors, according to them, play a significant foundational role in help children transit from everyday to scientific mathematics concepts. Philosophers and psychologists like Lakoff and Nunez have pointed out that one of ‘the eminent ways of visualization of mathematics concept is through metaphorical thought and exploring the metonymy of mathematical concepts’ (Nunez & Lakoff, 1999, p.79). As most of the mathematical concepts are founded on everyday metaphors, linguistic exploration of the source domain as well as the target domain along with their relations will provide the cognitive-linguistic foundation to the mathematical concepts. The target domain in mathematics like any other science discipline is understood in terms of its relations with the source domain (Nunez & Lakoff, 1999).

Lakoff and Nunez have borrowed the terms metaphors[1] and metonymy[2] from the linguistic and philosophical domain to explore the conceptual, semiotic and metaphorical foundation of mathematical concepts. Metaphor is considered as a ‘rhetorical convention’, and as a type of thought for the embellishment of language usage in descriptive writing and poetry (Finken, 2002). It is used to relate new experiences to prior understanding. The new experiences may contain elements that can be connected to the prior knowledge and also those which are entirely new. In recent years, several researchers (Lakoff & Nunez, 2000; Lakoff & Johnson (2003), Presmeg, 1992, 1997; Pimm, 1981, 1987; Panda & Cole, 2007; Rampal & Subramanian, 2011) have initiated research into exploring the metaphor and metonymy of mathematical concepts as cognitive constructs having specific cognitive structures. Metaphor is a figure of speech in which two unlike objects are compared by identification or by the substitution of one for the other (Finken, 2002), whereas metonymy here refers to change of name where a salient attribute is taken to stand for an entity (Finken, 2002). In multilingual and informal communities, folk games provide huge heuristic value if the everyday metaphors are carefully used in a classroom to create cross-linguistic registers for experiential mathematics (Panda, 2004; 2006; Panda & Cole, 2007).

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[1] Metaphor is a figure of speech in which two unlike objects are compared by identification or by the substitution of one for the other (Finken, 2002).

[2] Metonymy is a change of name where a salient attribute is taken to stand for an entity (Finken, 2002).

In algebraic mathematics, however, the metaphor and metonymy are more than linguistic devices; they structure human thoughts and therefore mathematical thinking (Zandieh & Knapp, 2006). Lakoff, Nunez, Johnson, Radford, Pimm and Presmeg have explicated the role of metaphor and metonymy in constructing everyday as well as the academic mathematical concepts (Font, Bolite, & Acevedo, (2010). Lakoff and Nunez (2000) have laid down the metaphorical underpinnings of many arithmetic and algebraic concepts. According to them, the metaphorical projection is the main cognitive mechanism that enables abstract mathematical entities through structuring bodily experiences. Because, 'image schema' drawn by the available metaphor and the metonymy help to build the ability of reasoning about mathematical concepts, these enable learners to make meaning of abstract mathematical concepts (Lakoff & Nunez, 2000). Both therefore advocate that the metaphoric or metonymic relationship is a way to develop cognitive access to a mathematical concept (Zandieh & Knapp, 2006).

Even though, both metaphor and metonymy are basic cognitive-social tools for learning (Finken, 2002), these are usually not used as pedagogic tool in school mathematics teaching. These cognitive-linguistic structures can be evoked in the elementary mathematics classrooms to introduce arithmetic (Panda, 2004; 2007) as well as algebraic concepts. Analysis of the NCF curriculum shows how the everyday metaphors can be used as a resource for introducing basic mathematics concepts in the Indian school mathematics classrooms (Panda, 2007; Panda & Cole, 2007; Mohanty, Panda & Pal, 2010; Panda, Mohanty, Nag & Biswabandan; 2011). The possibility of the same in the syllabus of higher classes would be limited. Analysis of the researches done in the area of teaching and learning of mathematics in Indian schools reveals that these researches do not consider metaphors and the metonymy as learning tools of mathematics, especially in the case of abstract mathematical concepts like the algebraic concepts introduced in the middle schools. The difficulties are found more in the middle school mathematics, especially in the algebraic mathematics (Subramaniam & Banargee, 2011; 2008). Common algebraic difficulties among middle school children are the inability to judge or comprehend the meaning of equality by using equal sign (=), failure in using arithmetic procedures in algebraic context, and understanding unknown numbers and variability in the algebraic problems (Subramaniam & Banargee, 2011, 2012).

Even though both metaphors and metonymies can be used as ‘mathematics teaching tools’ (Finken, 2002), those are usually not considered a teaching tool in the conventional mathematics classrooms. Perhaps these ‘figures of speech’ may be used in the elementary mathematics classrooms to introduce arithmetic concepts (Panda, 2006; 2007). When analyzing the National Curriculum Framework in India, 2005, there is a possibility to use metaphors as a teaching method to introduce basic mathematics concepts in the mathematics classrooms of Indian middle schools, whereas the possibility of the same in the syllabus of higher classes is limited. When analyzing the researchers who engaged in the teaching and learning of mathematics among Indian school children, the fact that they have not considered both metaphor and metonymy as learning tools of mathematics is very clear, especially in the case of abstract mathematics. But they have pointed out that even new textbook frameworks have not adequately addressed the issue of introducing abstract mathematics like symbolic algebra in the middle grades. They have also found out that greater difficulties in the school mathematics begin from middle school and those are mainly in algebra mathematics (Subramaniam & Banargee, 2011; 2008).

In this study, both metaphors and metonymy are used as cognitive constructs to address the difficulty of comprehensibility of middle school algebra. The present study therefore aims at explicating the metonymies and metaphors of algebraic concepts introduced in middle school algebra using pedagogic principle given by Davydov, Lakoff and Nunez and analyzing how these provide innovative pedagogic tools for teaching of algebra in middle schools. These tools are believed to provide multiple scaffolds to meaning making of variables and unknown numbers among the students. Introduction of the concept of variability in the middle school, creation of a shared meaning of unknown numbers and also the specific semiotic mechanism with which algebraic problems are solved can best be done by helping students work with the metaphorical basis of algebraic concepts and exploring the metonymy of the same.

I.1.1 An Overview of Major Cognitive Psychology Approaches to Mathematics Teaching and Learning

Most of the popular researches in psychology on the difficulty in mathematics teaching were explained by the social, biological and cognitive reasons. Literature in child psychology and the developmental theories were influenced heavily by phylogenetic and ontogenetic developmental accounts provided by Piaget and Vygotsky. Some of these literatures are discussed below to provide the theoretical foundation to this study.

Constructivism

According to Piaget (1970), the conceptual knowledge cannot be transferred from one person to another because it must be constructed from his or her own experience. In his theory, the concept of ‘genetic epistemology’ is explained as the discovering of the roots of various knowledge, including the scientific (Egodawatte, G. 2011). Later he explained the development of knowledge as a process of equilibration through assimilation and accommodation (Piaget, 1970).

In 1950s, mathematical learning was discussed within the framework of the process-object duality proposed by Piaget (as Tall et al. 2000 quoted by Berger, M. 2005). Here, the formation of a mathematical concept in students was explained by the schematic formation of different stages. The development of schema was explained through the concepts of assimilation[3] and accommodation[4]. But the central drawback of the Piagetian and neo-Piagetian theories is that they

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[3] People translate and encapsulate the transmitted information into a form they can understand. Cognitive process is the process by which a person integrates new perceptual matter or stimulus events into existing schemas (Wadsworth, B. J. 1996).

[4] Creation of new schema or modification of old schemes to understand new knowledge (ibid).

are rooted in a framework in which conceptual understanding is regarded as deriving largely from interiorized actions (Cobb, P. 1995). The predominant role of language, role of social regulation and the social constitution of the body of mathematical knowledge are not integrated into the theoretical framework.

I.1.2 Algebra in the school mathematics.

Traditionally mathematics is introduced in schools using the stage theory given by Piaget (Cai & Kunth, 2005). Hence, most school mathematics curricula separate the study of arithmetic, geometry and algebra; arithmetic being the primary focus of lower primary school mathematics, geometry and algebra being the primary focal point of middle and high school mathematics. This separation indicates that children have different cognitive stages to achieve these mathematical concepts. Theoretical framework of this notion is substantiated by Piaget. Constructivism in cognitive theories, typically follows Piaget in using biological metaphor and characterize mathematical learning as a process of conceptual reorganization. Here they assume that mathematical knowledge is created by students as they reflect on their physical and mental actions. In other words, mathematical learning of the students is basically explained through Piaget's theoretical concepts of assimilation and accommodation. Therefore, both pedagogy and curriculum for mathematics are developed in line with the Piagetian concepts of maturation and cognitive skills.

Few Vygotskian researchers like Davydove and Cobb questioned these traditional trends of mathematics in the school scenario. In Russia, Davydove theorized and proved that even elementary students can learn abstract mathematics like algebra in a pedagogically mediated environment. Based on his findings, he developed a curriculum for elementary students in Russia (Schimattaue, 2005). Later, Vygotskians like Schmittaue (2005) and Cai (2005) observed and analyzed the success of Davydov's Curriculum. Researchers in this arena found out that algebraic mathematics is introduced at the elementary schools in the countries like China, Singapore and South Korea (Cai & Moyer, 2007).

Traditionally, algebra is introduced in the middle school mathematics curriculum. A comparative study between the history of algebra curriculum and the enquiry into the development of algebra education in the present days showed that algebra mathematics in schools have not changed much over the years (Kieran, 1992). What school algebra still does is an elusive interplay of letters and numbers. Often, it is presented to students as pre-determined problems with strict rules. Traditional instruction doesn't leave a space in the classroom for the inputs of the students themselves. Teaching begins with the syntactic rules of algebra, which is not related to the real world of the students. This results in students failing to get the structure of algebra and also enjoying little satisfaction in practicing algebraic mathematics.

What is Algebra?.

From the time of Al-Khowarizmi, an Arab mathematician of the 9th century, algebra has been viewed as the science of equation solving (Kieran, 2004). Past twelve centuries, this view has not changed for the better. Algebra is still considered as a branch of abstract mathematics, involving manipulation of variables and numbers in the form of symbolic representations. In other words, algebra deals with objects of an indeterminate nature such as unknown variables and parameters (Banergee & Subramanian, 2012). The basic part of algebra is known as 'elementary algebra' and the abstract part of algebra is referred to as 'abstract algebra' or modern algebra (Cameron, 2008). Elementary algebra is essential for the study of mathematics, natural science, and engineering and is also applied in disciplines such as medicine and economics. However abstract algebra is used by mathematicians and physicists whose theories are proven by using mathematics.

According to Usiskin (1988), algebra has four fundamental conceptions. He conceptualized these differences according to the nature of variable combinations. The first conception considers algebra as generalized arithmetic. Here, variables are considered as pattern generalizer. In this conception, the key instruction for learners is "translate arithmetic rules and generalize". For example; the arithmetic expression such as ‘$2 \times -10 = -20$’ and ‘$5 \times -5 = -25$’ could be generalized to give properties such as ‘$x \times -y = -xy$’. Historically, algebra has been transformed into many other forms of mathematics such as calculus and analytic geometry owing to its power for generalizing arithmetic rules and regulations (Gunawardena, 2011).

According to the second conception, algebra is a study of procedures for solving particular kinds of problems. For example; consider the problem: when 19 is added to 9 times a certain number then the sum is 100, find out the number. When this problem is translated into algebraic language, it should be in the form of an equation ‘$9x+19=100$’ (solution of $x = 9$). Here variables are either unknown or constant and the key instruction for the learners is “simplify and solve”.

In the third conception, algebra is considered as a study of relationships among quantities. Here variables really tend to vary. For example, a formula for the parallelogram is “$A = b.h$”. This is a relationship between three quantities. As the variables A, b, and h can take any values, the feeling of the unknown is not established.

In the fourth conception, algebra is considered as a study of structures. According to this notion, variables are more than an arbitrary symbol. Here variables become an arbitrary object in a structure related by certain properties. These kinds of variables are generally found in abstract algebra. For example, consider the equation $5x^2+6ax–121a^2$. Here the conception of a variable is not represented in the same manner as in any previously discussed notions. The variable neither acts as an unknown, nor acts as an argument. Among these four fundamental conceptions, the first three are introduced in the middle school algebra and the fourth one is introduced in the higher secondary level.

Beginning of Algebraic Concepts in the School Mathematics

Conventionally mathematics curricula separate the study of arithmetic, geometry and algebra. Arithmetic being the primary focus of lower primary school mathematics, geometry and algebra being the primary focus of middle and high school mathematics. It indicates that children have different cognitive levels to understand these mathematical concepts. Here cognitive psychologists assume that mathematical knowledge is created by students as they reflect on their physical and mental actions. In other words, mathematical learning of the students basically explained through the Piaget's theoretical concepts assimilation[5] and accommodation.

But Vygotskian researchers had questioned these traditional trends of mathematics in the school scenario. In Russia, Davydove theorized and proved that even elementary students can learn abstract mathematics like algebra. Researchers in this arena found out that algebraic mathematics is introduced at the elementary schools in the countries like China, Singapore and South Korea (Cai & Moyer, 2007).

Comparison between the history of algebra curriculum and enquiry about the development of the algebra education in the present days showed that algebra mathematics in schools have not changed much over the years (Kieran, 1992). Still school algebra does an elusive interplay of letters and numbers. Often, it is presented to students as pre-determined problems with strict rules. For example, $x + 7 = 10$, the letter x is unknown but its value can be discovered by using the law of inverse[6]; then the given value of 'x' based on the solution is 3. Here, arithmetic procedures are used for solving algebraic problem. Therefore, arithmetic proficiency is necessary to excel in algebra (Booth, 1981). In other words, learners construct their algebraic notions on the basis of their previous experience in the arithmetic. Thus, algebraic system of learners receives structural properties which are associated with the number system (L. Liora & L. Drora. 1999).

Researchers like Matz (1980), Greeno (1982), Booth (1984), Kieran (1988) and Lins (1990) have discovered the close relationship between arithmetic expression and algebra. They attributed many of the fundamental difficulties of algebra to the

[5] People translate and encapsulate the transmitted information into a form they can understand. Cognitive process is the process by which a person integrates new perceptual matter or stimulus events into existing schemas (Wadsworth, B. J. 1996).

[6] The operation that reverses the effect of another operation

failure of the learners in identifying equivalent forms of an algebraic expression. Researchers in the area of teaching and learning of algebra have pointed out the importance of knowing the structure of arithmetic expressions to make sense of algebraic expressions and their manipulations (Erna. 1997).

Another group of researchers who were engaged in the algebra learning found out that learning algebra involves learning to read and use symbols in new ways (McCoy, 2000). But in this new way approach, symbols follow rules and regulations of arithmetic. In most of the cases, through instructions, students understand that variables represent numbers whose exact values are not specified, and they use variables appropriately. So the researchers in the field of algebra argued that algebra is developed from an exploration of quantitative relationships (Schmittau & Anne, 2004). Actions of numbers and concept of numbers instead of variables in algebra are also developed from the relationships between quantities (ibid). Therefore, most of the researchers who have focused on the algebraic difficulty among students, argued that the transition from arithmetic to algebra is the main cause of algebraic difficulties of students. But on the other hand, researchers like Davydov accommodate difficulties of algebra within the limitation of traditional design of teaching method (Minetola, 2002). According to them traditional instruction never leaves a space in the classroom for the students' own inputs. Here teaching starts with the syntactic rules of algebra, which is not related to the students' real world. It results in giving little satisfaction to students in practicing algebraic mathematics.

School Algebra and Difficulties among Students

"In school, algebra is often considered as a gateway to higher mathematics, not least because it provides the language in which mathematics is taught. Consequently, it is important that all students be given a genuine opportunity to learn algebra. Without this, they are cut off from many occupations, either because algebra is really used there or because it is specified as a preliminary qualification". (Stacey & Chick, 2004, p. 2)

From the observations of Stacey & Chick (2004), it is clear that learning algebra in school level has a crucial role in building a necessary ground for ongoing and future mathematics learning. Traditionally in school mathematics algebra is introduced in the middle school. Algebra is introduced through generalized arithmetic (Usiskin, 1988). It is often begun by either applying arithmetic rules to variables or through the introduction of variables whose values and quantities are unknown. For example, a) -10/-5 = 2; -20/-10 = 2 therefore '–x/-y = z', and b) 'x+7=10'. In the first example, rules and regulation of arithmetic are generalized into variables. Both the solutions of -10/-5 and -20/-10 denote the same number but express different relations. When it comes to algebra, the relational meaning is generalized into the concept of a function as the variables have not any particular values (Subramanian, K & Banerjee, R. 2004). In the second example, the letter x represents a quantity which is less than 7. Here, the value of x is determined by the relationship between two quantities which is given in the equation. Both the examples show the structural similarity between arithmetic and algebraic expressions.

In middle schools, students are trained to use arithmetic expression for learning algebraic expression (Linchevski & Livneh, 1999). But often, due to the absence of well-developed understanding of transformations in the numerical context, students start to use arbitrary procedures for simplifying algebraic expression. Then they may commit the same errors as in arithmetic context (Kieran, 1992). Studies show that in beginning algebra, students frequently face fundamental difficulties in understanding the subtle distinction between a variable and an unknown in an equation.

Even though the nature of algebraic mathematics in the school textbooks centers on arithmetic procedures, students who are proficient in arithmetic also face difficulty in dealing with algebra (Kieran, 1992). This difficulty among algebra beginners led the researchers to study more about algebra in the middle school. One of the early works in this area had been done through the Piagetian framework (Banergee, P. 2003). According to the Piagetian researchers, algebra beginners have same type of misconception and errors; they seem to be independent of geographical locations. For example, "*students in Fiji, Israel, and Japan experience similar error patterns as students in the United States and other countries*" (Reese, 2007). One of the algebraic hurdles among middle school students is the difficulty in performing arithmetic operation with algebraic problems (Booth, 1984; Stacey, 1994). For example 'x + y = xy'. Similarly, middle school students have a misconception about variables that different letters must represent different quantities (Kuchemann, 1981). For example, if students are told that rectangle *R* has side lengths '*a'* and '*b',* and then if asked whether *R* could be a square as w ell, it is highly unlikely that they say "yes" because the lengths of the sides are different (Reese, 2007).

Another misconception among beginners is the notion that a non-numerical answer cannot be possible in algebraic problems. They are under the impression that answers to every problems must be numbers (Booth, 1984). Thus the equal to sign (=) may be perceived as a unidirectional sign preceding a numerical answer (Kieran, 1981; Subrahmaniam & Banergee, 2008; Booth, 1984). For example, if students are told that team A scored 'n' goals and team B scored 'm' goals, they may obtain the correct algebraic expression for the total number of goals as 'n+m'. But the students might view the answer as incomplete or improper as the solution is not a number nor is it a single term (Matz, 1980; Booth 1984). Some studies have figured out the reason for this misconception in the insistence of the children on numerical or single term answers owing to prior experience in arithmetic. Another common misconception is that children consistently misinterpret letters as labels or units instead of variables (Reese, 2007). For example; a teacher use 10m to represent 10 meters but 10m has quite a different meaning in algebraic context. Often, students exhibit confusion in the use of letters. In the Piagetian framework, explanations of the algebraic difficulties were focused on the cognitive maturity of the children. All researches which were conducted in Piagetian framework emphasized that the introduction of algebra should be in 'formal-operational stage'[7]. Indeed most of the errors that students make in algebra can be seen in later years (Radford, 2001). Conflicts like these show that the conventional approaches to 'teaching and learning' algebra fail to produce a deeper and comprehensive understanding of algebra in the students.

Subsequent researches have moved out of the Piagetian framework because its inadequacy to analyze the nature of algebraic thoughts of the students and its development. This is reflected in the mistakes the students commit in the subsequent years. Therefore, divergent approaches to algebraic difficulties led to enquiries into the cultural-historical framework to understand algebraic difficulties. This framework focuses on the development of algebra with cultural artifacts and tools. It uncovers the concept of algebraic thinking. Since 1980, algebraic thinking was one of the most researched areas in mathematics education. In the discussions that happened in 1980s and 1990s, general consensus was achieved on two aspects; algebra deals with objects of indeterminate nature, such as unknown variables and parameters being the former, and the latter that such objects are dealt within an analytic manner (Radford, L. 2010). Researchers in the late 1980s had focused on the difficulties in movement from the arithmetic to the algebraic form of thinking.

I.1.3 Vygotskian Approach in the Teaching and Learning of Algebra

In 1950s, mathematical learning was mainly uncovered within the framework of process-object duality by Piaget (as Tall et al. 2000 quoted by Berger, M. 2005). The formation of a mathematical concept in students was understood by the schematic formation of different stages. The development of schema was explained through the concepts of assimilation[8] and accommodation[9]. This

[7] This is a developmental stage which was proposed by Piaget. It begins at age 11. When children enter into this age, they gain the ability to combine and classify items in a more sophisticated way and attain the capacity of higher order reasoning.

[8] People translate and encapsulate the transmitted information into a form they can understand. Cognitive process is one by which a person integrates new perceptual matter or stimulus events into existing schemas (Wadsworth, B. J. 1996).

framework suggests that conceptual understanding is regarded as deriving largely from interiorized actions (Cobb, 1995). Traditional design of the school curriculum framed in this paradigm, theoretically believed that learning is the mastery of practical skills and operations (Texas, 2003). Hence the instruction method is inadequate to provide assistance to overcoming the common difficulties in algebra. But Davydov, a trained Vygotskyan proposed a remedy for overcoming the limitations of the conventional method of teaching algebra (Minetola, 2002). His curriculum is built on the perspective in social constructivism which considers that knowledge production is a result of social interaction and that even higher mental processes are socially mediated. This perspective gives a framework to look at algebraic mathematics as both a cognitive activity which is constrained by social and cultural, and also as a cultural phenomenon which is constituted by a community of actively cognizing individuals (Cobb, P. 1995).

In his curriculum, he emphasized that sensorial experience of students from external world is important to build algebraic concepts. Within this perspective variables of algebra are more than just letters; these can help, explain and describe quantitative relationships which are grounded in the real world situations. For example, children relate five fingers to one hand and ten fingers to two hands. Without using any abstract symbolic notation, children are able to make a multiplicative relationship between fingers and hand that is the ratio of 5:1. So children may extend this multiplicative relationship to proportional relationships like five fingers are to one hand as ten fingers are to two hands or five fingers are to one hand as twenty fingers are to four hands (5:1::20:4) and so on. Here, the children intuitively become aware of the notion of the shifting unit – how a hand can be both one (hand) and five (fingers) at the same time. Here children learn to form another number through connecting one unit to the previous number learned. According to Davydov, the empirical aspects of the algebraic learning experience with number is that,

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[9] Creation of new schema or modification of old schemes to understand new knowledge (ibid).

"*In comparing many things having different quantities, the child singles out something similar or common in them — there turns out to be a separateness of every object from another, a certain spatial or temporal restrictedness about them. There is an individual object — and each object contains this sort of externally perceptible individuality or separateness. . . . Every individual object is a unit. A group of objects is a set of units (a collection of "individuals"). Above all, the child learns to single out in any observed object this peculiarity it has of being a separate entity, and of approaching groups of objects only as sets of units. In this way an abstraction of quantity is formed,"* (Davydov, 1972/1990 pp. 140-141)

The central concept of this perspective is the emergence of algebraic thinking. Schmittau (2005) observed that Davydov's three year elementary mathematics curriculum focuses on the algebraic structure and then applies algebraic understandings to concrete numerical instances. This pattern of learning activity for the development of algebraic thinking has a clear theoretical difference from the theoretical ground of the instruction of algebra in the study of arithmetic in the early grades. This organization of learning activity is formulated on the basis of Vygotsky's theoretical notion about the algebraic thinking early in the child's development. According to Vygotsky (1962), psychological phenomena occurs first on the social plane and then on the individual plane, therefore the intended 'theoretical thinking'[10] or algebraic understanding is attained through the appropriation of psychological tools in the form of specially designed schematics.

In detail, algebraic thinking is the specific way in which the students conceptually act, in order to carry out the actions required by the generalizing task (Radford, L. 2000). But the particular way in which the students conceptually act underpins the emergence of algebraic thinking in students and it was regulated by a socially established mathematical practice where the teacher plays a central role.

[10] Theoretical thinking = Scientific thinking (see foot note 8)

Therefore, the emergence of algebraic thinking is through the encounter between the subjectivity of the individual and the social means of semiotic objectification.

Semiotics of Algebra

According to the cultural-semiotic perspective of Vygotsky, signs are a constructive part of thinking. This is because meanings of signs are formed in the culture. In detail, cultural meaning of sign is mediated while the younger generations interact with other people in their culture. Cultural-cognitive approach considers both meaning making of sign and symbolic acts in the culture as semiotic activity. According to the perspective of cognitive psychology, semiotics used in the domain of mathematics helps to develop mathematical thinking (Radford, 2010; Schmittau, 2003, 2005). If the socio-cultural assumption is used to understand mathematical activity in the classroom, activities of mathematics can also be read as a special type of semiotic activity. This socio-cultural perspective gives new approaches on mathematics teaching and learning and it leads to both new conceptions of cognition and new views about knowledge and the cognizing subject.

One of the key peculiarities of this approach is that a semiotic platform always gives new doors and entrances to understand mathematical signs and formulas in a nonconventional way. This is the predominant logic behind using different semiotics in new trends among algebra teaching-learning groups. Traditionally, letters (like x, y, z), signs for operations (like +, -, ×) and sign of equality (=) have been considered as algebraic signs in school algebra. Hence, meanings of all these algebraic signs are made in schools via generalized arithmetic. In other words, conventionally algebra in the middle school is introduced as alphanumeric semiotic system (Steinbring, 2009). But researches after 1990 proved that there are other semiotic systems like language, diagram and patterns other than the alphanumeric to signify indeterminacy (Radford, 2000). These methods explicitly stated that no difference between known and unknown numbers, and letters have ever been a sufficient condition for thinking algebraically (Radford, 2010; Schmittau, 1993, 2003, 2004).

For example, (Ontario Curriculum Mathematics, 1997 cited by Radford, 2000); find out the value of sign '#' from the equation # + # + 4= 10 or find out the value of '▲' from the equation▲+ ▲ + ▲ + 10 = 19. Value of # in the first equation and value of '▲' in the second equation are '3'. This '*transitional language*' (Radford, 2004) approach, as alphanumeric approach for teaching algebra, relies on specific conceptions about what signs represent and the ways in which the meaning of signs are elaborated by the students.

Example 2: From the sequence pattern below, find out the number of patterns in the 50th figure. Fig (1) ♣♣♣, fig (2) ♣♣♣♣, fig (3) ♣♣♣♣♣, ……….., fig (50) ?

In this example only three figures are given. If someone wants to find out the number of patterns in the 50th figure, he/she should understand the relationship among patterns in the first three figures. The numbers of patterns increase by '1' from first figure to second figure is '1' and by '2' from first figure to third figure. If figure (1) is taken as the base, the changes in the figure can be expressed as 'Base, Base + 1, Base + 2'. Therefore the number of patters in the 50th figure is equal to 'Base + 49'. From the first figure, it is clear that number of patterns are 3. If the base is replaced by 3, the mathematical expression would be 3 + 49 = 52. Number of patterns in the 50th figure would be 52.

Meaning Making of Algebraic Signs

According to Johnson (1987), "*meaning is always a matter of human understanding, which constitutes our experience of a common world that we can make sense of. A theory of meaning is a theory of understanding. And understanding involves image-schemata and their metaphorical projections…..These embodied and imaginative structures of meaning have been showed to be shared, public and objective in an appropriate sense of objectivity*" (As Nunez, 1999 quoted Johnson, 1987, p-174).

From this perspective, the meaning making of algebraic signs depends upon an image schema and conceptual projections which are drawn by them. Algebra is then introduced in the classroom in the conventional manner, numbers are replaced by the variables of algebra, and signs of arithmetic procedures play the role of operation among these algebraic variables. Therefore, in this context, meaning of algebraic elements conveys that algebraic variables (letters like x, y, z) stand as a representative of numbers (Foster, 2007) whereon, often, imaginary schema of these variables are limited in the meaning of numbers. But introducing variables by using patterns give a different meaning to algebraic variables (Radford, 2004). This method helps learners imagine visually about variables. Similarly, equal to sign (=) in algebra makes a contradiction among algebra learners (Chae, 2000) because students use equal sign to indicate "the answer follows", in arithmetic classes. But in the context of algebra, equal to sign (=) means that expressions on the left and right side have the same value. This can be a stumbling block for the students who have learned that the equal to sign means "the answer follows". The usage of metaphors like balance scale (examples figure 1a and 1b) helps the students to understand how equal to sign is used in the equations and what kinds of operations on equations are permissible (Foster, 2007).

According to Lakoff and Nunez (2000) the metaphorical explanation draws an inferential structure from one domain to another. This inferential structure helps to create a conceptual blend between the new concept and the familiar one. Then the coordination of meaning of the new concept is based on common image schemata and forms of metaphorical thought (Lakoff and Nunez, 1997).

I.1.4 Metaphor and Metonymy of Algebra concepts

Researchers in the field of mathematics education have borrowed the term metaphor and metonymy from the literary domain. Both metaphors and metonymies are figures of speech. In a metaphor, two dissimilar objects are compared by identification or by the substitution of one for the other (Lackoff & Johnson, 1980). According to the literary description, metonymy is a change of name where a salient attribute is taken to stand for an entity (Beckson & Ganz, 1975). Here, the name of an object or an idea is substituted for another object or idea. Both metaphors and metonymies were used in the mathematics education with a concern over whether or not mathematics can be learned in the manner in which a language is learned.

Lackoff (2000), Nunez (2000), Johnson (1980), Press Meg (1997), Pirie (1989), Kieran (1989), Wacek Zawadowski (2000), Van Dormolen (1991) and Pimm (1987) theoretically proved that both metaphors and metonymies are basic tools for constructing meaning of abstract mathematics. All these researchers treated both metaphors and metonymies as cognitive constructs because these concepts play a remarkable role to structure our thinking (Zandieh & Knapp, 2006). Therefore, these concepts help develop mental or cognitive access to a concept. In most of the cases, metaphors originate from everyday experiences while metonymy is formed as a result of classroom instructions (Dogan-Dunlap, 2011 cited by Carrizales, 2011). When properties are projected from the first to the second domain, metaphors create a conceptual relationship between an initial or source domain (familiar one) and a final or target domain (the new or abstract one). Because metaphors link difficult ideas, they are essential for learners in terms of constructing meanings for mathematical entities (Lackoff & Nunez, 2000).

How do Metaphors and Metonymy Help in Algebraic Thinking?

According to Lackoff and Nunez (2000), algebra is about essence.

Algebra is the study of mathematical form or "structure." Since form (as Greek assumed) is taken to be abstract, algebra is about abstract structure. Since it is about essence, it has a special place within mathematics. It is that branch of mathematics that is conceptualized as characterizing essence in other branches of mathematics. In other words, mathematics implicitly assumes a particular metaphor for the essences of mathematical systems: The Essence of a mathematical system is an abstract algebraic structure (Lackoff & Nunez, 2000 p 110).

The field of algebra, therefore, is a magnificent case study of fundamental cognitive mechanisms in mathematical cognition. As Lackoff and Nunez pointed out, algebra is an abstract structure, and historically this abstract structure (algebra) is introduced through basic arithmetic structures such as addition, subtraction, multiplication, division and so on. All these basic arithmetic structures are metaphorically mapped. In other words, metaphorically mapped arithmetic concepts are used to build algebraic sense. To cite an example put forward by Lackoff &Nunez, consider the problem given below.

The numbers 1, 2 and 3 under the operation of addition modulo 3-that is, when the result of addition with these numbers (1, 2, 3) is equal to or greater than 3, the integer will be subtracted by 3. Thus 1 + 2 would normally be 3 but because of subtraction 3, 1 + 2 = 0. In the same way, 2 + 2 would be normally 4 but with 3 subtracted out, 2 + 2 = 1. Addition table for addition modulo 3 is given below.

Table: 1 addition table for Addition modulo 3

+	**0**	**1**	**2**
0	0	1	2
1	1	2	0
2	2	0	1

In this table, 0 is the identity element, since X + 0 = X, for each element X. For each element X, there is an inverse element Y, such that X + Y = 0. Thus, 0 is its own inverse, since 0 + 0 = 0, and 1 is the inverse of 2, since 2 + 1 = 0. The following laws are derived from this table.

Closure: The sum of every two elements is an element.

Associativity: X + (Y + Z) = (X + Y) + Z

Commutativity: X + 0 = X

Inverse: For each X, there is a Y such that X + Y = 0

The essence from the arithmetic structure is that set of elements, operation with addition modulo 3, and the stated arithmetic laws governing addition over these elements.

Algebra is not about particular essences; moreover it is about a general essence. By the assumption of Lackoff and Nunez, the general essence is called abstract because it indicates abstract elements, abstract operations and abstract laws. When the followed arithmetic structure is implemented into elements I, A, B with operation *, laws which have evolved here will be equal to laws which had evolved in the arithmetic operation.

Table: 2 Elements I, A, B with operation *

*	**I**	**A**	**B**
I	I	A	B
A	A	B	I
B	B	I	A

The laws holding with respect to this abstract table:

Closure: The operation * on any two elements of the set is an element of the set.

Associativity: A * (Y * Z) = (A * Y) * Z.

Commutativity: X * Y = Y * X.

Identity: There is an element I, such that X * I = X

Inverse: For each X, there is Y such that X * Y = I.

In this instance, it is very clear that abstract elements with an abstract operation * and the abstract laws are independent of arithmetic. This independent nature is obtained by the linking metaphorical role of addition modulo 3 (table-1).

Therefore addition modulo 3 is the source domain and the group of elements are the target domain. Details are given below:

Table 3: Details of Source domain and Target domain of Lackoff and Nunez.

Source Domain: Addition Modulo 3	**Target Domain: Algebra (elements)**
The set {0, 1, 2} of numbers	The set {I, A, B} of abstract elements
The binary operation +	The abstract binary operation *
The arithmetic identity element 0	The algebraic identity element I
Arithmetic closure law	Abstract closure law
Arithmetic associative law	Abstract associative law
Arithmetic identity: There is an element 0, so that X + 0 = X	Abstract identity: There is an element I, so that X * I = X
Arithmetic inverse: For each X, there is a Y so that X + Y = 0	Abstract inverse: for each X, there is a Y so that X * Y = I

The above presented is example of an algebra metaphor. Here, the abstract algebraic structure (the commutative group with three elements) is conceptually mapped by using the essence of addition modulo 3. Addition modulo 3 is a linking metaphor of algebra.

According to Pierie & Kieren (1989), a metaphor is a main learning mode for early works on a concept while metonymy is the working mode for formalization. Metaphors have several elements. The important elements are the target and the source. Target is the principle subject, concept or process to be shared. The source is the referent to which the target is being likened. The characteristics which are common in both target and source are called the ground and the characteristics which are not common among target and source are called tensions (Lackoff & Johnson, 1980). Consider the example, "Numbers are points on a line". The geometric conception of numbers is the target where the points on a line, the source (Lackoff & Nunez, 2000). In this example consider point B to be on the right side of the point A. In this case, number B would be greater than number A. Points to the left of the point mapped to the number zero are considered as negative numbers. Here, these two mapping are the connecting grounds. Usually tensions within the metaphor are not discussed in learning (Presmeg, 1998). In mathematical classrooms, metaphors are used as an interactive teaching and learning device and it can be used to make comparisons, to bridge two domains, to illuminate and to clarify.

For example: Consider the problem of an algebraic equation introduced through the balance scale metaphor. Three different coloured squares are placed on both sides of the balance scale (Fig 1a). The colours used are Purple (square P), Yellow (square Y) and Green (square G). The weight of squares to the left side of the balance scale is equal to the weight in the right hands side of the balance scale. Find out the weight of Y.

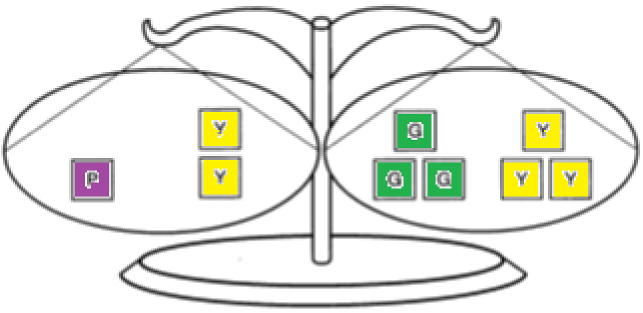


Fig (1a) and

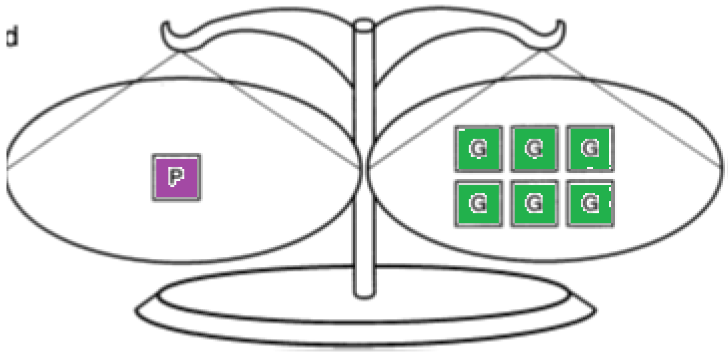


Fig (1b)

From figures (1a) and (1b), it is clear that weight of 1P + 2Y = weight of 3G + 3Y and weight of 1P = weight of 6G. If we use conventional algebra letters x, y, z instead of colors Purple, Yellow and Green the above equations would be, $x + 2y = 3z + 3y$ and $x = 6z$.

Step 1:

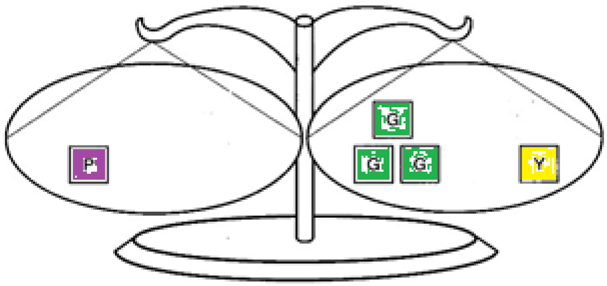


Fig (1c)

In step 1, two squares of Yellow colour are taken out from both side of balance scale as the weight of all the yellow squares are the same. Then the equation representation of the figure would be 1P = 3G + 1Y ($x = 3z + y$).

Step 2:

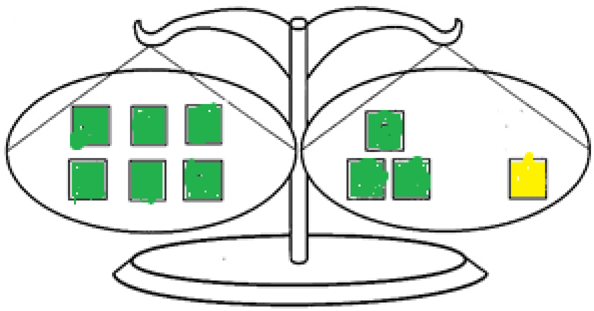


Fig 1(d)

From fig. (1b), it is clear that weight of P = weight of 6G. Therefore in step 2, left side of the balance scale is filled by six Green squares instead of the Purple Squares. So the equation of the balance scale in the step 2 would be 6G = 3G + 1Y ($6z = 3z + y$, in the conventional manner).

Step3:

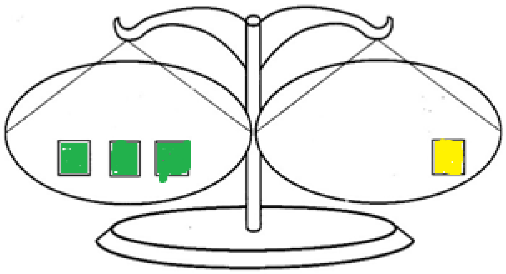


Fig 1(e)

In step 3, three squares named G are seen to have canceled each other from both sides of the balance scale. After the cancellation of the Green Squares, three Green squares in left side and one Yellow square in right side of the balance scale are what remain. This indicates that one Y is equal to three G. ie 1Y = 3G (y = 3z). If different steps in balance scale model of equation are expressed in way of conventional algebra problem, it can be written like this $x + 2y = 3z + 3y$ and $x = 6z$; then $x = 3z + y$; $6z = 3z + y$; $3z = y$ and respectively. In this example the balance scale model of equation is the source domain and conventional alphanumeric expression of the equation and its solution are the target domain. The balance scale metaphor helps to visualize algebraic expression as this metaphor effectively created an iconic representation of the algebraic equation.

Now consider the case of metonymy with another example.

Raju went to the market with Rs.500. He bought a whole bunch of bananas for Rs.150. He went back home in an auto rickshaw and paid Rs.20 rupees. How much money does he have remaining?

Step 1: How much money remains after buying bananas?

500 – 150 = 350

Step 2: How much money remains after paying for the ride back home?

350 – 20 = 330

Instead of indulging in two steps, we can find out the amount of money remaining by subtracting the total expense from the total amount of money Raju had.

ie. Total expense = **150 + 20 = 170**

Remaining = **500 – 170 = 330**

What do we see here?

$$\mathbf{(500 - 150) - 20 = 500 - (150 + 20)}$$

This calculation shows that instead of subtracting two numbers one after another we need to subtract their sum once. When this logic is expressed using letters, the expression of letters that follow would be $(x - y) - z = x - (y + z)$, for all numbers x, y, z, with $x > y + z$. In the above example, real life arithmetic problems are used to introduce the relationship of variables. Therefore in this example, real life arithmetic problem is a source domain and algebraic expression is the target domain. Arithmetic procedure is the connecting ground.

Metonymy is the basis of all mathematical symbolism (Finken, 2002). The function of metonymy is to facilitate condensation and referencing of objects (Lackoff & Nunez, 2000). In mathematics, both letters and symbols are metonymies for particular concepts. For example, let ‘y’ be an integer. Similarly let LMN be any triangle or let ‘XY’ be axis of any graph. Symbols which are represented as metonymies are often not arbitrary in nature; rather it is grounded in conceptual systems which are grounded in our physical systems (Lackoff & Johnson, 1980). For example,

The weight of 1 litter kerosene is 800 grams. What is the weight of 25 cubic centimeter of the Kerosene?   Note: 1 litter is 1000 cubic centimeter.

Step 1:

According to the proof **1 litter = 800 grams**

1 litter = 1000 cubic centimeter

Therefore **1000 cubic centimeter = 800 grams**

Then **1 cubic centimeter = 800/1000 grams = 0.8 gram**

25 cubic centimeter = 0.8 ×25 = 20 gram

If the volume of Kerosene in cubic centimeters is denoted by 'v' and the weight in grams by 'w', then the relation between these is that **w = 0.8 × v**

If the multiplication sign is removed, the equation would be, **w = 0.8v**

The relation between the weight (in grams) and volume (in cubic centimeters) of kerosene may be expressed in the form, **w/v = 0.8**

The meaning of this expression is that whatever be the quantity of kerosene, its weight in grams divided by its volume in cubic centimeters gives the constant 0.8.

In this example 'v' indicates volume of kerosene in cubic centimeter and 'w' indicates weight of kerosene in grams. Both 'v' and 'w' can be any quantities. According to the metonymy concept of algebra by Lackoff and Nunez (2000) both letters 'v' and 'w' are referred to as the fundamental metonymy of algebra. Because both letters represent any set of numbers in the number line and they facilitate the condensation and referencing of two physical properties. The central concept of both metaphor and metonymy is that these figures of speech help to draw an image schema. It helps the students to visually focus on the algebraic concepts. It helps the students to think visually. This thinking pattern helps students to overcome the traditional errors of algebra (Schmittau) as it facilitates in the pattern of thinking algebraically (Radford, 2010).

I. 2. Review of Literature

The Introduction to the thesis has set the broad view of the study. It suggests what the thesis purports to explore and the theories that will guide this study. This study is concerned with the teaching of algebraic concepts by using metaphors and metonymy of algebra. The literature review begins by reviewing the relevant studies of both algebraic difficulties among students and algebraic thinking in different themes, and drawing a theoretical support from the reviews for attaining substantial as well as pertinent questions to be researched.

I.2.1. Culture and Algebraic Thinking

Luis Radford (2000) conducted a longitudinal classroom research about students' algebraic symbolizations. The research was drawn from Vygotsky's historical-cultural school of psychology on the one hand, and from Bakhtin and Voloshinov's theory of discourse on the other. The research concerns were grounded in a semiotic-cultural theoretical framework in which algebraic thinking is considered as a sign-mediated cognitive praxis. Within this theoretical framework, the students' algebraic activity was investigated in the interaction of the individual's subjectivity and the social means of semiotic objectification. An ethnographic qualitative methodology ensured the design for the interpretation of a set of teaching activities. The discussion which was conducted among students was analyzed by discourse analysis. The results from the analysis throw light on the students' production of (oral and written) signs and their meanings as they engage in the construction of expressions of mathematical generality and on the social nature of their emergent algebraic thinking.

2.2. Primary School Mathematics and Algebraic Thinking

Jacob, et al. (2007) conducted a yearlong experimental study and thereafter presented the positive effects of a professional development of algebraic reasoning in the elementary students. 19 urban elementary schools, 180 teachers, and 3735 elementary students from one of the lowest performing school districts in California were selected for the study. Algebraic reasoning was generalized as arithmetic and the study of relations was used as the centerpiece for work. The teachers who participated generated a wide variety of student strategies, including those that reflected on the use of relational thinking, unlike the strategies generated by non-participating teachers. Students in the classes of the teachers who participated exhibited significantly better understanding of the equal to (=) sign and used strategies reflecting relational thinking considerably greater during interviews than the students in classes of non-participating teachers.

Algebra instruction has traditionally been postponed until adolescence because of historical reasons, and assumptions about cognitive development. Carraher, D. W. and team (2006) provided evidences that young students, aged 9-10 years, can make use of algebraic ideas and representations typically absent from the early mathematics curriculum and thought to be beyond the reach of students. This data were collected by a 30-month longitudinal classroom study. Four classrooms in a public school at Massachusetts, with students between Grades 2-4 were observed as part of the study. The data rejected the assumption that algebra is incomprehensive to the students till adolescence and also clarified that young students can integrate algebraic concepts and representations into their thinking. This particular research findings supported V.V. Davydov’s concepts teaching algebra in the early mathematics education.

Blanton & Kaput (2005) conducted a case study in order to examine how the classroom practices promote algebraic reasoning. The aim of the study was to explore the ways in which and to what extents a teacher was able to create a classroom situation that supported the development of algebraic reasoning in the students. They analyzed an year of classroom instruction to determine the robustness with which a teacher integrated algebraic reasoning into the regular course of daily instruction and its subsequent impact on the students' ability to reason algebraically. They took varieties of algebraic reasoning and techniques of teaching paradigms for supporting students’ algebraic development. They considered the diverse types of algebraic reasoning, their frequency and form of integration, and techniques of instructional practice that supported students' algebraic reasoning as a measure of the robustness of the capacity of a teacher to build algebraic reasoning. The results revealed that the teacher was able to integrate algebraic reasoning into instruction in planned and spontaneous ways that led to positive shifts in the algebraic reasoning skills of the students.

Jean Schmittau of the State University of New York and Anne Mortis of the University of Delaware (2004) carried out a comparative study between algebraic development in Davydov's elementary mathematics curriculum and the approach to algebra advocated by the National Council of Teachers of Mathematics in the US. This study revealed that there are striking differences between the two approaches to algebraic development. In the U.S curriculum, developing algebra took place as a generalization of number ,where as, in Davydov's curriculum the development of algebraic structure was formed through the relationships between quantities such as length, area, volume, and weight. The arithmetic of the real numbers follows as a concrete application of these algebraic generalizations. The instructional approach, while similar to constructivist teaching methodology, emanates from a very different theoretical perspective, namely, the findings of Vygotsky and Luria that cognitive development is enabled by overcoming obstacles for which previous methods of solution prove inadequate. In a study in which the entire three-year elementary curriculum of Davydov was implemented in a US school setting, children using that curriculum developed the ability to solve algebraic problems which were normally not encountered until the secondary level in the United States.

Research by Kaput and Blanton (2000) presented a scenario where third grade teachers reported their experiences of presenting a lesson algebraically. Students were given a problem: ▲ + ▲ = 6, ▲ + 9 = 12, ▲= ? Students were told that all the triangles had to be of the same value. After several attempts, students could find out that the value was three. Thereafter the teacher asked the students to suggest options that could replace the use of the empty triangles and some students suggested letters. The mathematics class proceeded with more problems involving large numbers and the students used calculators to verify quantities. This process incorporated the beginning of algebraic symbols and guided students in the process of thinking about patterns and making generalizations.

I.2.3 Algebra in Middle School

Asquith (2007) focused on the teachers' knowledge of the students' understanding of core algebraic concepts. In this study, knowledge of the 'middle school mathematics teachers', on the students' understanding of the equal to sign and variables was examined. Data were collected through interview technique from 20 middle school teachers. Interview was focused on the predictions of teachers about the student's understanding of the equal sign and variable. The teachers' predictions of students' understanding of variable aligned to a large extent with students' actual responses to corresponding items. In contrast, teachers' predictions of students' understanding of the equal sign did not correspond with actual student responses. Further, teachers rarely identified misconceptions about either variable or the equal sign as an obstacle to solving problems that required application of these concepts.

Malisani, and Spagnolo, (2007) analyzed whether the notion of "unknown" interferes with the interpretation of the variable "in functional relation" and the kinds of languages used by the students in problem-solving. They also tried to study the concept of the variable in the process of translation from algebraic language into natural language. To answer these concerns they presented two experimental studies. In the first one, they administered a questionnaire to 111 students aged 16-19 years. Drawing on the conclusions of this research they carried out the second study with two pairs of students aged 16-17 years. From these two experiments, they revealed that the solving procedures predominantly use natural language only. They followed the pace of spoken thought in which the semantic control of the situation was developed and took place. The students also used arithmetic language but not in a purely algebraic context. From this viewpoint, researchers concluded that, the individual development introduces some characteristic lines of the historical development.

Jeong Lim Chae (2005) conducted a study on 'sense making' of algebraic symbols as the part of an ongoing project, 'Coordinating Students' and Teacher's Algebraic Reasoning' (CoSTAR), funded by the National Science Foundation. The purpose of this study was to investigate how students constructed meaning for algebraic symbols and mathematical concepts with symbols in relation to narrative, tabular and graphical representations. This study focused on 4 seventh-grade students who were beginning to learn algebra with the mathematical context of representing changing situations with two variables in relation to each other. Data were collected from the videotaped interviews with the students based on classroom activities. The results of the data analysis mainly explained the process and the nature of students' referential relationships according to the three bi-directional ways of referential relationships centered on algebraic notations and revealed relevant issues for understanding students' referential relationships. The issues included students' appreciation of representing changing situations in various forms of representation in mathematics, their understanding of algebraic equations as an abstract form of representation, and their conceptions of variables and rates in the referential relationships. These issues also suggest instructional implications for teachers and mathematics educators to help enhance students' understanding.

Knuth, K. J and team (2005) studied the middle school students' understanding of two core algebraic ideas-equivalence and variable. And they analysed the relationship of their (equivalence and variable) understanding to performance on problems that require the use of these two ideas. Their analysis and findings suggested that students' understanding of these core ideas influenced their success in solving problems, the strategies they used in their solution processes, and the justifications they provided for their solutions.

Pyke, C. L. (2003) investigated the strategic representation skills of eighth-grade students while they were engaged in a set of tasks that involved applying geometric knowledge and using algebraic equations. The strategies studied were derived from Dual Coding Theory (DCT) (Paivio, 1971), and they were elicited with task-specific prompts embedded in an assessment developed for the study. The purpose of the study was to test a model that highlights strategic representation as a mediator of the effects of reading ability, spatial ability, and task presentation on problem solving. The proposed model was tested using the linear structural equations modeling approach to causal analysis and the data did not reject the model. The results showed that students' use of symbols, words, and diagrams to communicate their ideas, contribute in different ways to solving tasks and reflect different kinds of cognitive processes invested in problem solving.

In 1999, Liora Linchevski and Drora Livneh conducted a study to understand the relationship between algebraic and numerical context. Several researchers suggest that difficulties of students with the algebraic structure are in part due to their lack of understanding of structural notions in arithmetic. They assume that the algebraic system used by students succeeds structural properties associated with the number system with which students are familiar. This study explored this assumption. In an attempt to discover whether wrong interpretations of the algebraic structure found in an algebraic context occur in a purely numerical one, they individually interviewed 53 6^{th} grade students. The assessment revealed that difficulties of students with the algebraic structure are in purely numerical contexts.

I.2.4. Indian Researches in algebra learning

Rakhi Banerjee and K. Subramanian (2012) conducted a study among sixth grade students for over two years. It aimed at the evolution of a teaching approach over repeated trials in beginning algebra. The teaching approach focused on the structural similarity between the arithmetic and algebraic expression and it planned at supporting students in making a bridge from arithmetic to beginning algebra. Grade 6 students of two neighboring schools were selected for the study. One school taught in the vernacular medium while the other resorted to the use of English. The study was conducted over a period of two years. As part of the study three MSTs (Main Study Trial) were conducted, the first immediately after grade 5 examination, the second in the middle of grade 6 and the third after the completion of grade 6. The former trial focused on analyzing the level of knowledge and understanding of the students of the arithmetic rules and operations they have been learning until grade 5. They tested for the students’ understanding of equality/equivalence of expressions, understanding of rules, the procedures of evaluating/simplifying expressions, and the use of algebra to represent and justify/prove. The latter trials helped lay foundations of the above suggested concepts and thereafter employed them in the formation of a bridge for the smooth transition of the students from arithmetic to algebra. Analysis of the students' written and interview responses revealed that the teaching approach helped the students in creating meaning for symbolic transformations in the context of both arithmetic and algebra. And it also helped the children to make a bridge between arithmetic and algebra.

Rakhi Banergee, K. Subramanian and Shweta Naik (2008) conducted a study among 6th standard students to describe the evolution of a teaching learning sequence for grade 6 students who were beginning to learn algebra. The teaching learning sequence was designed to enable the students to make a transition from arithmetic to algebra by connecting their prior knowledge which related to arithmetic and its operations. Analysis of the repeated trials helped in the understanding of the nature of arithmetic and the tasks required in making a transition to algebra. And the structural concepts, 'term' and 'equality' enabled the students to develop a new symbolic system of algebra and its simple operations.

Subramaniam, K and Rani Banarjee from the Homi Bhabha Centre for Science Education and the Tata Institute of Fundamental Research, Mumbai respectively conducted a teaching intervention study in 2004 to explore the interconnection between the growing understanding of arithmetic expressions and beginning of algebra in the students. Three groups of sixth grade students were chosen. Among them, two groups received instruction in arithmetic and algebra, and one group received instruction of algebra mathematics without arithmetic. The groups who were instructed in arithmetic developed a strong understanding of algebraic mathematics. They performed better in algebra questions compared to the performance of the group who received algebra mathematics without arithmetic. Of the three groups, Groups A and C received instructions on algebra and arithmetic while Group B received no instructions on arithmetic. The groups which received instructions performed better in comparison to the other in the expression of verbal phrases and in the identification of algebraic terms. Group B which received no instructions on arithmetic employed a rule learnt in the context of the algebraic expressions to arithmetic expressions as well. Though the performance of groups which received arithmetic instructions was observed to be better than the other, the study does not claim evidence of arithmetic instruction improving performance of algebraic manipulation.

Most of the reviewed literatures are grounded in the Vygotskian theoretical structure except the researches that are done in Indian context. All the studies have violated the conventional understanding of the capacity of the students to think algebraically. And all studies acknowledged the profound role of the cultural artifacts of students in shaping algebraic thinking. Among the reviewed studies, researches which were focused on the algebraic reform in the context of elementary school suggests that generally, difficulties in the development of understanding of algebraic structures of the students in the elementary school are in purely numerical contexts. Although a few studies which were focused on the middle school algebra asserted that the difficulties of algebraic mathematics, among middle school students are related to equivalence and variable, while other researches in this domain addressed the transition difficulty of students from arithmetic to algebra. Middle school researches which were conducted among the Indian schools had only focused on the relationship between arithmetic operations and algebra, and how it can be used for algebraic teaching. No study has brought middle school algebraic difficulties beyond the alpha numeric issues. Moreover, no study has enquired the effectiveness of teaching middle school algebra via visualization.

I. 3. Statement of the problem.

Algebra is considered by many to be a “gatekeeper” in school mathematics because algebra is one of the foundations of abstract mathematics, so learning algebra is critical to further study in mathematics and it is essential for the future educational and employment opportunities (Knuth, Alibali, MicNeil, Weinberge, & Stephens, 2005). In schools, traditionally algebra is introduced in the middle school mathematics. In other words, historically in mathematics education, algebra follows arithmetic and geometry. Both arithmetic and geometric mathematics acquaint signs and their uses in mathematics. But even students who excel in arithmetic and geometric mathematics fail to perform in algebra mathematics (Kieran, 1992), especially students in grade 6 and 7 face stumbling blocks in acquiring algebraic concepts (Subramaniam, & Banargee, 2011; Radford, 2010, 2006, 2000; Presmeg, 1998).

But the demand for algebra in different levels of education is generally high (Usiskin, 1995). If students cannot properly understand algebraic concepts, it makes it further difficult for them to handle higher mathematics. Many studies revealed that students discontinue studying higher level mathematics because of their lack of success in algebraic mathematics (Bush, 2011). Therefore a detailed understanding of both learning and teaching algebra is very important. When analyzing the researches which engaged on the algebraic difficulties (Bush, 2011; Egodawatte, 2011; Banerjee, Subramaniam & Naik, 2008; Asquith, Stephens, Kunth &Alibali, 2007; Carraher, Schliemann, Brizuela & Earnest, 2006; Ferrin, Floden, McCrory, Burrill & Sandow, 2005; Sherin, 2002) one fact is evident. Often teachers and experts in education share a consensus that algebraic difficulties of beginners are due to the transition difficulty from arithmetic to algebra. This approach limits the understandings about algebraic difficulties within the alpha numerical issues. Consequently, teaching methods which are developed for solving algebraic difficulties mainly focus on bridging the gap between arithmetic and algebra (Radford, 2010). This conventional method perceives learning algebra as applying arithmetic operations in algebra mathematics. According to the socio cultural theory of Vygotsky, mathematical concepts are shaped through the direct or indirect interaction of the mind with the physical world. From this point of view, the traditional teaching method neglects the role of cultural tools, semiotics or artifacts within the physical world for developing algebraic concepts.

I. 4. Rational of the Study

Since antiquity, teaching algebra has been a difficult task (Lenart, 2004). Even though mathematics education has acquired new insights from cognitive revolution in psychology after later 1950s, the prevalently celebrated teaching method of algebra cannot properly solve students' algebraic difficulties. One of the profound reasons behind this is that very few studies have been conducted on both teaching and learning of algebra. Among them, most of the researches followed the popular constructivist framework of Piaget. Often, algebraic difficulties of students are accommodated within the boundaries of learning algebra (Lenart, 2004) and it is explained as the difficulty of transition of students from arithmetic to algebra. It implies that the objective of teaching algebra is to help the students reach a thorough level of understanding for handling equations with both constants denoted by numbers and unknown quantities denoted by letters. Conventionally algebra teaching method follows this. But by this method, algebraic difficulties of students are not fully solved (Schmittau, 2004). The access to instruction in algebra for every student is still a significant issue in schools (as Hernon, 2004 quoted Kaput, 1995). It highlights that more analysis is necessary in order to develop an understanding of the factors that help students to be successful in algebraic mathematics and how a teacher can properly assist in achieving this goal. Therefore, the present research focuses on the less discussed issues in the areas of learning and teaching algebra, paying focus on the technique of visualization of algebraic concepts to overcome difficulty in comprehension.

I. 5. Objectives:

In Indian middle schools, algebra is introduced to students through generalized arithmetic procedures. Arithmetic is tailored with algebra. But primary investigation which was conducted by the researcher in the middle schools of Kerala, India revealed that even students who excelled in arithmetic do not perform equally well in algebra. Kerala school Mathematics research explains this difficulty either by the specific transition difficulties that the students face in their movement from arithmetic to algebra, or because of the lack of pre-requisite conceptual and semiotic knowledge among students for learning algebra.

The present study, therefore, makes an attempt to examine if the usage of metaphors from everyday lives of children and the specific metonymies of algebra introduced in the line of the Davydovian approach will help children learn algebra better. It also attempts to analyse if the students benefit from a reverse movement in the discourse i.e. from algebra to arithmetic. The specific objectives are given below:

1) To study how algebra is taught in Government run middle schools in Kerala? What are the ways in which students deal with algebraic concepts and how they mathematize algebraic problems in the classroom.

2) To study the effectiveness of new pedagogic tools for teaching algebra where algebraic concepts and ideas are introduced through carefully designed activities by the researcher using Davydovian approach and the works of Lakkof and Nune. As a departure from the existing pedagogy, several relevant metaphors from everyday world of the students and the metonymy of formal algebra are used in a play way method.

3) To analyse and compare how the concept of unknown numbers, variability and relation between known numbers are introduced in a regular class vis-à-vis an experimental class where new pedagogic tools and materials are used.

4) To analyse how the necessary semiotic tools of middle school algebra are introduced in regular class vis-à-vis experimental class.

5) To compare the effectiveness of new pedagogy, teaching learning materials and CCE for algebra in regular and experimental class.

Research questions:

1. How do students translate word problems into algebraic equations in the regular Government run middle schools?

2. What is the metonymy of middle school algebra and how is this introduced to students in the middle schools in Kerala?

3. How do the students make meaning of 'algebraic signs' in the classroom when they encounter textbook problems on algebra?

4. How do middle school children construct unknown numbers and how do they abstract relationship between unknown numbers?

5. In the beginning of algebra lesson in the middle school, how is the concept of variability introduced to children by the teacher?

6. What everyday metaphors ground algebraic concepts? Do these metaphors scaffold student's learning of algebra better?

7. How do students mathematize algebraic problems in the classroom if algebraic concepts, ideas and relationships are introduced through metaphors and metonymy of algebra using pedagogic principles given by Davydov, Lakoff and Nunez?

8. Could semiotics be introduced better through these new pedagogic tools?

9. Do these pedagogic tools and materials provide a better understanding of middle school algebra? What kind of error analysis of students' works will help these teachers understand the learning trajectories of individual students?

METHOD

Algebra is considered by many to be a “gatekeeper” in the school of Mathematics because algebra is one of the foundations of abstract mathematics. So, learning algebra is critical to higher studies in mathematics and it is also essential for future employment opportunities (Knuth, et.al, 2005). But mathematicians and mathematics educators still tend to remain satisfied with their knowledge of mathematics and the general theories of mathematics education through classroom algorithmic practices, demonstration of mathematics models and working with heuristics of Eurocentric mathematics (Vergnaud, 1982). Mathematicians like Vergnaud (1982), Sfard (2002) and Cobb (2009) have emphasized the need of Mathematics education to be widely open to different ideas coming from other fields as well as communities other than the middle class Eurocentric communities. Indeed, this perspective highlights the nature of mathematics practices in a community and how the prior mathematical knowledge in children can be used for introducing mathematics in schools. Usually the stumbling blocks of school mathematics are not solved by this approach.

For example, primary investigation conducted by the researcher in the middle schools of Kerala revealed that all students who excel in arithmetic are not performing equally well in algebra. According to the findings of Kerala School Mathematics Research, this difficulty could be explained either by the specific transition difficulties of the students from arithmetic to algebra, or the lack of a pre-requisite conceptual and semiotic knowledge among students for learning algebra. Often this approach diminishes the importance of the epistemological basis of the background knowledge among learners for conceptualizing the algebraic concept. The present study makes an attempt to examine if the use of metaphors from everyday lives of children and the introduction of a specific metonymy of algebra using Davydovian approach would help children learn algebra better. It aims at inspecting if a reversal in the movement of discourse i.e. from algebra to arithmetic would benefit the students or not.

II. 1 The theoretical and the Methodological Paradigm of the Study

The cultural historical theory of Vygotsky provides the theoretical and methodological paradigm for this study. The present study employs the Vygotskian concept, 'Zone of Proximal Development' to understand how learning takes place in the classroom where algebra is introduced through metaphors and how the facilitation of the algebraic construction of students in the intervention class differs from that of the usual classroom approach which uses conventional method to teach algebra.

The Zone of Proximal development:

Lev Semenovich Vygotsky, who died 75 years ago, was concerned about how the human mind develops in a culture. He conceptualized the term, 'Zone of Proximal Development' (ZPD) to locate cognitive development and learning in a social interactive context of knowledge and tensions. He elucidated that the child follows the example of the adult and gradually develops the ability to do certain tasks without any help or assistance. He called the difference between what a child can accomplish with the help of an expert and what s/he can do in the absence of an expert. The Zone of Proximal Development is a metaphor for teaching and learning mediation that occurs between the learner and the more capable peer or adult guide (Cole, 1985). Researchers, who brought Vygotsky's theory into the context of education, used ZPD as a tool to understand how learning takes place in everyday social contexts and how everyday is brought both to create a context (and therefore a scaffold) and tensions in the learning of children.

The role of ZPD is also crucial in transcending the conceptual distance between knowledge that is embedded in everyday context and that which requires more abstract, context free and complex thinking. For Vygotsky, a key concept for the learning within the Zone of Proximal Development is the instruction which is preceded by mature abilities. The more experienced person takes responsibility for structuring the interaction and providing the necessary support until the learner is able to do the task independently (Cobb, 2000). In the classroom activity, teacher takes the role of the expert and helps the children to build concepts academically. According to Vygotsky (1987),

"The development of the scientific concept, a phenomenon that occurs as a part of the educational, constitutes a unique form of systematic co-operation between the teacher and the child. The maturation of the child's higher mental functions occurs in this cooperative process, that is, it occurs through the adult's assistance and participation". (Vygotsky, 1987: 168-169).

In this study, the theoretical concept of ZPD by Vygotsky is adopted to prove that both learning algebra and its conceptual development are a social collaborative activity. In this respect, both teaching and learning would be comprehended by using ZPD. The teaching method grounded in the Vygotkian theoretical concept-ZPD would employ the cultural tools and artifacts of students to prepare metonymy and metaphors of middle school algebra.

'Everyday concept' and 'Scientific concept':

According to Vygotsky, (1987) concept formation should be thought about at two levels, the 'scientific' level and the 'everyday' level. Scientific concept is different from everyday concept in terms of their acquisition. According to Vygotsky, "*everyday concepts are appropriated spontaneously by the child, through the social interaction that occurs in jointly undertaken activities, at his or her culture*" (Vygotsky, 1962). But scientific concept can only be acquired through deliberate and systematic instruction at the school setting (Wells, G. 1994). Vygotsky argues that the development of 'scientific concept' is through 'everyday concepts' which are constructed from culture. Similarly, scientific concepts prepare structural formations for the strengthening of the 'everyday concept' (Vygotsky, 1987). It reveals that both 'everyday' and 'scientific' concepts are appropriated from the culture in the course of specific forms of internal activity.

This understanding emphasizes the importance of linking the school tasks with what the child is already familiar with in the everyday context. Researchers who are remarkably engaged in mathematics education have found the teaching strategy which brings the ‘cultural semiotics’ of the student into the classroom more effective in the case of mathematical instruction. (Fraivilling, J. L et al. 1999). Vygotsky’s notion of concept formation in a play based context enabled the researchers to think out of the box. This exertion tremendously helped the researchers to contemplate on how everyday concepts and scientific concepts can be intertwined within the play based context, and how it can be used for building pedagogical approaches for mathematics education (Fleer, M & Ridgway, A. 2007).

In the present exploratory study, both every-day and scientific concepts are considered to prepare intervention programs of tutelage. Both metonymy and metaphors of middle school algebra were developed from the socio-cultural context of the students. It helps learners to build a bridge between the various ‘textbook algebra concepts’ to everyday concepts. According to Vygotsky’s theoretical concept, this method of teaching brings a child centered classroom.

II. 2 Nature of the Study

The study attempts to understand the effectiveness of new pedagogic tools of teaching algebra, where algebraic concepts and ideas are introduced through carefully designed activities by the researcher using Davydovian approach and the works of Lakoff and Nunez. The discursive tools were partly derived from the study by Panda and Cole (2007). The study also looks forward to discern how students abstract the relationship between unknown numbers and how they conceptualize variability. A qualitative research design was used to conduct the study. Lincoln (2011) identifies appropriateness of methods, strong orientation to everyday experiences of those under investigation, data collection in a natural setting, understanding rather than explaining the case and effects and open formulation of research questions as the characteristics of qualitative research methods. The present study was conceptualized within this paradigm.

II. 3 Method

For this study, the researcher used the multi modal method to collect data. Multimodal method is a qualitative method and is often used in cultural psychological work. In the study, interviews and observations were used for data collection. The researcher employed non-participant observation, interviews and analysis of 'textbook algebra' to collect relevant data for the study. The data were analyzed using content analysis technique.

II.3.1 Sample:

The 6th standard students were selected from an upper primary school in Calicut University, Thenjhippalam, Malappuram (district), Kerala by using purposive random sampling technique. Students were randomly divided into two groups; an experimental group and a control group. In the experimental group, algebraic concepts were introduced by using a new set of materials and pedagogic tools designed by the researcher using Davydovian and Lakoff's approach. On the other hand, algebraic concepts were introduced in the control group by using regular school methods.

II.3.2. Design of the Research

The researcher employed some of the principles of the 'between group design' of experimental method in this study. Various research activities, which depended on the objectives of the research that suited the requirements of different groups, were distributed among experimental groups and control groups. The teaching learning activities for the controlled group were based on the chapter on Algebra of the mathematics text book of class VI, while the activties for the experimental group were developed by the researcher.

i). Activities for Introducing Algebraic Concepts in the Control Group (activities are based on textbook instructions).

Chapter 10. *Akshara Ganitham* (Letter Math)

Addition and Subtraction

Mary is 4 years old while her brother John is 8. How old will Mary be after 2 years? How old will John be after 2 years? How old were they 3 years before?

Fill up the blanks in the table.

Table 4. Relationship between age of Mary and John.

Age of Mary	Age of John
1	
	6
	7
4	8
5	

In this problem, how do we find the age of John from the age of Mary?

We get John's age by adding 4 to the age of Mary.

This can be condensed as

Age of John = Age of Mary + 4

This can be further shortened by denoting the age of Mary by 'm' and the age of John by 'j'.

The relationship can be rewritten. The letter 'm' can be any number like 1, 2, 3 etc. With respect to the age of Mary, the age of John can be 5, 6, 7 etc. respectively

One problem: Many methods

The same problem can be expressed in many ways.

1. John is 4 years elder than Mary
2. Mary is 4 years younger than John.
3. The difference between the ages of John and Mary is 4 years.

When the relations are expressed using letters, many methods like the above can be followed.

If the age of Mary is indicated by 'm' and the age of John by 'j', the above relations become:

1. $j = m + 4$
2. $m = j - 4$
3. $j - m = 4$

In what ways can the relationship between the left and the right angles created on a line by another line be expressed?
Can they be represented using letters?

Another problem:

Look at the figure (Fig. 2)

Fig. 2

What is the relationship between the right and the left angles?

The sum of the right and the left angles is 180 degree. If the angle to the left is denoted by ‘l’ and the angle to the right by ‘r’, how can the relationship be expressed?

$$l + r = 180$$

Take a look at the figure below (Fig. 3a).

It is a 4 sided figure. This can be divided into 2 triangles by drawing a line connecting two of its corners (Fig. 3b).

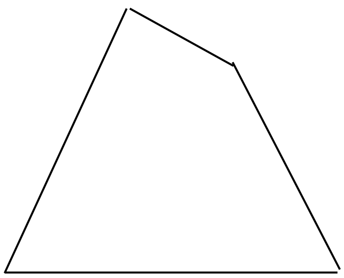


Fig. 3(a)

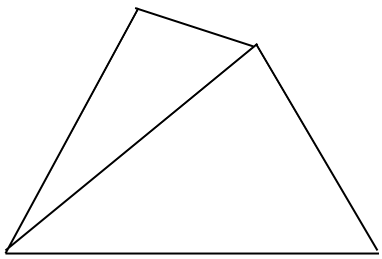


Fig. 3(b)

Consider a 5 sided figure (Fig. 3c). This can be divided into 3 triangles by drawing lines connecting one of its corners to two different corners (Fig. 3d).

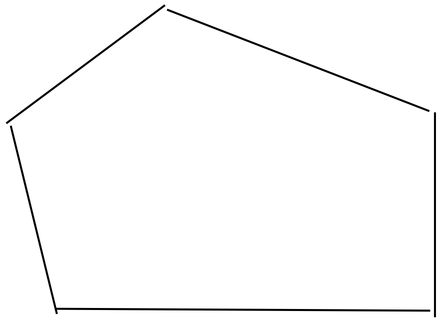


Fig. 3 (c)

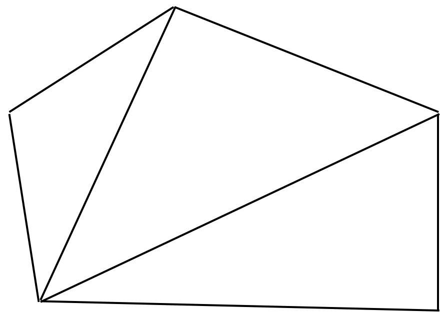


Fig. 3(d)

Now consider a 6 sided figure (Fig. 3e). This can be divided into 4 triangles by drawing lines connecting one of its corners to three different corners (Fig. 3f).

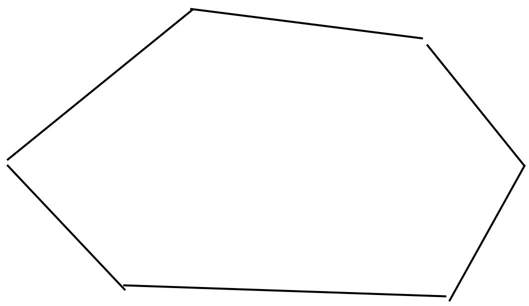


Fig. 3 (e)

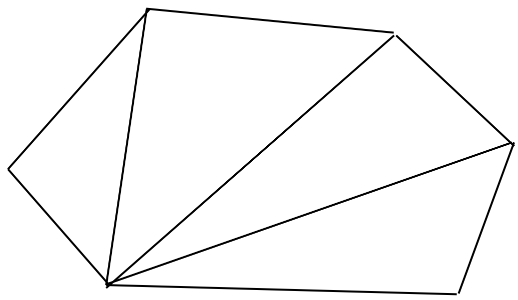
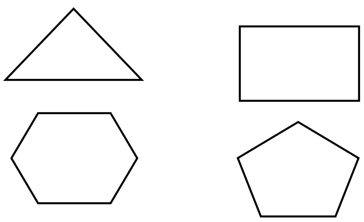


Fig. 3 (f)

Similarly, draw 7 sided and 8 sided figures. Draw lines connecting one corner to other corners and create triangles. Draw a table as shown below.

Regular Polygons

Polygons whose all sides and angles are equal are referred to as regular polygons.



Sides	Lines	Triangles
4	1	2
5	2	3
6	3	4
7		
8		

In a 12 sided figure, how many lines which connect on corner to the rest of the corners can be drawn? How many triangles can be created?

- What is the relationship between the number of sides and the number of lines?
- What is the relationship between the number of sides and the number of triangles?
- What is the relationship between the number of lines and the number of triangles?

If the number of sides is denoted by ‘s’, the number of lines by ‘l’ and the number of triangles b ‘t’, how can the relationship between them be expressed?

Look at what Sneha wrote,

- $s - 3 = l$
- $t + 2 = s$
- $t - l = 1$

Can the above set of relations be expressed in different ways? Try to find out.

***Akshara Gunanam* (Algebraic Multiplication)**

Rani is making triangles out of match sticks (Fig. 4).

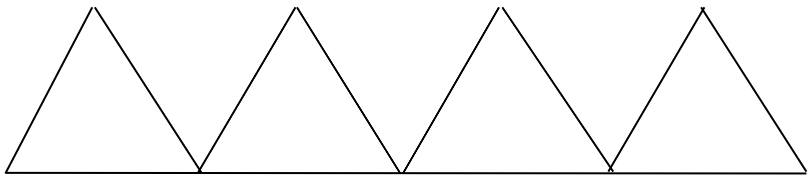


Fig. 4

How many triangles are there in the figure?

How many matchsticks were used to make the triangles?

How can it be calculated?

Is it to be calculated by adding 3+3+3+3 or by multiplying 3 by 4?

How many matchsticks are required to make 10 triangles?

In general, it can be said that the number of matchsticks is thrice the number of triangles.

How can it be represented using letters?

If ‘m’ stands for the number of matchsticks and ‘t’ stands for the number of triangles, what is the relationship between m and t?

$m = 3 \times t$

Generally when letters are used instead of numbers, the multiplication sign is avoided. Therefore the relationship $m=3 \times t$ is written as $m=3t$.

Q. All the students of a class buy pens worth Rs. 5 from a cooperative store. In the table below, enter the total amount of money spent by the students of each class.

Table 5. Relationship between the no. of students and total amount spent for pen

Class	No. Of students	Total Amount
6 A	34	
6 B	32	
6 C	36	

What are the different ways in which the relationship between the number of students and amount spent by them can be expressed? Express the same using letters.

Multiplication and Addition

Ravi has 3 ten rupee notes and 1 one rupee coin. Lissy has 5 ten rupee notes and 1 one rupee coin. How much money does Ravi have? How much money does Lissy have? How can it be calculated?

Likewise, if the number of ten rupee notes is 25 and one rupee coin is 1, what would the total amount of money be?

$(10 \times 25) + 1 = 251$

ii). Activities for the Experimental Group:

Intervention Program

Intervention program for teaching middle school algebra was developed by using both Davidovian and 'Lackoff's approach. Using Davydovian approach five activities were prepared to develop the 'basic Metonymy' for middle school algebra. Number of games were selected for introducing algebraic concepts and a series of activities (by using balance scale) was developed using both Davydovian principles and the approaches of Lackoff and Nunez.

A) Metaphor for middle school algebra.

Activity 1: Game -*Kothan Kallu*.

It is a game played using five similar stones. There are five steps in the game. For the successful completion of a game, a group would receive a maximum of five points. A person who is unable to complete the game in five steps will have to start from the beginning in his/her next chance.

The first step of the game, *Orukka* demands the player to place all the five stones on the floor and choose one out of them without disturbing the other four stones. The selected stone should be thrown up. By the time the stone comes down, the player has to get hold of any of the four remaining stones. This has to be repeated four times for the completion of the first step. If the person is unable to collect one stone from the ground by the time the first stone comes down, he fails and would be out. The next player gets the chance.

Second step, *Irukka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of two stones. And then the remaining two stones are also collected in the same fashion. If the player is unable to pick two stones by the time the first stone comes down, the player would fail. The next player gets the chance.

Third step, *Mukka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of three stones. The stone that is about to fall down has to be caught as well. The stone has to be thrown up again and by the time it comes down the remaining stone has to be collected. If the player is unable to pick two stones by the time the first stone comes down, the player would fail. The next player gets the chance.

Fourth step, *Nakka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of all the four stones.

Fifth step, *Thondi*, *Thappu Thalam Melam*: The player holds all the five stones inside the hand and throws one stone up into the air. By the time the stone comes down the player has to drop all the four stones on the floor and then catch the first stone inside his hand. This stone is thrown up again and the four stones are collected again. All the five stones have to be held inside the hands of the player. The player says *thappu thalam melam* aloud and shakes the stones in his hands and then throws all the five stones into the air and tries to collect them all in his/her outer hands. The stones are thrown up again and are to be held inside the palm. At the end, the player gets a point which equals the number of stones remaining in his hand. If the player fails to catch even one stone in his outer hand, he fails and will receive no points at all. The next person gets the chance. If he is able to catch at least one stone, the player can continue. The team which manages to reach the score that is fixed before the commencement of the game wins.

Activity 2: Game-*Nootamkolu.*

One long and ten short coconut leaflet midribs (*eerkilu*) are required. All the 10 short midribs should be of equal length. All the 11 midribs are to be dropped in the floor in such a manner that there should be at least one short midrib over the long one. Now, the player has to collect them one by one without disturbing the others. If any of the other midriffs is disturbed, the player loses. The player would get 10 points for the short ones and 100 points for the long one. The maximum score that can be attained is 200. A target score would be fixed beforehand and the team which meets the target first wins. The students got ready to play after the researcher completed explaining the rules of the games.

Researcher had used these games for formulating the equations by using the relationship between the number of games and the target point. By using these games, the researcher introduced the concept of unknown numbers and its calculation.

B) Activities for Developing Basic Metonymy of Middle School Algebra:

Activity 1: The researcher instructed the students to compare different things using length, volume, weight, composition or quantity. '*Manchadikkuru*[11]', '*panam kuru*[12]', '*cherupayar*[13]', '*Manal*[14]' and *'Chiratta*[15]*'* were used to compare things in terms of quantity.*'* and*'* were used to compare things according to both weight and size. Matching of things was done through two operations. These are: a) selecting the 'same things' according to a given parameter from the set and b) making groups on the basis of same things. [Assumption: This activity helps students to compare different things on the basis of length, volume, weight, composition or quantity]

Activity 2: In the second activity, students were asked to compare things such as '*manjadikkuru', 'panankuru, cherupayar and chiratta* according to the given parameters. They were encouraged and assisted to represent these comparisons using different signs of algebra such as <, > and =.

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[11] Red lucky seeds of *Adanthera Pavonia*. It is used as a small measure of weight

[12] Palm tree seed. It is used for different games.

[13] Green gram.

[14] Grain of sand. Granulated particle of the sand.

[15] Coconut shell- It is the hard shell of coconut. Often it is used as a measuring tool among adults and as playing tool among children.

Activity 3: In this activity, students were instructed to record the comparison of things in Malayalam alphabets. For example collection of '*manjadikkuru'* is represented by letters 'Ma', 'M' or 'm' and quantity of *'panamkuru'* by letter 'Pa', 'P' or 'p'. Comparison of both 'Ma' and 'p' can be expressed through Ma=p, Ma<p or Ma>p. Later, researcher asked the students to replace the letters by using different numbers and those were mathematically expressed as 1000 = 1000, 1000 < 2000 or 2000 > 1000 [Assumption: This activity helps the students in handling both symbols and algebraic variables contextually].

Activity 4: In this operation, the concept of addition in the context of letters was introduced. Students were asked to balance two different things with adding an adequate parameter. For example; we know from activity 2, Ma = Pa. If two hands '*Manjadikuru*' are added to 'Ma' then the quantity of 'Ma' would become greater than 'Pa'. ie. Ma+2 '*Manjadikuru'* > Pa. Later on, the researcher asked the students to cogitate on what should be done to equalize the quantity of Pa to Ma+2. The solution is the addition of 2 hands '*Manjadikuru'* to the quantity of 'Pa'. Therefore Ma+2 *Manjadikuru* = Pa+2 *Panamkuru*. Later pure numbers were used instead of objects. [Through this activity, the students can understand the idea of disturbance in equality and the preservation of it through addition].

Activity 5: In this activity, the concept of subtraction in the context of letters was introduced. Researcher asked the students to balance two things with different quantities by adding or subtracting an adequate quantity. For example; we know Ma<p from the activity 3. If the quantity 'Ma' is less than 'p' by two *chiratta manjadi* , then two '*chiratta manjadi'* can be added to 'Ma' for equalizing it to 'p'. i.e. Ma+2 '*chiratta manjadi'* = p. After mathematically expressing the same by using real numbers, students were taught that by subtracting 2 from 'p', the quantity would be equal to the quantity of 'Ma'. Then the expression of the earlier equation would change to Ma = p-2. Therefore, it can be stated that the difference in value between 'Ma' and 'p' is 2. 2 = Ma-p or Ma-p = 2. [This activity helps the students to relate the concept of subtraction with the concept of addition and helps them understand reduction or subtraction as another way to equalize relations].

Activity 6: Balance scale

Researcher used balance scale for equalizing the weight of *manjadikkuru, cherupayar* and *panam kuru* through this activity, the researcher introduced addition, subtraction and the equal to sign to the students.

For example: The researcher emptied the coconut shell full of green gram into one plate of the balance and then added a handful of red lucky seeds into the other. The plate with the green gram weighed greater than other. The researcher added another handful of red lucky seeds into the plate and asked the students to explain what the researcher had done. The plate with the green gram weighed greater than other.

II.3.3 Data Collection Techniques:

1) Interview:

Semi-structured Interview

“Semi-structured interview is a qualitative data collection strategy, in which, the researcher asks the informants a series of predetermined but open ended questions” (Mishler, 1991). It is a most widely used method of data collection in qualitative research in psychology, partly because the interview data compatible with several methods of data analysis (Lincoln, 2011). Because of the degree of the structure in this interview format, the resulting text is a collaboration of investigator and informant (Mishler, 1991). The semi structured interview provides an opportunity for the researcher to know how the participants’ think and talk about a particular aspect of their life experience. The question asked by the researcher functions as a trigger that encourages the participant to talk (Lincoln, 2011). Semi-structured interviews heavily depend on the rapport established between the interviewer and interviewee.

In the present study, the researcher used semi-structured interview as a data collection technique. This method was used to understand the necessary information, which cannot be attained by the classroom observation data.

Interview with teachers and students were followed as one of the techniques to collect data for the study.

Interview with mathematics teachers

i) Algebraic concept formation of the student

ii) The engagement of students in the algebraic problems: a comparative analysis between the students in both experiment and control group.

2) **Classroom Observation:** Non participant observation was used to collect data from the classroom activities and their engagements outside the classroom. Classroom activities were video recorded by the researcher.

Observation is a qualitative data collection strategy and it is especially appropriate for exploratory and descriptive studies, especially the ones that aim at the generation of interpretative texts. The methodology of observation seeks to uncover, make accessible and reveal the meanings (realities) people use to make sense out of their daily lives (Jorgensens, 1989). Observation involves i) the observer's immersion in the situation; ii) systematic, but unstructured observation, and iii) detailed recording of observations, generally from memory.

In this study the researcher used observation as a method to collect data. Rationale to select this method is to engage in naturalistic studies. Classroom observation was used for data collection. Non participant observation was used to collect data from the classroom activities in both control and experimental group. Classroom activities were video recorded by the researcher. Following activities were focused during the observation.

Classroom observation

1. Teaching style and method for introducing algebra

2. The interaction between learners and teachers

3. Behavioral pattern of the children such as gestures, facial expression, movements, and body language in general was observed.

4. Activities of the students within the classroom

5. The presence of mind of the students in learning algebraic concepts.

6. Engagement of the students within the classroom

II.3.4 Procedure for Data Collection:

Procedure for the collection of data was done in two stages. First stage involved three stages and second stage involved four steps. Both the first stage and the second stage were conducted among 6th standard students in the Calicut University Government School, Thenjippalam. Stage one was conducted among the controlled group and activities of the second stage was conducted among the experimental group.

Stage I: This stage was focused on the control group. Here, classroom activities in algebra and the hardships students face in understanding algebra resulting in poor performance were observed. Experience of students from the conventional algebra class was collected by means of interview. All these methods of data collection were executed through the following steps.

Step 1 – In activity one, chapter on algebra was introduced by the teacher. Teaching method and classroom activities were observed and video recorded by the researcher. Teaching style, the approach to doubts and performance of the students were observed from this class.

Step 2 - Mathematics teachers in the 6th standard were interviewed to understand the perception of the teachers on the performance of the students and their difficulties in the control group.

Step 3 - Selected students were interviewed to know the difficulties in dealing algebra mathematics and to understand engagements outside the classroom.

Stage II: In this stage, engagement of students with algebraic concepts, conceptions of teachers on the performance of students in algebra, the difficulties of students in performing algebra mathematics and everyday activities and experience of students are examined through two different steps.

Step 1 – In the first activity, algebraic concepts were introduced by the researcher to the experimental group through metaphors and metonymy of middle school algebra. Classroom activities were recorded by using video tape. The performance of the students was observed in general and the randomly selected students for the study were observed in particular.

Step 2 - In this activity, students were asked questions to explore their experience with algebra from the classes and to understand about the engagements of algebra outside the classroom.

II.3.5 Analysis of the Data

The content analysis technique was used for analyzing the data. This technique involved coding, categorizing (creating meaningful categories into which units of analysis can be placed), comparing (categorizing and establishing links between them), and concluding (Cohen, Manion Morrison, 2007). This technique helps to investigate and collect the data that involves the lives of individuals.

In this study both video and audio data were analyzed by using content analysis, and the performance of the students in dealing with problems on algebra was analysed by using error analysis. Following steps were done in the analysis part.

Step 1: Transcription of both video and audio data.

Step 2: Transcribed data was analysed by using content analysis.

Step 3: Performance and difficulties in algebraic concepts of selected students in both experimental and control group were analysed by using error analysis.

Step 4: Experimental and controlled group were compared by using various themes formed from the content analysis.

RESULT AND ANALYSIS

This chapter focuses on the necessary data and its analysis to find out the answers for the research questions that were raised in the first part of the research. This chapter is divided mainly into two parts. The former is the Result part and the latter, the Analysis part.

Result and Analysis

The Result part of the chapter deals with the data collected from the selected field of study. The resources of data include the class room teachings on the tenth chapter in their Maths textbook on Letter Maths; delivered to the students of Std. VI, Calicut University Government school, an interview in which Maths teachers who teach in Standard VI and Standard VII in the same school share their experiences and also a Teacher's Textbook which the teachers rely on for effective teaching. Towards the end, the response of students who attended a class which aimed at a different rendition of Algebra that they were familiar with through their textbook is also included in the results.

The Analysis part of the chapter attempts to analyse the data collected by the researcher through field work in order to find out the answers for the research questions which the researcher had raised. Research questions are analysed in the order in which they were presented in the initial part of the research.

Part I.

III. 1 Classroom Teaching of Letter Maths in Std.VI, Government School, Calicut University.

"We are going to learn a new chapter in today's class. The name of the chapter is Letter Math '*Aksharaganitham*' Everyone should pay attention and try to answer correctly." Having said so, the teacher began the class. In order to get going, the teacher enquired if every student remembered how to add and subtract and gave them a problem to solve. It was based on the relationship between the ages of two characters in the text book; Mary and John. If Mary and John were 4 and 8 years old respectively, how old will they be 2 years from now? The teacher, after having read out the question from the text book, looked at the students and repeated the question again. A number of answers were heard spoken out loud by the students but the teacher focused on just two answers: 6 and 10. She continued to question the students as to how they got the above answers. Most of the students replied that they added 2 to the ages of Mary and John. The teacher again asked them why they added 2 and not any other number. The students replied that the teacher had herself said so. She had asked them to find out the ages of Mary and John after two years. They also added that there existed a relationship between the number of years and age.

Next, the teacher went on to explain the relationship between the age of Mary and John. She asked them if the age of John was greater than that of Mary. The students nodded in agreement. She wanted to know by how many years John was older than Mary. After considering the response from the students, the teacher said, "John is four years elder than Mary. Isn't it? In other words, if we subtract 4 from John's age, we get the age of Mary, right?" The students agreed. Thereafter the teacher expressed the relation between the ages of Mary and John in the form of an equation on the black board.

Age of John = Age of Mary + 4

Age of Mary = Age of John – 4

In order to simplify the equation further, the teacher instructed the students to use the alphabet 'j' to denote John and 'm' to denote Mary where 'j' and 'm' stands for the initial letter of their respective names. She then modified the equations.

The first relation,

Age of John= Age of Mary + 4, changes to

$j = m + 4$

and the other relation,

Age of Mary= Age of John – 4, changes to

m = j – 4

Picture 1: Students listen and write the equation of the problem of John and Mary

Instead of reading out the relationship in terms of variables, as in j=m+4, the teacher insisted that the students read out the same using names.

After making sure that all the students understood the above equations, the teacher presented another example.

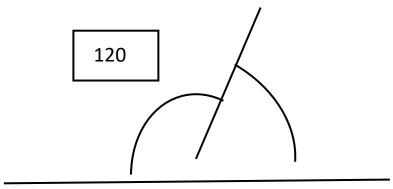


Fig. 5

The teacher drew a straight line on the board and another straight line which intersected the first. The second line created two angles on the first (Fig. 5). She suggested that the sum total of the two angles created by the intersecting line on the straight line is 180 degree. If the angle to the left is 120 degree, she asked the students to find out the measurement of the angle to the right. She divided the students into various groups and gave them time to find out the answer. She took care to see it that the students got enough time to discuss. “What is the measurement of the angle to the right?” she asked the students at the end of the discussion. “60 degree”, replied the students. The teacher congratulated them for finding out the right answer. She asked them to explain the relationship between the angle on the left and the right. Seeing students silent for a moment, the teacher asked them how they came up with the above answer. They replied that they subtracted 120 from 180. The teacher wanted to know what the numerals 120 and 180 refer to. She repeated the question to the majority of silent students and the other few who were attempting to answer the question.

Few responses were heard and the teacher repeated a correct answer that she happens to hear among the many answers. “Yes. The measurement of the angle to the left is 120 degree.” said the teacher to the students. “How about 180?” she asked again and answered to her own question. “180 degree is the sum total of the angle on the left and the angle on the right. Isn’t it?” The students who were hitherto answering in soft voices marked their agreement in loud voice. “If the sum of the angles on the left and right is 180, what is the relationship between the two angles?” The teacher continued. A student named, Alia Sultania suggested that if the measurements of the two angles were added, the sum would be 180. The teacher did not consider Alia’s response as the common idea of the group she belonged to. Instead she asked the whole class if they understood what Alia meant. The students nodded. The teacher expressed the above relation on the black board.

Left angle + Right angle = 180

She wanted to know if the above relation can be expressed using alphabets as in the earlier example of Mary and John. “Yes” replied the students. The teacher wanted to know how. The students formulated the below equation,

$i + v = 180$,

where ‘i’ stands for the Malayalam word ‘*Idathu*’ for Left and ‘v’ for the Malayalam word *‘Valathu’* meaning right.

The teacher insisted they use the English terms for the same and rewrote the equation as

$l + r = 180$

The teacher gave them a little time to ponder over the new equation and asked them if they understood the same. The students replied that they were clear. The teacher asked the next question. She wanted to know how to find out the measurement of the angle to the right and also how the students had earlier answered that the right angle measured 60 degree. Salman Faris, Alia Sultana and Dilshad replied that they subtracted 120 from 180. “If we subtract the measurement of the left angle from 180, we get the measurement of the right angle. Isn’t it so?” the teacher asked, after making modifications to the three answers she received. Then she wrote the same on the board and asked the students to write it down. After the students completed writing, the teacher posed the next question. “Can we express the same using alphabets like the earlier example?” This time, the students used the alphabets ‘l’ and ‘r’ instead of ‘i’ and ‘v’.

$r = 180 - l$

The students wrote in their notebooks and read it aloud.

If $r = 180 - l$, the teacher asked the students to formulate the equation to find out the left angle. Majority of the students replied that it can be found out by subtracting ‘r’ from 180. The teacher then wrote the equation of the same on the board.

‘l = 180 – r’

After having written so, the teacher asks if the above formula is correct or not. The students replied that the formula was correct. At the end of the class the teacher gave the students a few problems to solve from home.

In the following class, the teacher gave a brief summary of the lessons she had discussed in the former class and added that she would continue from the point she had stopped. She said that she was about to begin the new Maths lesson and drew a straight line on the board. She asked the students to name the thing she had drawn and the students replied that it was a line. She replied that the answer was correct and there after converted the line into a square and again asked the students to name the figure. The students replied that the new figure was a square. She congratulated the students. She asked the students if they knew how many sides a square had. All index fingers began to be pointed to the board as they counted the number of sides of the square and together shouted that the answer was four. The teacher repeated their answer and said that he square had four sides. However she added that she was going to add an extra side to the figure and drew a diagonal inside the figure, a straight line from one corner of the square to the other (Fig. 6).

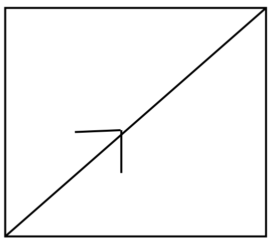


Fig. 6

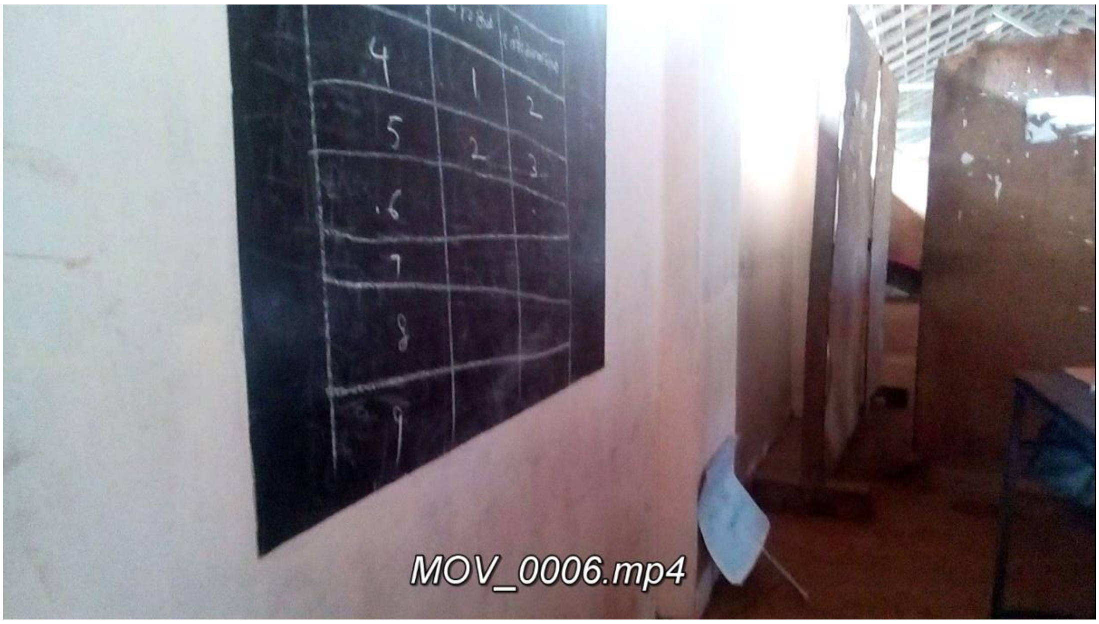
The students gazed at the diagram and the teacher asked them if they knew what shape the square was transformed into. They replied that the square has changed into triangles. “How many triangles?” the teacher continued. “Two” was the answer. The teacher praised the students and said that both their answers were correct and summed up the activity in a sentence.

“When a line was drawn inside a square which has four sides, it transformed into two triangles”. The teacher then presented before them another figure which had five sides (Fig. 7). She drew a five sided figure on the board and instructed the students to draw the same in their notebooks.

Fig. 7Picture 2: Students write the relationship between lines, sides and triangles.

The teacher asks them to count the number of lines she had drawn inside a five sided figure. The students replied that the teacher had drawn two lines inside it. The teacher asks them to find out the number of triangles which the two lines that were drawn from the two ends of the same side toward a common point created. The students replied that the lines created three triangles. If so, the teacher wanted to know how many triangles she would be able to create if she drew one more line in the figure. However his times she did not bother to diagrammatically represent the same. The students unanimously replied that she would create four triangles. The teacher again complimented the students.

Afterwards the teacher said that the rest of the activity would be carried out among groups and students were divided into groups based on their seating arrangements. Students sitting in two adjacent benches joined to form a group. The teacher then drew three columns on the board. The first column was titled ‘sides’; the second, ‘lines’, and the third, ‘triangles’.



Picture 3: Activity table drawn by teacher in the control group classroom

Table 6: Relationship between sides, lines and triangles.

Sides	Lines	Triangles
4	1	2
5	2	3
6	3	4
7	4	5
8	5	6
9	6	7

The first set of data that was entered was 4 in the first column, 1 in the second and 2 in the third. There after any one column would be left blank for the students to fill in. It might be one of the three columns. The duty of the groups was to find out the answers for the missing columns and complete the table. Once all the groups complete their task, students would be called out in a random fashion to fill in the table on the board and then explain the relationship between the number of sides, lines and triangles. With the help of the students, the teacher completed the table. None of the students who were chosen by the teacher entered a wrong answer in the table.

Picture 4: Table filled by the responses of selected students.

In the following class, the teacher gave a quick revision of the lessons she had taught in the class before. She told them to have another look at the table they had made. The teacher posed a new question.

If the number of sides of a figure is 12, how many lines and how many triangles can there possibly be in the figure. The class turned silent as the teacher repeated the question again. But there came no responses from the students. The teacher asked the students to take a closer look at the table and try to find out the answer. After studying the table for some time, few students (Alia, Saja and Salman) answered that if the number of sides is 12, the number of lines would be 9 and the number of triangles would be 10.

Picture 5: Teacher introduces mathematical operation based on triangle problems.

The teacher asked the opinion of the rest of the class. While a few continued to keep quiet, most of the students said that the answer was correct. The teacher asked the three to explain the method through which they got the answer. It was Faris who replied. He said that since there were already entries from 4 to 9 in the table, the answer for 12 was obtained by adding one to the successive entries from 9 to 12. The teacher was searching for other methods. Only Saja tried to answer. She was uncertain and therefore spoke softly. The teacher encouraged her to speak louder. She said that the number of lines can be obtained by subtracting 3 from the total number of sides and the number of triangles by subtracting 2 from the number of sides. The teacher was happy.

“Did everyone hear what Saja said? She said the right answer!” said the teacher. The teacher repeated Saja’s answer and added that the relationship between the number of sides, lines and triangles was the same. By subtracting 3 from the total number of sides, we get the number of lines and by subtracting 2 from the number of sides; we get the number of triangles. For further understanding, she gave a new problem to solve. The students were asked to find out number of the sides and triangles of a 15 sided figure.

The students wrote down the question in their notebook and tried to solve it. Some students peeped into their neighbour's book while a few discussed it among their friends sitting next to them. "Have you finished?" asked the teacher after around five minutes. Saja, Misbah, Alia Sultania, Faris and Misiria nodded. The teacher asked the others if they wanted more time. They responded that they would complete in a few minutes and after having completed informs the same to their teacher. The teacher looks at both the girls and boys and asks them to answer louder. Even now, it was Saja, Misbah, Alia Sultania, Faris and Misiria who showed enthusiasm to answer loud. But what marked the class response different in this instance was that, along with the loud voices of a few, there were also voices from the hitherto silent. The teacher asks them to explain the method through which they arrived at the answer. Most of the students replied that they followed the same method they had employed in the previous problem. They subtracted 3 from the number of sides to get the number of lines and subtracted 2 from the same to get the number of triangles. Though every student could not keep up to the pace of answering, most students participated in the process. Having said that the answer was correct, the teacher ended the day's class.

The following day, teacher greets the students and begins the class. She asked if she had assigned any works for the day or not. The students replied that they were not given any homework. She then asked if they thoroughly understood the relationship between the number of sides, lines and triangles. The students replied that they did. Students like Sachin, Sanal, Appu and Sunitha merely joined the others. The teacher divided the students into groups and wrote three questions on the board.

1. What is the relationship between the number of sides and number of lines?
2. What is the relationship between the number of sides and number of triangles?
3. What is the relationship between the number of lines and number of triangles?

The teachers asked the students to copy down the questions in their notebooks and try to answer them with the help of the problems they had solved in the previous class. The answers were to be written in the form of a sentence. Later the teacher encouraged them to rewrite the same using alphabets like they had done before. She explained how to do so and instructed the groups to formulate the same. The voices of the students who were absorbed in discussion are heard loud in the class while the teacher walked around the class. After some time, she asked the students if they had completed. She selected random students from various groups and told them to read out their answers. All the groups had got their answers right. The students replied that the number of lines would be 3 less than number of sides, the number of triangles would be 2 less than the number of lines and the number of triangles would be 1 greater than the number of lines.

The alphabets used by the different groups were distinct from one another. However there were similar trends among them. Most of the groups denoted the number of sides using '**v**'. '**v**' was used again to denote the number of lines. The letter '**t**' was used to denote the number of triangles. The teacher asked the reason for their choice of letters. The students replied that the first '**v**' comes from the Malayalam word "*vasam*" for sides, the second **'v'** from the word *"vara*" meaning lines and the third **'t'** from the word "*thrikonam*" for triangle. Another group (Alia, Faris and Saja) used **'s'** to denote the number of sides, 'v' for the number of lines and **'t'** for the number of triangles. The teacher was curious to know the reason for their different choice. It was Alia who replied. She suggested that they used the letter **'s'** to denote sides because the word 'side' was the English equivalent of the Malayalam term "*vasam*". The teacher smiled and asked the group why they did not consider using all terms in English. The class fell silent. The teacher assured the students that whatever they had written are right. However, in Letter Maths we employ English letters. She added that therefore it was advisable to use English words and their initial letters.

The teacher gave the English word for the terms in the problem. She said that sides denote "*vasam*", lines denote "*vara*" and triangles denote "*thrikonam*".

“Now rewrite your equations usings, l and t” said the teacher as she opened her text book.

Taking a look into the text book, the teacher wrote the equations listed below on the board.

$s - 3 = l$

$t + 2 = 5$

$t - l = 1$

The teacher asked if all the students had written the same. Some students nodded while the others copied the same into their notebooks.

In the next class, the teacher opened the text book and told the students that they would learn a new method. She instructed the students to open their texts books and look at a diagram. “Rani is making triangles out of match sticks” read the teacher. She explained to the students that a girl named Rani was making triangles. She made use of match sticks to do the same. The figure in the textbook is the triangle Rani made. “How many match sticks did Rani use to make a triangle?” asked the teacher. The students counted the number of match sticks in the figure and replied that Rani used 12 match sticks. The teacher wanted to know the minimum number of match sticks required to make a triangle.

Picture 6: Formation of the triangle using match sticks

Picture 7: Students listen to their teacher's lecture on 'letter-maths'

The students replied that three match sticks were necessary to make a triangle. The teacher asked the students to count the number of triangles in the figure. The students said that there were four triangles in total. The teacher represented the total number of match sticks used in the following fashion, on the board.

3+3+3+3 = 12

She then modified it to 3 x 4 = 12 and asked the students if there was anything wrong in doing so. Only a few students responded. The teacher repeated her question again louder and all students responded that the teacher was correct.

"In this case, the number of number of match sticks is three fold the number of triangles. That is why we multiplied the number of triangles by 3" said the teacher to the whole class.

"How can we represent the relationship between the number of matchsticks and triangles with the help of Letter Maths we have learned?" the teacher continued. She then added that she would use 'm' to denote the number of match sticks and 't' to denote the number of triangles. "How did we find out the total number of matchsticks?" asked the teacher. "We multiplied", a student replied.

“We did multiplication, didn’t we?” the teacher kept asking. While the students made various sounds in agreement, the teacher wrote the below formula on the board.

m = 3 x t

The teacher added that ‘3 x t’ can also be expressed as 3t. In order to make sure that the students have understood, the teacher gave another problem.

If the number of triangles is 10, how many matchsticks would be required?

Only a few students responded to the question. They said that the answer was 30.

“Now, you have to fill in the blanks of the table by yourselves.” Said the teacher and drew the following table.

Table 7: Number of students, classes and total amount of spent for pen

Class	No. Of Students	Total Cost
6 A	34	
6 B	32	
6 C	36	
6 D	30	

Students of a class buy pens worth Rs.5 from the school cooperative store. The students are expected to enter the total amount of money spent by each class in the third column. The teacher winds up the class instructing the students to complete the table before the next in the afternoon.

The teacher began the afternoon class asking the students if they had completed the table or not. The students replied that they had finished doing the work and took out their notebooks. The teacher asked select students to read out the answers and completed the table.

170, 160, 180 and 150 were the answers given by the students. The teacher asked the students to explain the method by which they got the answer. The students responded that they multiplied the total number of students in a class with the cost of a pen, i.e. Rs. 5. Even though all the students had got the answers right, not many responded to the teacher's question. The teacher told the students who had answered to convert the same into Letter math. The students took a few minutes and completed doing so. The letters they chose were different from one another. The teacher referred the text book and in order to bring uniformity insisted that the students use the variables in the text, i.e. 's' for the number of students, 'r' for the rate of a pen and 't' for the total amount of money.

$t = s \times r$

The teacher wrote on the board and the students copied it into their notebooks.

Picture 8: Students copy down the equations written by the teacher on the blackboard.

Thereafter the teacher asked the students if they have seen ten rupee notes and five rupee coins. All the students nodded. The teacher then went on ask another set of questions based on the information. The teacher asked them to find out the total amount of money a person possesses if s/he has 8 ten rupee notes and 4 five rupee coins. The students after little calculation replied that the person has Rs. 100 in hand. The teacher wanted to know how. Most students responded to the question. They said that they multiplied 8 by 10 and 4 by 5. Then they added the two figures and the final answer was 100. The teacher kept changing the number of notes in following problems. The students got all the answers correct. Later, the teacher asked the students to formulate an equation for the same using Letter Maths. She did not divide the students into groups. All students had to solve the problem on their own. The students who had expressed the relation in the form of a sentence could not formulate the same in terms of algebra with ease.

After they had completed, the teacher noticed that the students had used different alphabets. Therefore she asked them to follow common notations. She instructed them to denote the number of ten rupee notes using ‘t’, number of five rupee notes using ‘f’ and the total amount using ‘a’. The teacher observed that though the students know how to find out the correct answer, converting the same into algebra becomes difficult for most of them. She added that since the chapter titled ‘Letter math’ relies on English language, it was important that the students learn the English words of the objects that are employed in the problems. The teacher added that students would find ‘Letter Maths’ easy only if they know the English words of articles. Only then will they be able score good grades. Thereafter the teacher brought an end to the class as well as the chapter.

III. 2 Interview with Maths teachers

Maths teachers of Government School of Calicut University were interviewed by the researcher. In the interview, the general problems in teaching Maths as a subject as well as the particular problems that teachers come across while dealing with Letter math in the syllabus were discussed. As a teacher of Maths in the 6th and 7th grades, Ms. Shiby expressed her opinion that Maths is the most difficult subject to be taught in school. The reforms brought about in the text books or in the general syllabi do little help in reducing the difficulty of the students in learning the subject.

However the changes that have happened over the past five years cannot be neglected completely. Even then, the burden on the teacher is still heavy. There is lot more to teach than that can be completed in a single class. As a result of which most teachers are unable to follow the detailed method prescribed in the teaching manual, especially during the year ends. By the end of an academic year, the teacher would be deprived of the adequate time required to carry out the classes as laid out in the manual. That kind of a teaching happens mainly during the beginning of the year as there would be sufficient time available. Time depletes as the year comes to a close. It is in most cases the final chapters of a text that receives minimal attention. The chapter on like Letter math in the 6^{th} grade is one such kind.

As the same is introduced for the first time in sixth standard, it happens to be an area that should be given adequate attention. Even teachers themselves find it difficult to teach, says Ms. Shiby. Therefore they often teach the same in a rather vague fashion. Students do not attempt Letter math problems during exams. Even in the examination hall, it is noticed that the teachers have to give certain clues to the students to help them attempt questions on Letter Math. Due to such experiences, even teachers tend to believe that teaching algebra is futile. Teachers have a cluster meeting once in a year. The repeated complaints which maths teachers raise in the meeting are the difficulty to teach Letter Maths as well as the lack of time. It is during such meetings that the math-teachers at Government school, Calicut university, realize that this problem is universal. “Many cluster meetings have gone by but the difficulty associated with Letter Math still remain”, concluded Ms. Shiby.

“Issues on Letter Maths like the above are usually dismissed without further consideration in the cluster meetings.” says Ms. Maria. In Ms. Maria’s opinion, Letter Maths is more difficult for the students of Malayalam medium schools compared to the students of English Medium schools. The major problem which students encounter is that after solving a question from the text, they are able to solve only those questions which are similar to the textual problems. Questions different from the format of the text becomes difficult. There is a lack of clarity and students are doubtful of the lessons taught even in the previous classes.

Ms.Shiby observes that since the teachers are used to the situation, they develop little hope in the performance of the students. Even though the students speak out the answer loud in the form of sentences in the class, they find it difficult to express the same in terms of alphabets. Moreover Letter Maths hosts various kinds of problems ranging from the usual numeral to geometry and other mixed type which leaves the students confused. Ms. Maria added that the above was her personal opinion. “This is my personal opinion and the higher officials might not agree to this” said Ms. Maria with a smile. Yet another problem which teachers face is the difficulty to remember what has been taught and what the students have learned. This is mainly because of the frequent change in textbooks. By the time a teacher gets accustomed to a text book, it changes. A problem which usually aggravates with change in text books is that the volume of the things to be taught in a single class exceeds the optimum level. Teaching becomes strenuous.

“Usually teachers in the upper primary level are not specialised in the subjects they teach. This becomes a problem as Letter Maths turns into an extremely difficult area to be taught for teachers who have only a TTC” observed Ms. Maria who also added that she completed her MSc. Mathematics only after she joined the school.

As a response to Ms. Maria’s observation, Ms. Shiby says “It does not imply that the teachers are ignorant. But there are students who do not seem to understand no matter how many times we repeat the same. Majority of the students in our school are from Muslim and Tribal backgrounds. Most of them learn only things that appeal to them.” They shared the common opinion that even though the steps to be taken in teaching a chapter is enlisted in the teacher’s textbook, they are ineffective. The basic difficulties of the students remain. If the students are not taught well in the 6th grade, the teacher in the 7th grade has to suffer. Teachers who teach in both the grades understand the difficulty. Others do not seem to care. Rushing through the chapter leads to ineffective teaching worsening the problem. “New methods have to be introduced. Else the difficulty would persist. We are looking forward to it in the textbooks yet to come.” Concluded Ms. Shiby.

III. 3 Teacher's textbook

This unit contains the basics of algebra. The students have already learned few relationships between measurements in the previous classes. The area of a square is the product of its length and breadth. The volume of a cube is the product of its length, breadth and height. The perimeter of a square is twice the sum of its length and breadth. The perimeter of a triangle is the sum total of its sides. The students have learnt the above relations. In all the cases above, the magnitude might change but the relationship between the dimensions remain constant? Algebra works on this consistency. Algebra does not merely represent relationships using alphabets. It generalises relationships and with the help of alphabets and converts it into the language of maths. This unit focuses on the basics of the same. The unit gives emphasis on the generalisation of the relationship between dimensions and the conversion of the same into their algebraic forms. This enables the students to identify important elements and their features in order to create conclusions. The students get the chance to interpret and analyse relations expressed in terms of alphabets in different manners. It helps improve the students develop their communication skills as they try to explain the algebraic relationships in their mother tongue. The unit also provides scope for developing problem solving skills. The scope of algebra is also explored in the unit. The unit on algebra which begins as a relationship between dimensions evolves to analyse the logic between numerical relationships in Std. VII and thereafter to the solution of equations in Std. VIII. As a unit which introduces algebra, this is very important.

Table 8: Instructions of the teachers' textbook for introducing algebraic concepts.

Concepts	Method of instruction	Results
• The relationship between dimensions and magnitude can be expressed in different ways • Common relationships can be identified between different magnitudes and dimensions. • The relationship between dimensions and magnitude can be expressed using alphabets. • When problems are expressed using alphabets, there are some generally used methods. • Relationships that are expressed using alphabets can be interpreted in many ways.	• The age of the students, along with their problems related to geometry, figures and magnitudes are analysed and relationships between numbers and dimensions are formed. General ideas are developed and expressed in their mother tongue. • Ideas that arise out of practical situations can be expressed using alphabets and studied. • The relationships that arise out of practical situations can be analysed after being represented in their algebraic forms and interpreted and proved right.	• Assumptions on the relationship between dimensions and numbers can be formed and then interpreted. • The assumptions can be explained in one's language with clarity. • Magnitude /dimensions and their counts can be expressed using letters. • Relationship that is expressed using alphabets can be analysed and interpreted

Unit analysis **Time: 12 Periods**

Addition and Subtraction

This activity aims to express the relation between the ages of two people using letters in different ways. It discusses relationship expressed using alphabets. The relation developed should also employ the equal to sign and then analysed.

The teacher introduces the problems in the text book. The answers to the same have to be found out and written down.

- What is the age of Mary and John after 2 years?
- How old were they three years back?
- How old will John be when Mary is 40?

The table has to be completed by the students by themselves.

It has to be discussed in the class.

How did they find out the ages of Mary and John?

Points to be discussed:

- John is four years elder than Mary. In other words, if we subtract 4 from the age of John, we get the age of Mary.
- This has to be written as John's age= 4+ Mary's age
- This has to be rewritten as j= 4+m
- The magnitude denoted by the letter 'j' will always be 4 greater than the magnitude denoted by letter 'm'
- The different forms in which the same can be represented should be discussed and then written down.

$j = m + 4$

$m = j - 4$

$j – m = 4$

- The students should be made clear of the idea that all the letters denote numbers and that even if the magnitude changes, the relationship remains constant.

- In the relationship j = m = 4, instead of explaining that m is 4 added to j, it should be said that the age of John is 4 added to Mary.

The next is the problem related to the angle.

An angle is 120 degree. How much is the other?

Let the students answer.

? What is the relationship between the left and the right angles?

? In what other ways can it be expressed?

? How can it be expressed through letters?

The students should answer themselves and later discuss among groups and present the answers in the group. Discuss.

This aims to evaluate the ability to identify relationships, explain and interpret it in a language and then express it using letters.

Points to be discussed:

- The sum of the left and the right angles is 180 degrees.
- We get the measurement of the left angle by subtracting the measurement of the right angle from 180.
- We get the measurement of the right angle by subtracting the measurement of the left angle from 180.
- Using letters, this can be expressed as below

l + r = 180

$180 - l = r$

$180 - r = l$

***Aksharalokam* (The Letter World)**

The teacher draws a four sided figure on the board and draws lines connecting one of its corners to other corners in every possible fashion.

? What is the number of sides?

? How many lines connecting corners are drawn?

? How many triangles are formed?

The students are asked to answer the above questions.

Thereafter the students are instructed to draw figures of 3, 4, 5, 6, 7, 8 sides respectively and tabulate the number of sides, lines and triangles. They are asked to find out the number of sides, lines and triangles in a 12 sided and 15 sided figure. The table has to be evaluated and the relations explained.

1. The number of sides and the number of lines
2. The number of triangles and the number of sides.
3. The number of lines and the number of triangles.

The answers are to be written down in notebooks

1. The number of lines is 3 less than the number of sides.
2. The number of sides can be obtained by adding 2 to the number of triangles.
3. We get 1 by subtracting the number of the lines from the number of triangles.

The relationships are then expressed using letters.

$s - 3 = l$

$t + 2 = s$

$t – l = 1$

The above relations are to be studied and new relations developed. These are to be later expressed in language and then using alphabets.

For e.g.: We get 3 by subtracting the number of lines from the number of sides.

$s – l = 3$

Likewise,

$s = 3 + l$

$s – 2 = t$

$s – t = 2$

$t – 1 = l$

$l + 1 = t$

These relations have to be explained in language. Make the students understand that even if the number of sides, lines and triangles change, the relationship remains the same. Make sure that the explanation does not sound too mechanical.

The relationship t-1=l should be explained as the number of lines is one less than the number of triangles. The students should be asked to explain the remaining relations in the same manner.

The ability of the students to identify relations, explain; analyse and create inferences, develop ideas to express relations using letters and their communicative skills for the same has to be focused.

Aksharagunanam (Letter Multiplication)

Hitherto relationships employing addition and subtraction were generalised and expressed in language and letters. Relationships which involve multiplication and division would be dealt with in the subsequent section. The examples in this section discuss letters that are multiplied and divided by only one number. Multiplication and division involving more than one number is discussed in the later sections.

After solving the problem related to triangle and match sticks, the students would come up with the idea that the number of matchsticks is thrice the number of triangles. The discussion should help the students formulate the relation, m = 3t. The discussion should also lead them to formulate the relationship to find the number of triangles which is one third the number of match sticks. i.e. t = m/3.

The remaining problems should be explained in language and expressed using letters in different forms. (Let the students choose the alphabets)

The ability to identify, explain and interpret relationships, express relations using letters, form inferences and put the ideas into practice should be analysed.

The understanding of the students to express relations using letters should be analysed.

Multiplication and Addition

This section discusses problems which involve both addition and multiplication. (e.g. 5a+b)

The ability to identify and explain relationships, to form inferences and evaluate relationships and express those using letters should be analysed.

The students explain the method used to calculate the amount of money Ravi and Lissy possess. Attention should be paid to explain the method used.

Ravi → 3 should be multiplied to 10 and 1 added to it (1 added to 3 times 10)

Lissy →5 should be multiplied to 10 and 1 added to it (1 added to 5 times 10)

If there are 25 ten rupee notes and 1 one rupee coin,

25 should be multiplied to 10 and 1 added to it.

After explaining a number of similar problems, the teacher poses a question.

? If there are many ten rupee notes and 1 one rupee coin, how can the same be expressed?

The number of notes should be multiplied by 10 and 1 added to it. The students should be made clear that the number of notes might change, but numbers 10 and 1 are constant.

Since there are many ten rupee notes, the same can be expressed using letters. Let the number of ten rupee notes be't'. The relation would be $10t + 1$. Give students time to find the relation themselves. The students should be made clear that this is one added to ten times the number of ten rupee notes.

If there are 5 ten rupee notes and 2 one rupee coins and 8 ten rupee notes and 7 one rupee coins, ask the students to express the same using words.

- 2 is added to 5 times 10
- 7 is added to 8 times 10

The teacher can ask questions after giving a couple of examples.

? If there are many ten rupee notes and 1 one rupee coin, what would the total amount of money be?

It would be 1 added to ten times the number of ten rupee notes.

? How can the relation be expressed using letters?

Let the students find out (10t+c)

This should be interpreted as a number added to 10 times another.

The answers of the following questions should be found out, explained using words and then expressed using letters.

The relationship in letters should be interpreted using numbers as well.

The following questions can also be used to analyse the unit.

I

1. What is the perimeter of a triangle whose sides are 3m each? What will the perimeter be if the sides are 10 meters each?
2. What is the basic relation between the sides and perimeter of a triangle whose all sides are equal?
3. What are the different ways in which the relationship can be expressed?
4. Express the relationship using letters.

II

Look at the triangles made by using match sticks. Tabulate the number of triangles and matchsticks in each of the figures.

1. How many matchsticks are required to make 12 triangles?
2. What is the relationship between the number of triangles and number of match sticks?
3. How can the relationship be expressed if the number of triangles is denoted by‘t’ and the number of match sticks by ‘s’.

Brief Analysis: Class in which Algebra is taught with the help of the Textbook

1. The method by which algebra is introduced in the class:

The teacher presents algebra in the class by explaining the problems in the textbook and by making the students solve them. The teacher begins with an introduction to the same and there after instructs the students to do the math by themselves. The teacher asked a few students to tell their present ages, their age ‘n’ years from now and to deduce a relationship between the two. The frame of all the problems posed by the teacher was either addition or subtraction, which the students were very much familiar with. While the students were used to solving problems which involved numbers, the teacher instructs them to use the names of students who told their ages in the class instead and thereafter to use only the first alphabet of their names.

The teaching of ‘algebra-maths’ continued with problems from the textbook from different spheres like the relationship between the number of lines and triangles, the relationship between the number of matchsticks and triangles formed with them, the relationship between the number of currency notes and their value etc. The teacher followed the same pattern throughout. She began with making the students solve problems which contained only numbers and later demanded them to replace the numbers with the names of objects, shapes etc. and thereafter to replace the same with the first letter of the names. This is the method by which the teacher introduced ‘letter maths’ in class. It was the initial letter of the names of the objects in English that was used every single time.

The method of abbreviation adopted by the teacher which made use of English alphabets to introduce the concepts of the unknown and variability, acted as an obstacle for the comprehension of algebraic concepts in the abstract manner in the students. This is because through the method adopted by the teacher, algebra is introduced as a mathematical problem that involved mathematical operations and English language alphabets. Instead of looking at the letters as non-number symbols that denote uncountable or immeasurable objects or quantities, letters are understood as mere abbreviations, i.e. the initial alphabet of the object or subject names in English

2. The participation of the teacher and students in class:

It was observed that not all students were actively engaged in solving problems and saying the answers out loud in a class which was led by a teacher who resorted to rely solely on the textbooks. Many students were seen to merely write down things which they heard in the class. The teacher had the final word in all that took place in the class from start to end. Few students enthusiastically followed the teacher's words and read out the answers. The teacher was also seen to modify answers so that they suit the model presented in the textbook. Many students were reluctant to answer as they feared of being wrong. The teacher did not discuss wrong answers. The students considered those answers which the teacher stressed and repeated to be right. All students were seen to participate in the process of repeating an answer stressed upon by the teacher. In cases were the students were uncertain of the answers, their voices were found to be low. When the students answered in low voices, the teacher was seen to choose the right answer from the responses and went on to discuss it. He/she paid no attention to the wrong responses. It was also noted that only a few students responded to the teacher's questions.

3. The ways in which the students handled problems:

'Letter-maths' problems which were represented using numbers were comprehensible to at least a few students. However they experienced difficulty when the teacher demanded them to replace numbers with letters even after the teacher provided them with enough models. It was observed that when the students were asked to express relations using alphabets, they began with the Malayalam words of the objects and then proceeded to the English alphabet whose sound matches with the vernacular. The teacher considered this method to be erroneous and that deeply troubled the students. Not all students were familiar with the English words and therefore viewed 'letter-maths' as an obstacle. This was a problem which was faced by majority of students in the class. Inability to understand the logic behind solutions when letters were used instead of numbers was yet another common issue. More over most of the problems in the text book which the teacher explained were of the same nature. As a result of which when names in ‘letter-maths’ problems were replaced by objects or by shapes, students were left confused. Two or three were exceptional as they were successful in formulating equations in Malayalam. However they were dismissed by the teacher with the suggestion that they failed to find the English word and its initial letter. Students were able to come up with right answers when they worked together as a group. But not all students in a group participated in the activity and remained silent. It was observed that most groups were led by a student whose answer was approved by the rest. Therefore, all students who were part of a group did not try and find out answers. Also, every group had a student who performed well in class.

4. The method by which the teacher checked the algebra comprehensibility of the students:

The teacher presented examples from the textbook followed by questions of the same type to be solved by the students in groups. After the students arrived at the right answers, the teacher explains the steps by which the result was obtained. Having finished, he/she enquires if all the students had understood the same. If the response from the students is positive, the teacher proceeds to the next problem.

5. The metaphors and metonyms used by the teacher to explain algebraic mathematics:

The teacher used examples from everyday life which employed numbers.

For example: How old will a student be ‘n’ years from today? What is the relationship between the matchsticks used and the triangles formed with them? What is the relationship between the cost of a pen and the number of students in a class? What is the relationship between the value and number of currency notes?

The letters used to represent the names of objects and people were always the initial letter of the English words. These letters serve as the metonymy of algebra in the mathematical operations. The short form of relations and also equations were written using the same letters (abbreviation). No students were seen to use examples of letters of their choice. Those who used letters different from those in the textbook were made to make necessary changes. The teacher approved only one form of equation as correct.

III. 4 Teaching Algebra in the Experimental Group.

The Experimental group selected was another class room in the same school in which the data collection was held. The students of this group had not studied algebra hitherto. It was the researcher who introduced the concept of algebra in the class for the first time. Like the class chosen to be taught the textual concepts, this class also had Hindu, Muslim and Tribal students.

Day 1

The researcher began the class asking the students to name the different games they knew. There were numerous answers like cricket, football, police and thief, hide and seek, kitchen and cooking etc. There were also games which both girls and boys could play together. *Kothan Kallu*[16] *and Nootam Kolu*[17] were among them. Since both the teams could play it together, the researcher asked the students if they were interested to play the game. The students were amused and readily agreed. The researcher informed the students that they would play one game after the other. The students agreed with a nod.

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[16] It is a local game of the children of Kerala. It is played by five similar stones. There are five steps in the game such as orukka, irukka, mukka, nakka and thondi, thappu thalam melam.

[17] It is a play which uses one long and ten short coconut leaflet midribs. All the 10 short midribs should be of equal length. All the 11 midribs are to be dropped in the floor in such a manner that there should be at least one short midrib over the long one.

The researcher began with the game called *Kothan Kallu*. He made sure that all the students were aware of the rules of the game and explained it for the knowledge of the whole class.

Collect five similar stones. There are five steps in the game. For the successful completion of a game, a group would receive a maximum of five points. A person who is unable to complete the game in five steps will have to start from the beginning in his/her next chance.

The first step of the game, *Orukka* demands the player to place all the five stones on the floor and choose one out of them without disturbing the other four stones. The selected stone should be thrown up. By the time the stone comes down, the player has to get hold of any of the four remaining stones. This has to be repeated four times for the completion of the first step. If the person is unable to collect one stone from the ground by the time the first stone comes down, he fails and would be out. The next player gets the chance.

Second step, *Irukka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of two stones. And then the remaining two stones are also collected in the same fashion. If the player is unable to pick two stones by the time the first stone comes down, the player would fail. The next player gets the chance.

Third step, *Mukka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of three stones. The stone that is about to fall down has to be caught as well. The stone has to be thrown up again and by the time it comes down the remaining stone has to be collected. If the player is unable to pick two stones by the time the first stone comes down, the player would fail. The next player gets the chance.

Fourth step, *Nakka*: The player drops all the five stones on the floor and picks one without disturbing the other. The stone is thrown up in the air and by the time it falls down, the player has to get hold of all the four stones.

Fifth step, *Thondi* , *Thappu Thalam Melam* : The player hold all the five stones inside the hand and throws one stone up into the air. By the time the stone comes down the player has to drop all the four stones on the floor and then catch the first stone inside his hand. This stone is thrown up again and the four stones are collected again. All the five stones have to be held inside the hands of the player. The player says *thappu thalam melam* aloud and shakes the stones in his hands and then throws all the five stones into the air and tries to collect them all in his/her outer hands. The stones are thrown up again and are to be held inside the palm. At the end, the player gets a point which equals the number of stones remaining in his hand. If the player fails to catch even one stone in his outer hand, he fails and will receive no points at all. The next person gets the chance. If he is able to catch at least one stone, the player can continue. The team which manages to reach the score fixed before the commencement of the game wins.

Having explained the rules of the game, the researcher begins the game. The students got ready. Two groups were selected using lottery method; one of boys and the other of girls. Both the teams got ready to play. The boys' team won the toss. 20 was fixed as the target score. The boys began to play. In the first chance they got 3 points. They failed in the second chance. The girls' team began to play. They received five points each in the first and the second chances but could not clear the third. Boys continued playing and gained five points but failed in the next chance. The girls played again and gained another ten points. The total score reached the target. The class declared the girls' team the victors. The class applauded their victory.

As the first game came to an end, the researcher began the next game. The name of the game is *Nootam Kolu*. The researcher explained the rules of the game. Even though every student knew the rules of the games, the researcher had to make sure everyone was clear.

One long and ten short coconut leaflet midribs (*eerkilu*) are required. All the 10 short midribs should be of equal length. All the 11 midribs are to be dropped in the floor in such a manner that there should be at least one short midrib over the long one. Now, the player has to collect them one by one without disturbing the others. If any of the other midriffs is disturbed, the player loses. The player would get 10 points for the short ones and 100 points for the long ones. The maximum score that can be attained is 200. A target score would be fixed beforehand and the team which meets the target first win. The students get ready to play after the researcher completed explaining the rules of the games.

The students suggested that they divide into two groups like before; a girls' group and a boys' group. Participants were selected through lottery method. Boys expressed the confidence that they would win this time. They won the toss but allowed the girls to play first. This decision might have been an outcome of their previous failure. 500 was fixed as the target score. The girls' team could attain only 10 points in the first chance. The boys' team which began the game with enthusiasm also could not gain much. They got 20 points. Chances kept shifting between the two teams. The rest of the class cheered up the participants and also observed if any participant broke the rules. At the end, it was again the girls' team who won the game. The boys appeared dejected and challenged the girls that they would defeat them in the games to come.

After both the games came to an end, the students had two tables in their notebooks. One was that of *Kothan kallu* and the other of *Nootam kolu.* The researcher then raised a question related to the rules of both the games and the data they entered in the two tables. The target score set for *kothankallu* was 20 and it was the girls' team who won. The researcher asked the students the number of steps through which the girls' team met the target. The students replied that they took 5 steps. The researcher then asked the students the minimum number of steps required to reach the target if they were able to get full score in every step. The students replied that they needed only 4 steps. When they were asked the reason for their response, the students replied that they would get 5 points for the successful completion of a game. Hence only 4 games were necessary to obtain 20 points. The researcher asked the students the total number of steps required to meet 20 points if one managed to get only 1 point in a step. They replied that a total of 20 steps were required. The researcher modified the target score to 25 and repeated the question. The student seemed doubtful and in return asked the researcher to let them know the score obtained by the player in every step. They wanted to know if the player scored maximum or not. The researched clarified the question and added that the player obtained the maximum score in every step. Even before the researcher completed the question, the students shouted the answer loud. The answer was 5. The researched instructed them to write down the questions and answers in their notebooks.

The students wrote, 5 games = 25 points, in their notebooks.

Then, the number of steps required to meet 20 points if one scored 4 in a step was expressed as

5 games = 20 points

The researcher asked whether both the games were similar. The students disagreed with a smile. The researched marked their response and wanted to know the point of difference as they had written ‘5 games’ in both the equations. Shyam, a student seated in the back row answered that if both the set of games were similar; the score should also be different. It was not the case in this instance. All the students supported Shyam. The researcher asked them to differentiate between the two games and Akhila, another student replied that since the points of the games were different, the games were also different. She explained her point with the example of the game they had played. She suggested that though both boys and girls played the same number of steps, the girls managed to score 20 while the boys could not. Taking cue from the example, the researcher explained that even if the number of games on the left hand side is similar, a close look at the right hand side would make it clear if two relations were same to one another or not. The researcher proceeded to the next question. The students seemed satisfied at the researcher’s agreement to their opinion and eagerly waited for the next question. Now that they had found out the number of steps required to score 20 if each step carried 5 points, 4 points and 1 point respectively, the researcher asked the students to write down the number of steps required to score 20 if the steps carried 3 points and 2 points respectively, in their notebooks. The researcher instructed to write it in the decreasing order from 5 to 1.

(5 points in a game) 4 games = 20 points

(4 points in a game) 5 games = 20 points

(3 points in a game) 7 games = 20 points

(2 points in a game) 10 games = 20 points

(1 points in a game) 20 games = 20 points

Picture 9: Students in the experimental class complete the table for '*kothan kallu'*

After the students completed the table, the researcher asked the students the relationship between the number of games and the target point. The students replied that the point acquired in a single game decided the number of games to reach the target score. The researcher asked the students the number of games required to meet a target score of 40, if each game carried a maximum of 5 points.

The students readily answered that 8 games were necessary. The researcher demanded the students to write down the number of games required if the target score is increased by 5 from 20.

The researcher demonstrated it by writing the below on the board.

4 games = 20 points

5 games = 25 points

The students were instructed to write down the rest.

6 games = 30 points

7 games = 35 points

8 games = 40 points

After the students had completed the table ranging from 4 games to 8 games on the left hand side, the researcher asked the students to trace the relationship between the number of games and target score. After a minute of silence, the students suggested the number of games increased by 1 and the target score increased by 5 every time. The researcher asked the students if the continuity of the data entered in the table helped them to answer quickly. The students agreed that the calculation was easier. The researcher asked the students to calculate the total score if the number of games played was 10. The students answered that the score would be 50.

However all the students had not responded, when the researcher asked them to explain the method by which they found out the answer, a section of the students replied that they multiplied 10 by 5 while the others replied that they added 2 to the left hand side and 10 to the right hand side. The researcher asked the students if the second method was practical always. Few students expressed the opinion that it was while few others disagreed. The researcher then instructed them to find the score if the number of games played is 100 by the same method. The students looked amused and made noises. Jasmine, a student said the method does not work for all problems. Most students supported Jasmine. When the researcher asked the reason for the change in opinion, Akhila suggested that multiplying 10, the number of games in this instance by 5, the maximum score in a game would provide them the maximum score, 50. The example put forward by Akhila cleared the students of their remaining doubts.

The whole class shouted the answer to the previous question posed by the researcher aloud. While a few replied that the answer could be found out by multiplying 100 by 5, others suggested the answer to be 500. The researcher welcomed all the responses and proceeded to the next question. If the number of games is 1000, what would the maximum score be? The students looked startled at the figure and there was a commotion in the class. The researcher asked the students to maintain silence. He suggested that instead of using the number 1000, if the number of games is expressed as '*thone*[18]' (what would be the score? The students responded that the score would also be lot/many (*thone*). The researcher agreed to their opinion with the clarification that the score would be 5 times the number of games played and made the students to repeat the same after him and then applauded the students. The researcher asked the students the maximum score if they had played a *thoni*[19] full of games. Sooraj Shyam replied with a smile that a boat full of games has to be multiplied by 5. The students had a fun time as they heard the metaphor 'boat full'. The researcher asked the students to find similar examples.

Here an abstract non-counting symbol replaces an exact quantity. And by using it, they formulated an operation and arrived at a solution. This indicates that the students could use their mathematical acquired knowledge in a non-numerical context. It helped develop a concrete (physical quantity) to abstract (non-quantity) mathematical thinking among the students. The cognitive move from concrete to abstract enabled the students to think algebraically.

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[18] A lot/ many

[19] Boat

Swaravi suggested that a ship full of points can be obtained from a *kappal*[20] full of games. The students then gave examples like a *puzha*[21], *kadal*[22], bus, lorry, *vimanam*[23] etc to indicate things of greater magnitude.

Having understood that the students got the logic of the problems right, the researcher raised the next question. The examples put forward by both the students and the researcher present objects which are big in terms of its size. How can the relation between a '*kappal'* (ship) full of games and a '*kappal'* (ship) full of points be expressed? The students seemed doubtful. The researcher encouraged the students to represent *kappal* by using the Malayalam alphabet ക (ka) or the English alphabet 'k'. The points can be expressed as '5 X ക (ka)' or '5xk'. The students shared a common doubt. They asked if they could use any alphabet or language of their chose. The researcher assured that it was a matter of personal choice and that they had the freedom to choose any letter or alphabet. The students began writing their answers. Some of them were seen discussing it among their neighbours. Finally, every one completed writing. The students read out their answers. Most of them had used English letters. The letter chosen was the English equivalent of the initial letter of the Malayalam words. Only few students used Malayalam letters to create the equation. The equations written in English and Malayalam were both equally accepted.

Day 2

The researcher begins the class from the point he ended the previous class. The students had formulated equations in both English and Malayalam. All the equations were based on the assumption that the maximum score in a game is 5.

[20] Ship

[21] River

[22] Sea

[23] Aeroplane

The researcher asked the students whether the equations undergo any changes if the score is changed to 4. Most of the students were silent and therefore the researcher repeated the question. Few students asked whether changing the numeral from 5 to 4 was sufficient or not. The researcher wrote the equations suggested by the students on the black board.

$$\left.\begin{array}{r} k = 5 \text{ x } k \\ ka = 5 \text{ x } ka \end{array}\right\} \quad \text{in the case of 5}$$

ka = 4 x ka ______in the case of 4

When the students who were silent till now saw the equation on the board they were no longer puzzled. The researcher instructed the students to modify the equations they had formulated. The researcher then went close to each student and examined if every student was able to bring the necessary changes and ensured that no student got stuck in between. After the students completed making changes, the researcher asked them if the equation would change or not provided the number changes from 4 to 3. The students together said aloud that the equations would change. The researcher demanded the students to explain the change. The students confidently replied that the number in the equations would change from 4 to 3. The researcher found students speaking to one another in a bench and questioned them about the matter of their discussion. A student, Sooraj replied that he had been explaining the change from 4 to 3 in the equation to his friend. The researcher brought their attention back to the classroom discussion and instructed the students to write down the equations with 3 as the maximum score. Shreya enquired if they could use the same alphabets or not. The researcher clarified that they could use any letter of their choice. Once all the students had completed writing down the equations, the researcher asked them to closely examine the three equations.

ka = 5 x ka (5 points in a game)

ka = 4 x ka (4 points in a game)

ka = 3 x ka (3 points in a game)

The researcher asked the students to trace similarities or differences if any in the set of equations.

Picture 10: Students write algebraic equations in the experimental class .

It was observed that the students made changes to the equations they had written after a few moments of analysis. Taking into consideration the changes suggested by the students with respect to the change in the number, the researcher modified the equations further. If the maximum score in a game is further changed to 2 and thereafter to 1, taking into account the fluctuating numerical value in the equations, the researcher rewrote the equations as below,

ka = 5 x ka changes to

ka = changing score x ka

Few students seemed puzzled at the change brought about in the equation. The researcher asked the students if the equation can be expressed in the above manner or not. Since the maximum score associated with each game changes from 5 to1, the students agreed that the changing score can be expressed in the manner in which the researcher had done. The researcher instructed the students to make the same changes in their notebooks. It was at that time another discussion was observed among the girls. The researcher asked them the subject of their discussion. They hesitated to answer. The researcher insisted them to speak up. Akhila raised a doubt. She wanted to know if it was possible to write ‘score in each game’ instead of ‘changing score’ in the equation. The researcher left the doubt to the scrutiny of the whole class. Few students seemed to agree to Akhila’s opinion. The researcher replied that the equation can be written in the manner Akhila suggested.

The researcher asked the students which among the two methods of expression was easier. They replied that the second was comparatively easier. In this manner, the whole class agreed to the change suggested by Akhila and brought about changes in their notebooks. The researcher examined the students modifying their equations and randomly picked a few and made them read out their equations. After 7 students read out their equations the researcher suggested that they condense the equations further. Sooraj had a smile when he shared his opinion that if the present equations were made short, it would be easier to read. The researcher backed the opinion of Sooraj and suggested that they condense the phrase ‘score in each game’ using alphabets of their choice. The students completed the task in a few minutes. There after the researcher asked Sooraj to read out his equation.

k = p x k , (read out Sooraj).

In the same fashion, the researcher randomly selected students and made them read their equations aloud. It was observed that the letter ‘p’ was used by all the students in the class to denote the score associated with each game even when different alphabets were used to denote the game. The researched shared his observation with the class. The students responded that they used ‘p’ because it stood for the term 'points' and therefore was more relatable. The reason was common to all. The students continued reading out their examples. Other students did not find the examples difficult as the student who read out the equations resorted to reading his/her example first in the form of a sentence followed by the equation. The students expressed interest in listening to the examples of the others and all the students completed reading out their examples. After the students had completed, the researcher asked the students if everyone one of them understood the relationship between the metaphors used by the students to denote large quantities and the alphabets used to express the same. It was observed that the students found the equations easier as they used names of familiar things to formulate their equations. The researcher ended the class after being convinced that the students could write equations and condense them further with ease.

Students denoted large quantities using symbol words like *kappal*, *kadal*, *puzha* etc. It is further replaced by nonsensical symbols like '*ka*', k, or *pa*, p. The students did not hesitate to consider these as a representational quantity term and used it in place of the meaningful word and continued with mathematical operations using these symbols. Through the establishment of this process and through similarly designed mathematical activities, letters '*ka*, k, *pa* and p become placed as the metonymies of algebraic discourse. This then opens the scope for conceptually as well as historically placing the shared metonymies of school mathematics algebra. The symbolic resources of natural play way method work as extensional resources for establishing the semiotics of algebra which is technically called metonymy.

<u>Day 3</u>

The next class was based on the game '*Nootam kolu'* which the students had played before. The researcher reminded them of the method they had resorted to formulate equations based on '*Kothan kallu'* and found that the students clearly remembered the same. The researcher asked the students if they were aware of the maximum score a player could get in '*Nootam kolu'*. The students replied that 200 was the maximum one could attain. The researcher repeated the set of questions which he had raised based on *Kothan kallu.* The researcher questioned the students on the number of games a player will have to play to obtain 1000 points if he manages to score 200 in each game. The students replied within a few minutes that the player would require 5 games.

This time the researcher did not formulate questions of the earlier kind in which the score kept changing while target score remains fixed. The researcher asked the students to find out the maximum score a person would obtain in 10 games if he manages to obtain the maximum score in each game. The students replied that the player would receive 2000 points in 10 games. The researcher asked the students if both the methods, i.e. the method of '*Kothan kallu'* and '*Nootam kolu'* were the same. The students nodded in agreement. Thereafter the researcher instructed them to use familiar objects to denote big numbers (increasing score in this case) as they had done before in the previous game. As instructed, the students formulated sentences and equations based on the game '*Nootam kolu'* and the maximum score in each game. To indicate the increasing score in '*Kothan kallu'*, the students had used examples like river, sea, stars, sky etc. Similar examples were used in '*Nootam kolu'* as well. The students read out their examples. Few students had borrowed examples used by their friends in the earlier game. Interestingly some students claimed the rights to a few examples. However, it can be concluded that all the students employed examples of one kind or the other to represent the relationship between the number of games and the scores.

'Akasatholam Nootam kolu kali' (sky high *Nootam kolu* game) = 200 x *'Akasatholam Nootam kolu kali'* (sky high *Nootam kolu* game)

AN = 200 x AN

Ananya formulated an equation as shown above. She had used two alphabets (A and N) in the same. The researcher asked her the reason for the choice. She replied that 'AN' stood for the initial letters of '*Akasatholam Nootam kolu kali'* and that it was easier to understand. Few other students were also seen to have used two alphabets. The other students seemed content with the explanation given by the students who used two alphabets. According to them letters were only techniques to help them understand better the relations in sentences. The number of games did not matter anymore. The students had clearly understood that multiplying the number of games by 200 was the required method.

Picture 11: Students write down equations using their own metaphors and metonymies.

After having completed writing, Aswanth, one among the students raised a doubt. He wanted to know if the game score could be obtained by multiplying it with 100, provided the score in the stage is 100. The researcher assured Aswanth that his observation was correct. Thereafter the researcher asked the students to substantiate the argument. Aswanth replied that he had suggested so because of the similarity in method, which he had observed, this game shared with the former game, '*kothankallu'*. Researcher took into consideration the observation and inference suggested by Aswanth. The researcher asked the rest of the class if any of them had the same idea in mind. A number of students marked their agreement to this. Ananya observed that while the maximum score in '*kothankallu'* was 5, the same in '*noottankolu'* was 200. The researcher asked the students if like in the game of '*kothankallu'* where scores ranging from 1 to 5 could be earned successively, scores from 1 to 200 could be possible or not in the case of '*nootam kolu'*. The students marked their disagreement in unison.

The researched asked for a reason and the students replied that the minimum score possible in '*nootam kolu'* was 10. Thereafter the researcher encouraged the students to find out the scores possible in '*nootam kolu'*. The students read aloud the multiples of 10 ranging from 10 to 200. The researcher subsequently wanted to know if the students could express the relationship they had already devised in a different manner without employing numbers. The students then proceeded to create an equation. They modified their equations by replacing '200 points' by 'p'. All the students were seen to have used the same alphabet, **AN= p X AN**, to cite the example of Ananya. On the request of the researcher, the students read aloud the possible values of 'p' in the game of '*nootam kolu'* (10, 20, 30 ...200) and in the game of '*kotha nkallu'* (1, 2, 3, 4, 5). The researcher posed the next question. He wanted to know if the same variable 'p' could be used in both sets of equations, as the value of 'p' differs in both the games. Interestingly, Shyam responded with a counter question. As the game score varies even within a game, he questioned the validity of the researcher's query. The researcher answered that he has a lot to learn from them and went on to ask another question. He asked if any of the students had used the same letter to denote the number of levels in both the games. Few students like Ashique who had written '*kadalolam kali'* (sea full of games) to indicate both '*nootam kolu'* and '*kothan kallu'*, Fathima who had denoted the same by '*vanolam kali'* (sky high of games) and Abhinav who had written *'kayalolam kali'* (lake full of games) raised their hands. They also added that the number of games in both the cases is the same. Ashique suggested that when he used the term *'kadalolam'*, he had in mind both the games. The rest of the class also had the same opinion.

The researcher then paid attention to the similarity in the variables the students had used and questioned them if the variable 'k', which was used by Ashique to denote '*kadalolam kali'* and by Abhinav to denote *'kayalolam kali'* was the same or not. Among the responses, Ashique's and Abhinav's were heard the loudest. They replied that the 'k' in both the cases were different as they stood for two different entities which were clearly distinguishable. Akhila backed their argument by suggesting that there existed no ambiguities. The researcher agreed to their opinions and concluded the day's class.

Day 3

The students were amused by the properties the researcher had brought to the next class. The researcher placed a common balance, few pouches and coconut shells he had in hand on the table and stood facing the students. The researcher asked the students why there was a look of wonder in their faces. The students pointed to the pouches and wanted to know what was inside them. Few others pointed to the balance and questioned what it was for. The researcher replied that they would come to know everything as the class progresses. Few students also attempted to stand up and thereby have a closer look at the items. After making all the students settle down in their seats, the researcher took the coconut shells in his hands. He enquired about their familiarity with the object. The students replied that they use coconut shells to play and also burn them in the kitchen to cook. Few added that at times of a power failure, coconut shells were burnt in order to press clothes and went on to elaborate how the burnt shells were put in traditional metallic iron boxes and the heated boxes in turn used to iron out clothes. Few others stated that they were ignorant of the same. The researcher intervened by suggesting that the discourse has brought to light yet another use of coconut shells which was new to many.

After having said so, the researcher placed three shells on the table and instructed the students to closely examine them. The students were expected to compare the sizes of the three shells and find out if they were similar or not. Standing up and tilting their heads, the students examined the shells and answered that the shells were not similar. On the researcher's question, they responded that the second shell was of a different size. The researcher then called upon Hassan to choose a shell which would match the size of the other two, from among the remaining in the sack. Hassan did as instructed. The rest of the class approved Hassan's selection.

The researcher asked him to cross check the possibility of an even more perfect choice. After another round of examination, Hassan confirmed that there was none. Having completed the task, Hassan went back to his seat while the researcher got ready to carry on with the next stage of his activity. The researcher emptied the contents of the pouches into the three coconut shells. The researcher asked the students what the coconut shells now contained. The students replied that they held red lucky seeds (*manjadikkuru*), palm tree seeds (*panankuru*) and green gram (*cherupayar*). Few students referred to palm seeds as fox-droppings (*kurukkan kashtam*). The rest of the students who called it *panankuru* disagreed to the other name and added that these grow on palm trees. The others retaliated by asserting that they used the term *kurukkan kashtam* to indicate palm seeds. The researcher intervened with the suggestion that though varieties of names were used, the thing remains the same.

Picture 12: A cup of *Manjaadikkuru* (red lucky seeds)

The researcher took the palm seeds in hand and went near the students and asked them to identify the object. While a group referred to it as *panankuru*, the other group insisted on calling it *kurukkan kashtam*. The researcher suggested that he approved both and proceeded towards the coconut shells. He pointed to the three shells, each comprising of red lucky seeds, palm seeds and green gram and instructed them to find out which shell contained the greatest quantity. The students responded with a doubt. They wanted to know if by quantity the researcher meant count or not. The researched answered that he had meant the same. The students posed another doubt. They wanted to know if by count the researcher meant an approximate figure or the exact. The researcher struck back with a counter question. He asked them if it was possible to get an exact count or not. The students replied that though it was possible, it would be a time consuming process. Therefore both the students and the researcher came to a consensus to use an approximate figure. The researcher directed them to write down their responses in their notebooks.

Picture 13: Researcher ask the students to compare '*manjadikkuru'* and '*panamkuru'*

After they completed doing so, the researcher made them read out their answers. Most of the students had the opinion that the shell which comprised of green gram had the greatest quantity while a few opined that the shell with the red lucky seeds had the greatest. The researcher wanted to know the basis upon which they came to the conclusion. The students replied that size was their tool. They substantiated their argument by suggesting that as green gram was smaller in size as compared to red lucky seeds and palm seeds, it would be more in number. The researcher validated their logic. He then asked the latter section of students who had chosen red lucky seeds to be their answer to justify their choice. Even they responded with the same logic of size. The former section who chose green gram disagreed to the same.

The researcher gave a handful of both green gram and red lucky seeds to those students who had chosen the latter as the answer. The students compared the size of both and shared the opinion that their assumption had gone wrong and corrected themselves. After the whole class came to a consensus that the answer to the researcher's earlier question was green gram, the researcher posed the next question. He wanted to know if there existed a relationship between the quantity and size. The students replied that with the increase in size, quantity decreases and vice versa. The researcher instructed them to formulate an equation to show the relationship.

Decrease in size = Increase in quantity

Smaller = Greater

The researcher thereafter directed them to formulate equations to demonstrate a relation between the size of each commodity and its quantity.

Green gram (*cherupayar*) = More in number (*kooduthal ennam*)

Red lucky seeds (*manjadikkuru*) = Less in number (*kuravu ennam*)

Palm tree seeds (*panankuru*) = Least in number (*ettavum kuravu ennam*)

The students were then instructed to express the same using variables. It was observed that this time, majority of the students had used Malayalam alphabets to express the same.

che = *koo*

ma = *ku*

pa = *e ku*

The researcher enquired the reason why most students had chosen Malayalam alphabets to English. The students replied that it left them little less confused. Few students were seen to use have earlier used ‘k’ to denote both *kooduthal* (more) and *kuravu* (less) and then replaced it with *koo* and *ku*. Few others were seen to have used the English alphabets c, m and p to respectively denote green gram (*cherupayar*), red lucky seeds (*manjadikkuru*) and palm tree seeds (*panankuru*). Even they replaced the English alphabets with the Malayalam alphabets, *che*, *ma* and *pa*.

After the students had written down the equations, the researcher presented another question. He asked the students how they could possibly collect palm tree seeds which equals in number to the number of green gram in a coconut shell.

Students: By taking many shells full of palm tree seeds, we can balance the number of green gram and palm tree seeds.

The researcher asked if by many more shells of palm seeds, the students meant to add more to the one already in hand or not. Without much confusion, the students nodded in agreement. The researcher then enquired if they could express the same in the form of a sentence.

One of the students required a clarification. He wanted to know if the researcher expected them to express the words uttered by him in the form of a sentence or not.

Researcher: “Didn’t you people mention before that in order to balance the number of green gram and palm seeds, we need to take many a shells of palm seeds for a single shell of green gram?”

Students (together): “Yes”

Researcher: “Even I said the same. Many more shells of palm seeds have to be added to the one shell already in hand.”

Students: “Yes. More shells have to be added to the already existing.”

Researcher: “You have to express this addition in a sentence form.”

The students wrote the same in their notebooks and read them aloud. The researcher then demanded them to condense the sentences further into equation form.

As addition figures in the relationship, the researcher asked the students how they would express addition in a sentence. While most students raised the doubt if they could possibly use an alphabet to denote the same, few others asked if they were permitted to use the plus sign ‘+’ to indicate the same. The researcher wanted to know which method kept them at ease. The students replied that the usage of a sign made things more comprehensible. Hence it was decided upon majority to employ the plus sign in the equation.

ku.pa + *pa* =

ku.pa + *pa* = *che*

ku.pa + *pa* = *koo*

These were the three forms by which the students had expressed the relationship. The first group left the right hand side (RHS) of the equation blank as they could not figure out how to abbreviate the same. The second group used *che* to indicate green gram equal in number to the number of palm seeds. By *koo* the third group meant the count obtained after adding more palm seeds to the earlier which is equal to the number of green gram. The researcher observed that none of the equations were wrong. Considering the case of the first group of students, the researcher made the general comment that they should not have left the RHS blank and instead should have tried formulating a short form which is comprehensible to them. He added that the abbreviations used by the other two groups were correct; *che* stood for a lot of green gram which is equivalent in number to that of palm tree seeds added thereafter. *koo* on the other hand indicated that there were more green gram and was therefore acceptable. After stating his assumptions on the short forms employed, the researcher asked the students the basis on which they had used the same. The first group replied that they had used *che* as it was the sum total of all the palm seeds which now equals the number of green gram. The second group suggested that since the addition of palm seeds to the earlier yielded more palm seeds, they used the short form, *koo*.

The researcher then gave the students the task to balance the number of red lucky seeds and green gram.

"Sir, do we employ the same technique?" a student from the front row raised the doubt.

"Yes" replied the researcher to the whole of the class.

Few minutes later, the students began to talk to one another and there was a commotion in the class.

"Did every one finish?" the voice of the researcher was heard above the rest. Seated in their own positions, the students answered that they did. Few others nodded. The researcher randomly chose a girl and asked her to read aloud the answer the girl next to her had written in her notebook. Varsha read out what Ananya had written down. Ananya wrote that adding more number of red lucky seeds to the ones already in the shell gives us equal number of red lucky seeds and green gram and had formulated the short form of the same. The researcher asked Varsha if her answer was the same or not. Varsha replied that there were slight differences. She read out "adding more red lucky seeds to the original makes the number of red lucky seeds and green grams equal". Varsha observed that the difference was only in the choice of words and not in their sense. The researcher asked Ananya and the rest of the class if Varsha's argument was correct. They responded that it was right. Towards the end of the discussion, the researcher wrote the equations of the students on the board.

ku.ma + ma = che

ku.ma + ma = koo

None of the students failed to fill the RHS of their equations this time. After writing down the two equations, the researcher added a third one.

ku.ma + ma = ku.pa + pa

Having written this, the researcher asked if the above equation was right or not. No students seemed ready to answer. They remained silent for some time and later began to talk among themselves. The researcher therefore divided the students into groups according to their benches and gave them a little more time to find out the answer. The students began to discuss among their fellow members of the group until the researcher announced that they had run out of time. He then demanded the students to answer. Shyam, a student from the back row suggested that the equation the researcher had written was correct as adding more red lucky seeds to the original as well as adding more palm seeds to the original yielded exactly the same result. Both stood for green gram. The researcher congratulated Shyam for the response and asked the rest of the class for disagreements if any. Ummukolsu, a student from the third row replied that their answer was different. The researcher asked her to read out their answer. She replied that adding more red lucky seeds to the original increases its number. In the same manner adding more palm seeds to the original also increases its number. Hence they can be equated.

The researcher observed that the response was correct. He raised the counter question of what balances their number. If there was no green gram, adding more palm seeds would provide us with an increase in number of palm seeds. Same would be the case with red lucky seeds. However the two entities would not be equal in number to be equated. After giving it a little thought, Ummukolsu and the others agreed that it was the green gram which made equal the number of palm seeds and red lucky seeds.

Researcher: “Imagine that Ummukolsu has in hand equal number of palm seeds and red lucky seeds. However this number is not equal to the number of green gram. Is the number of palm seeds and red lucky seeds still the same?”

Ummukolsu: “Can you repeat the question again?”

The researcher thus repeated the question with necessary pauses.

Ummukolsu: “Number of Palm seeds = Number of Red lucky seeds

Hence they are equal in number.”

Researcher: “Why?”

Ummukolsu: “The number of palm seeds and red lucky seeds may not be equal to the number of green gram. But they are still equal. “

Researcher (to the rest of the class): “Is it correct.”

Students (most of them): “Yes”

Researcher: “All the students are really smart. You are able to answer correctly regardless of the twists and turns I bring about in a question.”

The researcher ended the day’s session after appreciating the class.

Day 4

The researcher carried along a balance scale to the next class as well. The students became amused and restless. They stared at the balance which was kept on the table. The researcher asked the students if they were familiar with the object or not. They responded that it was a familiar object. The researcher proceeded to ask what the object was meant for. The students replied that it was used to weigh things and added that they have seen the grocer using it. There came many responses from the students. The researcher intervened with the comment that since the students knew what the instrument was, they would witness the use of the same in their classroom. He emptied the coconut shell full of green gram into one plate of the balance and then added a handful of red lucky seeds into the other. The plate with the green gram weighed greater than other. The students closely observed the procedure. The researcher added another handful of red lucky seeds into the plate and asked the students to explain what the researcher had done. They described in detail what the researcher had done to both the sides of the balance. After the students had finished, the researcher added another handful of red lucky seeds into the plate, as a result of which the second plate weighed greater than the first. Sounds of surprise emerged from the students as the plates shifted positions. The researcher enquired what had happened. The students replied that the plate which had been heavier in the beginning rose up while the other came down. The researcher asked the reason behind him doing so. The students remained silent for a moment and later replied that the researcher had done so in order to balance the weight of green gram and red lucky seeds.

Picture 14: Introducing mathematical operations using balance scale.

“Yes, you are right. I tried to balance the weight of green gram and red lucky seeds.” replied the researcher. “What steps did I follow to do the same?” he continued.

“You first added a handful of red lucky seeds into the plate (pointing to the respective arm of the scale). Thereafter you repeated the procedure.” Students replied.

Researcher: “And what did you observe?”

Students: “The plate with the red lucky seeds came down.”

Researcher: “Does both the plates weigh the same?”

Students: “No”

Researcher: “How can they be balanced?”

Students: “Remove a portion of red lucky seeds from the plate”

The researcher did as per the instructions by the students. There was a movement again in the balance. The plate with the red lucky seeds rose a bit. The students shouted in unison that removing a little more red lucky seeds would balance both the arms. The researcher again followed the instruction of the students. Both the arms became almost equal this time. The students suggested the same. The researcher wanted to know how the students came up with the conclusion. While one group pointed towards the needle in between the arms and observed that both the arms carried equal weight provided the needle rests parallel to the ground, the other group suggested that both the arms carried equal weight when both the plates are in the same level. The researcher assured both the groups that they were right. He still had in hand the red lucky seeds he had removed from the plate at different points in time. He showed the same to the students and asked how much would they come up to in total. The students answered in many a figure.

Researcher: “By quantity, I did not mean the count. Is this quantity equal to the one that I initially added in the scale?”

Students: “No. It is less than that.”

Researcher: “Then let us refer to this little quantity in my hand as a small portion.”

The students readily agreed. The researcher asked the students to write down the relation between red lucky seeds and green gram in their notebooks in the manner in which they had formulated relations between red lucky seeds, green gram and palm seeds in the earlier class. Few students raised the doubt if they were to follow the method they had earlier employed to balance the number of green gram and red lucky seeds in coconut shells. The researcher nodded and then demanded the students to begin. Adequate time was given while the researcher engaged in observing each student of the class. The students were seen talking to each other in between and then getting back to their task. Few students appeared to have completed the work. The researcher asked the whole class if they had finished or not. Few students replied that they required more time. After the whole class had finished, the researcher chose a student from the front row to read his answer aloud.

“One coconut shell of green gram = 3 handful of red lucky seeds, a little deducted from the whole”

He read.

The researcher asked the rest of the class if their responses were similar to the above or not. While a few expressed their agreement, few others marked their disagreement. The researcher coaxed them to read out their response.

“Red lucky seeds in Sir’s hand + red lucky seeds in hand + red lucky seeds in hand, red lucky seeds that were removed = one coconut shell full of green gram” read out a student.

After listening to the two sets of responses, the researcher went on to analyse the same. He asked the class to identify remarkable differences, if any between the two sets of responses. The students expressed their opinions.

In the first short form, the process of adding a handful of red lucky seeds thrice was clubbed together or was not written separately. However, in the second short form, with the employment of the plus sign (+) the three stages were distinctly outlined. The researcher accepted their opinion but with the disappointment that none of the students employed the sign to denote the process of removing a portion of red lucky seeds from the plate to balance the weight. He then asked the students to identify the sign that they had learned in the mathematics classes that would cater to their demand. No responses were heard from the students and therefore the researcher repeated the question. He offered them the clue that they had employed the plus sign (+) in place of addition while the process they were referring to was subtraction. As a response to the hint, few students suggested that they should use the minus sign (-). Few students remained silent. The researcher questioned those students if the rest of the class were right or not in using the minus sign (-). They replied that the class was right. The researcher demanded justification. Sreehari stood up and answered:

“I have 5 mangoes in hand, out of which 2 are removed. I still have 3 mangoes. We get 3 by subtracting 2 from 5. In the same manner, red lucky seeds were removed from the set. Hence minus sign (-) can be used.”

The researcher asked the reason behind his silence even though he knew the answer. Sreehari smiled and kept quiet. The researcher then enquired if any others could offer a different explanation. However, the students expressed solidarity to the opinion of Sreehari. Thereafter, the researcher instructed the students to reorganise their equations using the minus sign (-). Sooraj wanted to know which among the two equations they were expected to use. The researcher suggested them to use the one of their choice, preferably the one they had written themselves. In few minutes time, the students rewrote the equations using minus sign (-).

“One coconut shell green gram = 3 handful red lucky seeds – 1 fistful red lucky seeds”

"Red lucky seeds in Sir's hand + red lucky seeds in hand + red lucky seeds in hand – 1 fistful red lucky seeds that were removed = one coconut shell full of green gram"

The researcher ended the class after examining the new equations of the students. He informed the students that he would meet them in the afternoon and left the class.

In the afternoon session, the researcher summarized the activities they had performed in the morning session. He picked up the balance that he had placed in a corner of the class. He placed it on the table and added red lucky seeds into one plate and palm tree seeds into the other. The latter had to be added four times in order to balance both the plates. The students were not the least amused this time. They observed it as a continuation of the earlier activity. Thereafter the researcher removed the palm tree seeds from the left plate of the balance and added a handful of green gram into the empty plate. He also added a portion of the palm seeds he had just emptied into the left plate. Both the plates became balanced. The researcher then asked the students if they properly saw both the activities. The students nodded. The researcher then demanded the students to point out differences between the two activities. Varada stood up and suggested that in the first activity the researcher added palm tree seeds four times into a plate in order to make it equal to the red lucky seeds in the other plate, while in the second activity green gram was added first and followed by palm tree seeds. This differs from the first in the fact that it cannot be expressed in the manner in which addition of palm tree seeds alone is done. Instead it has to expressed as below,

Green gram + palm tree seeds = red lucky seeds

The researcher agreed to the opinion of Varada and added that this problem arises because both the commodities were different. He then demanded the class to express both the activities in short form.

1. $pa + pa + pa + pa = ma$

   $4pa = ma$

2. $che + pa = ma$

The researcher closely examined the equations of the students. He asked the class if they had any doubts regarding the formulation of the same. They replied that the process was clear. He insisted that they repeat what they had understood. The students replied that only homogenous substances can be written together while heterogeneous objects have to be written in separation.

As a continuation of the class, the researcher summoned a few students to the table and instructed them to separate green gram, red lucky seeds and palm tree seeds and drop them back into their respective pouches. He let them go back to their seats after they completed their tasks. Thereafter the researcher randomly chose a boy and a girl to come near the common balance on the table. Ananthakrishnan and Anjana were respectively the chosen students. The researcher instructed Anjana to fill the right plate of the balance with green gram using her right hand while Ananthakrishnan was made to fill the left plate of the balance with red lucky seeds using his right hand. The researcher then asked if the weight on both hands were equal or not. As it was not, Ananthakrishnan was asked to add a few more red lucky seeds in the left plate using his right hand. The students replied that a balance could not be achieved even this time. Finally both green gram and red lucky seeds became equal after Ananthakrishnan added the latter three times again using his right hand.

Picture 15: Students practically formulating algebra equations using balance scale.

The researcher suggested the students to verbally express the process they had just witnessed.

Green gram in Anjana’s hands = Five times the red lucky seeds in Ananthakrishnan’s hands

The researcher agreed to the relation and thereafter called upon two more students to the balance. As many volunteered to come, he chose the student who rose up first and another who rose up last. There was a look of hope in the others who sat with their eyes glued to the balance on the table. Both the students were instructed to add a few green gram in the right plate of the balance a single time, using their right hand, adjacent to the green gram which Anjana had added earlier. As both did as instructed, the harmony between the arms of the balance was once again lost. The researcher asked the students to suggest a method to achieve the harmony back. The students suggested that red lucky seeds be added again. Using his right hand, the researcher added a handful of red lucky seeds to the plate into which

Ananthakrishnan had already added his share. The students indicated that the weight was still not equal. It was when the researcher added the red lucky seeds for the third time that both the plates became balanced. The researcher directed the students to verbally express the activities carried out by himself and earlier by Anjana, Ananthakrishnan and the other students.

Anjana + Ajanya + Sooraj = 5 times Ananthakrishnan + 3 times Sir

The students wrote down relations as above.

“What is the green gram in the balance equal to?” asked the researcher.

“It is equal to a handful each of Anjana, Ajanya and Sooraj” they replied.

“All right, then what is the red lucky seed which now equals the green gram also equal to?” went on the researcher.

“It is equal to five handfuls of Ananthakrishnan and three handfuls of Sir” they replied.

Pointing towards the left arm of the balance which held red lucky seeds, the researcher raised a question. He wanted to know why the students did not resort to saying eight handfuls of red lucky seeds when the material added by Ananthakrishnan and himself was one and the same. The students responded with a smile that it would a mistake to do so. The researcher demanded justification.

Students: “Though both parties used their right hand to add the seeds into the balance, it cannot be clubbed together. This is because the sizes of both hands are different as they belong to two different people. The hand of the teacher is bigger than that of the student. Hence it is not possible to group them together.”

The researcher had another question in mind. He asked the students why they did not combine the quantity in the case of green gram provided that all the three were students with little hands. The students teased the researcher for his ignorance and attempted to clarify their point.

“The quantities of green gram that can be held in the hands of all the three students are different. Therefore, it does not make any difference if they belong to the same class or have small hands.” replied the students.

The researcher pretended that he did not understand the logic. Suchitha demanded the three students to show their hands to the researcher and there after requested the researcher to take a look at it and compare their sizes. The researcher expressed his satisfaction in the method employed and sent back all the students to their respective seats. After the class had settled, the researcher offered an explanation.

“It is not just the nature of the material that obstacles the process of combining them together but also their quantities. Even homogenous objects which vary in their quantities or which are provided to us in ambiguous quantities cannot be grouped together.

Since two different items, green gram and palm tree seeds were added to balance the weight of a third different item, red lucky seeds in this instance, the materials had to be separated using the plus sign (+) in the first activity.”

On the other hand, since the sizes of the hands were different leading to difference in quantities, the plus sign ought to be employed in the second activity as well. It would have been possible to add them together if the quantity that can be held in all hands were known to be the same. This is precisely the reason why the amount of red lucky seeds which was dropped in the balance by Ananthakrishnan could be added together and expressed as five handfuls. The amount of seeds he would add every single time would be the same. Hence they can be added. Since the quantity that can be accommodated in the hands of the researcher was different, it has to be separated using the plus sign (+). ”

The students compared the quantities of uncounted objects like red lucky seeds, green gram, and palm tree seeds. In order to express the relationship between the objects mathematically, they employ the initial alphabets of the word names in Malayalam. The students derived the answers to the questions of the researcher through the conjoining of these letters and their already acquired algorithmic knowledge. Through this method which applies non number symbols and algorithmic knowledge without employing numbers, the students were seen to move from one stage of mathematical learning to the higher level. This growth was achieved as a result of the development of the ability of the students to move from concrete thinking (numbers) to abstract thinking (non-number symbols).

The abstract thinking aided by non-number symbols and mathematically acquired signs promoted the development of algebraic concepts in the students. The concepts thus developed were further metaphorically extended using the discourse pertaining to the balance scale in order to ensure the development of algebraic thought in the students. This paves way to the expected treatment of algebraic problems in the abstract manner.

After offering the above explanation, the researcher proceeded to a few questions.

Researcher: "I take a handful red lucky seeds. Do you know how many there are?"

Students (most of them): "No."

Students (few of them): "There are many."

Researcher: "All right. As we are are not sure of the count, let us denote it as x. The amount of red lucky seeds in a handful is x. I take four handfuls of red lucky seeds and drop them in a plate. Let us denote this new quantity using y. Can you convert the same into an equation?"

Students: "Do we follow the method we earlier resorted to?"

Researcher: "Yes."

Without taking much time, most of the students wrote,

$4x = y$

The researcher asked the remaining students if they had difficulty in understanding as they still did not complete. They responded that they took some time to write down the question, hence the delay. They also wrote the same equation, $4x = y$.

There after the researcher proceeded to the next question.

Researcher: "Let p stand for a handful of red lucky seeds by Ananthakrishnan. He drops 6 handfuls of the same into the plate. Let us denote this by q. Convert the same into equation."

$6p$ = q, wrote the students.

Researcher; "If 4 handfuls of mine equal 6 handfuls of Ananthakrishnan, which among the equations, $y = 6p$ or $y = q$ is correct?"

The students readily approved both the equations. The researcher demanded explanation. They replied that $6p$ denoted six handfuls of red lucky seeds Ananthakrishnan could grab which equals q, the total quantity. This is also equal to y. Hence both the equations are right.

Researcher: "In that case, is $6p$ and q equal to $4x$?"

Students (with a smile): "Yes"

Researcher: "Even though $4x$, y, $6p$ and q denote the same quantity, we have used different alphabets to indicate the same. Can you now eliminate the alphabets and then express the same using only numbers? How much red lucky seeds will it come if we add 4 handfuls of mine? How much will there be if 6 handfuls of Ananthakrishnan is also added? I shall give you an example."

15 + 15 + 15 + 15 = 60, the researcher wrote on the board.

Researcher: "If a handful of mine contains 15 red lucky seeds, four handfuls will contain 60 seeds. Let us imagine that a handful of Ananthakrishnan contains 10 red lucky seeds. By the way, how many handfuls of seeds did he grab?"

Students: “Six”

Researcher: “Therefore 10 should be added six times”

10 + 10 + 10 + 10 + 10 + 10 = 60, the researcher wrote again.

Researcher: “Isn’t both the quantities equal?”

At this point, a student showed her disagreement. She believed that a single handful would itself contain around 60 red lucky seeds. She was skeptical of the count. The researcher assured her that 60 was only a hypothetical figure and that the exact count was unknown. Thereafter he instructed the students to make relations using different numbers. The students organised themselves into groups and completed the task in a few minutes. The researcher requested a group to read out their answer after all the groups had completed. Their response was as stated below.

Red lucky seeds collected by sir would be 4x3=12

Red lucky seeds collected by Ananthakrishnan would be 6x2=12

The researcher learnt that two other groups had also made use of the same numbers. There were five more groups remaining. The researcher demanded one group to read out their answer. Their answer is stated below.

Red lucky seeds collected by sir would be 4x300=1200

Red lucky seeds collected by Ananthakrishnan would be 6x200=1200

It was observed that of the 5 groups, 4 groups chose the same set of numbers. Therefore the researcher demanded the final group to read out their answer, which is also stated below.

Red lucky seeds collected by sir would be 4x30=120

Red lucky seeds collected by Ananthakrishnan would be 6x20=120

After all the groups had finished, the researcher commented that all the groups had got the answers right. Thereafter he asked the students to formulate relationships with green gram and palm tree seeds using some other alphabets. This task was presented as a home work. He also asked 4 out of 8 groups to specify the relationship between the numbers and alphabets in their equations. On the final note that the students would remain in the present groups even in the next class, the researcher ended the day's class.

<u>Day 5</u>

In the following class, the researcher examined if all the groups had completed the task assigned. It was observed that all the groups had done the homework. The researcher randomly chose students to read aloud their answers. The 4 groups who were asked to specify the relationship between the numbers and alphabets in their equations read out their examples. What was peculiar of these four groups was that the numbers chosen were all even and small. There figured both even and odd, and big (>10) and small numbers in the examples of the other groups.

Example I:

1 plate of sand = x

25 plates of sand = y

$25x = y$

1 plate of gravel = a

50 plates of gravel = b

$50a = b$

25 plates of sand = 50 plates of gravel

$25x = 50a$

$y = b$

Example II:

1 coconut shell of dhal = y

12 coconut shells of dhal = z

$12y = z$

1 plate of gooseberry= n

7 plates of gooseberry = m

$7n = m$

12 coconut shells of dhal = 7 plates of gooseberry

$12y = 7n$

$z = m$

While a set of students made use of objects like fruits, vegetables, tamarind seeds etc, few others were seen to have used minute objects like sand and gravel. The students had successfully completed their tasks. All the examples were read out in the class. The objects used appeared familiar to the whole class. The students showed no wonder or discomfort towards the relationships framed between the different objects. After completion, the researcher ended the class and bid farewell to the students.

Brief Analysis: Class which Introduced Algebra Differently from the Usual Treatment of Traditional Textbooks

1. The method by which algebra is introduced in the class:

   The researcher introduced algebra concepts through games like *noottam kolu* and *kothan kallu*. The students were made to express the relationship between the score they obtained in the games and the number of times each game was played. Apart from deducing relationship using the maximum score possible in both the games, the students were asked to formulate relationship between different scores and the number of steps required to meet the scores and thereafter create equations. Gradually the researcher demonstrated the use of a quantity which the students were familiar with which was different from usual numbers and expressed equations between the score and number of games using the new quantity. Students resort to the use of word symbols (*kappal, kadal* and *puzha* etc) to express the magnitude of the game scores. Games act as vehicle that helps transport the students from numerical thinking to abstract thinking that is not number bound. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather natural development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers.

The students were then asked to formulate equations using first letters of the object. The replacement of physical quantities (eg: 5000,10000) by metaphorical word symbols like '*kappal*' by 'ka' or 'k' and by 'x' or 'y' or 'z' subsequently occurs due to the human abilities for symbolization and the cultural scaffolding of classroom mathematical discourse by the pedagogically mediated intervention of the researcher. Algebra thinking is essentially theoretical. It develops through a cultural-cognitive process of the kind discussed above. Expressing quantity concepts including relationship through algebraic operations develop gradually. The researcher brought about no changes to the Malayalam letters used by the students.

Using quantities like red lucky seeds, palm tree seeds and green gram in coconut shells, the students were made to compare different quantities and find out which was greater or lesser without using numbers. The students compare the quantities of uncounted objects like red lucky seeds, green gram, and palm tree seeds. In order to express the relationship between the objects mathematically, they employ the initial alphabets of the word names in Malayalam. The students derive the answers to the questions of the researcher through the conjoining of these letters and their already acquired algorithmic knowledge. Through this method which applies non number symbols and algorithmic knowledge without employing numbers, the students are seen to move from one stage of mathematical learning to the higher level.

With the use of a balance, the students were taught to represent equations employing the plus sign (+), minus sign (-), and equal to sign (=). All the equations were formulated without relying solely on numbers.

2. The participation of the teacher and students in class:

From the beginning, all the students actively participated in the class. They expressed interest in both playing the games and answering questions related to the same. Examples, images, short forms and letters that came up as a part of teaching algebra were all suggested by the students themselves. The teacher resorted to creating a frame which accommodated varieties of answers with his questions. The need to stick on to a single and uniform answer was ruled out. The researcher dealt with the problems in such a way that ensured every student understood the logic behind the problems. A discussion procedure mostly accompanied the process of finding out the answer. Discussions were held even when the problems were solved by the students alone or in groups. All students showed willingness to read out their answers loud. It shows that students are confident in their answers. Students were randomly chosen to read their answers. Other students were also seen to refute responses. This gives students an opportunity to understand other possible answers on the same concept. If the students have objection in the answers, the researcher utilized this space for a creative discourse for scaffolding the algebraic concepts. The researcher however did not dictate answers. Instead he tried to create connections between the varieties of responses that came up from the students. This discourse leads them to acquire more clarity on the algebraic concepts.

3. The way in which the students handled problems:

   After playing games, the students learned mathematical expressions in the form of sentences through the relationship between the number of games played and the score obtained. Thereafter they studied how to convert verbal relationships into algebraic expressions. When examples were explained, the students themselves suggested the nature of the calculations. This happened even when the students employed metaphors in place of numerals. It shows that an abstract non-counting symbol replaces an exact quantity. Through non-counting score of games and other objects (‘*manjadikkuru’, ‘panamkuru’ and ‘cherupayar’*), the students were able to imagine mathematical situations and solve them better without the use of numbers. Similarly, when students use symbol words (eg: k, ക derived from ‘*kappal’* full of games) they did not doubt to take it as a representational quantity term and used it in place of the meaning full word and continued with the mathematical operation using this symbol. Through the establishment of this process and through similarly designed mathematical activities, letters ‘ka, k, pa and p become placed as the metonymies of algebraic discourse. The students handled those situations which involved sentences and their algebraic expressions with much ease unlike the usual classroom experience which dealt with numbers alone. Through group discussion and discourse with the teacher, students developed better algebraic thought.

4. The method by which the teacher checked the algebra comprehensibility of the students:

The children’s games, their scores, and the mathematical operations they made use of to arrive at the scores, served as the context that facilitated the growth of abstract thought in the students. Doubts related to the metaphorical ‘word symbols’ and associated metonyms (non-number symbols) used by the researcher were raised. It was the students who clarified the doubts of the researcher and the researcher understood the level of comprehension of the students through the same. The researcher made sure that he discussed all the responses from the students. A close examination of the erroneous responses of the students helped them analyse the mistakes they had committed by themselves while also allowing the rest of the class to think dynamically. Situations like these served as the platform for the development of divergent and abstract thinking in the students. It was the participation of the students that made it clear that they were able to handle algebraic concepts in the flexible and abstract format.

In another words, students who failed to get the answers right were made to understand their mistake for themselves and get a better idea of the concept and its logic. The researcher helped them to conceptually reach the correct answers approved by the whole class.

Researcher asked the students to compare the quantities of uncounted objects like red lucky seeds, green gram, and palm tree seeds. Researcher observed how the students logically talked about quantities in abstract terms and how they carried out comparison of quantities in the light of mathematical operation. He also checked if the students were able to express the examples in algebraic form without the help of the teacher. He also checked if the students were still able to solve problems provided there was a change in the background of the questions.

5. The metaphors and metonyms used by the teacher to explain algebraic mathematics:

   The relationship between the points scored and number of games played were explained using the games suggested by the students: *kothan kallu* and *nootam kolu*.

   In order to express the idea of greater points, the 'word symbols' suggested by the students such as a boatful of points, sky high points, sea full of points, lake full of points etc. were maintained. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather unconscious development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers. Word symbol is replaced by non-number symbols derived from the symbol words. Students did not hesitate to use non-number symbols as representational quantity terms. They used it in place of the meaningful word and continued with the mathematical operation using this symbols. Through this process non-number symbols get placed as a metonymies of algebraic discourse. The metonymies used were both the Malayalam alphabets and the English alphabets chosen by the students.

Mathematical concepts were introduced using red lucky seeds, palm tree seeds and green gram apart from the use of numbers. Mathematical calculations involving the initial use of numbers followed by objects was not the method resorted by the researcher. The method suggested by the researcher involved the use of objects alone and comparisons among the same. With the use of balance, the students could also witness the mathematical calculations in operation. Algebraic relations involving objects like red lucky seeds, palm tree seeds and green gram were formulated and the students were taught the logic behind the same. The palms of the students and also the researcher were used to demonstrate the difference in scales of measurement. Moreover, this was used to formulate various equations. The research made use of the balance scale to visually demonstrate the algebraically understood variables and their mathematical operations. By this method, the researcher succeeded in metaphorically extending the context of the already acquired algebraic knowledge and the non-counting symbols of the students.

DISCUSSION

This chapter discusses the facts of findings recorded in the previous chapter on results with respect to the objectives of the study. The research questions put forward in the beginning of the study are divided into five segments and discussed thereafter. The first pertains to the control group that deals with the method of teaching algebra in government run middle schools. The second revolves around the experimental group, a classroom in which algebra is taught using a novel pedagogical method carefully devised by the researcher. The third is a comparative discussion on how students in the classroom in government run middle schools, which uses the conventional methods of algebra teaching and learning and students in the classroom where algebra is taught using the method developed by the researcher, comprehend the concept of unknown numbers in algebra. The fourth section focuses on identifying the semiotic tools employed in both the control and experimental groups and analyses the usage of the same in both the classrooms. The thrust of the fifth and final segment is on a comparative analysis of the pedagogical methods, teaching-learning materials and Continuous and Comprehensive Evaluation method.

IV. 1 Algebra Taught in the Control Group-Government run Middle School

A. The Teaching Method of the Teacher

The teacher introduced algebraic concepts to the students of standard VI in Government run middle schools, selected as the comparison group, with the help of the chapter on "Letter Maths" in the Mathematics textbook of Std. VI. The teacher presented problems on algebra with reference to the numerical problems on addition and subtraction in the textbook. The first problem was based on the ages and its relationship between two children. For better understanding and involvement of the students, the teacher went on to use examples from the class. Thereafter numbers were replaced by alphabets in the subsequent algebraic problems. She demonstrated how numbers can be substituted by letters using the example of the relationship between the ages of Mary and John in the textbook.

She introduced an equation incorporating numbers and letters. The method employed by the teacher in transforming a mathematical problem into an equation, in accordance with the textbooks, was through expressing the relationship between the ages of Mary and John mathematically. It was the question of the teacher, as to what figure ought to be added to the age of Mary in order to obtain the age of John, that generated the answer 4. At the point where it is understood that John and Mary are two children of different ages, the students also begin to realize that 4 is no longer a number alone but also stands for 4 years. Taking advantage of this understanding, the teacher creates an equation in which the initial letters of the ages of both, Mary and John, 'M' and 'J' respectively is used instead of their full names while the numerical value, 4 in this instance remains unaffected. The teacher reverses the relationship between Mary and John and instructs the students to create an equation accommodating the change, drawing reference from the equation they just came across. She checks whether all the students were able to formulate equations or not.

The teacher proceeded to the next problem after refreshing the earlier lessons on the laws regarding sum of angles in a triangle, and sum of angles formed on a line by another line with intersects the same. She brushed up the rules that the sum of angles of a triangle is 180 degrees, and the sum of angles formed upon a line by another intersecting line is also 180, and moved on to the next problem in algebra. She demonstrated the use of signs of addition (+) and subtraction (-), along with alphabets, through the problems on angles. The method used by the teacher in the solution of problems on triangles was the same as that of the first question.

> Having given the value of one of the two angles, she instructed the students to find out the unknown value. However it was seen that the shift from the problems on age to that of angles was not smooth for the students. Even though they knew that the sum of angles is 180 degrees, a law which they had already learnt in their geometry classes, and that they were provided with the value of one of the two angles, the students stumbled upon the idea of converting it into an equation and finding out the unknown value. It was

observed that the teacher carried out the rest of the discussion in the class revolving around the responses of a few smart students alone.

Even in the case of the problem on angles, the teacher employed the same method of using the initial letters ‘l’ and ‘r’ to denote the left angle and right angle and went on to formulate the equation. The students succeeded in formulating equations of the above fashion. However, instead of employing the letters ‘l’ and ‘r’, the students used ‘e’ and ‘v’, which stood for the terms *edathu* and *valathu*, the Malayalam equivalents for left and right respectively. This was a deviation from the standards set by the textbook, closely followed by the teacher. She insisted that they use ‘l’ and ‘r’, instead of ‘e’ and ‘v’. The problem in approach lies in the fact that the teacher paid undue importance to the model laid out by the text and failed to analyse whether the algebraic thought developed by the students were right or wrong. According to this approach the answers of the students qualifies to be correct only if they toe the lines of the textual models. The teacher strived hard to help the students fit into this mould.

The method of abbreviation adopted by the teacher which made use of English alphabets to introduce the concepts of the unknown and variability, acted as an obstacle for the comprehension of algebraic concepts in the abstract manner in the students. This is because through the method adopted by the teacher, algebra is introduced as a mathematical problem that involved mathematical operations and English language alphabets. Instead of looking at the letters as non-number symbols that denote uncountable or immeasurable objects or quantities, letters are understood as mere abbreviations, i.e. the initial alphabet of the object or subject names in English. Abstract thinking in the students should necessarily be developed through the objects themselves that are represented by the abbreviations. If not the thought delivered by the object and the thought delivered by the abbreviations will be one and the same.

For example, to represent the ages of John and Mary, letters ‘j’ and ‘m’ were used. In the context of this situation, the concept of age of two persons always proposed a number. More over this being the age of mortal men, the possible choice of numbers was certainly limited. In the first place the students were expected to think about this limitation in the numerical sense followed by the process of replacement of this numerical thought by an abstract thought employing letters that was characterized by algebraic symbols and the concept of the unknown.

Algebraic concepts if introduced through objects that are immeasurable or easily not countable would actually enable the students to think beyond the limits of numerical values taken into account the familiarity of the students with numbers. However, all the word symbols presented by the teacher were confined to the realm of the numbers.

For example, in the problem of the angles, it is understood that the sum of the left and the right angle will not exceed 180 degrees. In this case, the students would surely be limiting their thinking to this particular figure or value. The employment of letters instead of numbers becomes meaningless in the actual sense. This prevents the growth of mathematical thinking beyond numbers in the abstract level in the students. This becomes evident through the subsequent activities of the classroom. All the algebraic problems were introduced in the same method.

This was seen to repeat in the problems on relationship between sides, angles, triangles etc. The equations formulated by the students using English alphabets, similar in pronunciation with the initial letter of their Malayalam equivalents were discarded as incorrect. Only equations that employ English alphabets corresponding to English words were accepted as correct.

The textual approach followed by the teacher leads to a misconception in the students that algebraic equations are an amalgamation of numbers and English letters, which in turn stands for English words. The students begin to internalize the notion that algebra is such equations which are a combination of the English language and numbers. This perception of the students tampers the real line of thought proposed by algebra. Mere formation of equations, undue necessity of accommodation of English letters into equations, and numerical values in the midst of letters tends to repel students from the very idea of algebra with the passage of time. In the interview conducted, teachers reported that even students who answer promptly in class were seen to make mistakes in the examinations. The interview also brought to light the fact that there is a general lack of clarity among the students in their comprehension of the concepts of algebra.

Having completed the problems on addition and subtraction, the teacher moved on to the problems on multiplication in Letter Maths. In order to introduce multiplication in Letter Maths, the teacher made use of a picture of a triangle made out of match sticks, by a girl named Rani. Using the example of the match stick triangle, the teacher made the students write down the answers for her questions on how many sticks were used for the creation of a single triangle and the number of triangles that can be made by select number of match sticks. Thereafter the teacher instructed the students to identify the mathematical operation in action and to transform the operation into algebraic equation. This was the method employed by the teacher to introduce multiplication in algebra. In order to obtain the total number of match sticks used, the students multiplied the total number of triangles to the number of matchsticks required to make a single triangle. She thereafter asked the students to resort to the use of letters instead of numbers. Similar to the former instances, the letters used by the students were not uniform. Hence the teacher insisted that the whole class use the equation from the textbook, m=3 x t and added that 3 x t can also be expressed as 3t and demanded the class rewrite the equation. She thereafter copied a table from the textbook onto the black board and asked the students to fill the empty spaces. This was her method of measuring the level of comprehension of the students. The details of the table on the board were read out in the form of verbal question by the teacher. The question focused on identifying the total number of students in the A, B, C, and D divisions of Std. VI, and the total money spent by the students to buy pens worth Rs.5 each. The students were expected to fill the blanks with the total amount spent on pens. In this case, the students multiplied two figures. Thereafter, instead of numbers, they were required to use letters to represent either objects or subjects. The letters continued to be the initial letters of English words.

The method employed by the teacher to introduce algebra in the class room was the primary use of pure numerical problems followed by the replacement of the same by alphabets. The three basic arithmetic operations; addition, subtraction and multiplication were introduced in the classes on algebra with reference to lessons taught in earlier classes, numerical to be specific. Students with a relatively lesser competence in these operations would continue to experience difficulty in understanding algebraic concepts. In the actual sense, the teacher explained algebra exclusively using numbers and numerical operations. Verbal questions and geometrical figures act as aids to achieve the result.

It is through the conversion of mathematical questions in the form of sentences into equations employing letters instead of numbers that the teacher presents and teaches lessons of algebra in the class.

B. Comprehension of Students

Though all students listened to teacher's lectures attentively and tried to respond promptly, the course of the class was steered by a few bright students. The attention and consideration of the teacher was limited to this minority group. This created a division among the class in terms of the degree of activity; fairly active and poorly active. Like any other class, it is the construction of this division that act as a prominent barrier in understanding algebra. As a result, in the students, the abstract thought put forward by algebra and also the ability to think mathematically remain under the shade of skepticism and doubt.

Through the lectures of the teacher, the students managed to build the idea that a variable is any unknown quantity in mathematical problems represented by a letter. They begin to set laws regarding the usage of variables in algebra. The primary rule they tend to internalize is that the variable should be the initial letter of the name of the object/subject it stands for. Second, the variable should relate to a subject/object name in English. Third, the English letter variable should stand for an English word which is the direct translation of the subject/object name from their vernacular to English. The common mistakes made by the students, the silence they observe against the queries of the teacher, and the general slowness in solving problems justify the above observations. It is these observations that hinder their easy movement from one type of algebraic problems to another. The students were unable to absorb the essence of algebra conceptually. Even though there were efforts on the part of the teacher to split the whole class into different groups to ensure equal participation of all students alike, this strategy also seems to fail to achieve results.

Algebra remains outside the realm of comprehension in many students. The teacher raised questions regarding the problems she presented in order to make sure all students understood the solutions to problems and responded positively in class. Only a few students were seen to respond to most of the questions, while the rest merely repeated the responses they just heard. However, even the group of actively responding students observed silence when the teacher introduced various other examples of algebra. The reason behind the silence was that the answers they had in mind required the assistance of English translation of familiar object names in Malayalam. This was evident in problems on the relationship between sides, lines, triangle etc. which were first introduced through diagrams and later equated without the diagram in place. They were found to be stuck at the phase where letters were required to establish relationship. They stumbled at their limited ken in English language. They knew not what lines, sides, triangles etc. were called in English. This happened to be their greatest challenge and mental block in understanding the concepts of algebra.

However, never did they share this difficulty with their teacher. Nor did the teacher realize it. Instead, the teacher moved on to the judgment that algebra is a tough area for the students. A transmission difficulty that seem to interfere in the understanding of the students, who had dealt with mathematical operations in numbers with ease but those with letters with great strain, happened because the existing method of teaching merely transposes letters over numbers. In the conceptual sense, algebraic mathematics gives importance on a mathematical engagement between variables, beyond numbers. Unfortunately, the focus of the teachers happens to be concentrated on generating right answers which are in agreement with the textual models, instead of the formation of logic in students. The textbook that the teacher relies on serves to be the agency that decides right and wrong.

IV. 2 Algebra Taught in the Experimental Group

A. The Teaching Method of the Researcher

The researcher taught algebra in Std. VI, the class chosen as the experimental group, with the help of two games suggested by the students themselves. The researcher selected two games that can be played by both boys and girls from their list of favorite games. The researcher thereafter made the students play the selected games in the classroom. The researcher ensured involvement of each student by making them take part in the games of *nootankolu* and *kothankallu*. As all students actively participated in the games, there was no division in terms of participation in the class. The divisions with respect to the degree of involvement therefore became insignificant in this context. The researcher declared the team which attained the target point first as the winning team. The girls' team was declared first winners. Having announced the first set of winners, the researcher encouraged the students to estimate the total number of games to be played in order to meet the target point. The researcher kept asking questions related to the games by changing the target points of each games and keeping the maximum possible score constant and also vice versa. The students were seen to readily respond to the questions raised by the researcher, while the researcher continued raising new questions.

It was observed that the students had not even a slight difficulty in estimating the maximum points and number of games. In order to find the answers, the students employed the mathematical operations they had learnt in the earlier classes. The researcher did not direct the students to follow any set methods to arrive at the answers. To make certain that the students were properly aware of the mathematical operations they had put to use, the researcher even asked for alternate methods if any to carry out the same. In this manner, the researcher succeeded in developing a discussion culture in the classroom. He went on to use ‘word symbols’ like sea, river, sky, mountain etc. as metaphors to represent large quantities beyond the limit of measurement. The researcher suggested a metaphorical ‘word symbol’ and the students contributed the rest. They never failed to estimate the number of games and the maximum score possible in each game with the help of these metaphors. This became possible because the strategy of the researcher was not mere replacement of a number by an object. Instead, he intended to establish an imaginative connection between the games played and the points scored, relying deeply on the logic of this relationship. This strategy bore fruit because the students were familiar with the games they chose to play and the method of calculation of game scores. The metaphors that facilitated thought process beyond numerals were also self-chosen and thus familiar. The researcher never imposed a single correct answer over the whole class. Instead he helped students retrospect their thoughts and findings through counter questions until they ended up creating logical connections by themselves. As the relationship between the number of games and the score attained was expressed in the form of long sentences, the researcher encouraged the students to shorten the same by converting them into equations. It was at that point that the students made use of letters for the first time. The researcher maintained the concepts of the students even while formulating equations. As there were no restrictions on the use of letters or language, English and Malayalam in this case, the students appeared relaxed and efficiently formulated equations.

To sum up the strategic plan of the researcher, he began with involving students in the class via games and establishing a relationship between the number of games and score. By maintaining the same relationship, the researcher made possible the conversion of a mental image in the form of a sentence into an equation with the help of metaphors (e.g. *kapalolam kali* meaning a ship full of games), and went on to condense the equations further by employing alphabets (e.g. 'ka', or 'ക' or 'k') chosen by the students themselves. Through the mathematically oriented axiomatic discourse, students were contextually encouraged to think without numbers. In the practice, an exact quantity is replaced by a non-counting symbol ('*thone'*) and students still engaged an operation involving this quantity and arrived at a solution. Later the non- counting symbol ('*thone*') is replaced by symbol words like '*kappal' (*meaning ship). Along with this even when the symbol words were substituted by different nonsensical symbols like 'ka' or ക, students did not experience doubts to it take as a representational quantity term and used it in place of the meaningful word. The students continued with the mathematical operation using this symbols.

Through the adoption of games as a means of teaching, the researcher helped the students understand the logic to think beyond numbers and to mathematically express their thoughts with the help of non-number symbols (letters).

Here, the students resort to the use of word symbols (non-number) to express the magnitude of the game scores. Games act as vehicle that helps transport the students from numerical thinking to abstract thinking that is not number bound. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather unconscious development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers. It is the already acquired mathematical knowledge of the students put to operation that actually made this transition from numbers to non-number word symbols (ship, sea, river, sky) meaningful. The children's games, their scores, and the mathematical operations they made use of to arrive at the scores, served as the context that facilitated the growth of abstract thought in the students. From the perspective of the students, the word symbols used to denote game scores along with the abbreviations they eventually employed to denote the same, radically changes their existing realities, i.e. mathematical operations are used alongside numbers. This change actually runs deep into the basic structure of the perception of the students and gets reflected in their thoughts. This change that the student is unaware of act as the fertile ground for the germination of further algebraic concept seeds in the subsequent algebra teaching and learning sessions.

In the second stage, the researcher demanded the students to compare quantities using three materials; red lucky seeds, palm tree seeds and green gram. The researcher asked the students to compare three coconut shells full of each of the three materials and thereafter estimate the greatest quantity in terms of number. The students did not require thinking twice before answering that the coconut shell with the green gram had the highest number of seeds. In this manner, even without knowing the exact quantity of materials, the researcher enabled the students to compare different quantities using materials from their daily lives, and to form equations based on the relationship between the size and number of these familiar objects. The researcher proceeded to ask the students to suggest methods to make the number of green gram in the coconut shell equal to that of the number of palm tree seeds. The students responded that the number of green gram and palm seeds can be made equal by adding more palm seeds to the coconut shell. The researcher demanded to express the procedure in abbreviation using letters. It was evident form the equations formulated by the students that they preferred the use of the plus sign (+) to denote addition of more palm seeds to the existing quantity. Here even if the students face difficulty to think about the exact quantity (of different seeds) without employing numbers they compared different quantities using their already acquired algorithmic knowledge. And mathematically expressed the relationship between the quantities using abstract non-number symbols (*ku.pa + pa = che).* Use of first letter of the object names in Malayalam language by the students to represent a quantity and subsequent presentation of the textbook symbols as an “agreed upon” symbols by a community of formal mathematicians help children understand the social as well as the intellectual history of the evolution of metonymy of algebra.

The researcher also observed that the students used different letters to form equations, discussed the reasons of their choice with the whole class, and the researcher assured the students that any letter can be used to condense sentences and relations into the format of an equation.

For example: In the equations representing the process of equalizing the above mentioned quantities, it was observed that there were two major types of relations.

ku.pa + *pa* = *che*

ku.pa + *pa* = *koo*

In both the equations, the letters to the left of the equations were the same but the letters to the right were different. The researcher demanded the students to explain the choice of the letters. The former set of students responded that they used *che* to denote *cherupayar* (green gram) and as the addition of *kure* (more) *panamkuru* (palm tree seeds) to the existing quantity of *panamkuru* (palm tree seeds) would make both quantities equal. The second set of students responded that they used *koo* to express *kooduthal* (a lot) *panamkuru* (palm tree seeds) as *kure*(more) *panamkuru* (palm tree seeds) were added to the existing quantity of *panamkuru* (palm tree seeds).

The class on algebra thus moved on with discussions and interactions between the students and the researcher. The researcher raised a number of questions while the students responded with answers and counter questions. The researcher also introduced the logic that a measure of red lucky seeds that is equal to a coconut shell full of green gram is also equal to the measure of palm tree seeds used in the earlier equations, and formulated a new relationship between the materials. The researcher equated quantities of red lucky seeds and palm tree seeds.

The comparison between different seeds (green gram, red lucky seeds, palm tree seeds) were introduced as an extension of the abstract thought developed in the context of the students' games. In order to compare quantities, the students were unaware of the exact counts. The comparison between such quantities whose counts are difficult to estimate was made possible using the commonsensical logic of the relationship between size, numbers and quantities. The comparison between the three objects in the coconut shells were mathematically expressed using non number symbols (ചര, മ, പ), and the abstract idea pertaining to the quantities. The non-number symbols chosen by the students were the initial letters of the names of the objects in Malayalam. The researcher employed this method in agreement with the conventionally followed “agreed upon” method of text books that rely on the community of formal mathematicians for the introduction of the concept of variability in algebra.

In the abstract sense, the comparisons between quantities were carried out using the symbols that served as abbreviations that stood for the chosen objects. The students made use of their already acquired algorithmic knowledge for the comparison of quantities as well as in their equalization. When mathematical operations are carried out along with abbreviations that are non- counting symbols, letters cease to be mere non-number symbols and are transformed into signs that stand for abstract quantities. This change in perception that results from the process of formation of equations helps letters to acquire a new dimension shaking itself free from its exclusively linguistic associations wherein a letter is merely an alphabet. Technically this perception becomes established as a semiotics of algebra, referred to as metonymy.

The researcher introduced the concept of variables in algebra and the operations carried out in the formation of equations, with the help of examples of familiar images and objects. The method adopted gave ample space to the play of imagination. The researcher also maintained space for discussion in the class so that the students could raise questions and also share their opinions.

It was through the help of a common balance that the researcher introduced the concept of mixing of two different substances together, in order to make the quantity of the mixture equal to that of a third substance. The researcher demonstrated the equations they had dealt with using the metaphor of a balance scale.

The researcher visually expressed the procedures of addition and subtraction, and the usage of the same in the equations, with the help of the balance and objects. Through the activity, the researcher explained why homogenous substance can be added together in an equation while the addition of heterogeneous substance is possible only with a plus (+) sign.

The research made use of the balance scale to visually demonstrate the algebraically understood variables and their mathematical operations. By this method, the researcher succeeded in metaphorically extending the context of the already acquired algebraic knowledge and the non-counting symbols of the students. A culturally mediated algebraic thinking emerges as a result of this visualization process.

The researcher analysed the level of comprehension of the students through discussions, the questions he raised in the class and examining the responses he received for the students. In this fashion, the researcher discovered the solutions to the problems faced by the students, in understanding concepts, from within themselves. In order check the accuracy of logic formation of the concepts they dealt with, the researcher discussed every single response of the students. Instead of the researcher explaining the logic behind the diverse responses, he made the students explain their concepts by themselves. Since the researcher paid no stress on a single correct response, the students were not afraid to voice their responses and they were freed of the fear of going wrong. This confidence aids the formation of concepts and logical understanding in the students. The positive relationship between the researcher and the students blurred the hierarchies of performance and activity of the students in the class.

B. Comprehension of the Students

From the beginning, all the students actively participated in the class. They expressed interest in both playing the games and answering questions related to the same. Examples, images, short forms and letters that came up as a part of teaching algebra were all suggested by the students themselves. The teacher resorted to creating a frame which accommodated varieties of answers with his questions. The need to stick on to a single and uniform answer was ruled out. The researcher dealt with the problems in such a way that ensured every student understood the logic behind the problems. A discussion procedure mostly accompanied the process of finding out the answer. Discussions were held even when the problems were solved by the students alone or in groups. All students showed willingness to read out their answers loud. Students were randomly chosen to read their answers. Other students were also seen to refute responses. The researcher raised a number of questions so that the students learned with greater clarity. The researcher however did not dictate answers. Instead he tried to create connections between the varieties of responses that came up from the students.

In the context of the children's games, the word symbols used to represent large quantities is one that points out the abstract thoughts of the students. As the word symbols used by the students were different from each other, the letters used by them in formulating equations were the abbreviations of the corresponding words. In this manner, all students made use of a unique nonsensical symbol to represent the high scores of each game. The context of estimation of game scores along with the acquired algorithmic knowledge of the students that helped them arrive at the game score, prevented the rejection of the arbitrary symbols (abbreviations) in the mathematical discourse of the classroom. In the context of the games where the students of the same class made use of different non-number symbols to denote game scores, while also accepting the other non-number symbols used by the rest of the class through classroom discourse, the algebraic thought that the same number can be expressed using different symbols developed in the students. This moved the comprehension of the students forward from concrete thinking to abstract thinking.

Through the everyday concepts of the students that were used in the mathematically oriented axiomatic discourse of the classroom the researcher enabled the students to comprehend the non-number symbols as algebraic variables and the calculations that involved these variables as algebraic equations. This becomes clear from the examples suggested by the students and those that they wrote down in their notebooks. A close examination of the erroneous responses of the students helped them analyse the mistakes they had committed by themselves while also allowing the rest of the class to think dynamically. Situations like these served as the platform for the development of divergent and abstract thinking in the students. It was the participation of the students that made it clear that they were able to handle algebraic concepts in the flexible and abstract format.

The students were seen to be totally involved in the method of using games in teaching. As a result, the method of the researcher to introduce the concepts of algebra appeared easy for the students. The general belief with respect to the difficulty in comprehension of algebra in students is that the difficulty lies in the transition from numbers to letters. This is because of the fact that algebra is still wrongly taught as a replacement of letters by alphabets. However, with the use of games, students have proved themselves efficient to think beyond numbers. In all the activities they took part in, they were able to carry out mathematical operations without the use of numbers whenever and wherever required, showing that they are able enough to understand abstract concepts and carry out its operations. It is this ability that initiated the comprehension of algebraic thought. Through the examples they presented, it was evident that the students imbibed the abstract thought of algebra effortlessly.

The researcher introduced the algebraic concepts through the metaphors and metonymies suggested by the students thus depriving them of any difficulty pertaining to familiarity. The students had even converted few sentences presented by the researcher into other formats that suited their comfort level.

For example, after having completed writing, Aswanth, one among the students raised a doubt. He wanted to know if the game score could be obtained by multiplying it with 100, provided the score in the stage is 100. The researcher assured Aswanth that his observation was correct. Thereafter the researcher asked the students to substantiate the argument. Aswanth replied that he had suggested so because of the similarity in method, which he had observed, this game shared with the former game, '*kothan kallu'*. Researcher took into consideration the observation and inference suggested by Aswanth. The researcher asked the rest of the class if any of them had the same idea in mind. A number of students marked their agreement to this. Ananya observed that while the maximum score in '*kothan kallu'* was 5, the same in '*noottam kolu'* was 200. The researcher asked the students if like in the game of '*kothan kallu'* where scores ranging from 1 to 5 could be earned successively, scores from 1 to 200 could be possible or not in the case of '*nootam kolu'*. The students marked their disagreement in unison.

The researched asked for a reason and the students replied that the minimum score possible in '*nootam kolu'* was 10. Thereafter the researcher encouraged the students to find out the scores possible in '*nootam kolu'*. The students read aloud the multiples of 10 ranging from 10 to 200. The researcher subsequently wanted to know if the students could express the relationship they had already devised in a different manner without employing numbers. The students then proceeded to create an equation. They modified their equations by replacing '200 points' by 'p'. All the students were seen to have used the same alphabet, **AN= p X AN**, to cite the example of Ananya. On the request of the researcher, the students read aloud the possible values of 'p' in the game of '*nootam kolu'* (10, 20, 30,...200) and in the game of '*kothan kallu'* (1,2,3,4,5). The researcher posed the next question. He wanted to know if the same variable 'p' could be used in both sets of equations, as the value of 'p' differs in both the games. Interestingly, Shyam responded with a counter question. As the game score varies even within a game, he questioned the validity of the researcher's query. The researcher answered that he has a lot to learn from them and went on to ask another question. He asked if any of the students had used the same letter to denote the number of levels in both the games. Few students like Ashique who had written '*kadalolam kali'* to indicate both '*nootam kolu'* and '*kothan kallu'*, Fathima who had denoted the same by '*vanolam kali'* and Abhinav who had written *'kayalolam kali'* raised their hands. They also added that the number of games in both the cases is the same. Ashique suggested that when he used the term *'kadalolam'*, he had in mind both the games. The rest of the class also had the same opinion.

All these instances present the picture that the students participated in a learning method in which they were active participants. Through the discussions carried out, the students realized that there are numerous paths to learn certain concepts and they were given the assurance that they need not stick to a single method. They also succeeded in understanding the logic behind various other methods adopted by different students in the same class.

IV. 3 Comparison between the Comprehension of the Concepts of the 'Unknown' and the 'Concept of Variability' in the Students of the Control Group and Experimental Group

A. Comprehension of the Students in the Control Group

The students of the Government run School became acquainted with the concept of algebra through the lectures of the teacher that were based on the textbook. It was through numerical problems that the teacher introduced algebra in the class. For example, in the problem of the angles formed upon a line, the value of the angle to the left was given. The students were expected to find out the value of the angle to the right. The notion of this uncertainty in numerical problems was used to present the idea of the unknown in algebra. It is the answers, of the questions in the sentence format that act as the vehicle that carry students through the concept of the unknown number in algebra.

The problem on the ages of John and Mary laid the foundation of variables in algebra. Students recognized that letters can also be used to represent numerical values meaningfully. It was through the same problem that the students encountered the concept of variability for the first time. In the actual sense, the intention of the teacher was to impart knowledge on the concept of variability with the support of equations. In the subsequent problems on the relationship between sides, lines, triangles, left and right angles etc. the variable chosen by the teacher was always the initial letter of the words. It was the transition from numbers to letters that the teacher referred to as the concept of the unknown. In other words, the concept of the unknown was explored in the class through letters. The teacher established the idea of variability through the transformation of sentence structures into equations consisting of numbers and alphabets. It is rather unsure as to what majority of students grasped the ideas of the unknown and variability presented by this design. The numerical values associated with variables become limited to a few in the examples proposed by the teacher in the class. This curbs the development of algebraic thought as something beyond mere numbers. The concept of the unknown in algebra becomes obscure and misrepresented. If the students were able to digest the concepts of algebra offered by the textbook, explained and exemplified by the teacher, they would not have encountered any difficulty in the movement from one type of algebraic problem to the other. This proves that instead of the formation of abstract concepts, the cognitive framework of the students become influenced by another concrete pattern of thought. The difficulties of comprehension in the students reported by the teachers justify the findings of the researcher. The unwritten law that variables ought to be initial letters of words in English, keenly followed and periodically reaffirmed by the teacher, leads to obstacles in the formation of equations in students. Stress on the use of English alphabets made students internalize the notion that algebra is a kind of mathematics where mathematical operations use English letters as an operation and not numbers. As this difficulty obstructs the formation of mathematics concepts in students, learning the thoughts propounded by algebra becomes impossible.

For example, in the case of explaining the relationship between sides, lines and triangles, the alphabets used by the different groups were distinct from one another. However there were similar trends among them. Most of the groups denoted the number of sides using '**v**'. '**v**' was used again to denote the number of lines. The alphabet **'t'** was used to denote the number of triangles. The teacher asked the reason for their choice of letters. The students replied that the first '**v**' comes from the Malayalam word "*vasam*" for sides, the second **'v'** from the word *"vara*" meaning lines and the third **'t'** from the word "*thrikonam*" for triangle. Another group (Alia, Faris and Saja) used **'s'** to denote the number of sides, 'v' for the number of lines and **'t'** for the number of triangles. The teacher was curious to know the reason for their different choice. It was Alia who replied. She suggested that they used the letter **'s'** to denote sides because the word 'side' was the English equivalent of the Malayalam term "*vasam*". The teacher smiled and asked the group why they did not consider using all terms in English. The class fell silent. The teacher assured the students that whatever they had written are right. However in Letter Maths we employ English letters. She added that therefore it was advisable to use English words and their initial letters.

The teacher gave the English word for the terms in the problem. She said that sides denote "*vasam*", lines denote "*vara*" and triangles denote "*thrikonam*". It shows that the mathematically oriented axiomatic discourse which happened in the algebra class doesn't resort to the use of knowledge or tools from the culture of the students in the introduction of algebraic symbols and concepts. Teacher served as a mere agent who transmitted the information in the textbook to students. The students remained in the class as recipients who recorded the information they received. By this method of teaching, mathematical imagination as well as visualization (ability to think without numbers which is necessary for algebraic thought) of the students are not meaningfully developed. Therefore the students are not moved from their concrete thinking (i.e. arithmetical thinking) to abstract thinking (i.e. algebraic operation).

B. Comprehension of the Students in the Experimental Group

The researcher introduced the concept of the unknown in the class through two games and their target points. He offered the students freedom to choose any method to calculate the target points of the games they selected to play, provided that the student scored the maximum possible points in each game. The student identified the different paths to arrive at the answers and thereafter the researcher presented a few metaphors that facilitated the mathematical operations without numbers possible with ease. While the researcher suggested a metaphor to denote size, the students proposed the rest. They carried out necessary operations with respect to the context of the games they took part in and formulated equations of the same.

The students never experienced the feeling of the impossible happening when they multiplied the maximum points to the metaphor of the sea to indicate a sea full of game points. The same effortlessness was observed in the cases of *kothan kallu* and *nootam kolu*.

The metaphors used by the researcher and the students such as sky, lake, ship, sea etc. attributed the feeling of largeness and enormity in the students. This feeling of vastness beyond the limit of calculation, association with the metaphors, helped students form proper connections between the metaphor and game points, the number in this instance. This was possible because the students became able to think mathematically beyond numbers. This was exactly the primary requirement of the concept of the unknown in algebraic thought. Variables were introduced as a means to break down the lengthy metaphorical relationship between games and score. The variables were in no manner similar to the ones students were familiar with in the government run middle school classes. Variability was introduced through relationships like a lake full of game points represented as 'game points of a single game X a lake full of games'

In other words, variability was explained through the use of metaphors that represented the unknown, but did not stand for numbers alone.

In this way, students were introduced to algebraic symbols, concepts and operations using their own everyday concepts in a mathematically oriented axiomatic discourse. In order to teach the concept of variability and equations, the researcher made use of the examples suggested by the students themselves rather than imposing textual examples on them. The method adopted was not limited to the substitution of numbers by letters.

This was the reason why the students were able to follow logic to carry out the operations in *nootam kolu*, with the help of the methods they made use of in the context of the *kothan kallu*.

After the completion of the games, the researcher aimed to explain the concepts of variables and variability through the comparison of familiar objects like red lucky seeds, palm tree seeds, green gram. The students became conscious of the relationship between the size and number of objects and the volume they occupy through the examples of objects in the coconut shell and used logic to form assumptions regarding the same.

This logic helped the students to point out the coconut shell with the maximum number of seeds, even without knowing the exact figure. This logic cleared the way for proper understanding of the concepts of the unknown and variability in algebra. The children initially struggled to think about quantities without employing numbers. But the use of common balance helped them to talk about quantities using abbreviations like the first letter '*che*' derived from the word '*cherupayar'* (green gram) to represent the given quantities in different coconut shells. The children compared these quantities using their already acquired algorithmic knowledge and talked about the quantities using abstract non-number symbols.

The erroneous comparison by some students provided opportunities to the teacher to introduce pedagogically the concept of 'unknown' and 'variability'. It also helped students to naturally reflect on the errors and engage meta-mathematically with the process that led to erroneous comparisons or equations. The concept of 'variability' that an abstract symbol can represent different quantities depending on the mathematical context was not fully derived by the students from their everyday observations. The teacher handled the problem of translating the mathematical concepts as visualized by the students while playing the games '*kothan kallu'* and '*noottan kolu'* into mathematical operation.

Every single example in the control group began with numbers and the algebraic meaning of the unknown was seen to be left unattended. In the process of making equal the number of green gram which is more in quantity compared to that of red lucky seeds and palm tree seeds, the students resorted to the use of addition and subtraction along with the employment of variables. Both the objects referred to in the equations formulated were selected and also the variables were chosen by the students themselves.

IV. 4 Semiotic Tools of the Control and the Experimental Group

A. Semiotic Tools in the Control Group

Signs, in mathematics can be described as a language that makes possible the manipulation of the ideas embedded in numbers. Every sign has a meaning. Every sign has a distinctive function. Human beings as a society have agreed through experience that a child understands the different mathematical signs and puts them to practice in his/her everyday lives. Numbers and signs therefore should be looked at as entities that are not constrained to the boundaries of a class room. It is the teacher or the educational system in practice that design the mode of perception of numbers and signs in the students either by the total negation of the practical knowledge of the student or partially considering the same. The situation of the class on algebra is no different. The students made use of the already learned algorithmic knowledge obtained through five years of schooling, to comprehend algebra as well. The signs that have already learnt in the text books of former grades are the signs of addition (+), subtraction (-) and multiplication (X). The teacher uprooted this system of knowledge and planted it in the different soil of algebra. Signs were put to use in the context of letters, instead of familiar numbers. Even if the students were able to identify connection between operations of numbers and signs in their daily lives, they failed to do so in practical situations. This is because of the problem in the examples suggested by the teacher to introduce signs. The nature of mathematical problems was exactly the same as those that they had come across in the previous standards.

For example, the problems used to introduce algebra in the Std VI, such as the problems related to sides and lines, relationship between angles of a triangle, relationship between the left and right angles formed upon a line, the match stick triangle made by Rani, etc.

These examples and illustrations, acts as mere reminders of the concepts they have already come across in the earlier classes. This memory fails to connect itself with the social, cultural, practical and historical knowledge achieved by the students in their growth and evolution. The process of blending numbers and letters is conceived as meaningless. By this method students internalize the notion that algebra is a kind of mathematics where mathematical operations use letters and not numbers. Therefore algebraic symbols and concepts remain a textbook bound system of knowledge. The insistence on the part of the teacher to use English words and letters drive the students further away from the concepts.

Textbook becomes the center of knowledge in the algebra classes. The teacher's choice of non-number symbols (technically metonymy of algebra), concepts (formation of equation by use of already acquired mathematical operations) and correct answers is dependent solely on the textbook. It was observed that much of the equations and non-number symbols (i.e. variables) suggested by the students were readily rejected without any chance of consideration. Therefore the students were left with no chance to establish relationships between their cultural artifacts and tools, and the concepts of algebra. A bridge between algebra and the everyday concepts of the students fail to be established, thus leaving algebra, nothing short of a scientific concept in comprehension. Teaching and learning algebra becomes a routine. This leads to a number of concerns in the teacher such as the students being able to deal algebra in class one and unable to do so in the next, students finding it impossible to understand the concepts in spite of the number of times they repeat the concepts, and the generalized comment that algebra is difficult for most students. The teachers tend to be unaware of the idea that a non-spontaneous concept in students is better introduced with the help of the institutions that cater to the needs of their spontaneous concepts.

The semiotics tools employed in the algebra classes in the government run schools was unfamiliar and strange to the students. The lived in experiences of the students, the situations they confront in their daily lives, and the cultural tools in their mother tongue are not entertained in the class as that is not part of the rigid structure of the textbook. Such knowledge, presented as distanced from that they had acquired so far, ends up being eliminated out of their systems. They do not find their way into their memory and thought process. This kind of a scientific knowledge has only a short life expectancy.

B. Semiotic Tools in the Experimental Group

Metaphors and metonymies specially developed by the researcher were used to introduce the concepts of algebra like variables, variability and equations in the class. The researcher enlightened the students with the help of a model on the use of metaphors to denote the abstract numbers and mathematics operation, metonymies to understand the significance of variables etc. The class was however was not dependent on the researcher’s models. Students suggested individual metaphors. The variables chosen were also the clear discretion of the students. The researcher provided only a method and not a fixed pattern.

The researcher suggested metaphors like sea full of games, lake full of games, sky high of games, etc. He went on to guide students form equations to convert algebraic thought developed by them and understand the practical applications of the same. The researcher also relied on the operations the students had learnt before. They were however contextually used by the students out of their logic alone and never out of compulsion or direction. The researcher cannot totally neglect the knowledge of the students, which they have acquired through years of schooling. The researcher provided space to the students to use these pre acquired knowledge while also offering new insights into the use of non-number symbols (variables) and signs. Students were given He helped the students understand the true purpose of signs and variables.

Students were given opportunity to use their own word symbols (*kappal, kadal, puzha* etc.) to denote huge quantities and it was replaced by non-number symbols (i.e. first letter of the object names in Malayalam). Students did not find it difficult to take them as a representational quantity term and used it in place of the meaningful word and continued to the mathematical operation using this symbol. Through this similarly designed mathematical activities non-number symbols (of students own) get placed as a metonymy of algebra.

While introducing variables, the researcher also paid attention to help students express their concepts in the form of the sentences, without the loss of clarity and logic and to further condense the same to the format of equations. He explained how equations can be used to express complex and abstract thoughts and their relations with absolute ease. As a result of which, the letters used to express short forms was freed from all kinds of linguistic restrictions.

The researcher demonstrated addition, subtraction and multiplication with the help of a balance. The researcher helped the students understand the concept of addition as no different from the idea it puts forward through language and signs, and its practical sense through demonstration. The researcher added substances to one side of the balance to show how the two plates balance themselves. The concept of subtraction and the real practical meaning of the minus (-) sign was also explained with the help of the balance. The researcher also made sure that the students carried out addition and subtraction without numbers using objects like red lucky seeds, palm tree seeds and green gram. The idea of equalizing and the meaning of the equal to (=) sign was also demonstrated with the balance. It was achieved by making equal the weight of the substances in both the plates of the balance. He explained the necessity of a mathematical sign, the equal to sign (=) through the activity of making the number of green gram equal to the number of palm tree seeds and red lucky seeds put together. This activity was also used to express the addition of two different variables, denoting different objects.

Even the language of equations was amended in the class. The class therefore experienced the movement from conventional methods of explanation, such as examples and illustrations, to the imaginative and visualized method of teaching employing word symbols, non-number symbols (metonymies) and balance scale. Through these metaphors and metonymies, the students experienced clarity in thought and expression of concepts. To think beyond numbers is easily said. Putting the same to practice has always pestered academics. In the class, in order to help students think of quantities rather immeasurable, he used metaphors like boat full. Such metaphors boost imagination in the students and help them think of the uncountable even without being conscious of the effort. The metaphors used by the researcher also did not point to any fixed answers. Answers shall vary according to people and situations. It is at this point that the students involved themselves in mathematical activities devoid of numbers, algebraically. After the example of the boat, the metaphors suggested by the students all stood for large non number objects that came out of their imagination. Thinking beyond numbers became effortless as word symbols (metaphors of large quantity) used was familiar to the students. Hence the algebraic knowledge attained by the students through the use of familiar metaphors and comfortable abbreviations, became part of their everyday lives as they were able to relate to every bit of it. This then opens the scope for symbolic resources of natural play way method work as extensional resources for establishing semiotics of algebra which is technically called metonymy. The linguistic resources required to scaffold these symbols, concepts and operations as seen in this teaching method are metaphorical in nature. The whole discourse in the class operates at the second and third order of symbolization. The replacement of physical quantity by a metaphorical word symbol by abbreviation of that word and by ‘x’ or ‘y’ or ‘z’ subsequently occurs due to the human abilities for symbolization and the cultural scaffolding of classroom mathematical discourse by the pedagogically mediated intervention of the researcher. Thus algebra perceived as a scientific concept was converted into an everyday concept through the intervention of the researcher. The new encountered concept found proper connections with the already acquired mathematical concepts. This enhanced abstract thinking and the developed concepts stayed deep within the mathematical thought of the students.

IV. 5 Pedagogy, Teaching – Learning Materials and ‘Continuous and Comprehensive Evaluation’: Comparison between Regular Class and Experimental Class

A. Regular Class

The teacher explained algebra based on the chapter on ‘Letter Maths’ in the Mathematics textbook of Std.VI. The teacher referred to a ‘Teacher’s Text Book’ to form the structure of the class she was about to finish regarding what to commence teaching with, the areas to be taught, the method of evaluation of the comprehension of the students, etc. The teacher follows every single guideline of the textbook and the course of the class room was directed by the same in the teacher- centric classroom. The level of comprehension and involvement of the whole class was judged through the responses of the active and bright students of the class alone. The inactive and average students of the class were neglected and their responses rejected. The teachers in middle school usually do not hold a graduation degree in their concerned subjects of teaching. Any teacher can be a mathematics teacher in middle school. As a result, their dependency level on the teacher's manual to deal with concepts like algebra is relatively high. This was the sole reason of their obsession with the methods and techniques suggested by the textbook. This adversely affects concept comprehension and development in the students. Students tend to depend on their teacher or any other student in the classroom to solve problems. In the absence of either the teacher or a classmate, many students find it impossible to solve mathematical problems. The very purpose of the class becomes futile.

Another issue lies in the undue stress on the analysis and opinions of the teacher which outweighs the opinions and suggestions of the students. This negatively affects the participation of the students. Instead of being spaces where exchange of ideas between the teacher and the students take place, classrooms are reduced to spaces where the teacher channelizes the students to follow definite patterns and methods of the textbook. This trend divides the classroom into a bunch of excellent students, average students and poor students. The teacher becomes an agent of the textbook, one which determines right and wrong in the classroom. Hence the ambiance of the regular class promoted divisions in terms of comprehension.

This trend draws students backwards from learning and understanding mathematical concepts. Apart from the binaries of right and wrong, the students also begin to internalize the misconception that classroom knowledge is scientific while practical knowledge lies in the world outside. This textbook oriented method of pedagogy that equips students to think and comprehend algebraic concepts in concrete terms alone. The examples and solutions presented in the textbook to introduce algebra make use of the existing familiarity of the students with numbers. As a result of which the word symbols as well as the non-number symbols that they employ to cultivate abstract thought work within the limits of this numerical knowledge only. This number bound concrete thought adversely influences and hinders the development of abstract mathematical thought in the students. It becomes pedagogically impossible to inculcate the abstract thought that is vital to the comprehension of algebra. This issue goes unaddressed and the difficulty the students face is conveniently interpreted as the difficulty in transition from number oriented mathematical thinking to variable oriented mathematical thinking.

Hence learning becomes mechanical and serves the purpose of passing examinations alone. Students who work hard to meet this requirement are termed studious and the rest as lazy. The irony lies in the fact that even studious students do not understand the concepts clearly. If not, the Mathematics teachers who were also students at some point in their lives would have been able to present concepts they now deal with through methods that would enhance comprehension and thereby free the disciple of its difficulty pertaining to the teaching and learning of concepts. They would not have resorted to pedagogical tools and methods totally ignoring the cultural knowledge of their students.

Single textbook method along with common evaluation system rules out diversity in the classroom. The evaluation methods of the teacher are also based on the guidelines of the textbook. After introducing an algebraic concept, the teacher moves on doing similar problems and depending on the constraints of time, assign homework. The home works are looked at and ‘corrected’ in the following class. The evaluation does not cover the areas of the choice of techniques, the level of understanding, doubts remaining, etc.

The splitting of the classroom into small groups was another means of evaluation. In the context of the regular class, instructing the students to fill the blanks on the board that pertained to a problem on algebra, and calling out the next student if the first fails were the other evaluation techniques of the teacher. This method can be used to understand the number of students who could answer right and the number of students who could not answer right alone. This fails to touch upon areas like understanding why students made answers wrong or what factors affect the understanding of concepts in the students. The problem with the existing method of evaluation is that it identifies which student follows the patterns of the textbook and arrives at the right answers created by the textbook and which student does not. This has been going on for generations. The teachers put the blame on the students. Unfortunately, there has been no retrospection, to this date, regarding the teaching methods that have left students miserable for ages. The problem has to be tackled at the root level, the pedagogical level to be precise.

B. Experimental Classroom

The pedagogical methods adopted by the researcher depended on the games played by the students and also objects from their surroundings such as red lucky seeds, palm tree seeds and green gram. While dealing with the algebraic concepts, the researcher paid attention to the level of comprehension of the students and made sure that every student could understand and relate to the concepts he dealt with. The metaphorical 'word symbols' used by the researcher proved the same. The researcher succeeded in ensuring the active participation of the students in the learning process with the help of cultural tools and artifacts pertaining to them. The idea of participation was not limited to students responding to the questions of the researcher. They were seen to raise doubts and queries on the methods adopted by their friends, upon which they had issues on clarity. They made proper use of this discussion environment in play in the classroom in order to make logical connections. Freedom to voice opinions, space for discussion regarding clarity and high degree of participation of the students, characterized the method of teaching adopted by the researcher.

Through the mathematically oriented discourses of the above method the students were able to analyse examples other than those relating to the concepts of the abstract. At the same time, this divergent opportunity helped the students to critically analyse the examples proposed by the students. The researcher made use of this discourse to introduce the concepts of unknown, variability and the formulation of the equations that represent the relationship between the former two. The researcher adopted a pedagogy method that also focused on making them understand their mistakes. The alternative pedagogy method devised by the researcher focused not merely on making the students arrive at the right answers but envisioned them to arrive at efficient comprehension of abstract algebraic thought. The alternative pedagogy method scaffold this indispensable abstract thought.

All the students enjoyed equal status and consideration in a class where there were no restrictions upon right and wrong and linguistic excellence. The employment of familiar games and objects and exploration of diverse means of logic formation helped the students feel more comfortable and confident. The method of the researcher was a shift away from the teacher-centric method adopted in regular class to a student-centered approach. Discussion was the key to the introduction of the algebra concepts and the researcher played the role of an agent in the transmission of these ideas. His focus was on the formation of the algebraic thought in the students than making them merely involved in mathematical activities. The researcher introduced the concept of algebra paying focus on the crucial elements of meaningful visualization of algebraic concepts with the help of metaphors and metonymies. This facilitates the development of an inclusive concept in which algebra and everyday experiences of the children are intertwined.

The method of evaluation of the researcher was the analysis of the right and wrong responses of the students. The reasons raised by the students individually and then as part of groups, in arriving at answers were analysed logically. Another method of evaluation was the analysis of counter questions raised by the students. In the case of group activities, responses of each group were studied and analyzed by the rest of the group. This was another technique of evaluation. Evaluations were always centered up on the concept of logic formation. He also analysed the mistakes of the students. Comprehensive evaluation was done to ensure that the students understood every body’s logics clearly. Keen observation was paid on the ability of the students to transport themselves from one algebraic concept to the next with the help of pre acquired knowledge. Evaluation was also done on the equations formulated by the students.

To sum up, the crest of evaluation was on the development of algebraic thought in the students. It also aimed to examine the degree of comprehension of the students through metaphors, metonymies and equations they themselves proposed. Evaluation moved from one student to the next, ensuring participation of each students and providing everyone an equal space for expression of opinions and concerns. As the researcher discussed all the metaphorical word symbols and non-number symbols of the students, the arbitrariness of the symbols used was lost and all the symbols were accepted by the whole class. The researcher’s method deviated from the teacher’s method on the lane that his method was not confined or limited to a single correct answer.

SUMMARY AND CONCLUSION

Since antiquity, teaching algebra has been a task difficult to achieve (Lenart, 2004). Even though mathematics education has acquired new insights from cognitive revolution in psychology after later 1950s, prevalently celebrated teaching methods of algebra to this date cannot properly solve students' algebraic difficulties. One of the profound reasons behind this is that only a few studies had focused on teaching and learning of algebra. Among them, most of the researches followed the popular constructivist framework of Piaget. Quite often, algebraic difficulties of students are accommodated into the boundaries of learning algebra (Lenart, 2004) and it is explained as the difficulty of the students in the transition from arithmetic to algebra. It implies that the task of teaching algebra is to help the students to reach a sufficient understanding level for handling equations with both constants denoted by numbers and unknown quantities denoted by letters. Conventional algebra teaching methods follows this pattern. By this method, the algebraic difficulties of students are not fully solved (Schmittau, 2004). The access to instruction in algebra for every student is a cause of concern in schools (as Hernon, 2004 quoted Kaput, 1995). It highlights that more analysis is necessary in order to develop an understanding of what factors help students to be successful in algebraic mathematics and how a teacher can properly assist in achieving this goal. Therefore the present research has focused on the less discussed issues in the areas of learning and teaching algebra and it also studied the technique of visualization of algebraic concepts to overcome the difficulty in comprehension.

The cultural historical theory of Vygotsky has provided the theoretical and methodological paradigm for this study. The present study has employed the Vygotskian concepts such as 'Zone of Proximal Development', scientific concept and everyday concept to understand how learning takes place in the classroom where algebra is introduced through metaphors and how the facilitation of the algebraic construction in students in the intervention class differs from that of the usual classroom approach which uses conventional method to teach algebra.

The researcher used ‘between group design’ of experimental method. Therefore various research activities, which depended on the objectives of the research that suited the requirements of different groups, were distributed among experimental groups and control groups. The research activities of the controlled group were based on the chapter on Algebra of the mathematics text book of class VI. The research activities of the experimental group were based on the activities devised by the researcher based on the approach of Davydov and ‘Lackoff. Using Davydovian approach six activities were prepared to develop the ‘basic Metonymy’ for middle school algebra. Few games were selected for introducing algebraic concepts and a series of activities (by using balance scale) was developed using both Davydovian principles and the approaches of Lackoff and Nunez

Brief Analysis of control group: Class which Teaches Algebra with the Help of a Mathematics Textbook

1. The method by which algebra is introduced in the class:

The teacher presented algebra in the class by explaining the problems in the textbook and by making the students solve them. The teacher began with an introduction to the same and there after instructed the students to do the math by themselves. The teacher asked a few students to tell their present ages, their age ‘n’ years from now and to deduce a relationship between the two. The frame of all the problems posed by the teacher was either addition or subtraction, which the students were very much familiar with. While the students were used to solving problems which involved numbers, the teacher instructed them to use the names of students who told their ages in the class instead and thereafter to use only the first alphabet of their names.

The teaching of ‘algebra-maths’ continued with problems from the textbook from different spheres like the relationship between the number of lines and triangles, the relationship between the number of matchsticks and triangles formed with them, the relationship between the number of currency notes and their value etc. The teacher followed the same pattern throughout. He/She began with making the students solve problems which contained only numbers and later demanded them to replace the numbers with the names of objects, shapes etc. and thereafter to replace the same with the first letter of the names. This is the method by which the teacher introduced ‘letter maths’ in class. It was the initial letter of the names of the objects in English that was used every single time.

The method of abbreviation adopted by the teacher which made use of English alphabets to introduce the concepts of the unknown and variability, acted as an obstacle for the comprehension of algebraic concepts in the abstract manner in the students. This is because through the method adopted by the teacher, algebra is introduced as a mathematical problem that involved mathematical operations and English language alphabets. Instead of looking at the letters as non-number symbols that denote uncountable or immeasurable objects or quantities, letters are understood as mere abbreviations, i.e. the initial alphabet of the object or subject names in English.

2. The participation of the teacher and students in class:

It was observed that not all students were actively engaged in solving problems and saying the answers out loud in a class which was led by a teacher who resorted to rely solely on the textbooks. Many students were seen to merely write down things which they heard in the class. The teacher had the final word in all that took place in the class from start to end. Few students enthusiastically followed the teacher’s words and read out the answers. The teacher was also seen to modify answers so that they suit the model presented in the textbook. Many students were reluctant to answer as they feared of being wrong. The teacher did not discuss wrong answers. The students considered those answers which the teacher stressed and repeated to be right. All students were seen to participate in the process of repeating an answer stressed upon by the teacher. In cases were the students were uncertain of the answers, their voices were found to be low. When the students answered in low voices, the teacher was seen to choose the right answer from the responses and went on to discuss it. She paid no attention to the wrong responses. It was also noted that only a few students responded to the teacher’s questions.

3. **The ways in which the students handled problems:**

'Letter-Maths' problems which were represented using numbers were comprehensible to at least a few students. However they experienced difficulty when the teacher demanded them to replace numbers with letters even after the teacher provided them with enough models. It was observed that when the students were asked to express relations using alphabets, they began with the Malayalam words of the objects and then proceeded to the English alphabet whose sound matches with the vernacular. The teacher considered this method to be erroneous and that deeply troubled the students. Not all students were familiar with the English words and therefore viewed 'letter-maths' as an obstacle. This was a problem which was faced by majority of students in the class. Inability to understand the logic behind solutions when letters were used instead of numbers was yet another common issue. More over most of the problems in the text book which the teacher explained were of the same nature. As a result of which when names in 'Letter Maths' problems were replaced by objects or by shapes, students were left confused. Two or three were exceptional as they were successful in formulating equations in Malayalam. However they were dismissed by the teacher with the suggestion that they failed to find the English word and its initial letter. Students were able to come up with right answers when they worked together as a group. But not all students in a group participated in the activity and some remained silent. It was observed that most groups were led by a student whose answer was approved by the rest. Therefore all students who were part of a group did not try and find out answers. Also, every group had a student who performed well in class.

4. **The method by which the teacher checked the algebra comprehensibility of the students:**

The teacher presented examples from the textbook followed by questions of the same type to be solved by the students in groups. After the students arrive at the right answers, the teacher explains the steps by which the result was obtained.

Having finished, he/she enquires if all the students had understood the same. If the response from the students is positive, the teacher proceeds to the next problem.

5. **The metaphors and metonyms used by the teacher to explain algebraic mathematics:**

The teacher used examples from everyday life which employed numbers.

For example: How old will a student be 'n' years from today? What is the relationship between the matchsticks used and the triangles formed with them? What is the relationship between the cost of a pen and the number of students in a class? What is the relationship between the value and number of currency notes?

The letters used to represent the names of objects and people were always the initial letter of the English words. Through the mathematical operation these letters become metonymy of algebra. The short form of relations and also equations were written using the same letters (abbreviation). No students were seen to use examples of letters of their choice. Those who used letters different from those in the textbook were made to make necessary changes. The teacher approved only one form of equation as correct.

Brief Analysis of experimental group: Class which Introduced Algebra Differently from the Treatment of Traditional Textbooks

1. **The method by which algebra is introduced in the class:**

The researcher introduced algebra concepts through games like *noottam kolu* and *kothan kallu*. The students were made to express the relationship between the score they obtained in the games and the number of times each game was played. Apart from deducing relationship using the maximum score possible in both the games, the students were asked to formulate relationship between different scores and the number of steps required to meet the scores and thereafter create equations. Gradually the researcher demonstrated the use of a quantity which the students were familiar with which was different from usual numbers and expressed equations between the score and number of games using the new quantity. Students resort to the use of word symbols (eg. *Kappal, kadal* and *puzha* etc.) to express the magnitude of the game scores. Games act as vehicle that helps transport the students from numerical thinking to abstract thinking that is not number bound. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather unconscious development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers.

The students were then asked to formulate equations using first letters of the object. The replacement of physical quantities (5000, or 10000) by metaphorical word symbols like '*kappal*' by 'ka' or 'k' and by 'x' or 'y' or 'z' subsequently occurs due to the human abilities for symbolization and the cultural scaffolding of classroom mathematical discourse by the pedagogically mediated intervention of the researcher. The algebra thinking is essentially theoretical. It develops through a cultural-cognitive process of the kind discussed above. Expressing quantity concepts including relationship through algebraic operations develop gradually. The researcher brought about no changes to the Malayalam letters used by the students.

Using quantities like red lucky seeds, palm tree seeds and green gram in coconut shells, the students were made to compare different quantities and find out which was greater or lesser without using numbers. The students compare the quantities of uncounted objects like red lucky seeds, green gram, and palm tree seeds. In order to express the relationship between the objects mathematically, they employ the initial alphabets of the word names in Malayalam. The students derive the answers to the questions of the researcher through the conjoining of these letters and their already acquired algorithmic knowledge. Through this method which applies non number symbols and algorithmic knowledge without employing numbers, the students are seen to move from one stage of mathematical learning to the higher level.

With the use of a balance, the students were taught to represent equations employing the plus sign (+), minus sign (-), and equal to sign (=). All the equations were formulated without relying solely on numbers.

2. The participation of the teacher and students in class:

From the beginning, all the students actively participated in the class. They expressed interest in both playing the games and answering questions related to the same. Examples, images, short forms and letters that came up as a part of teaching algebra were all suggested by the students themselves. The teacher resorted to creating a frame which accommodated varieties of answers with his questions. The need to stick on to a single and uniform answer was ruled out. The researcher dealt with the problems in such a way that ensured every student understood the logic behind the problems. A discussion procedure mostly accompanied the process of finding out the answer. Discussions were held even when the problems were solved by the students alone or in groups. All students showed willingness to read out their answers loud. It shows that students are confident in their answers. Students were randomly chosen to read their answers. Other students were also seen to refute responses. This gives students an opportunity to understand other possible answers on the same concept. If the students have objection in the answers, the researcher utilized this space for a creative discourse for scaffolding the algebra concepts. The researcher however did not dictate answers. Instead he tried to create connections between the varieties of responses that came up from the students. This discourse leads them to acquire more clarity on the algebra concepts.

3. The way in which the students handled problems:

After playing games, the students learned mathematical expressions in the form of sentences through the relationship between the number of games played and the score obtained. Thereafter they studied how to convert verbal relationships into algebraic expressions. When examples were explained, the students themselves suggested the nature of the calculations. This happened even when the students employed metaphors in place of numerals. It shows that an abstract non-counting symbol replaces an exact quantity. Through non-counting score of games and other objects (‘*manjadikkuru’, ‘panamkuru’ and ‘cherupayar’*), the students were able to imagine mathematical situations and solve them better without the use of numbers. Similarly, when students use symbol words (eg: k, ക derived from ‘*kappal’* full of games) they did not doubt to take them as a representational quantity term and used it in place of the meaning full word and continued with the mathematical operation using this symbol. Once such a process is established both through similarly designed mathematical activities, non-number symbols get placed as a metonymy of algebra. The students handled those situations which involved sentences and their algebraic expressions with much ease unlike the usual classroom experience which dealt with numbers alone. Through group discussion and discourse with the teacher, students developed better algebraic thought.

4. The method by which the teacher checked the algebra comprehensibility of the students:

The children’s games, their scores, and the mathematical operations they made use of to arrive at the scores, served as the context that facilitated the growth of abstract thought in the students. Doubts related to the metaphorical ‘word symbols’ and associated metonyms (non-number symbols) used by the researcher were raised. It was the students who clarified the doubts of the researcher and the researcher understood the level of comprehension of the students through the same. The researcher made sure that he discussed all the responses from the students. A close examination of the erroneous responses of the students helped them analyse the mistakes they had committed by themselves while also allowing the rest of the class to think dynamically. Situations like these served as the platform for the development of divergent and abstract thinking in the students. It was the participation of the students that made it clear that they were able to handle algebraic concepts in the flexible and abstract format.

In another words, students who failed to get the answers right were made to understand their mistake for themselves and get a better idea of the concept and its logic. The researcher helped them to conceptually reach the correct answers approved by the whole class.

Researcher asked the students to compare the quantities of uncounted objects like red lucky seeds, green gram, and palm tree seeds. Researcher observed how the students logically talked about quantities in abstract terms and how they understood comparison of quantities in the light of mathematical operation. He also checked if the students were able to express the examples in algebraic form without the help of the teacher. He also checked if the students were still able to solve problems provided there was a change in the background of the questions.

5. The metaphors and metonyms used by the teacher to explain algebraic mathematics:

The relationship between the points scored and number of games played were explained using the games suggested by the students: *kothan kallu* and *nootam kolu*. In order to express the idea of greater points, the 'word symbols' suggested by the students such as a boatful of points, sky high points, sea full of points, lake full of points etc. were maintained. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather unconscious development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers. Word symbol is replaced by non-number symbols derived from the symbol words. Students used the non-number symbols as a representational quantity term. They used it in place of the meaningful word and continued with the mathematical operation using this symbols. Through this process non-number symbols get placed as a metonymy of algebra. The metonyms used were both the Malayalam alphabets and the English alphabets chosen by the students.

Mathematical concepts were introduced using red lucky seeds, palm tree seeds and green gram apart from the use of numbers. Mathematical calculations involving the initial use of numbers followed by objects was not the method resorted by the researcher. The method suggested by the researcher involved the use of objects alone and comparisons among the same. With the use of balance, the students could also witness the mathematical calculations in operation. Algebraic relations involving objects like red lucky seeds, palm tree seeds and green gram were formulated and the students were taught the logic behind the same. The palms of the students and also the researcher were used to demonstrate the difference in scales of measurement. Moreover, this was used to formulate various equations. The research made use of the balance scale to visually demonstrate the algebraically understood variables and their mathematical operations. By this method, the researcher succeeded in metaphorically extending the context of the already acquired algebraic knowledge and the non-counting symbols of the students

Major Focuses of the Discussion

The discussion is centered upon the objectives and research questions of the study. These can be broadly divided into five sections.

The emphasis of the first section is on how the concept of algebra is introduced in the control group. This is further divided into two: the method adopted by the teacher to teach the concepts of algebra and the level of comprehension of the students. In the control group, the teacher introduces algebraic concepts making use of the problems presented in the textbook without bringing about changes in the method of solution of the same. The teacher solely depended on the chapter named "Letter Maths" in the mathematics textbook of Standard VI. The teacher resorted to the use of numbers and their operations, such as addition and subtraction, which the students were already familiar with, in order to introduce the novel method of algebra. The teacher proceeded to letter maths after referring to the concept of the unknown to denote answers the students were yet to figure out and went on to substitute the unknown with English alphabets. In this manner, the teacher substituted the names of objects, persons, shapes, etc. with the initial alphabet of their English names and developed a method that proceeded to create combinations of two and three non-number symbols. The different stages of the method adopted by the teacher to teach algebra, tends to come a full circle once the formula involving letters has been made. The teacher relied on the ‘Teacher’s textbook’ for the above methods.

However, with the method adopted, it was evident that all the students were unable to comprehend algebra in the same manner. It was observed that when English alphabets were used instead of numbers, the students ranging from the bright to the least responding were equally doubtful. The major difficulty the students faced was that it was mandatory that they made use of the initial letter of English words while many were not familiar with the English names of the objects to be formulated into equations. Students were seen to use the English equivalent of the initial letters of Malayalam words as the variable. This method was dismissed as wrong by the teacher. This served as an obstacle in the proper comprehension of algebra in its very basic stage itself. The early formed assumption that algebra was a combination of numbers and English alphabets, along with the dependence on the English language which they are uncomfortable with, add to their difficulty in understanding the next stages of algebra. Instead of looking at the letters as non-number symbols that denote uncountable or immeasurable objects or quantities, letters are understood as mere abbreviations, i.e. the initial alphabet of the object or subject names in English. Abstract thinking in the students should necessarily be developed through the objects themselves that are represented by the abbreviations. If not the thought delivered by the object and the thought delivered by the abbreviations will be one and the same. Algebraic concepts if introduced through objects that are immeasurable or easily not countable would actually enable the students to think beyond the limits of numerical values taken into account the familiarity of the students with numbers. However, all the word symbols presented by the teacher were confined to the realm of the numbers. The students would surely be limiting their thinking to this particular figure or value. The employment of letters instead of numbers becomes meaningless in the actual sense. This prevents the growth of mathematical thinking beyond numbers in the abstract level in the students.

In the second section of the discussion, the focus of the researcher is on the introduction of the algebraic concepts in the experimental group and the comprehension of the students with the methods employed. The researcher made use of two games which the students were familiar with, namely, *Nootankolu* and *Kothankallu* to invite the attention of the students in order to introduce the concept of algebra in the class. The researcher employed these games to help the students understand the concept of the unknown and variable, which are crucial to the understanding of algebra. Games act as vehicle that helps transport the students from numerical thinking to abstract thinking that is not number bound. That the students were able to use word symbols instead of large numbers to express the same activities they spontaneously carried out while trying to attain the target game scores (number of games x maximum score of each game = total score), helps in the rather natural development of the abstract thought wherein mathematical operations are made possible outside the context of the numbers.

In order to make sure that the students understood the concepts beyond numbers, like the unknown, the researcher actively engaged the students by splitting them into groups and making them come up with examples of their own. It was through discussion in the class that it was decided whether the examples put forward by the students were conceptually right or not. The employment of this method helped in the development of the concepts beyond numbers instead of the conventional method adopted by the teacher that merely substituted numbers with letters. Eventually, the concepts beyond figures developed in the thought of the students along with the usage of already acquired mathematical operations, helps them formulate equations that are quite lengthy. It was at this point that the researcher introduced the concept of algebraic equations by further simplification of the lengthy equations formulated by the students. From the perspective of the students, the word symbols used to denote game scores along with the abbreviations they eventually employed to denote the same, radically changes their existing realities, i.e. mathematical operations are used alongside numbers. This change actually runs deep into the basic structure of the perception of the students and gets reflected in their thoughts. This change that the student is unaware of act as the fertile ground for the germination of further algebraic concept seeds in the subsequent algebra teaching and learning sessions.

The researcher did not insist on the usage of English alphabets (non-number symbols) as variables, nor did he disapprove of their use to formulate equations in the classroom. In order to present algebraic problems, the researcher utilized things from the everyday lives of the students, their games, play things (seeds) etc. The non-number symbols chosen by the students were the initial letters of the names of the objects in Malayalam. The researcher employed this method in agreement with the conventionally followed "agreed upon" method of text books that rely on the community of formal mathematicians for the introduction of the concept of variability in algebra. In the abstract sense, the comparisons between quantities were carried out using the symbols that served as abbreviations that stood for the chosen objects. The students made use of their already acquired algorithmic knowledge for the comparison of quantities as well as in their equalization. When mathematical operations are carried out along with abbreviations that are non- counting symbols, letters cease to be mere non-number symbols and are transformed into signs that stand for abstract quantities. This change in perception that results from the process of formation of equations helps letters to acquire a new dimension shaking itself free from its exclusively linguistic associations wherein a letter is merely an alphabet. Technically this perception becomes established as a semiotics of algebra, referred to as metonymy.

The teaching method of the researcher ensured active participation of the students and paid ample attention to the doubts and queries raised by them. Therefore the students enjoyed better clarity of thought while dealing with algebraic concepts. The students being able to find examples for themselves, being able to analyze examples and answers of fellow students, and also being able to explain why answers went right or wrong proves the efficacy of the pedagogy method of the researcher. In this manner, the researcher's method relied primarily on the students paying due consideration to their socio- cultural capital in order to foster novel algebraic concepts depending on the pre requisite knowledge (mathematical knowledge earned from previous classes and knowledge from everyday lives) of the students. In other words, students were introduced to algebraic symbols, concepts and operations using their own everyday concepts in a mathematically oriented axiomatic discourse.

The third section discusses how the students of the control group and the experimental group understand the concepts of the unknown and variability. In the control group the teacher introduced problems involving numerals and instructed the students to solve the questions presented. According to the teacher, the concept of the unknown stood for the answers which were not known at the moment and that needed to be found out. To carry forward the idea of the unknown, the teacher presented many questions in the descriptive form exactly as they were presented in the textbook. In the text book oriented method of the teacher, only English alphabets (initial letter of the English words in translation) were used to denote the concept of variability. The teacher also insisted on the use of the exact same English alphabets that were used in the solutions of the textbook. In this instance, fluency in the English language acts as the greatest barrier for the students to arrive at the correct answer. In most cases, the students instead of translation of words from Malayalam to English, transliterate names of objects/persons of the algebraic problems from Malayalam to English and prefer to choose the initial alphabet of the transliterated word, the letter in this case would be likely equivalent to its corresponding sound in Malayalam. This method rules out the possibility of the answers of the students from coming anywhere close to the answers presented in the text books. This adversely affects the comprehension of the students of the algebraic concepts that would follow. The conventional pedagogy method fails to consider the reasons why the students tend to make mistakes. It is the undue importance that the teacher devotes to the textbook that actually creates the binary of the right and the wrong in the students.

However in the experimental group, it was through games and metaphors from day to day lives that the researcher introduced the concept of the unknown. The researcher encouraged the students to come up with various other metaphors apart from the ones he himself had presented in the class. It was through the logical analyses of the same that the researcher introduced the concept of the unknown in class. The researcher's method rejected the notion of a single correct answer format. He examined how logically close or apart the students were from the concepts he introduced. The researcher's method to introduce the concept of variability enabled students to raise doubts, support/ reject the answers of the fellow students, and also encouraged them to create different answer formats. It was through the employment of a common balance that the researcher illustrated the transformation of numerical problems into their algebraic counterparts. The researcher did not insist on the usage of any particular language to formulate equations. As a result, the students formulated algebraic equations with ease and efficiently comprehended the logic behind the different problems. As the classes were carried out in the form of an open discussion, each student seemed to be totally engaged in the activity by voicing their opinions and anxieties in the classroom.

The fourth section focused on the semiotic tools of teaching algebra concepts. The students made use of the knowledge on mathematical operations and signs which they gained through five years of schooling, to comprehend algebra as well. The signs had already learnt the signs of addition (+), subtraction (-), and multiplication (X) in the text books of former grades. The teacher uprooted this system of knowledge and planted it in the different soil of algebra. Signs were put to use in the context of letters, instead of familiar numbers. Even if the students were able to identify connection between operations on numbers and signs in their daily lives, they failed to do so in practical situations. This is because of the problem in the examples suggested by the teacher to introduce signs. The nature of mathematical problems was exactly the same as those that they had come across in the previous standards. The semiotics tools employed in the algebra classes in the government run schools was unfamiliar and strange to the students. The lived in experiences of the students, the situations they confront in their daily lives, and the cultural tools in their mother tongue are not entertained in the class as that is not part of the rigid structure of the textbook. Such knowledge, presented as distanced from that they had acquired so far, ends up being eliminated out of their systems.

They do not find their way into their memory and thought process. This kind of a scientific knowledge has only a short life expectancy.

Metaphors and metonymies specially developed by the researcher were used to introduce the concepts of algebra like variables, variability and equations in the class. The researcher enlightened the students with the help of a model on the use of metaphors to denote the abstract quantities, metonymies to understand the significance of variables etc. Thinking beyond numbers becomes effortless as metaphorical word symbols used were familiar to the students. Hence the algebraic knowledge attained by the students through the use of familiar word symbols and comfortable non-number symbols (i.e. metonymies), became part of their everyday lives as they were able to relate to every bit of it. Thus algebra perceived as a scientific concept was converted into an everyday concept through the intervention of the researcher. The researcher made use of the balance scale to visually demonstrate the algebraically understood variables and their mathematical operations ('+', '-', and 'x'). By this method, the researcher succeeded in metaphorically extending the context of the already acquired algebraic knowledge and the non-counting symbols of the students. A culturally mediated algebraic thinking emerges as a result of this visualization process.

The focal point of the fifth section is the pedagogy, teaching and learning materials, and the continuous and comprehensive evaluation of the regular and the experimental class. It looks into how these factors aid in the formation of concepts of a new area like algebra in the students. Unlike most states of India, the syllabi of the government run schools of Kerala undergo continuous periodical changes. These changes brought about to text books and pedagogy always offer a student centric approach to education. However in mathematics classrooms, particularly while dealing with algebra, the objective of student centric teaching remains an unachievable aim. In the classrooms, while students are being taught new concepts, the difficulties they face when being exposed to something new is left unattended. It is due to this reason that in a regular classroom, even a well performing student is unable to comprehend things in its total sense. This reason is something which the teachers themselves accept in the interviews conducted.

Though textbooks and the examples put forward in the textbooks undergo periodical changes, it is always the teacher's manual which most teachers blindly rely on. This offers the suggestion that the changes in syllabi do not reflect in the method of pedagogy in classrooms. In this instance, teaching/learning materials stands for merely the textbooks and the examples within. Especially while dealing with areas like geometry, the pre-existing knowledge of the students serve to be crucial. In the classes on algebra, it was observed that the involvement and participation quotient of the students tend to drop in the subsequent classes. There was a stark distinction between the students of the class as one group of students who were able to arrive at the right answers and the other group which depended on the former group to arrive at the right answers. Even though the teacher divided the class into small groups, rest of the students of a group depended on the one student who the teacher approached for right answers in a select group. The difficulties and doubts of the rest of the students of a group seem to drown when the teacher approves the right answer of a select group.

The teacher finally enquired if all the students understood the concepts or not. All students were seen to reply in the positive though most have not understood much of what has been taught in class. The reluctance of the students to voice their inability of comprehension throws light upon the lack of liberty and democracy in conventional classrooms. This stage is crucial with respect to the skill of a teacher. The teacher should efficiently observe his students, try to imagine and think from the perspective of the students, and clarify doubts as an open discussion in the classrooms, thereby making concept formation less burdensome. However the teacher carries out the continuous evaluation of her students bearing in mind and making use of the knowledge they might have acquired in their previous standards, along with the recent knowledge on algebra they have been exposed to. This practice actually limits the very objective of continuous and comprehensive evaluation of the student's progress.

The continuous and comprehensive evaluation fails to unify the cultural and social knowledge of the students, and also overcome the obstacles the students face while dealing with a new concept like algebra. Be it in terms of pedagogy or study materials, educational modes undergo regular changes and reforms. However, the difficulties which many students suffer from are left unaddressed. The teachers tend to express their helplessness by suggesting that their wholehearted efforts do not reap results. This show how miles apart the students are from the scheme of education they are part of. The non- students (adults) who formulate syllabi are heavily influenced by difficulties which they assume to be experienced by the student community. The possible solutions developed by the adults to the possible problems supposed by adults themselves do not in any matter help with the real problems of the students. It does not accommodate the community of students as a whole.

From the experimental pedagogy method carried out by the researcher in the experimental group it was clear that every student being different each doubt reflected the thought of a particular student and so each individual response needed to be counted. By the method which encouraged dialogue on how an answer went wrong and how another proved right, the students experienced better understanding of concepts and also became confident to carry out diverse and individual methods to arrive at an answer. This is the method that actually promoted the student-centric pedagogy proposed by the existing syllabi. In the experimental classroom, the researcher’s mode of intervention was different. He became invisible at times, became a child the other time, and served as the voice of conceptual clarity through discussion of concepts of algebra to help students arrive at a conclusion comprehensible to all, the other times. In the method of pedagogy adopted by the researcher in the experimental classroom, the influence of the teacher by no means interfered with the autonomy of the students.

The teaching/learning materials of the experimental classroom comprises of the examples, images, metaphors, and games which the students themselves chose from their everyday lives or immediate environments. This helped them to be comfortable in the classroom, guaranteed participation in activities and made everyone enjoy a sense of inclusiveness in the classroom. There were no answers which did not carry the essence of the students, nor were there any discussions which were not connected to the day to day lives of the students. Though algebra as a concept was unfamiliar to them, each student on their own discovered a familiar route which was unique to each one of them. The course of the experimental classroom was steered forward by the researcher by solving the difficulties that arise while dealing with a scientific concept like algebra among the students themselves, through discussions on why and how an answer went wrong, and through continuous and comprehensive evaluation of the knowledge attained by the students from the previous classes with the backdrop of games they were familiar with.

A Mathematics Classroom that Accommodates the Students Completely

In a conventional classroom where textbooks serve as the repository of information/knowledge and teachers the agents who facilitates the movement of knowledge into the students, the role of the students is limited to that of a mere receiver obliged to receive anything that reaches them. The pedagogy of the regular classrooms continues to carry forward the assumption that the students are ignorant of the things the teacher teaches in the mathematics classroom. Though the State Council for Educational Research and Training (SCERT), Government of Kerala proposes student centric pedagogy in classrooms, the case of the mathematics classroom has undergone no evolution over the decades. It still revolves around the teacher, the text book and the students as it had always been in the past. By holding cluster meetings in the state level, the state actively tries to understand the myriad difficulties faced by the students in the process of learning. The mathematics teachers unanimously approve of the difficulty they experience, which is the inability of the students to understand the concepts. Though the cluster meetings devote a considerable stretch of time to discuss the difficulties, the absence of effective analyses of the blockages experienced by the students result in new text books being printed in which only the examples, figures, values and illustrations undergo change. The real change has to occur in the classrooms, where textbooks and textual examples are final. From being the agent who passes on textual information to the students, the teacher should become a facilitator for the students to attain new knowledge, helping them knock down the hurdles they are bound to witness by being empathetic and supportive to the students. In that case, the students and the teachers will never be at odds from one another. Though by practice it is the teachers who stand while the students are seated in a classroom, in the psychological sphere the students believe themselves to be on their feet standing and submissive while the teacher is viewed as being seated and dominant. In such a scenario, the students will be insecure to voice their doubts and opinions and further more point out to the inefficacy of the explanations of the teacher. This picture of a mathematics classroom has to change for the better, especially one in which algebra is dealt with.

A Mathematics Classroom where the Students can Discover Themselves

In a classroom that progresses on its reliance of a textbook, it is difficult for the students to identify themselves. This depends on the mode which the teacher adopts to present mathematical problems and their examples to the students. Coherence and comprehension can be guaranteed if teachers make use of examples suggested by the students themselves rather than imposing textual examples over their line of thought, and by the adoption of games or objects which students of the state are familiar with. When knowledge from the social and cultural environments of the students makes their entry into the classrooms, each student of a classroom becomes a part of the proliferation of knowledge that occurs. Lest knowledge becomes limited in range to mere information, that which do not carry the essence of the individual. The community of students would suffer from the degradation of being reduced to mere recipients of information that they are bound to remember and recollect. In this instance, students are likely to forget the information they had taken trouble to memorize very soon after they appear for the examinations. For the students, retaining until being examined becomes the only objective for storing information. The students become conditioned to store and delete.

In order to surpass this impediment, classrooms should develop an ambience that accommodates each student and necessitate their total involvement. Classrooms should cease to being centers of approval of the right and the rejection of the wrong. Each student ought to be made an active participant of the discussions in the classrooms. Every Mathematics classroom should therefore ensure to maintain the culture of open and democratic discussions.

Mathematics Concepts with Creative Discussion

Students are seen to face difficulties right at the beginning of learning a new concept like algebra. This was evident in the conventional classrooms. The involvement and interest the students exhibited in the first class suffered a considerable set back in the subsequent classes. This shows that the students find themselves distanced from the class. It is here that discussion becomes important. As in the experimental classroom where the teacher mediates and carries forward the discussion proposed by the students, the scope for creative discussion develops in the classroom. It is the teacher who has to carry forward the discussion in such a way so that it gets directed to the concepts to be taught thus aiding the students to understand concepts better. This ensured mathematics classrooms will turn out to be creative spaces where the unification of thoughts, imagination, and everyday knowledge that exists beyond the four walls of a classroom takes place.

The Importance of an Algebra Classroom which Nourishes Imagination

The primary difficulty which algebra poses to the students is the inability to cultivate thoughts that move beyond numbers. The algebraic variables like x, y, and z are taught and therefore grasped by the students to be substitutes for some numerical values. They understand the concept of the unknown in connection with a number whose value is not known at the moment. They comprehend algebraic variables as letters which stand for unknown numbers. This kind of pedagogy entraps students to think around numbers alone making abstract thinking nearly impossible. This is in contrast with the very objective of algebra which proposes mathematical thinking not confined to numbers. The pedagogy of conventional scheme serves as the greatest obstacle for the achievement of this aim. The development of thought beyond numbers in students becomes essential. Algebra lessons should cater to this development of the abstract thought that would facilitate the solving of mathematical problems without the use of numbers. It is in this respect that the need for a new method of pedagogy of algebra that promotes imagination and creativity in the students, and that which helps them imagine without reliance on numbers becomes crucial. The findings of the experimental group where pedagogy was based on metaphors as well as metonymies suggested by the students themselves, points to the indispensability of the need for revision of the curriculum and pedagogy

Inevitable Need for Mathematics-knowing Teachers in Middle School

In Kerala, Teacher Training Course (TTC) serves to be the qualifying course for teachers who choose to teach till Standard VII. By the time students reach middle school, the number of subjects increases, as a result of which these teachers (upper primary) are forced to take up specific subjects to teach. As the basic qualification to pursue TTC continues to be Std. XII, it is highly unlikely that any of these teachers have a Bachelor's Degree in the concerned subjects they teach. This applies to mathematics as well. The teachers in most cases have knowledge in mathematics that they acquired during their schooling till Std. XII.

From the point of view of a debate, it can be argued that in order to understand, analyze and teach from grade five to grade seven, this knowledge would suffice. Unlike other subjects, deep subject knowledge in mathematics on the part of the teacher can solve a lot of comprehension related problems in the students. A change in the treatment of the subject is paramount. Only then can the classroom discourse travel further from the information presented in the textbooks alone. The doubts of the students will not be limited to mere stares nor will the ego of the teacher be hurt by questioning. Creative solutions to the difficulties of the students can be obtained in cluster meetings.

Above all, the students would understand lessons with greater clarity of concepts. Otherwise, every concept which the teacher finds incomprehensible in the text book will remain incomprehensible to the students as well. Concepts which the teacher understands partially will also not be communicated to the students in completion. Teachers will turn out to be solely accountable for the inability of the students to understand the different concepts of algebra. Finally, for the students in their growth from primary classes to higher levels, mathematical concepts would remain episodes laboriously learned, memorized, recollected, yet forgotten very much like forgotten dreams.

LIMITATIONS OF THE STUDY

1) **Constraint of time:** Research period includes time for formulating questions, reviewing literature, designing the study, developing research tools, collecting data in the field, analysing the data for emerging themes and then analysing the themes to develop new understandings. A qualitative research is characterized by emerging rather than an existing static data and therefore it is difficult to assign a fixed time period within which the data can be collected and analysed. The limited time period imposed constraints on utilizing the richness of data for bringing out more themes.

2) **Considering response of all students:** Responses of all students in both experimental and control group were not fully discussed in the research. Responses and performance of the students were organized and discussed by the theme of objectives only.
3) **Observer's effect:** Classroom observations were an important source of data since the process of concept building using socio-cultural resources, the performance of students and the role of teacher in the mathematics class could only be observed in a classroom setting. Researcher's presence in the classrooms was therefore essential. However, it was observed that the researcher's presence in the class during the beginning of the fieldwork affected the interaction between the students and the teacher and between the students. The students tended to be quieter and occasionally distracted. Once the researcher's presence had become regular and a rapport had been formed between the researcher and the students and the researcher and the teacher, the interactions in the class became more spontaneous and the participants in the class felt freer to express their emotions, both positive and negative.
4) **Lack of Post- effect study:** The study has not attended to the effects of the experimental approach in the students of the experimental group who would eventually have become part of the regular class. It does not focus on whether or not the students of the experimental class would be able to cope with the conventional approach of the regular classroom, or look at them through a different light.

SUGGESTIONS FOR FURTHER RESEARCH

1) A longitudinal study with a larger sample can be undertaken with at least two to three time line data for the confirmation and generalization of the current findings.

2) An extensive research can be done on the basis of the present study to understand the mathematics curriculum and the existing teacher training programs in the Kerala school education.

3) An extensive research can be made in understanding the concept formation of the algebra mathematics.

4) Studies on the scope and applicability of linguistic techniques such as metaphors and metonymies in the pedagogy of number systems, dimensions, geometry etc. can be undertaken.

5) Studies can be undertaken to understand the importance of imagination in mathematics learning.

6) Studies can be undertaken to understand how a mathematics curriculum can be shaped to increase the creative nature of the children.

7) An extensive research can be done to understand the importance of inclusive education in the mathematics classroom.

8) An extensive research can be carried out on the neuro-psychological effects of metaphors and metonymies in the pedagogy of algebraic mathematics in the students.

REFERENCES

Amit, M., & Klass-Tsirulnikov, B. (2005). Paving a Way to Algebraic Word Problems Using a Nonalgebraic Route. *Mathematics Teaching in the Middle School*, *10*(6), 271- 276.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. *Educational researcher*, *25*(4), 5-11.

Arzarello, F. (2006). Semiosis as a multimodal process. *RELIME. Revista latinoamericana de investigación en matemática educativa*, *9*(1), 267-300.

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning*, *9*(3), 249- 272.

Banerjee, R. (2011). Is Arithmetic Useful for the Teaching and Learning of Algebra?. *Contemporary Education Dialogue*, *8*(2), 137-159.

Banerjee, R., & Subramaniam, K. (2012). Evolution of a teaching approach for beginning algebra. *Educational Studies in Mathematics*, *80*(3), 351-367.

Banerjee, R., Subramaniam, K., & Naik, S. (2008). Bridging arithmetic and algebra: Evolution of a teaching sequence. In *International group of the psychology of mathematics education: Proceedings of the Joint Meeting of PME* (Vol. 32, pp. 121-128).

Barber, H. C. (1925). Real improvement in algebra teaching. *The Mathematics Teacher*, *18*(6), 364-374.

Bazzini, L. (2001). From grounding metaphors to technological devices: A call for legitimacy in school mathematics. *Educational Studies in Mathematics*, *47*(3), 259-271.

Belkhir, J., Yarnevich, M., Shirley, L., & Charlemaine, C. (1995). Mathematics for all children: A multicultural race, gender & class analysis. *Race, Gender & Class*, 125- 160.

Berger, M. (2005). Vygotsky's theory of concept formation and mathematics education. In *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Bergen, Norway,* Vol. 2, 153-160.

Bills, L., Wilson, K., & Ainley, J. (2005). Making links between arithmetic and algebraic thinking. *Research in Mathematics Education*, *7*(1), 67-81.

Bishop, A. J. (1991). *Mathematical enculturation: A cultural perspective on mathematics education* (Vol. 6). Springer.

Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 412-446.

Booth, L. R. (1988). Children's difficulties in beginning algebra. *The ideas of algebra, K-12,* 20-32.

Booth, L. R., & Johnson, D. C. (1984). *Algebra: Children' Strategies and Errors: A Report of the Strategies and Errors in Secondary Mathematics Project*. Nfer Nelson.

Bose, A., & Subramaniam, K. (2011). Exploring school children's out of school mathematics. In *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 177-184).

Bourdieu, P. (1990). *The logic of practice*. Stanford University Press.

Boyer, C. B., & Merzbach, U. C. (1991). *A history of mathematics*. Wiley.

Bråting, K., & Pejlare, J. (2008). Visualizations in mathematics. *Erkenntnis*,*68*(3), 345-358.

Britt, M. S., & Irwin, K. C. (2008). Algebraic thinking with and without algebraic representation: a three-year longitudinal study. *ZDM*, *40*(1), 39-53.

Bush, S. B. (2011). *Analyzing common algebra-related misconceptions and errors of middle school students.* (Order No. 3502269, University of Louisville). *ProQuest Dissertations and Theses,,* 372. Retrieved from http://search.proquest.com/docview/969319136?accountid=142596. (969319136).

Cai, J. (2005). The Development of Students' Algebraic Thinking in Earlier Grades from Curricular, Instructional and Learning Perspectives. *ZDM* , 1-4.

Cai, J., & Moyer, J. (2008). Developing algebraic thinking in earlier grades: Some insights from international comparative studies. *Algebra and algebraic thinking in school mathematics*, 169-182.

Cameron, P. J. (1998). *Introduction to algebra*. Oxford University Press.

Carraher, D. W., & Schliemann, A. D. (2002). Chapter 8: Is Everyday Mathematics Truly Relevant to Mathematics Education?. *Journal for Research in Mathematics Education. Monograph*, 131-153.

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics education*, 87-115.

Carrizales, R. (2011). *Cognitive constructs in linear algebra; metaphors, metonymies, modes.* (Order No. 1518191, The University of Texas at El Paso). *ProQuest Dissertations and Theses,* ,123.Retrieved from: http://search.proquest.com/docview/1039154212?accounti d=142596. (1039154212).

Chae, J. L. (2005). Middle school students' sense-making of algebraic symbols and construction of mathematical concepts using symbols.

Chatters, A. W., & Hajarnavis, C. R. (1998). *An introductory course in commutative algebra*. Clarendon Press.

Civil, M. (1998, April). Bridging in-school mathematics and out-of-school mathematics: A reflection. In *Proceedings of the Annual Meetings of the AERA*.

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 16-30.

Cobb, P. (1995). Cultural tools and mathematical learning: A case study.*Journal for research in mathematics education*, 362-385.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational psychologist*, *31*(3-4), 175- 190.

Cobb, P., Yackel, E. & McClain, K. (2009). *Symbolizing and Communicating in the Mathematics Classroom*. Mahwah: Lawrence Erlbaum Associates, Inc.

Cobb, P., Yackel, E., & McClain, K. (Eds.). (2000). *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design*. Routledge.

Cole, M. (1971). The Cultural Context of Learning and Thinking: An Exploration in Experimental Anthropology. *American Journal of Society*, 80(1),285-286.

d'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, *5*(1), 44-48.

Daniels, H., & Wertsch, J. V. (2007). *The Cambridge companion to Vygotsky*. Cambridge University Press.

Davydov, V. V. (1988). The concept of theoretical generalization and problems of educational psychology. *Studies in East European Thought*, *36*(3), 169-202.

Davydov, V. V., & Kerr, S. T. (1995). The influence of LS Vygotsky on education theory, research, and practice. *Educational Researcher*, *24*(3), 12-21.

Day, R., & Jones,G. A. (1997). Building Bridges To Algebraic Thinking.*Mathematics teaching in the middle school*, *2*(4), 208-12.

Denzin, N. K., & Lincoln, Y. S. (2011). *The SAGE handbook of qualitative research*. Sage.

De-Oliveira, L. C., & Cheng, D. (2011). Language and the Multisemiotic Nature of Mathematics. *Reading Matrix: An International Online Journal*,*11*(3).

Devrim, U. Z. E. L., & Uyangor, S. M. (2006). Attitudes of 7th class students toward mathematics in realistic mathematics education. In *International Mathematical Forum* (Vol. 1, No. 39, pp. 1951-1959).

DiMaggio, P. (1997). Culture and cognition. *Annual review of sociology*, 263-287.

Dixit, S., & Mohanty, A. (2009). Development of historical understanding among 9-to 14- year old children. *Psychological Studies*, *54*(1), 54-64.

Donald, M. (1991). *Origins of the modern mind: Three stages in the evolution of culture and cognition*. Harvard University Press.

Dörfler, W. (2008). En route from patterns to algebra: Comments and reflections. *ZDM*, *40*(1), 143-160.

Drouhard, J. P., Panizza, M., Puig, L., Radford, L., Chacón, A. M. A., Bagni, G. T., & Bardini, C. Working Group 6. Algebraic Thinking, 631.

E Nisbett, R., & Norenzayan, A. (2002). Culture and cognition. *Stevens' handbook of experimental psychology*.

Edwards, L. D. (2011). Embodied cognitive science and mathematics. *Proceedings of PME 35*, *2*, 297-304.

Edwards, T. G. (2000). Some" Big Ideas" of Algebra in the Middle Grades. *Mathematics Teaching in the Middle school*, *6*(1), 26-31.

Egodawatte, G. (2011). *Secondary school students' misconceptions in algebra.* (Order No. NR77791, University of Toronto (Canada)). *ProQuest Dissertations and Theses,* 215. Retrieved from: http://search.proquest.com/docview/919730022?accountid=142596. (919730022).

English, L. D., & Sharry, P. V. (1996). Analogical reasoning and the development of algebraic abstraction. *Educational Studies in Mathematics*, *30*(2), 135-157.

Fauzan, A., Slettenhaar, D., & Plomp, T. (2002). *Traditional mathematics education vs. Realistic mathematics education: Hoping for changes*.

Ferrini-Mundy, J., Floden, R., McCrory, R., Burrill, G., & Sandow, D. (2005). Knowledge for teaching school algebra: Challenges in developing an analytic framework. *American Education Research Association. Montreal, Quebec, Canada*.

Finken, T. M. R. (2002). *Patterns of metaphor use in algebra reform curriculum classrooms.* (Order No. 3058402, The University of Iowa). *ProQuest Dissertations and Theses,*,179-179p.Retrievedfrom http://search.proquest.com/docview/287848656?accountid=142596. (287848656).

Font, V., Bolite, J., & Acevedo, J. (2010). Metaphors in mathematics classrooms: analyzing the dynamic process of teaching and learning of graph functions. *Educational studies in Mathematics*, *75*(2), 131-152.

Font, V., Godino, J. D., Planas, N., & Acevedo, J. I. (2010). The object metaphor and synecdoche in mathematics classroom discourse. *For the Learning of Mathematics*, *30*(1), 15-19.

Frances, R. (1997). What Does Algebraic Thinking Look Like and Sound Like with Preprimary Children?. *TEACHING CHILDREN MATHEMATICS*, 297.

Gerdes, P. (1997). On Culture, Geometrical Thinking and Mathematics Education. In Powell, A. B & Frankenstein, *Ethnomathematics* (pp. 223-247). New York: New York Press.

Ginsburg, H. P., & Allardice, B. S. (1984). Children's difficulties with school mathematics. In B. Rogoff and J. Lave (Eds), *Every Day Cognition: Its Development in Social Context* (pp. 194-219). Cambridge, MA; Harvard University Press.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in mathematics*, *39*(1- 3), 111-129.

Greer, B. (2009). Culturally Responsive Mathematics Education. Routledge;New York.

Greer, B., Mukhopadhyay, S., Powell, A. B., & Nelson-Barber, S. (Eds) (2009). *Culturally Responsive Mathematics Education*. New York: Routledge.

Gregg, D. U., & Yackel, E. (2002). Helping Students Make Sense of Algebraic Expressions: The Candy Shop. *Mathematics Teaching in the Middle School*, *7*(9), 492-97.

Hershkowitz, R., & Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of socio- mathematical norms. *Educational Studies in Mathematics*, *39*(1-3), 149-166.

Hodgkin, L. (2005). *A History of Mathematics: From Mesopotamia to Modernity: From Mesopotamia to Modernity*. Oxford University Press.

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for research in mathematics education*, 258-288

Jorgensens, D. L. (1989). *Partecipant Observation: A Methodology for Human Studies*. Sage.

Joseph, G. G. (2011). *The crest of the peacock: Non-European roots of mathematics*. Princeton University Press.

Karp, A., & Vogeli, B. R. (2010). Russian mathematics education: history and world significance. *AMC, 10*, 12.

Kieran, C. (1988). Two different approaches among algebra learners. *The ideas of algebra, K-12*, 91-96.

Kieran, C. (1992). The learning and teaching of school algebra.

Kieran, C. (2004). The core of algebra: Reflections on its main activities. In *The Future of the Teaching and Learning of Algebra The 12 th ICMI Study* (pp. 21-33). Springer Netherlands.

Kieren, T. E. and S. E. B. Pirie (1991). Recursion and the mathematical experience. Epistemological foundations of mathematical experience. L. P. Steffe. New York.

Kiong, P. L., & Yong, H. T. (2001). Scaffolding as a teaching strategy to enhance mathematics learning in the classrooms. In *Proceeding of the 2001 Research Seminar in Science and Mathematics Education*.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2011). Middle school students' understanding of core algebraic concepts: Equivalence & variable. In *Early Algebraization* (pp. 259-276). Springer Berlin Heidelberg.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2011). Middle school students' understanding of core algebraic concepts: Equivalence & variable. In *Early Algebraization* (pp. 259-276). Springer Berlin Heidelberg.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence &Variable1. *Zentralblatt für Didaktik der Mathematik*, *37*(1), 68-76.

Kozulin, A. (2004). Vygotsky's theory in the classroom: Introduction. *European Journal of Psychology of Education*, *19*(1), 3-7.

Kriegler, S. (2007). Just what is algebraic thinking. *Introduction to algebra: Teacher handbook*, 7-18.

Kriegler, S. (2007). Just what is algebraic thinking? *Submitted for Algebra ic Concepts in the Middle School [J1, A Special*.

Lakoff, G. and M. Johnson (1980). Metaphors we live bv. Chicago, The University of Chicago Press.

Lakoff, G. and R. E. Nunez (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. Mathematical Reasoning: Analogies. Metaphors, and Images. L. D. English. Mahwah, NJ, Lawrence Erlbaum Associates: 21-89.

Lakoff, G. and R. E. Nunez (2000). Where mathematics comes from: How the embodied

Lamon, S. J. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education*, 170-193.

Lazar, N. (1934). Psychology vs. Tradition in the Teaching of Algebra. *The Mathematics Teacher*, *27*(1), 53-59.

Lefebvre, P. (1969). Mathematical Structures and THE ROLE of ALGEBRA in School Mathematics. *The Mathematics Teacher*, *62*(8), 673-678.

Lénárt, I. (2004). What Is Wrong with the Teaching of Algebra. *For the Learning of Mathematics*, *24*(3), 30-32.

Lennes, N. J. (1909). Modern tendencies in the teaching of algebra. *The Mathematics Teacher*, *1*(3), 94-104.

Lerman, S. (2002). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. In *Learning discourse* (pp. 87-113). Springer Netherlands.

Lew, H. C. (2004). Developing algebraic thinking in early grades: Case study of Korean elementary school mathematics. *The Mathematics Educator*, *8*(1), 88-106.

Linchevski, L. (1995). Algebra with numbers and arithmetic with letters: A definition of pre- algebra. *The Journal of Mathematical Behavior*, *14*(1), 113-120.

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational studies in mathematics*, *40*(2), 173-196.

Lindblom, J., & Ziemke, T. (2002). Social situatedness: Vygotsky and beyond.

Lins, R. C. (1992). *A framework for understanding what algebraic thinking is* (Doctoral dissertation, University of Nottingham).

MacGregor, M., & Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, 449- 467.

MacGregor, M., & Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, 449-467.

MacGregor, M., & Stacey, K. (1997). Students understanding of algebraic notations 11– 15. *Educational studies in mathematics*, *33*(1), 1-19.

Malisani, E., & Spagnolo, F. (2009). From arithmetical thought to algebraic thought: The role of the "variable". *Educational Studies in Mathematics*, *71*(1), 19-41.

Matusov, E. (2011). Irreconcilable differences in Vygotsky's and Bakhtin's approaches to the social and the individual: An educational perspective.*Culture & Psychology*, *17*(1), 99- 119.

Matz, M. (1980). Towards a computational theory of algebraic competence. *Journal of Mathematical Behavior*, *3*(1), 93-166.

Mishler, E. G. (1991). *Research interviewing*. Harvard University Press.

Moffett, P., & Corcoran, D. An evaluation of the implementation of Realistic Mathematics Education (RME) within primary schools in the North and South of Ireland Final Report.

Mohanty, Ajit K., Panda, M. and Pal, Rashim. (2010). Language policy in education and classroom practices in India: Is the teacher a cog in the policy wheel? In Ofelia Garcia & Kate Menken (eds.), *Negotiating language policies: Educators as policy makers.* London: Routledge.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research?. *Educational studies in mathematics*, *61*(1-2), 219-245.

Moseley, B., & Brenner, M. E. (1997). Using Multiple Representations for Conceptual Change in Pre-algebra: A Comparison of Variable Usage with Graphic and Text Based Problems.

Mowat, E., & Davis, B. (2010). Interpreting embodied mathematics using network theory: Implications for mathematics education. *Complicity: An International Journal of Complexity and Education*, *7*(1).

Nathan, M. J., & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction*, *18*(2), 209-237.

National Curriculum Framework (2005). National Council for Educational Research and Training. New Delhi: NCERT.

Nelissen, J. M. C. (1999). Thinking skills in realistic mathematics. *Teaching and learning thinking skills*, 189-213.

Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics*, *70*(2), 159-174.

Noss, R. (1986). Constructing a conceptual framework for elementary algebra through Logo programming. *Educational Studies in Mathematics*, *17*(4), 335-357.

Novitasari, I. (2007). Realistik Mathematics Education (RME): Pendekatan Pendidikan Matematika Dalam Konsep dan Realitas. *Jurnal Pemikiran Alternatif Pendidikan*, *12*.

Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education.*Educational studies in mathematics*, *39*(1-3), 45-65.

Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education.*Educational studies in mathematics*, *39*(1-3), 45-65.

Panda, M. & Cole, Michael. (2007b). As-if Discourse, Inter subjectivity and Mathematics Learning in Schools: Swapping between two Discourses. Conference Proceeding of the 2nd Socio-cultural Theory in Educational Research and Practice Conference, 10- 11 September 2007, University of Manchester, UK.

http://www.education.manchester.ac.uk/research/centres/lta/LTAResearch/Sociocultu ralTheoryInterestGroupScTiG.

Panda, M. (2004). Culture and Mathematics: A Case Study of Saoras. In Karuna Chanana (ed.). *Transformative Links between Higher Education and Basic Education: Mapping the Field*. New-Delhi, SAGE.

Panda, M. (2006). Mathematics and Tribal Education. *Economic and Political Weekly*. No.2, Vol. XLI, Jan 14-26. No. 1, Vol 1, p. 8

Panda, M. (2007a). Saora Culture, *As-if* Discourse and Mathematics Learning. In Gang Zheng, Kwok Leung & John Adair (Eds.). *Perspectives and progress in contemporary cross-cultural psychology*. Beijing: China Light Industry Press.

Panda, M. (2009). Language matters, so does culture: Beyond the rhetoric of culture in multilingual education. In A. K. Mohanty, M. Panda, T. Skutnabb-Kangas & R. Phillipson, (eds). *Multilingual Education for Social Justice: Globalising the Local*. New Delhi: Orient Blackswan, 295-312.

Panda, M., Mohanty, A. K., Nag, S. & Biswabandan, B (2011). Report of the Longitudinal Study of MLE and non-MLE schools in Andhra Pradesh and Odisha, *Swara*, Vol. 6-7.

Pirie, S. and T. Kieren (1994). "Beyond Metaphor: Formalising in Mathematical Understanding within Constructivist Environments." For the Learning of Mathematics 14(1): 39-43.

Powell, A. B., & Frankenstein, M. (Eds.). (1997). *Ethnomathematics: Challenging Eurocentrism in mathematics education* (p. 63). Albany, NY: State University of New York Press.

Presmeg, N. C. (1998). Metaphoric and metonymic signification in mathematics. *The Journal of Mathematical Behavior*, *17*(1), 25-32.

Puig, L. (2004). History of algebraic ideas and research on educational algebra. Retrieved from https://www.uv.es/Puigl/icme-10.pdf

Pyke, C. L. (2003). The use of symbols, words, and diagrams as indicators of mathematical cognition: A causal model. *Journal for Research in Mathematics Education*, 406-432.

Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational studies in mathematics*, *42*(3), 237-268.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical thinking and learning*, *5*(1), 37-70.

Radford, L. (2010). Algebraic thinking from a cultural semiotic perspective. *Research in Mathematics Education*, *12*(1), 1-19.

Radford, L. (2010). Signs, gestures, meanings: Algebraic thinking from a cultural semiotic perspective. In *Proceedings of the Sixth Conference of European Research in Mathematics Education (CERME 6)* (pp. 33-53).

Radford, L. (2012). On the development of early algebraic thinking. *PNA*,*6*(4), 117-133.

Radford, L. (2014). Towards an embodied, cultural, and material conception of mathematics cognition. *ZDM*, *46*(3), 349-361.

Radford, L., & Puig, L. (2007). Syntax and meaning as sensuous, visual, historical forms of algebraic thinking. *Educational Studies in Mathematics*,*66* (2), 145-164.

Ramanujam, R. & Subramaniam, K. (Eds.) (2012). *Mathematics Education in India: Status and Outlook* , Mumbai: Homi Bhabha Centre for Science Education (TIFR). https://mathedu.hbcse.tifr.res.in/wp-content/uploads/2014/01/INP- Book_Mathematics-Edu-in-India_2012_KS-RR.pdf.

Rampal, A. & Marker, K. (2011). Embedding Authenticity and Cultural Relevance In The Primary Mathematics Curriculum. 12th International Congress On Mathematical Education, 2011.

Raymond, A. M., & Leinenbach, M. (2000). Collaborative action research on the learning and teaching of algebra: A story of one mathematics teacher's development. *Educational Studies in Mathematics*, *41*(3), 283-307.

Schmittau, J. (1993). Connecting Mathematical Knowledge: A Dialectical Perspective. *Journal of Mathematical Behavior, 12*(2), 179-201.

Schmittau, J. (1993). Vygotskian Scientific Concepts: Implications for Mathematics Education. *Focus on Learning Problems in Mathematics*, *15*(2), 29-39.

Schmittau, J. (2003). Cultural-historical theory and mathematics education. *Vygotsky's educational theory in cultural context*, 225-245.

Schmittau, J. (2004). The Developmemt of Algebra in the Elementary Mathematics Curriculum of V.V.Davydove. *The Mathematics Educator*, 60-87.

Schmittau, J. (2004). Vygotskian theory and mathematics education: Resolving the conceptual-procedural dichotomy. *European Journal of Psychology of Education*, *19*(1), 19-43.

Schmittau, J. (2005). The Development of Algebraic Thinking: A Vygotskian Perspective. *ZDM* , 16-22.

Schwartz, F. R. (1997). what does algebraic Thinking Look Like and Sound Like With Primary Children. *Teaching Children Mathematics* , 296-300.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and instruction*, *20*(2), 119-150.

Soares, J., Blanton, M. L., & Kaput, J. J. (2006). Thinking Algebraically across the Elementary School Curriculum. *Teaching Children Mathematics, 12*(5), 228-235.

Spielhagen, F. R. (2006). Closing the achievement gap in math: The long-term effects of eighth-grade algebra. *Journal of Advanced Academics*, *18*(1), 34-59.

Stacey, K. & Chick, H. (2004). Solving the problem with algebra. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The Future of Teaching and Learning of Algebra*. *The 12th ICMI Study* (pp. 1-20). Boston: Kluwer.

Stacey, K., Chick, H., & Kendal, M. (Eds.). (2004). *The future of the teaching and learning of algebra: The 12th ICMI study* (Vol. 8). Springer.

Steele, D. F., & Johanning, D. I. (2004). A schematic–theoretic view of problem solving and development of algebraic thinking. *Educational Studies in Mathematics*, *57*(1), 65-90.

Steele, D. F., & Johanning, D. I. (2004). A schematic–theoretic view of problem solving and development of algebraic thinking. *Educational Studies in Mathematics*, *57*(1), 65-90.

Steffe, L. P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education*, *25*(6), 711-733.

Stephens, A. C. (2006). Equivalence and relational thinking: Preservice elementary teachers' awareness of opportunities and misconceptions. *Journal of Mathematics Teacher Education*, *9*(3), 249-278.

Stetsenko, A., & Arievitch, I. (2002). Teaching, learning, and development: A post-Vygotskian perspective. *Learning for life in the 21st century: Sociocultural perspectives on the future of education*, 84-96.

Streefland, L. (1986). Rational analysis of realistic mathematics education as a theoretical source for psychology. Fractions as a paradigm. *European Journal of Psychology of Education*, *1*(2), 67-82.

Streefland, L., & van den Heuvel-Panhuizen, M. (1998). Uncertainty, a metaphor for mathematics education?. *The Journal of Mathematical Behavior*, *17*(4), 393-397.

Subramaniam, K. (2003). Elementary Mathematics: A Teaching-Learning Perspective. *Economic and Political Weekly*, 3694-3702.

Subramaniam, K., & Banerjee, R. (2004). Teaching arithmetic and algebraic expressions. In *Proceedings of the 28th Conference of the International* (Vol. 3, pp. 121-128).

Subramaniam, K., & Banerjee, R. (2011). The arithmetic-algebra connection: A historical-pedagogical perspective. In *Early Algebraization* (pp. 87-107). Springer Berlin Heidelberg.

Subramaniam, K., & Bose, (2012). A. Measurment Units and Modes: The Indian Context. 12th international congress on Mathematics Education.

Subramanian, A. R. (2012). Transforming the Elementary Mathematics Curriculum: Issues and Challenges . In K. S. Ramanujam, *Mathematics Education in India: Status and Otlook* (pp. 63-88). Mumbai : Homi Bhabha Center for Science Education .

Tall, D. O. (2007). Embodiment, symbolism and formalism in undergraduate mathematics education. *David Tall Home Page*.

Tonneson, V. C. (2011). *Teacher instructional practices designed to meet the individual learning needs of mathematically gifted/talented students in middle school algebra I.* (Order No. 3451828, The College of William and Mary). *ProQuest Dissertations and Theses,*, 424. Retrieved from http://search.proquest.com/docview/863625706?accountid=142596. (863625706).

Treffers, A. (1993). Wiskobas and Freudenthal realistic mathematics education. In *The legacy of Hans Freudenthal* (pp. 89-108). Springer Netherlands.

Tunks, J., & Weller, K. (2009). Changing practice, changing minds, from arithmetical to algebraic thinking: an application of the concerns-based adoption model (CBAM). *Educational Studies in Mathematics, 72*(2), 161-183.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. *The ideas of algebra, K-12*, *8*, 19.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. *The ideas of algebra, K-12*, *8*, 19.

Usiskin, Z. (1995). Why is algebra important to learn. *American Educator*, *19*(1), 30-37.

Van Amerom, B. A. (2003). Focusing on informal strategies when linking arithmetic to early algebra. *Educational Studies in Mathematics*, *54*(1), 63-75.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In *Encyclopedia of mathematics education* (pp. 521-525). Springer Netherlands.

Victoria, R. J. (2007). Professional Development Focussed on Children's Algebraic Reasoning in Elementary School. *Journal For Research in Mathematics Education*, 258-288. (Schliemann, 2002).

Vygotsky, L. S. (1980). *Mind in society: The development of higher psychological processes*. Harvard university press.

Wadsworth, B. J. (1996). *Piaget's theory of cognitive and affective development: Foundations of constructivism*. Longman Publishing.

Wagner, R. (2013). A Historically and Philosophically Informed Approach to Mathematical Metaphors. *International Studies in the Philosophy of Science,27* (2), 109-135.

Walqui, A. (2006). Scaffolding instruction for English language learners: A conceptual framework. *International Journal of Bilingual Education and Bilingualism*, *9*(2), 159-180.

Wertsch, J. V. (Ed.). (1986). *Culture, communication and cognition: Vygotskian perspectives*. CUP Archive.

Windsor, W. (2010). Algebraic thinking: A problem solving approach. *Shaping the future of mathematics education*, 665-672.

Windsor, W. (2010). Algebraic Thinking: A Problem Solving Approach.*Mathematics Education Research Group of Australasia*.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities—A Contemporary Journal*, *3*(2), 49-60.

Yackel, E. (1997). A foundation for algebraic reasoning in the early grades. *Teaching children mathematics, 3*, 276-281.

Yerushalmy, M. (2006). Slower algebra students meet faster tools: Solving algebra word problems with graphing software. *Journal for Research in Mathematics Education*, 356-387.

Zack, V., & Graves, B. (2002). Making mathematical meaning through dialogue:"Once you think of it, the Z minus three seems pretty weird". In*Learning Discourse* (pp. 229-271). Springer Netherlands.

Zandieh, M. J., & Knapp, J. (2006). Exploring the role of metonymy in mathematical understanding and reasoning: The concept of derivative as an example. *The Journal of Mathematical Behavior*, *25*(1), 1-17.

Zazkis, R., & Liljedahk, P. (2002). Generalization of patterns: The tension between algebraic thinking and algebraic notation. *Educational studies in mathematics*, *49*(3), 379-402.