COLOR IMAGE THRESHOLDING USING ENTROPIC APPROACH

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MASTER OF TECHNOLOGY
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BY
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CERTIFICATE

This is to certify that the dissertation entitled "**COLOR IMAGE THRESHOLDING USING ENTROPIC APPROACH**" being submitted by Mr. **Nikhil Kumar Rajput** to the School of Computer and Systems Sciences, **Jawaharlal Nehru University**, New Delhi, in partial fulfillment of the requirements for the award of the degree of **Master of Technology** in **Computer Science and Technology**, is a record of bonafide work carried out by him under the supervision of **Prof. Karmeshu.**

This work has not been submitted in part or full to any university or institution for the award of any degree or diploma.

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DECLARATION

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The matter embodied in the dissertation has not been submitted for the award of any other degree or diploma in any university or institute.

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Dedicated to

My country

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Nikhil Kumar Rajput

Abstract

A new framework based on Tsallis entropy is proposed to threshold color images. Using this framework different colors of the images are considered separately unlike the method due to Otsu which is currently in vogue. Our methodology is found to perform better than the Otsu method in most of the images that we have considered. Further the parameter q of the Tsallis entropy is found to be different from unity. This indicates the role of non-extensive entropies and their relevance in image analysis

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Chapter-1
Introduction

An image is representation of an object which is basically sampled quantized function of two dimensions. This sampled quantized function has been generated by optical means sampled in equally spaced rectangular grid pattern with gray level intensity values quantized in equal intervals. Thus we can say that, an image is a rectangular grid of pixels. It has a definite height and a definite width counted in pixels. Each pixel is square and has a fixed size on a given display. A color image, on the other hand has extra 32 bits in contrast to gray scale image. Here 8 bit each for red, green and blue colors and the remaining 8 bits to the transparency.

An Image can be considered as a composition of object and its background. This results in varying gray level of pixels, which can be captured by an image histogram. This feature of image histogram can be utilized to categorize the image into its object and background component. This is an important problem and is called image thresholding. Thresholding is the process of converting an image into its equivalent thresholded (or binary) image depending on the threshold value. The basic problem is to obtain an optimal value of the threshold.

1.1 Image histogram

Image histogram [1] refers to a histogram of the pixel intensity values and is used to cast image statistics in an easily interpreted visual format. Thus histogram is a graph showing the number of pixels in an image at each different intensity value found in that image. For an 8-bit grayscale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels in terms of grayscale values. It may be pointed out that one can adopt two procedures viz, it may simply be a picture of the required histogram in a suitable image format, or it may be a data file of some sort representing the histogram statistics.

There are various applications of Image histogram for example in Image capturing, improving visual appearance of image and in determining what type of processing has been applied to the image. Now most of the digital cameras come with the facility of plotting the histogram along with its analysis. For the purpose of illustration we consider in Figure 1(a) the image of JNU library building. The corresponding gray level histogram is given in Figure 1(b). In a similar manner histograms of color images, either in terms of red, green and blue channels can be obtained, or can be represented in 3-dimensions, so that the three axes representing the red, blue and green channels.

Fig 1.1(a) JNU Library

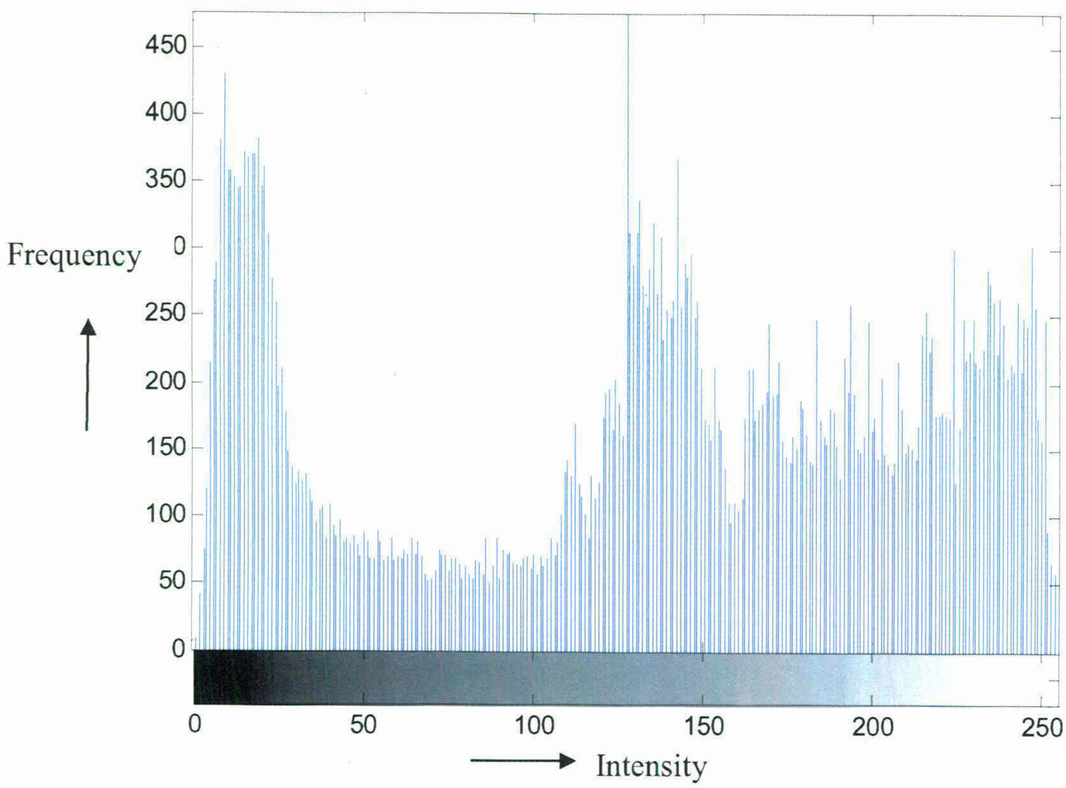


Fig 1.1(b) Histogram corresponding to Fig 1.1(a)

One of the important problems in image analysis is related to image segmentation which is discussed in subsequent subsection.

1.2 Image Segmentation

Image segmentation is a key step in image analysis. In this process, we divide an image into its components and thus enabling distinction of objects of interest from its background. The basis of image segmentation is based on some properties of the image such as color, intensity or texture. Each of the pixels in a region is similar to some characteristic properties mentioned above. There are several schemes of image segmentation, which are listed as follows –

- Region based segmentation
- Boundary based segmentation
- Template matching
- Texture based segmentation
- Thresholding

In **Region based method**, we partition the image into connected regions such that neighboring pixels of similar intensity levels are grouped together. Adjacent regions can thus be merged by using some criterion involving properties like homogeneity, sharpness of region boundaries etc.

The well known region based image segmentation model was introduced by An and Chen[2]. They decomposed image into a set of two regions viz Ω and Γ where Ω represent the bounded open set and Γ represent the smooth edges through which the regions separate.

In order to overcome some of the limitations of region based methods, **boundary based methods** were proposed to look for implicit boundaries between regions. The commonly used boundary based segmentation detection methods which are known as ridge detection and edge detection. Ridge detection follows the peaks (local maxima) in the original image. This segmentation scheme has been used by Wang [3] for lung tissue images.

When objects do not have large enough ridges at the region boundaries, the gradient operator can be used to enhance the boundaries. Edge detection is quite similar to ridge detection method except that ridges are tracked in gradient space instead of image space. The gradient space can be computed by simply applying the gradient operator to the entire image. The two dimensional gradient operator is defined as

$$\nabla I(x, y) \equiv i\frac{\partial}{\partial x}I(x, y) + j\frac{\partial}{\partial y}I(x, y) \qquad (1.1)$$

where I(x,y) is a function of two dimensional intensity levels.

Texture as an image feature is very useful in many image processing and computer vision applications. In texture classification and segmentation, the objective is to partition the given image into set of homogenous textured regions. Manjunath, Haley and Ma [4] used Gabor function and complex sinusoid having the one-dimensional form

$$g_S(x, w_c, \sigma) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{\left(\frac{-x^2}{2\sigma^2}\right)}\, e^{jw_c x} \qquad (1.2)$$

Gabor functions are used as complete, albeit non-orthogonal basic sets. Figure 1.2 shows some textured images as taken from [4]

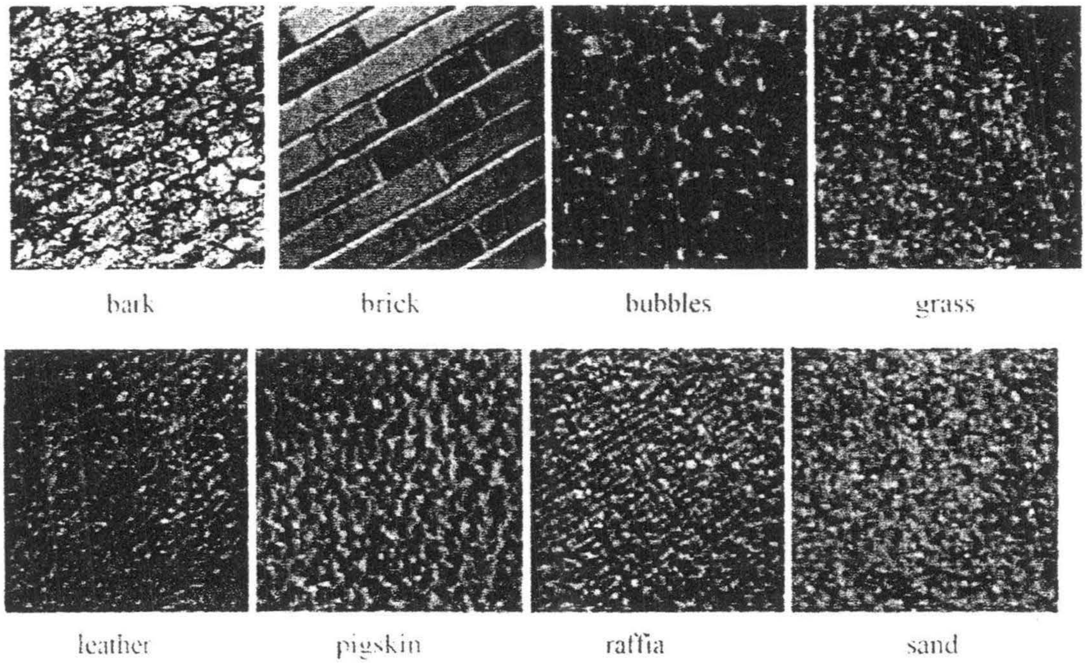


Fig 1.2 Textures (source Manjunath et al.[4])

An important problem in image analysis is the detection of the presence of an object in a scene. This problem can be solved by using a special type of an image segmentation technique .An a priori knowledge about the detected object (a template) is used to identify its location in a given scene. This method is termed as template matching based image segmentation.

Another technique which can be employed for finding matches of a searched pattern is based on image correlation technique. Here we attempt to locate the pattern w(x,y) of size JxK within an image F(x,y) of a larger size MxN.

The thresholding method of segmentation is described in detail in the following subsection.

1.3 Image Thresholding

Segmentation has always been a central problem in Image processing, which distinguishes the object from its background. This is done by clustering pixels on the basis of color, texture or intensity. There are various methods that have been proposed for segmentation like histogram method, edge detection method, region growing method, level set method, graph portioning method and watershed transforming methods. One of the methods of image segmentation is known as image thresholding and it requires the intensity values of the pixels.

Figure 1.2 shows the thresholding process, with background and object components (as adapted from [Matlab])

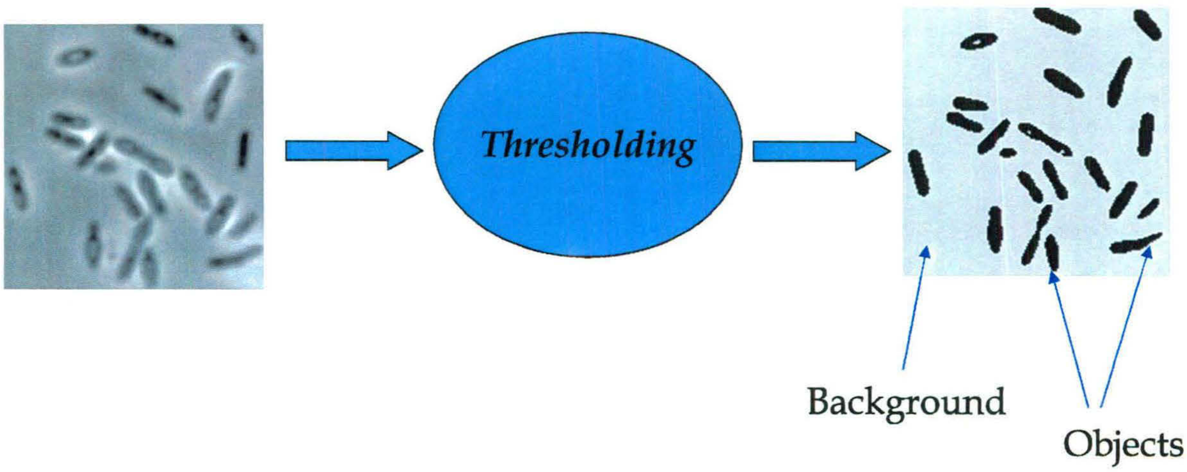


Fig 1.3 Thresholding process (image source[Matlab])

The variation of pixel intensity values can be used to transform the image to a binary (or thresholded) image using the concept of thresholding [1]. If the intensity value of the original image at (i,j)th pixel is denoted by a_{ij} and the binary image generated through thresholding by b_{ij}, then the transformed image can be represented as

$$b_{ij} = \begin{cases} 1 & if\ a_{ij} \geq T \\ 0 & if\ a_{ij} < T \end{cases} \tag{1.3}$$

where T is the threshold value.

The objective of image thresholding is to segment a given image so that the object is more distinguishable from its background. This has been an active area of research in image processing and several methods have been proposed as listed below [5]:

- Histogram shape based methods
- Clustering based methods
- Entropy based methods
- Object attribute based methods

One of the widely used approaches is based on the concept of entropy. The rationale for using the entropy framework is due to the fact that it provides a measure of uncertainty [6] of random distribution of pixel intensities.

1.3.1 Histogram shape based Thresholding methods

In this method, features captured by image histogram like the peaks, valleys and curvatures are analyzed to obtain the threshold value. Rosenfeld and Torre [7] proposed a method based on the concavities of the histogram vis-a vis its convex hull. He pointed out that the deepest concavity points become the candidates for the thresholding.

Sezan [8] proposed peak and valley thresholding method in which he carried out the peak analysis by convolving the histogram function with smoothing and differencing kernel .

Using a simple functional approximation to the PMF consisting of two step function shape modeling, Ramesh, Yoo and Sethi [9] proposed a new thresholding method. In Figure 1.3, the image histogram based thresholding process is depicted.

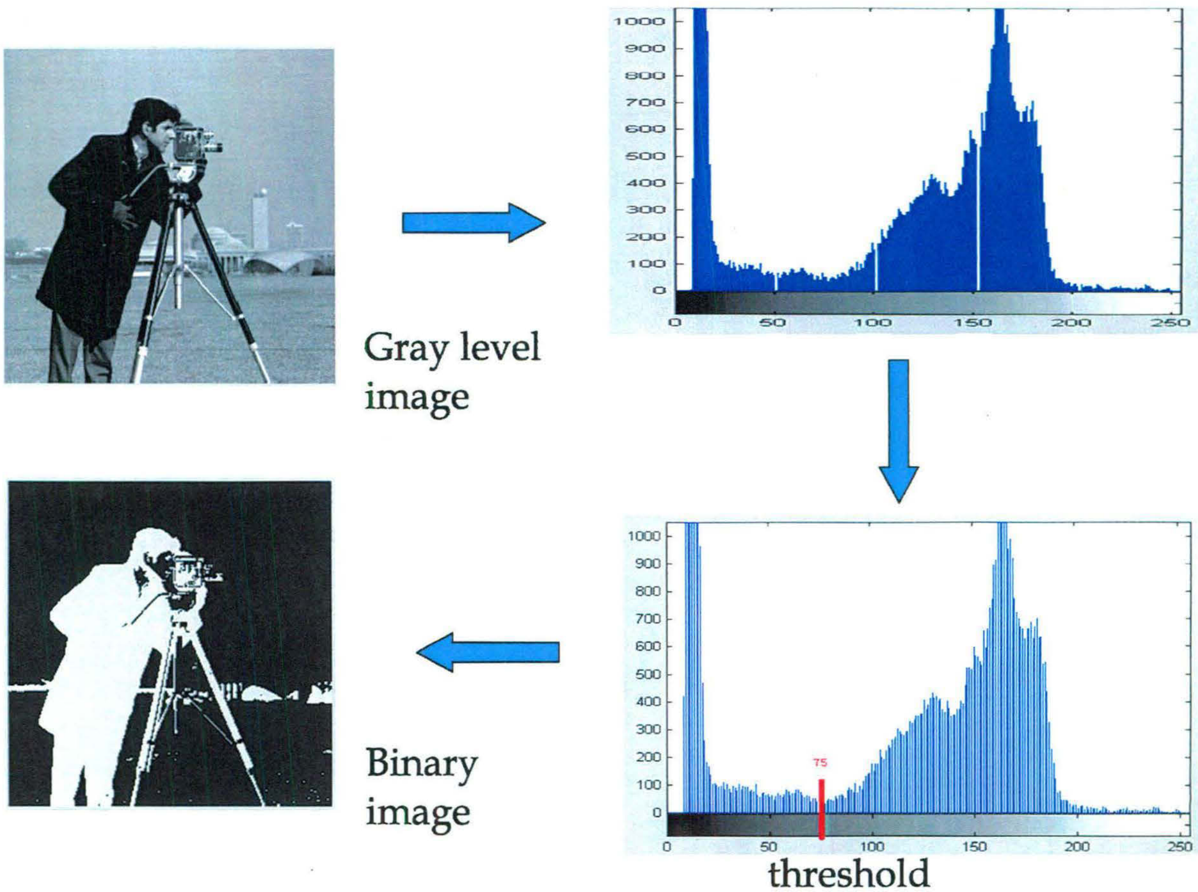


Fig 1.4 Thresholding based on histogram of image
(image source[MATLAB])

1.3.2 Clustering based Thresholding methods

The pixel intensity levels are sampled into two clusters as a background and foreground (object) or alternatively modeled as a mixture of two Gaussians. Ridler and Calvard [10] used two class Gaussian mixture models for his iterative scheme of cluster based image thresholding method.

In the context of a new clustering threshold method Otsu [11] suggested a procedure to obtain an optimal threshold value by minimizing the weighted sum of within class variances of the foreground and background pixels.

In minimum error thresholding method Lloyd [12], assumed that an image can be characterized by mixture distribution of foreground and background pixels. Lloyd [12] used an iterative search to minimize the total misclassification error assuming equal variance Gaussian density function.

Another framework based on fuzzy clustering thresholding assign fuzzy clustering membership to pixels depending on their differences from the two class means. This framework was adopted by Jawahar, Biswas and Ray[13].

1.3.3 Object Attribute based methods

These methods are based on similarity measures like fuzzy shape similarity, edge coincidence, gray level moments of gray level and binarized images.

Moment preserving thresholding method as suggested by Tsai [14] considers the gray level image as the blurred version of an ideal binary image.

In topological stable-state thresholding, Russ [15] found that thresholding level is adjusted at a point where the edges and shape of the object get stabilized.

S.K Pal and Rosenfeld [16] generalized the concept of fuzzy geometry in his 'enhancement of fuzzy compactness thresholding method'.

In the context of document binarization, Liu, Srihari and Fenrich[17] have considered document image binarization based on texture analysis.

The method based on entropy of the image has been discussed in detail in subsequent chapters.

Chapter-2
Entropy and Image

An Image can be characterized through the probability distribution of pixel intensity values, and entropy provides information content in the image. Different entropic measures exist in literature. The well known entropy measure is due to Shannon[6] which is widely used in information theory. Other entropic measures which can be regarded as generalization of Shannon's entropy are referred to as Renyi[6] and Tsallis[18] entropy. Here entropy is also called as parametric entropy which contains an additional parameter. In the limiting case both entropies tend to Shannon entropy. Besides these entropic measures we have also discussed here the cross entropic measure

2.1Entropy

Consider a random experiment, which can be described by a probability distribution $P=\{p_1,p_2,\ldots,p_n\}$ with possible outcomes $X=\{x_1,x_2\ldots,x_n\}$ such that $P(X=x_i)=p_i$, $i=1.2\ldots n$

It can be viewed as an ensemble $(X,p)=\{(x_1,p_1),(x_2,p_2),\ldots,(x_n,p_n)\}$

2.1.1 Shannon Entropy

Shannon [6] defined the entropy as

$$H_n(p_1,p_2,\ldots,p_n)\equiv H_n(P)=-\sum_{i=1}^{n} p_i \log_2 p_i \qquad (2.1)$$

We take $0\log_2 0=0$. The unit of information is bit. It can be checked that the maximum value of entropy is obtained when $p_1=p_2\ldots= p_n=1/n$ such that $H_{max}=\log_2 n$. The minimum value of entropy is 0 when one of the outcomes is certain to occur, say in the case of probability distribution $\{1,0,\ldots,0\}$. Entropy can be regarded as a measure of uncertainty.

Since images can be characterized through probability distribution, it is therefore appropriate that entropy can be meaningfully employed to characterize images.
Now as X is a random variable with probability mass function p_x , the eq() can be expressed as

$$H(X) = E[-\log p_X],$$

where E denotes the expectation.

H(X) can be interpreted as the average information provided by a realization of the experiment. Alternatively, one can say H(X) is average uncertainty removed or average amount of information gained by observing the outcomes of X. For n=2, we get Shannon's entropy function

$$f(p) \equiv H_2(p,1-p) = -p \log p - (1-p)\log(1-p) \qquad (2.2)$$

This can be interpreted as entropy of a Bernoulli variate X with parameter p and $H_2(p,1-p)$ attains its maximum value for p=1/2 . Thus

$$H_2(1/2,1/2) = 1 \text{ bit}.$$

This is known as the normalization condition.

The behavior of H2(p,1-p) is pictorially represented in Figure 2.1 . As expected as H2(0,1) and H2(1,0) are equal to zero and the maximum value of H2(p,1-p) is attained at p=1/2.

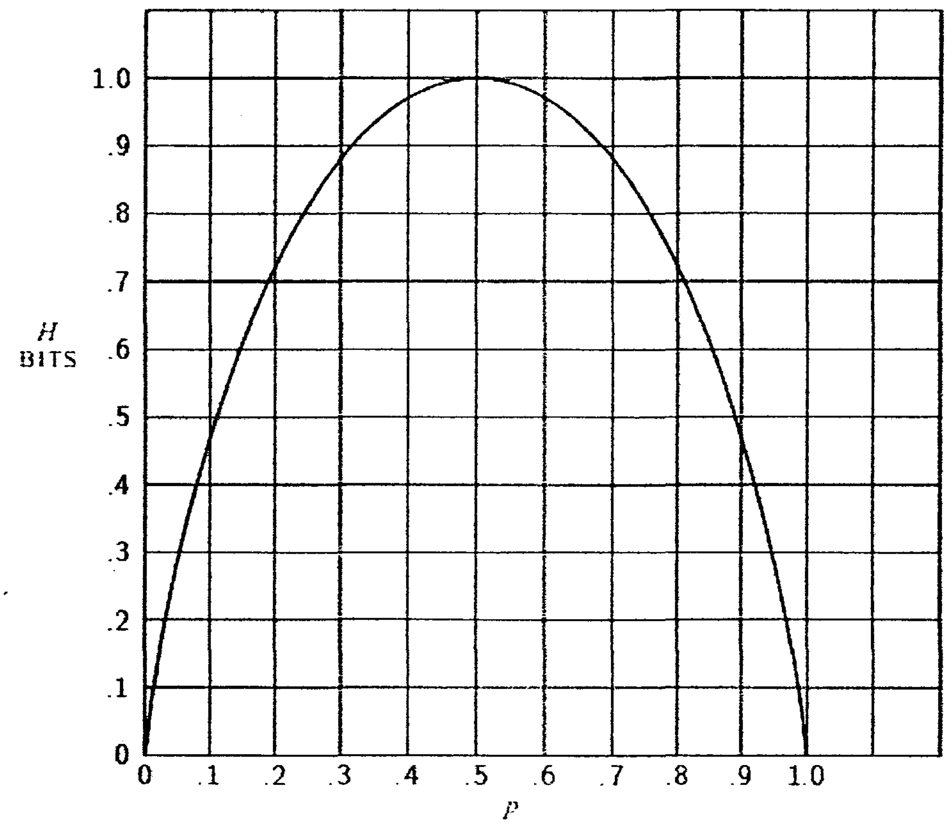


Fig 2.1 Entropy in case of two probabilities p and (1-p)

2.1.2 Joint and Conditional Entropies, Mutual Information

Let us consider a probabilistic system which can be described in terms of joint random variables X and Y defined in the same probability space. The joint entropy is given as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} \log p_{ij} \qquad (2.6)$$

where $p_{ij} = p(x_i,y_j) = P(X=x_i, Y=y_j)$, $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$ is the joint probability mass function. For convenience we write $P(X=x_i) \equiv p_i$ and $P(Y=y_j) \equiv q(y_j) \equiv q_j$. When X and Y are statistically independent, $p_{ij}=p_i q_j$, and it is easily seen that

$$H(X,Y) = H(X) + H(Y). \qquad (2.7)$$

From general considerations, statistical dependence of random variables X and Y results in reduction of their joint entropy. Such a situation makes it relevant to introduce conditional entropy $H(X|y_j)$ representing the amount of uncertainty about X when outcome y_j is observed; symbolically

$$H(X|y_j) = -\sum_{i=1}^{n} p(x_i|y_j) \log p(x_i|y_j). \qquad (2.8)$$

The conditional entropy H(X|Y) is defined as the amount of uncertainty remaining about X given prior knowledge of Y. It is obtained by averaging the above equation over all y_j's. Thus

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} q(y_j)p(x_i|y_j) \log p(x_i|y_j)$$
$$= -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,y_j) \log p(x_i|y_j) \qquad (2.9)$$

Similarly, we define

$$H(Y,X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{p}(\mathrm{x_i}, \mathrm{y_j}) \log \mathrm{q}(\mathrm{y_j}|\mathrm{x_i}) \qquad (2.10)$$

It can be observed that

$$H(X|Y) = H(X,Y) - H(Y), \qquad (2.11)$$

and

$$H(X|Y) \leq H(X) \qquad (2.12)$$

with equality when X and Y are statistically independent.

Further, we can visualize the random variables X and Y as input variable and output variable respectively in a system. For a stochastic system, H(X) represents the uncertainty about input X. H(X|Y) gives the conditional entropy of random variable X when the output Y is observed. The difference can be regarded as a measure of dependence of X and Y and

$$I(X;Y) = H(X) - H(X,Y) \qquad (2.13)$$

is called mutual information between X and Y

From eqn (2.9),(2.11) and (2.13) , we find

$$I(X;Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{p}(\mathrm{x_i}, \mathrm{y_j}) \log \frac{\mathrm{p}(\mathrm{x_i},\mathrm{y_j})}{\mathrm{p}(\mathrm{x_i})\mathrm{q}(\mathrm{y_j})}, \qquad (2.14)$$

The mutual information discussed above have the following properties

(i) Symmetry – The mutual information between the two random variables is symmetric in nature as

$$I(X;Y) = I(Y;X)$$

(ii) Non-negativity- The value of mutual information between the variables X and Y is always greater than zero

$$I(X;Y) \geq 0,$$

The equality holds when random variables X and Y are independent i.e., $p(x_i,y_j)=p(x_i)*p(y_j)$. It is straightforward to see from eqn 2.14 that I(X;Y)=0.

The above framework can be extended to continuous random variables and the mutual information can be defined as

$$I(X;Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\log\frac{f_X(x|y)}{f_X(x)}\,dx\,dy \qquad (2.15)$$

where f(x,y) is a joint probability density function and $f_X(x)$ is the marginal density function and $f_X(x|y)$ is a conditional density function of X for a given Y.

2.1.3 Renyi Entropy

Different entropic measures generalizing Shannon's entropy have been proposed. One of the important and well-known measures is due to Renyi [6] who defined entropy of order α as

$$H_{\alpha}(P) = \frac{1}{1-\alpha} \ln \sum_{i=1}^{n} p^{\alpha} \,, \qquad \alpha \neq 1, \alpha > 0 \qquad (2.16)$$

where α is a parameter. This entropy is known to possess additive property implying that the sum of entropies for independent systems equals the total entropy of the entire system i.e.,

$$H_{\alpha}(P \cup Q) = H_{\alpha}(P) + H_{\alpha}(Q)$$

The advantage of this entropy is that there exists a parameter α that can be adjusted to satisfy certain requirements of the problem. Using method of Lagrange's undetermined multiplier, Renyi's entropy also assumes maximum value when all probabilities are equal.

It can be seen from eq (2.15) that Shannon entropy is recovered as a special case as $\alpha \to 1$.

Renyi broadened the framework to enable characterization of information measure for generalized probability distributions. Taking weight of a distribution as $W(P) = \sum_{k=1}^{n} p_k$,a distribution having weight $W(P)<1$ is called incomplete distribution. Defining the mean value of entropy,

$$H_{n+m}(P \cup Q) = H_{n+m}(p_1,p_2,\dots,p_n, q_1,q_2,\dots,q_m)$$

$$= \frac{W(P)Hn(P)+W(Q)Hm(Q)}{W(P))+W(Q)} \qquad (2.17)$$

and with appropriate postulates, Renyi determined entropy Hα(P) of order α(as given by eqn 2.15. It may be pointed out that Renyi's entropy is additive but does not satisfy recursivity.

This entropic measure has been used by Prakash [19] in the context of image thresholding. In his M.Tech dissertation work, he used Renyi entropy as a measure of uncertainty and used it for multilevel thresholding for different images.

2.1.4 Tsallis Entropy

This entropy has recently been proposed to deal with systems which are non-extensive in nature. Such systems are typically observed when one deals with systems having multi-fractal structure. It is natural that images which posses such features should be characterized through this non-extensive entropy. In contrast to other known entropies, Tsallis entropy [18] does not possess the additivity property. In terms of non-extensive parameter q, Tsallis defined this entropy as,

$$S_q = \frac{1-\sum_{i=1}^{k} p_i{}^q}{q-1} \qquad (2.18)$$

In the limit q→1, this entropy also reduces to Shannon's entropy.

An important aspect of the Tsallis entropy is that it is nonextensive in contrast to Shannon and Renyi entropies. Further Tsallis entropy is non-additive in nature.

For two statistically independent systems, the entropy of the whole system is defined by the following rule

$$S^q(A+B)=S_A^q+S_B^q+(1-q)S_A^q.S_B^q \qquad (2.19)$$

which is called as pseudo additivity property due to the extra term viz. $(1-q)S_A^q.S_B^q$. It can be seen that for q→1, it reduces to

$$S^q(A\cup B)=S_A^q+S_B^q \qquad (2.20)$$

One can define different variants of extensivity as follows

Subextensive entropy (q<1)

$$S^q(A+B)< S_A^q+S_B^q \qquad (2.21)$$

Extensive entropy (q=1)

$$S^q(A+B)= S_A^q+S_B^q \qquad (2.22)$$

Superextensive entropy (q>1)

$$S^q(A+B)> S_A^q+S_B^q \qquad (2.23)$$

Chapter-3
Entropy Based Thresholding Methods

The problem of thresholding arises due to the fact that the histogram associated with images is bimodal, and in general can be multimodal. This enables us to obtain an optimal threshold value for extracting object from the background. One of the widely adopted approaches is based on entropy framework.

In this class of methods, we use the entropy of the distribution of the gray scale levels of the image to calculate the threshold value. We now describe some of thresholding methods based on above mentioned entropies

3.1 Method based on entropy of histogram

On the basis of the Shannon entropy of the histogram of the image, Pun [20,21] proposed two algorithms to calculate the threshold.

(a) In the first method, he calculated the optimal threshold value by maximizing the posteriori entropy H, given by-

$$H=H_b+H_w$$

where $H_b=-\sum_{i=0}^{t} p_i \log_2 p_i$ and

$$H_w=-\sum_{i=t+1}^{n} p_i \log_2 p_i \qquad (3.1)$$

H_b and H_w are the entropies of the background and object, respectively.

For maximizing H, Pun[20] proposed an evaluating function f(t) which is given as

$$f(t)=\left[\frac{H_t}{H_T}\frac{\ln p_t}{\ln \max(p_0,...,p_t)}+\left\{\frac{1-H_t}{H_t}\right\}\frac{\ln(1-p_t)}{\ln \max(p_{t+1},...,p_n)}\right] \quad (3.2)$$

where

$$H_t=-\sum_{i=0}^{t} p_i \log_2 p_i \ , \quad (3.3)$$

$$H_T=-\sum_{i=0}^{n} p_i \log_2 p_i \quad (3.4)$$

$$p_t=\sum_{i=0}^{t} p_i$$

(b) In the second method, the threshold value depends on an anisotropic coefficient,

$$\propto=\left[\frac{\sum_{i=0}^{m} p_i \log_2 p_i}{\sum_{i=0}^{n-1} p_i \log_2 p_i}\right] \quad (3.5)$$

The threshold value t is chosen such that

$$\sum_{i=0}^{t} p_i=\begin{cases}1-\propto & if\ \propto\leq 1/2\\ \propto & if\ \propto\geq 1/2\end{cases} \quad (3.7)$$

Pun[21] investigated a few images and found that the second method performs better than the first method.

3.2 Image thresholding using Renyi entropy

Sahoo, Wilkins and Yeager[22] proposed a method to find the threshold value using Renyi's entropy. The Renyi entropies of the object($H_o^{\propto}$) and its background($H_b^{\propto}$) are given by

$$H_o^{\propto}(t) = \frac{1}{1-\propto} \ln \sum_{i=0}^{t} \left[\frac{p_i}{p_t}\right]^{\propto} \quad (3.8)$$

$$H_b^{\propto}(t) = \frac{1}{1-\propto} \ln \sum_{i=t+1}^{n} \left[\frac{p_i}{1-p_t}\right]^{\propto} \quad (3.9)$$

Sahoo et al[22] calculated the threshold value by maximizing the sum of above two entropies i.e., $(H_o^{\propto}(t) + H_b^{\propto}(t))$

For a particular value of $\propto$, the threshold $t^*(\propto)$ can be obtained from

$$t^*(\propto) = \underset{\alpha}{\operatorname{argmax}} \left(H_o^{\propto}(t) + H_b^{\propto}(t)\right) \quad (3.10)$$

An illustration of Renyi entropy based image thresholding is shown in the Figure 3.1(a) and Figure 3.1(b). The image is of UofL campus.

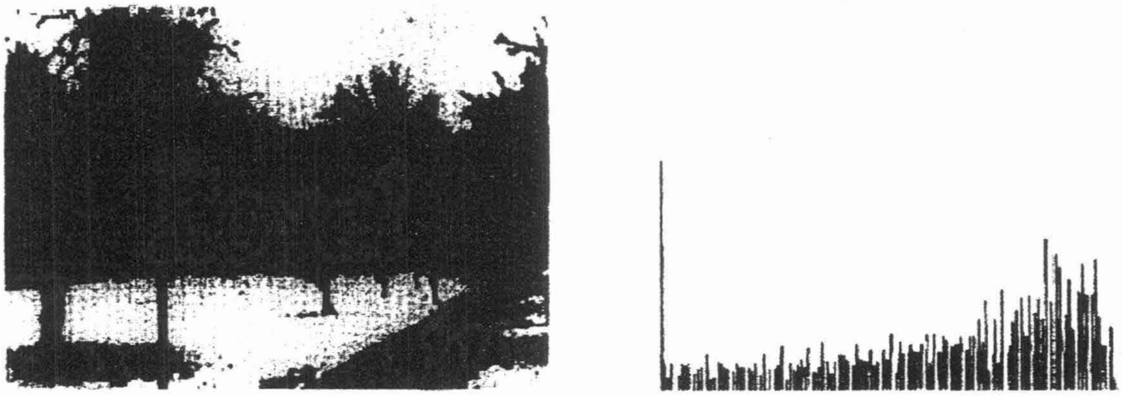


Fig 3.1(a) Original image of UofL campus and its histogram [Sahoo et al.(22)]

Fig 3.1(b) Thresholded at t*=155 and at t*=177 [Sahoo et al(22)]

3.3 Image thresholding using Tsallis entropy

Using Tsallis entropy Albuquerque, Esquef, Gesualdi and Albuquerque[23] proposed a method to obtain an optimal threshold value for different values of q. The entropies of object and background are

$$S_o^q = \frac{1-\sum_{i=1}^{t}(\frac{p_i}{p_o})^q}{q-1} \tag{3.11}$$

$$S_b^q = \frac{1-\sum_{i=t+1}^{n}(\frac{p_i}{p_b})^q}{q-1} \tag{3.12}$$

Unlike Shannon and Renyi entropies, the Tsallis entropy is pseudo additive in nature. Hence, the sum of the above two entropies comes out to be

$$S^q = S_o^q + S_b^q + (1-q)S_o^q . S_b^q \tag{3.13}$$

Now the optimal threshold value

$$t_{opt}^* = \underset{q}{\text{argmax}} \, (S^q)$$

$$= \underset{q}{\text{argmax}} \, (S_o^q + S_b^q + (1-q)S_o^q . S_b^q) \tag{3.14}$$

Albuquerque et al[23] demonstrate the influence of parameter q in the thresholding. Figure 3.1 shows the Tsallis entropy based image thresholding with varying q parameter. However, these authors point out that they could not find evidence which relate q and image categories.

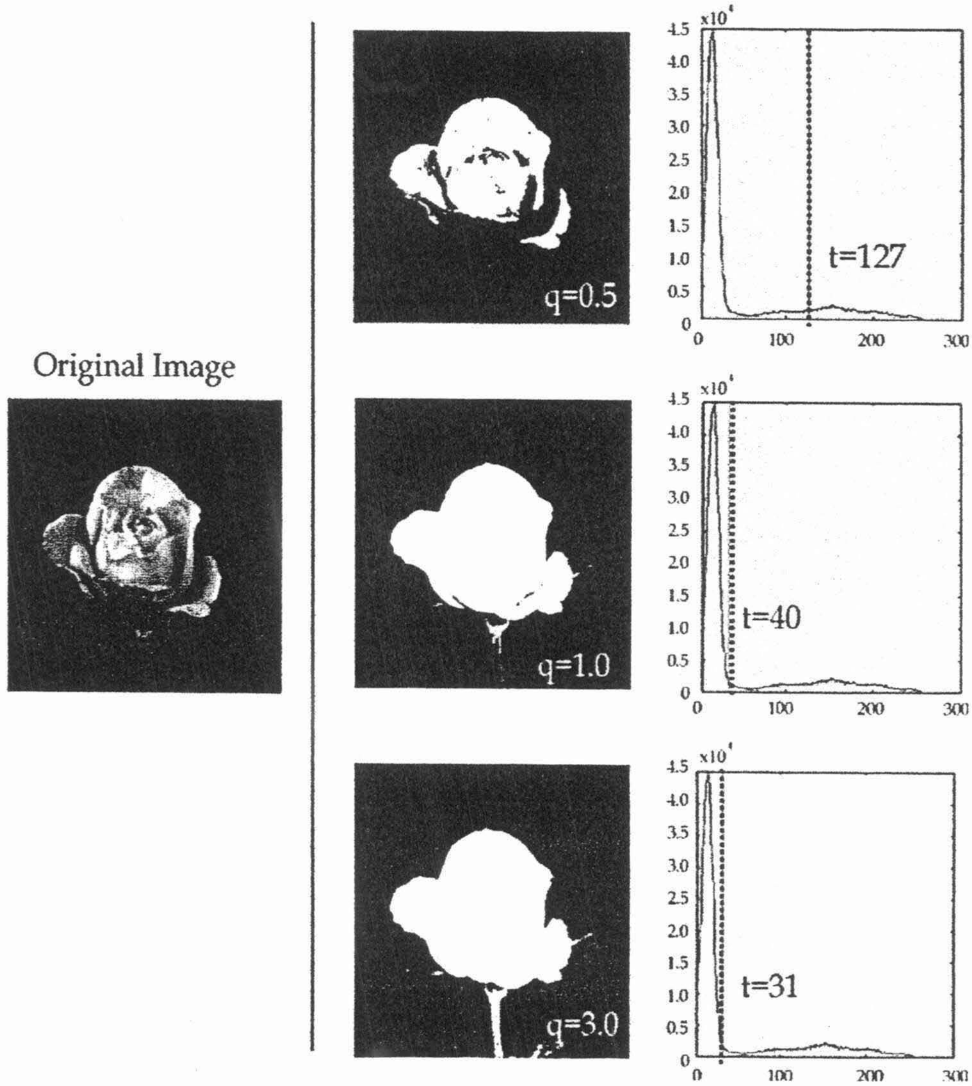


Fig 3.2 Tsallis entropy based image thresholding [Albuqurque et al.(23)]

Chapter-4
Color Image Thresholding Using Tsallis Entropy

Celenk and Haag [24] proposed a method to apply a gray level thresholding to a color image. In this method, each color component image is considered and analyzed on the same footing as that of a gray scale image. The advantage is that each color is thresholded separately. These thresholded images are then fused together to obtain the final image by means of majority-voting scheme.

Recently Du and Chang [25] also examined the issue of thresholding by extending gray-level method to red, green and blue color such that each colored pixel (r,g,b) can be represented by three code word. They defined t_r,t_g and t_b as the threshold values for red, green and blue colors and expressing (i,j)th pixel as a triplet (r_{ij},g_{ij},b_{ij}). The thresholding of each pixel in color image can be done as follows:

$$
\widetilde{r_{i,j}} = \begin{cases} 1 & r_{i,j} > t_r \\ 0 & r_{i,j} \leq t_r \end{cases}
$$

$$
\widetilde{g_{i,j}} = \begin{cases} 1 & g_{i,j} > t_g \\ 0 & g_{i,j} \leq t_g \end{cases} \qquad (4.1)
$$

$$
\widetilde{b_{i,j}} = \begin{cases} 1 & b_{i,j} > t_b \\ 0 & b_{i,j} \leq t_b \end{cases}
$$

Du and Chang [25] calculated these threshold values by within-class and between class distances by calculating the mean of clustered class of the three colors.

Following their approach, we first separated the color image into three components images for red, blue and green colors as depicted in Figure 4.1. For these images, we calculated the threshold values and generated the binary images corresponding to them. These separate binary images are then combined to obtain the final image by one of the following two methods:

(i) ORing of the component thresholded images..

(ii) Using three level thresholding.

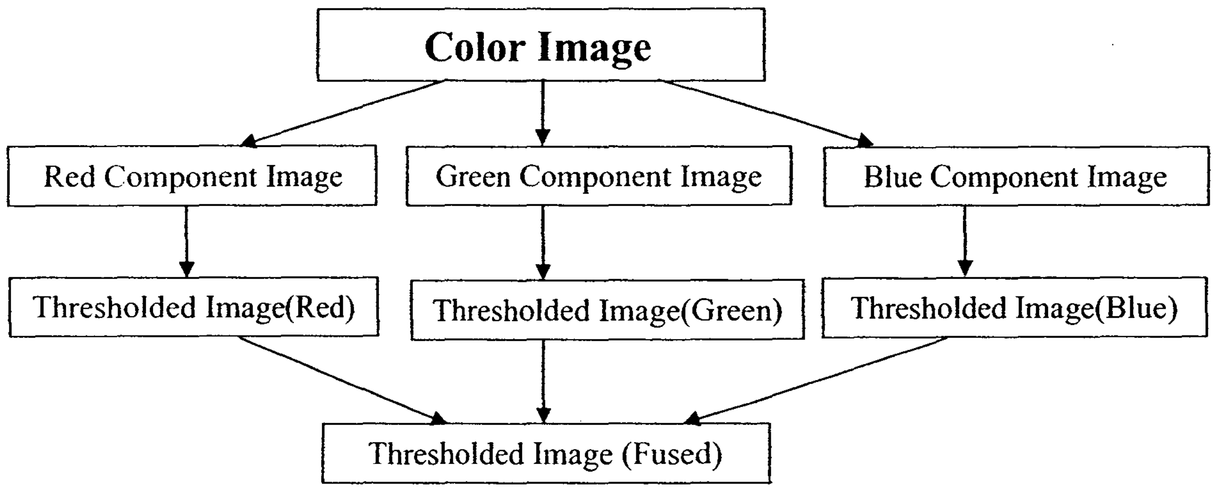


Fig 4.1 Outline of steps for color image analysis

Extending the framework, the entropies of the components (RGB) of color images are calculated as follows:

The entropies for object and background for different colors can be obtained . For the object part the entropies for red, green and blue colors are defined as

$$S_{Ro}^{q} = \frac{1-\sum_{i=1}^{t_r}\left(\frac{p_i}{p_{ro}}\right)^q}{q-1}$$

$$S_{Go}^{q} = \frac{1-\sum_{i=1}^{t_g}\left(\frac{p_i}{p_{go}}\right)^q}{q-1} \qquad (4.2)$$

$$S_{Bo}^{q} = \frac{1-\sum_{i=1}^{t_b}\left(\frac{p_i}{p_{bo}}\right)^q}{q-1}$$

Similarly for background part, the entropies for respective colors are

$$S_{Rb}^{q} = \frac{1-\sum_{i=t_r+1}^{n}(\frac{p_i}{p_{br}})^q}{q-1}$$

$$S_{Gb}^{q} = \frac{1-\sum_{i=t_g+1}^{n}(\frac{p_i}{p_{bg}})^q}{q-1} \qquad (4.3)$$

$$S_{Bb}^{q} = \frac{1-\sum_{i=t_b+1}^{n}(\frac{p_i}{p_{bb}})^q}{q-1}$$

By using argmax function, the optimal threshold values can be calculated as

$$t^*_{r_{opt}} = \underset{q}{\text{argmax}}\ (S^q_{Ro} + S^q_{Rb} + (1-q)S^q_{Ro}.S^q_{Rb})$$

$$t^*_{g_{opt}} = \underset{q}{\text{argmax}}\ ((S^q_{Go} + S^q_{Gb} + (1-q)S^q_{Go}.S^q_{Gb}) \qquad (4.4)$$

$$t^*_{b_{opt}} = \underset{q}{\text{argmax}}\ (S^q_{Bo} + S^q_{Bb} + (1-q)S^q_{Bo}.S^q_{Bb})$$

These three calculated optimal threshold values can be used to obtain the final image.

4.1 Implementation and Analysis of proposed methods

In relation to the above mentioned methods we will now describe the methodology for combining the component color images to generate the final thresholded image .

In the first method , we perform following operations

1. Generate three separate images corresponding the three threshold value calculated i.e., t_r, t_g, t_b
2. Take OR of the three generated binary images to give the final thresholded image.

Football Image: Illustrative Example

For the purpose of illustration we consider the '**football**' image. Figure 4.2(a) is the original color football image and Figure 4.2(b) represents corresponding histogram.

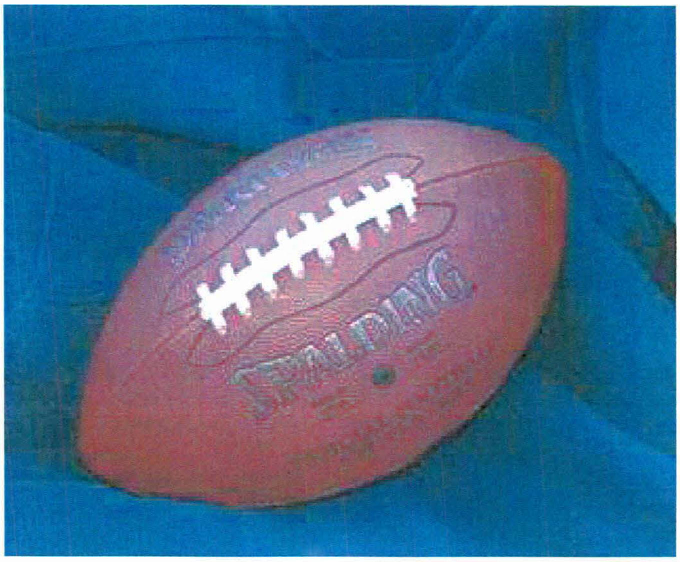


Fig 4.2(a) Color football image

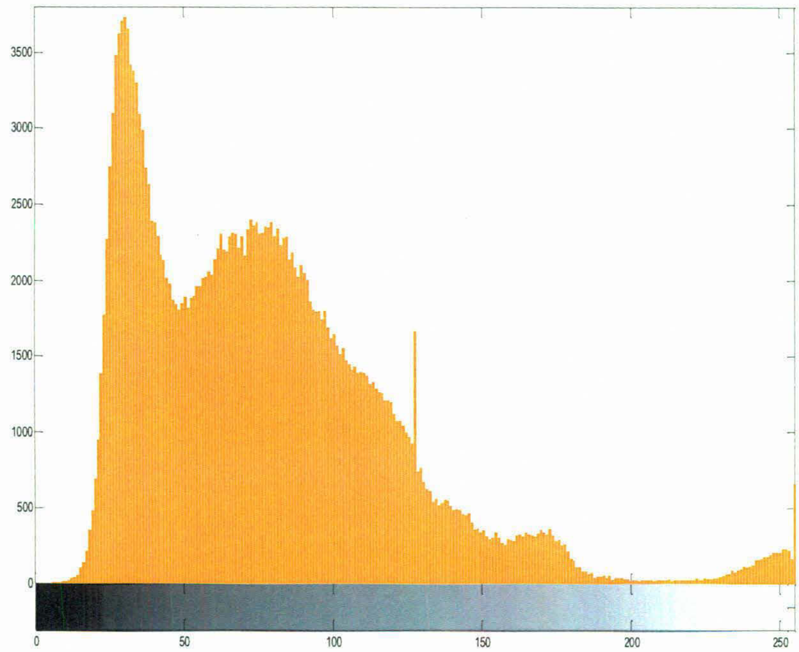


Fig 4.2(b) Histogram of football image

We first divide the original image into three component images corresponding to red, green and blue colors. The three component images are shown in the following figures. Figure 4.3(a) and Figure 4.3(b) represent the red component image and its histogram. Figure 4.4(a) and Figure 4.4(b) represent the green component image and its histogram. Similarly Figure 4.5(a) and Figure 4.5(b) represent the blue component image and its histogram.

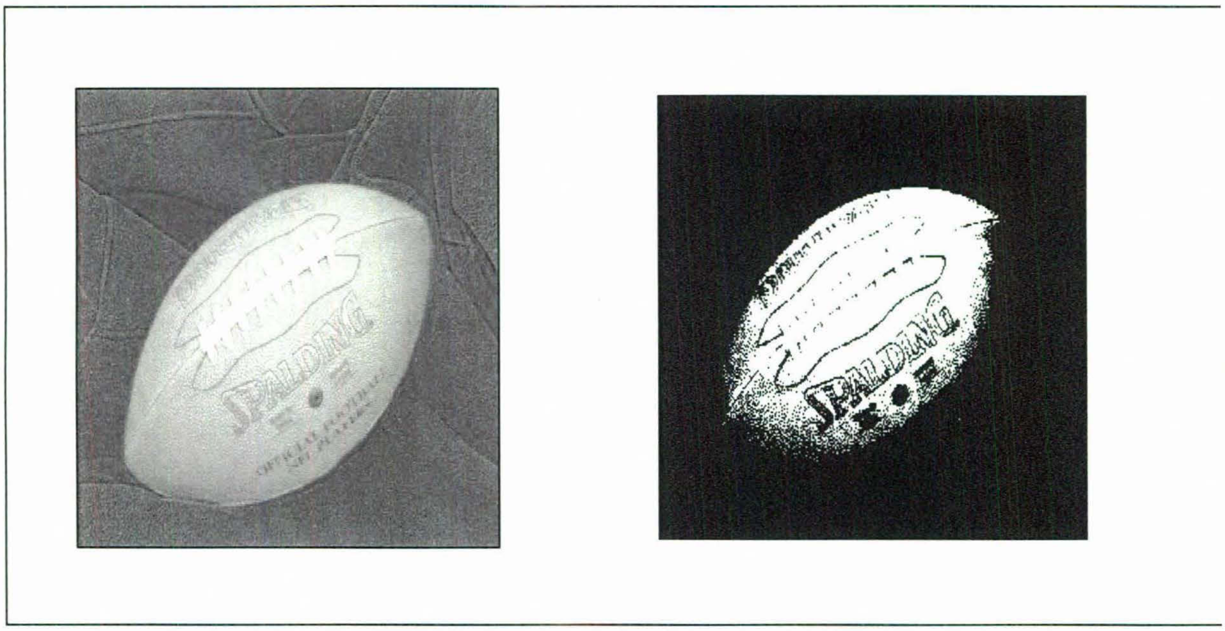
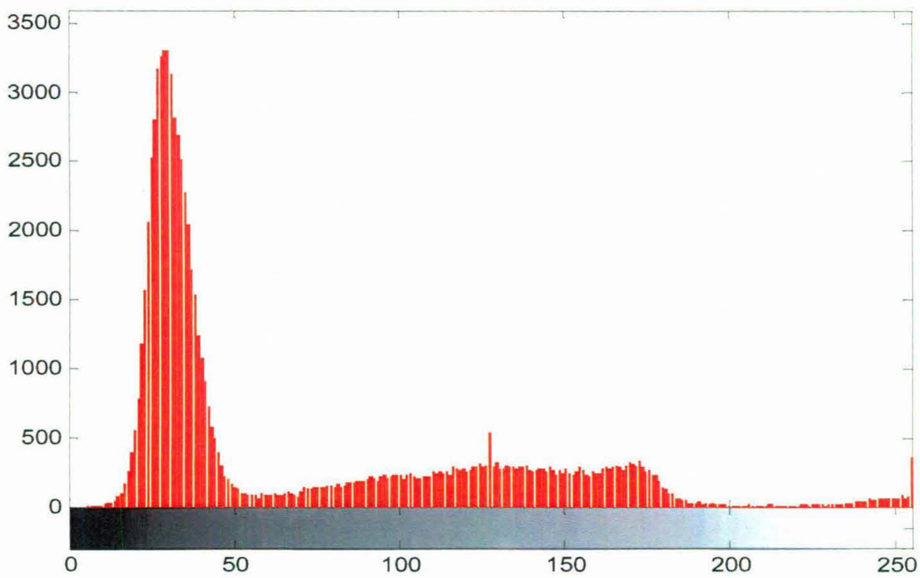


Fig4.3 (a) Red Component image and its thresholded image (above) and Fig 4.3(b) corresponding histogram (below)



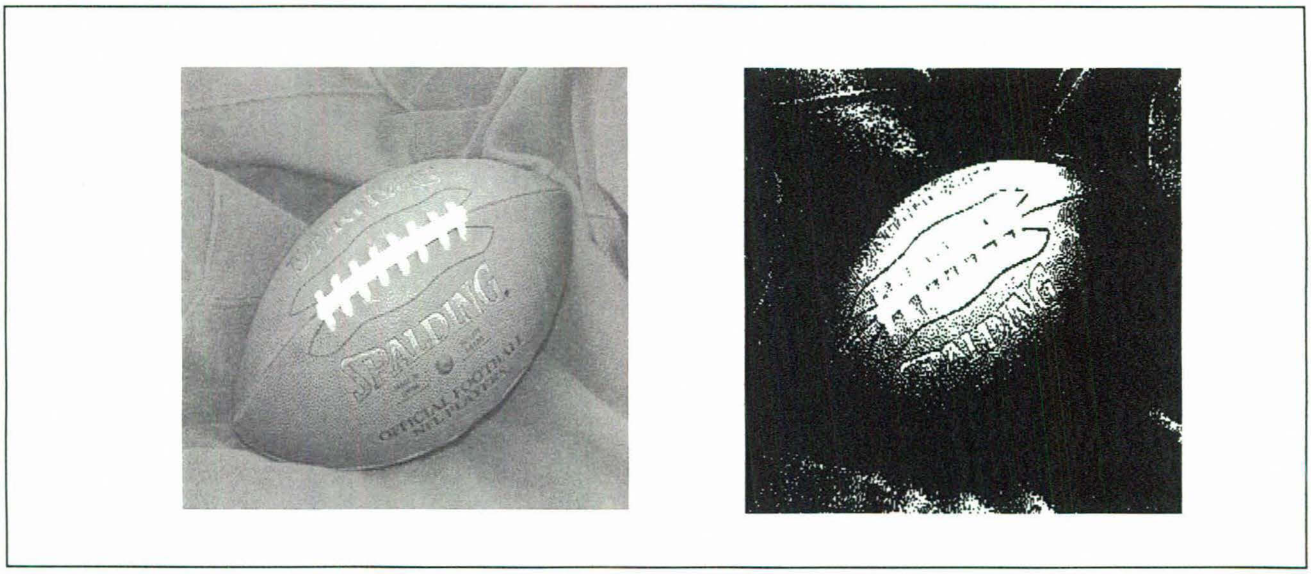


Fig4.4 (a) Green Component image and its thresholded image

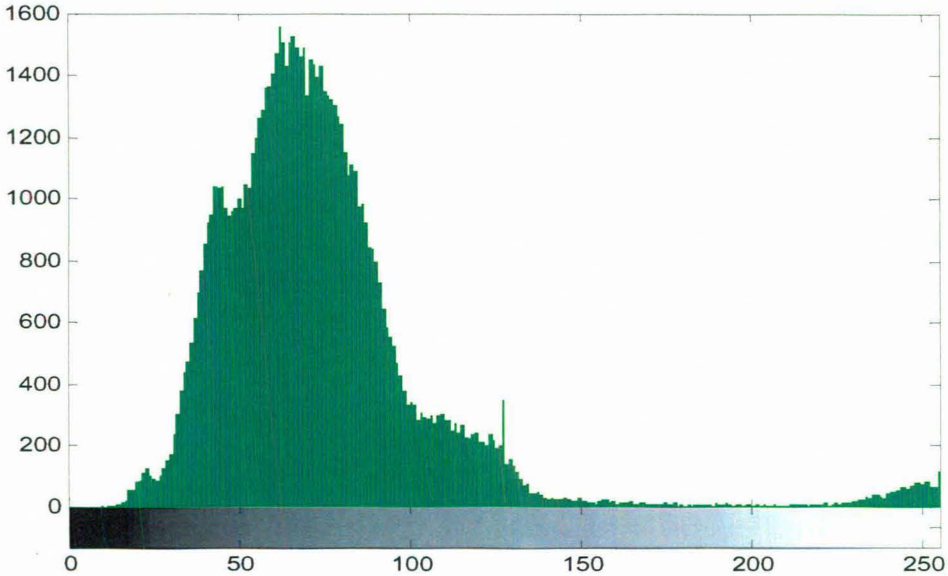


Fig4.4 (b) Histogram corresponding to Green component image

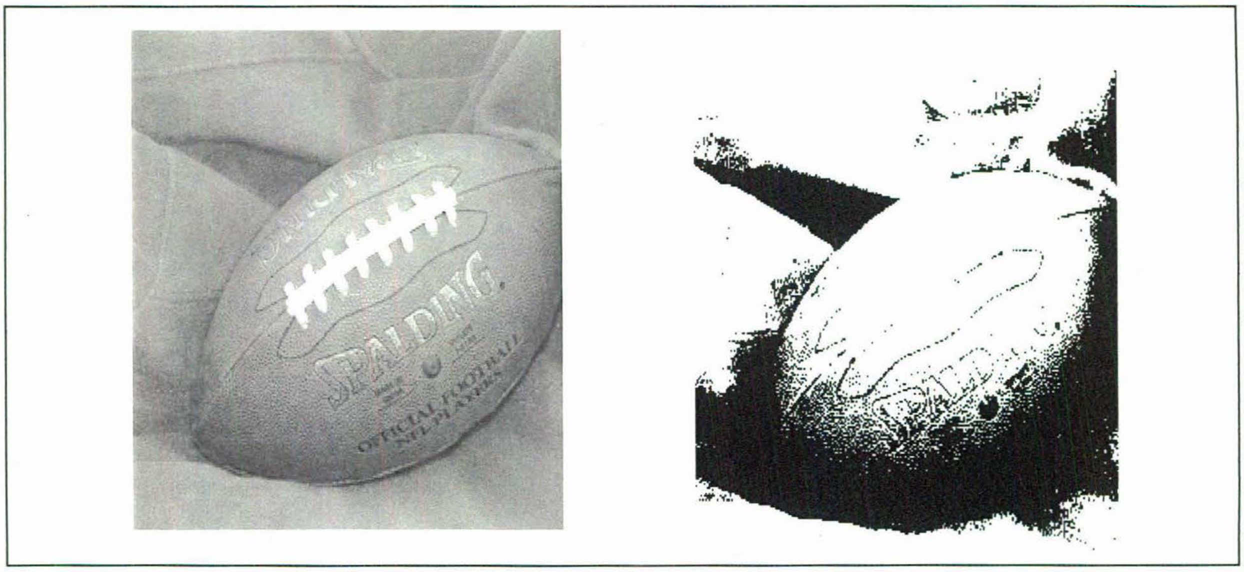


Fig 4.5(a) Blue Component image and its thresholded image

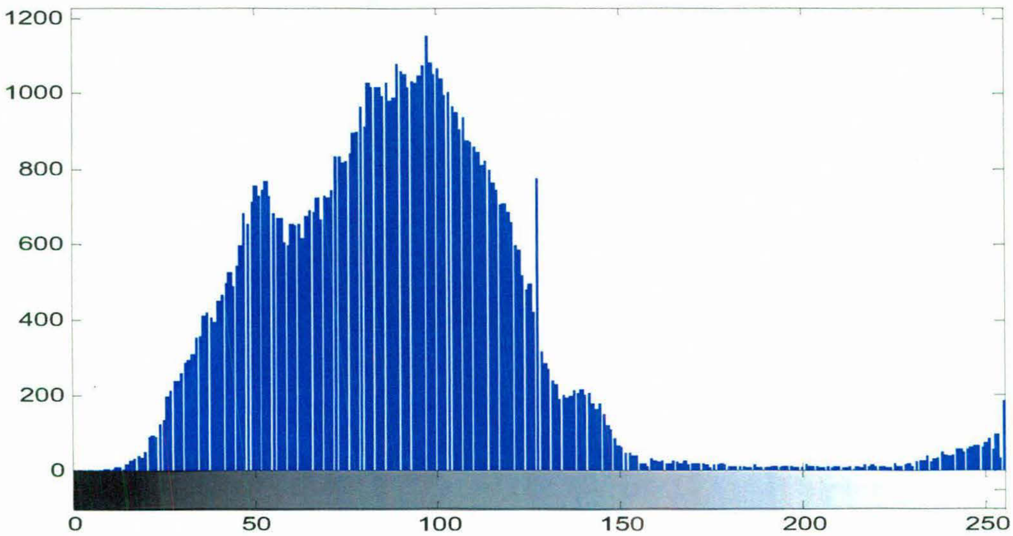


Fig4.5 (b) Histogram corresponding to Blue Component image

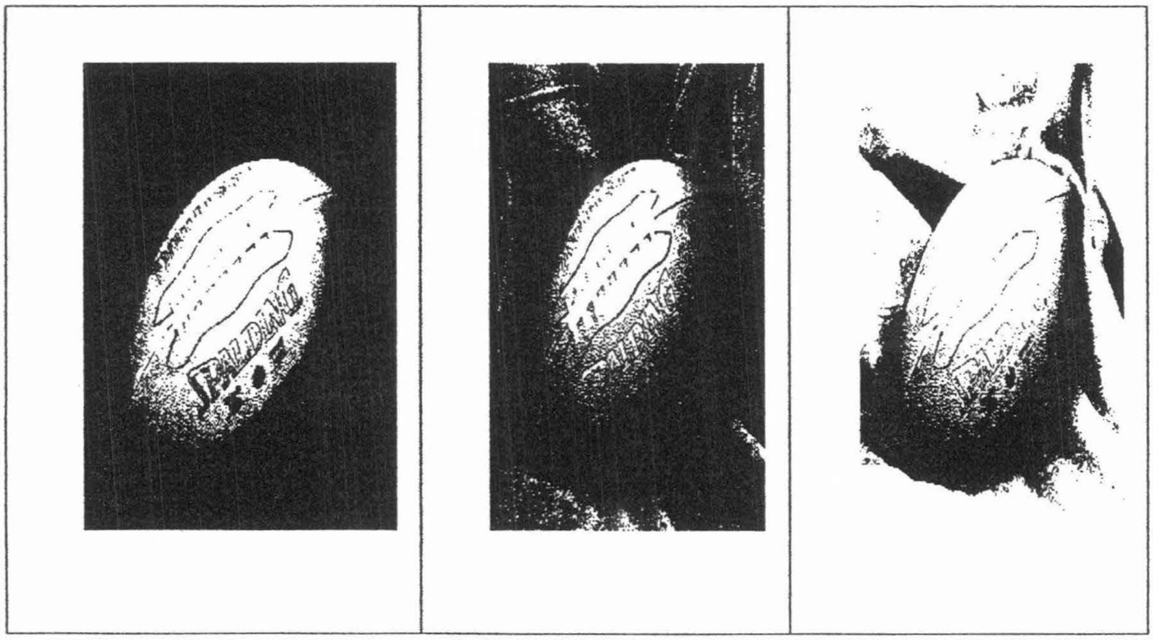


Fig4.6 (a) The three thresholded images corresponding to red green and blue component images respectively

The three thresholded images for red, green and blue (Figure 4.6(a)) are combined together to form the final thresholded image (Figure 4.6(b))

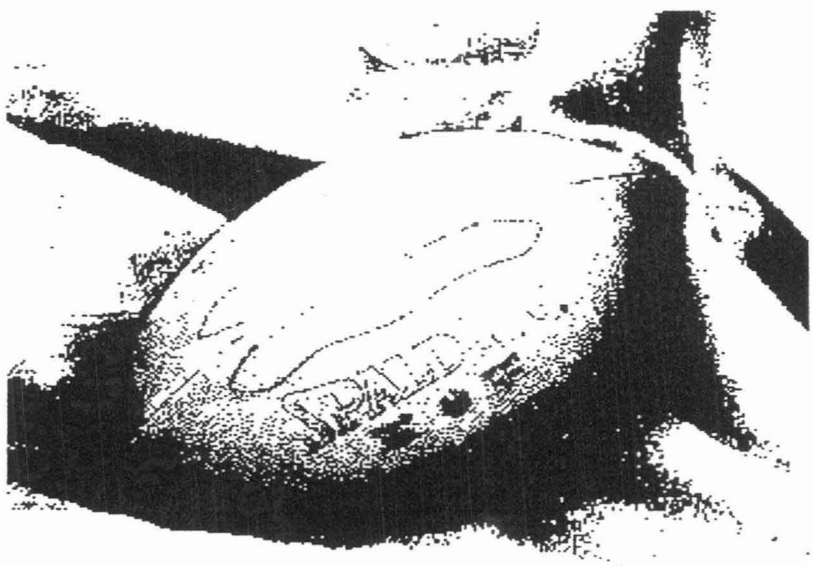
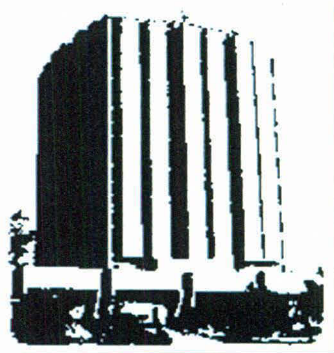


Fig4.6 (b) Final thresholded image

Comparitive Study of Our Method with Otsu's method:JNU Library image

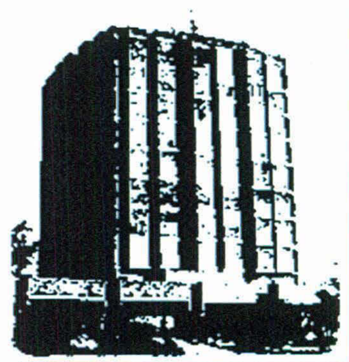
For demonstrating the efficacy of proposed method we consider a different image.

We can see the difference between the widely used method (Otsu) and the method proposed in the Figure below.



Our method

JNU Library



Direct method

Fig4.7 JNU Library and its thresholded image(q=0.8)

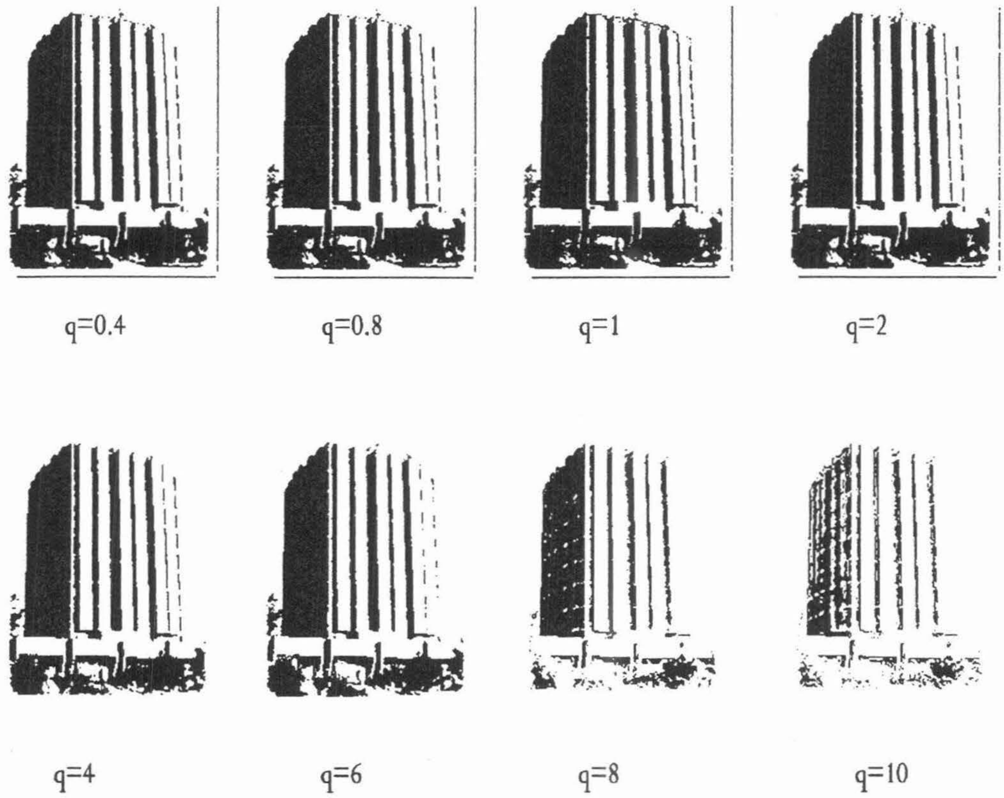


Fig4.8 JNU Library and its thresholded image with varying parameter q

Figure 4.8 shows the variation of thresholding with q parameter of Tsallis entropy. Based on our analysis we find that the quality of thresholded image is better for the values of the parameter q lying between 0.3 to 0.8. However the quality deteriotes when one considers high values of q(>3) as well as low values of q(<0.2).

In the second method, we generate the final thresholded image by using three level thresholding, in following way

1. Calculate the maximum and minimum threshold value out of the three threshold values.

2. Suppose the three threshold value be t_{max}, t and t_{min} , the three levels are chosen as

   I. First level- between threshold value t_{min} and t

   II. Second level- between threshold value t and t_{max}

   III. Third level- value above t_{max}

3. Corresponding to these three levels, we generate a thresholded image.

For illustration of this method, we have considered three images 'Lena' image, 'Football image' and 'Tile Julia' image which are thresholded using three threshold values. The ranges between these threshold values are used to determine the threshold levels.

Lena image:Illustrative example

The original 'Lena' image is shown in Figure 4.9(a), and corresponding thresholded image using three level thresholding is depicted in Figure 4.9(b).

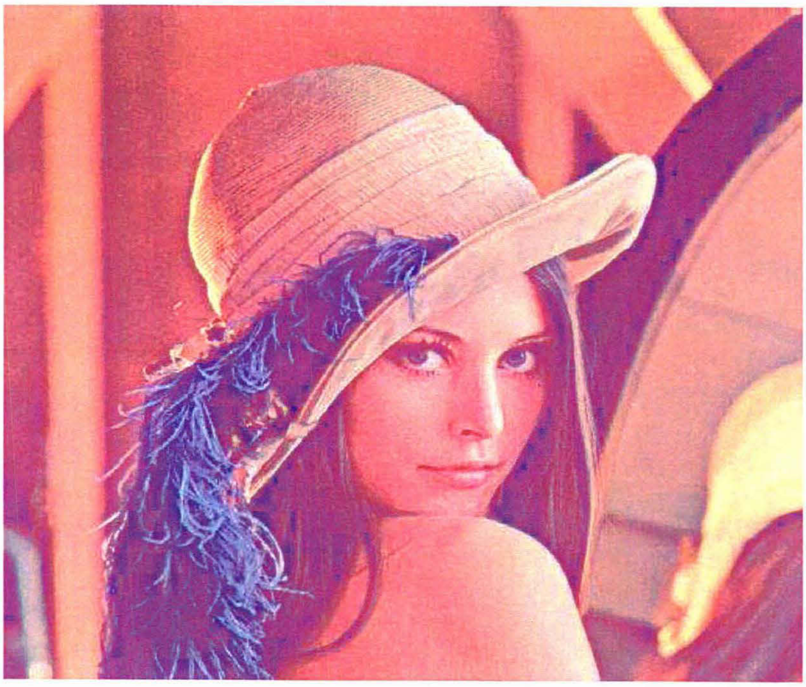


Fig 4.9(a) Lena(original) image

Fig 4.9(b) Thresholded Lena image

Football image:Illustrative example

Here we consider the 'football' image for illustrating the second method. Figure 4.10(a) is the original image and Figure 4.10(b) is the thresholded image using three threshold levels.

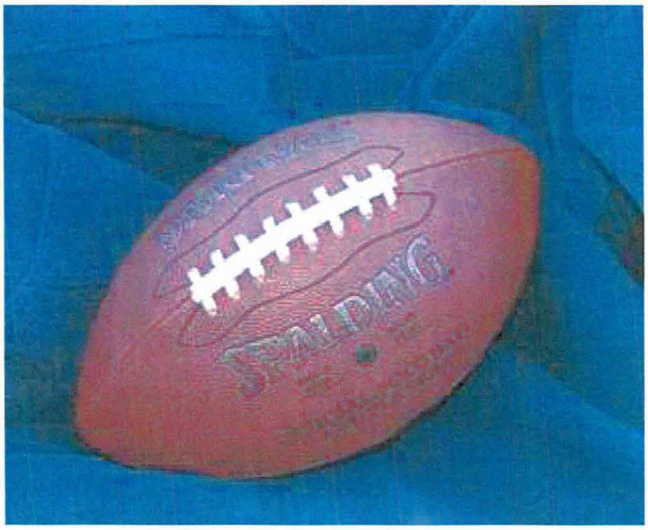


Fig 4.10(a) Football image

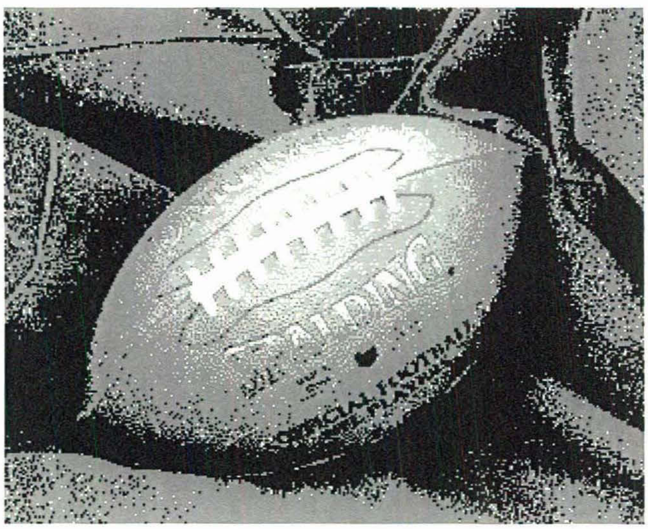


Fig 4.10(b) Thresholded Football image

Comparative Study:Tile Julia Image

For further comparison of our method with that of method due to Otsu[11], we use the 'Tile Julia' image which is thresholded using three level thresholding . The image is also thresholded using Otsu[11] method. One can easily see the difference between the two thresholded images.

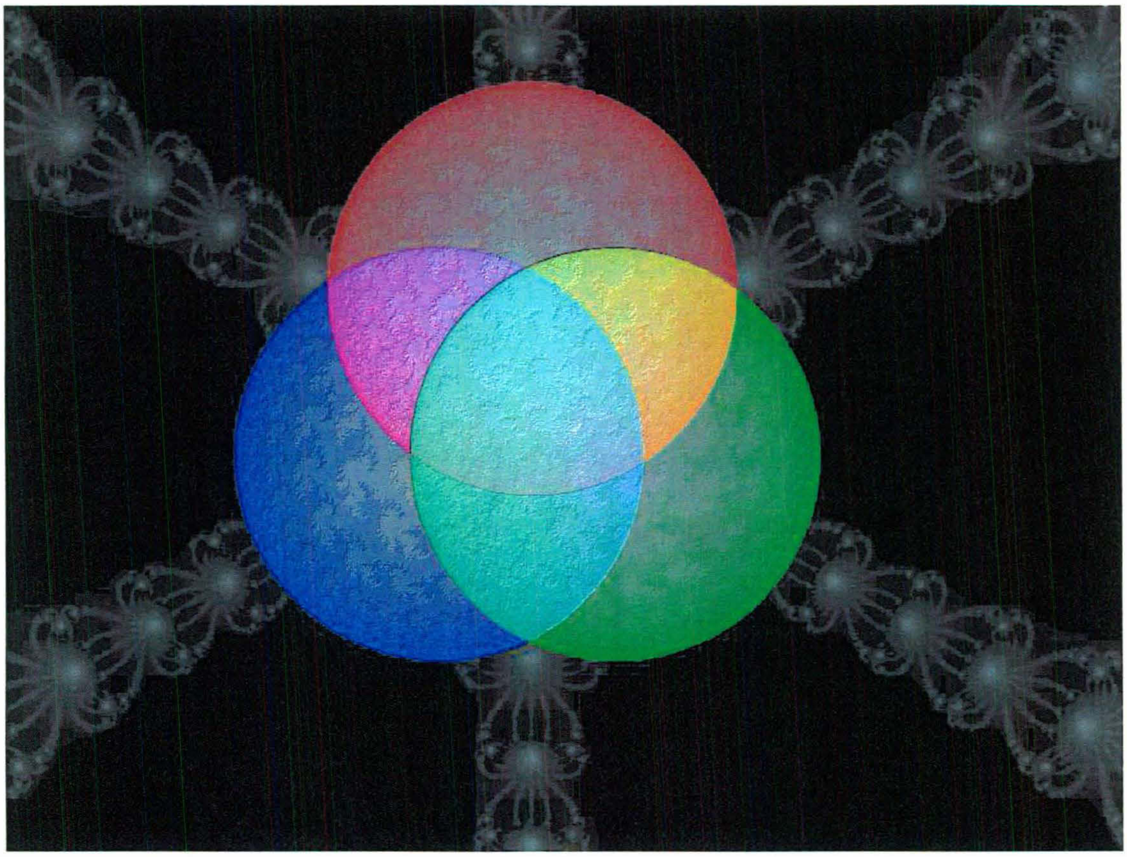


Fig 4.11(a) Tile Julia image (image source [26])

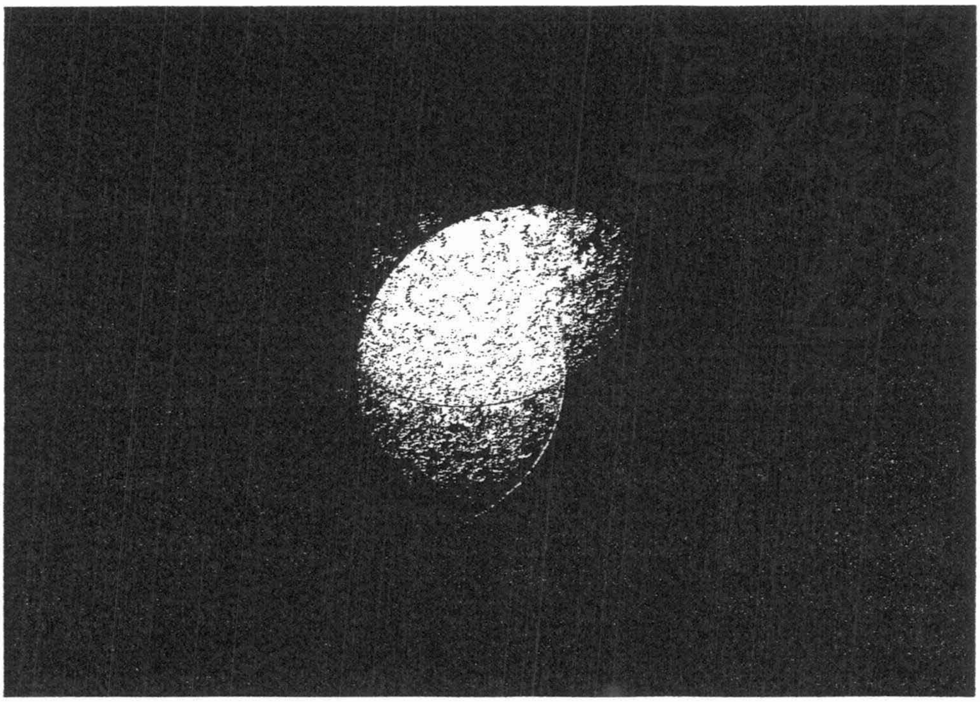


Fig 4.11(b) Threshold Tile Julia image (Direct method)

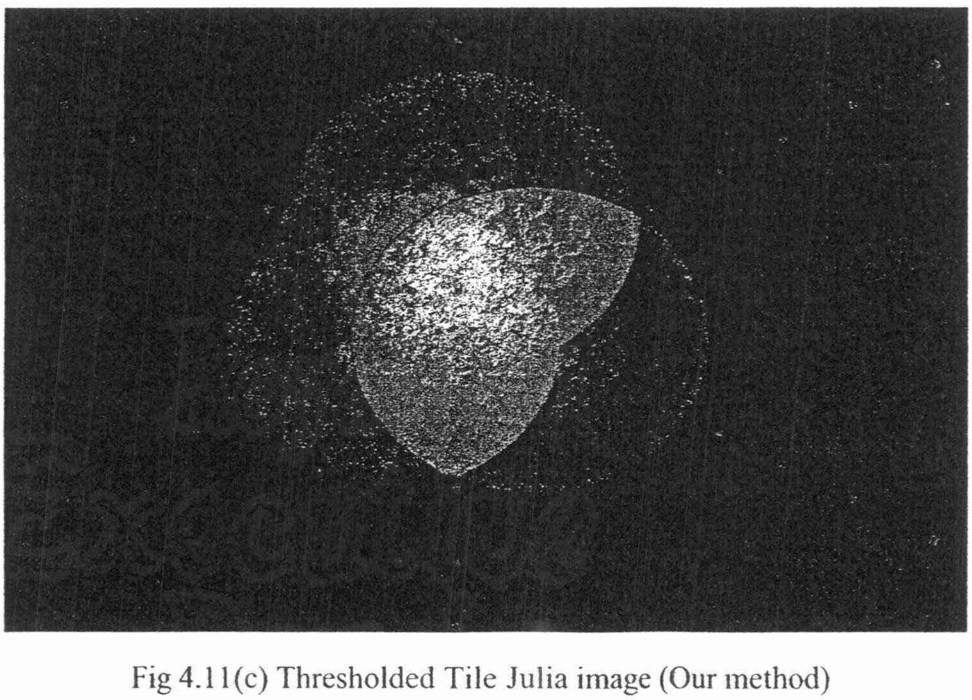


Fig 4.11(c) Thresholded Tile Julia image (Our method)

We note that for this image our proposed method outperforms the method due to Otsu [11]. The reason for the efficacy of our method is that it separately deals with the colors of the images. In contrast to this Otsu method treats all the colors in the domain of gray levels.

Chapter-5
Conclusion and Future Work

We have propose a new procedure for thresholding of color images and find that this procedure performs better than the existing techniques. Here we have given two methods for thresholding a color image. Both these methods use three threshold values. The advantage of using these proposed methods is that the information about the color pixels is taken into account. This information content pertaining to different colors when one applies gray level thresholding algorithms to the color images. The methods are based on entropy of the images which is considered to be an appropriate measure of uncertainty. Accordingly the entropy easily maps the variability of the histogram of the image pixel intensity values.

The use of Tsallis entropy in image analysis seems to have better potential than that of Shannon entropy. More analysis in this aspect is needed. If this holds then it raises a more fundamental issue which relates to the relevance of non extensive entropic methods over those of extensive entropic methods.

In passing we note that proposed methods are particularly more suitable for the color images .As a part of future enquiry the suitability of these methods for different color combination need to be investigated.

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