Some Aspects of Bank Runs

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Girijesh Kumar Tiwari

CENTRE FOR STUDIES IN DIPLOMACY, INTERNATIONAL LAW AND ECONOMICS
SCHOOL OF INTERNATIONAL STUDIES
JAWAHARLAL NEHRU UNIVERSITY
NEW DELHI - 110067
INDIA

2003

JAWAHARLAL NEHRU UNIVERSITY

Telegram : JAYENU
Telex : 031-73167 JNU IN
Telephones : 667676 / 418
667557/
New Delhi-110067.

Certificate

This is to certify the dissertation entitled "Some Aspects of Bank Runs", submitted by me in partial fulfillment of requirements for the award of MASTER OF PHILOSOPHY has not been previously submitted for any other degree of this or any other University.

Girijesh Kumar Tiwari
(GIRIJESH KUMAR TIWARI)

We recommended that this dissertation be placed before the Examinees for evaluation.

GURBACHAN SINGH
(SUPERVISOR)

ALOKESH BARUA
(CHAIRPERSON)

Chairperson
Centre for Studies in Diplomacy
International Law and Economics
School of International Studies
Jawaharlal Nehru University
New Delhi-110067

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All the mistakes that remain in the dissertation are mine.

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Contents

Chapter I

Introduction

A bank run is an event in which depositors other than those with immediate needs (some or all of them) demand redemption of their demand deposits. When only some of these depositors demand redemption, we call it a partial run. When all of such depositors demand redemption, we call it a complete run. This phenomenon may be confined to a bank or may prevail in the entire banking sector. These are two different issues, but in much of the literature, a single bank has been used as a representative bank (representing the entire banking sector). Hence this difference is not relevant in much of the literature. In this dissertation, I also have used a single bank as a representative bank.

From the earliest times, banks have been plagued by the problem of bank runs in which many or all the banks' depositors attempt to withdraw their funds simultaneously. Because banks issue liquid liabilities in the form of deposit contracts, but invest in illiquid assets in the form of loans, they are vulnerable to runs that can lead to closure and liquidation. Prior to the twentieth century, banking panics occurred frequently in Europe and United States. Panics were generally regarded, as a bad phenomenon and central bank intervention to eliminate this was required as essential. Not only in developed countries, but also in developing countries, bank runs and financial stability had been an issue of great economic importance.

Many economists have viewed runs as irrational, examples of manias or mob psychology (e.g. Kindleberger, 1978). The more recent works consider a run as a rational behavior on the part of an individual depositor.

Recent work on bank runs provides two classes of explanations for runs. One view sees runs as a manifestation of an inherent problem in banking. If banks offer to honour the liabilities of depositors on first come first serve basis, then, since their assets are illiquid and they cannot possibly honour everybody's liabilities immediately, those depositors who arrive late will suffer losses. In this view, runs are not irrational because if each depositor believes that other depositors will demand redemption, for whatever reason, then she also had better demand redemption. According to this view, runs are self-fulfilling prophecies. This is popularly known in the literature as sunspot view (e.g.-Diamond and Dybvig, 1983). The other view ascribes bank runs to business cycles (Allen and Gale, 1983). According to this view, when depositors receive signals that returns on banks' investments are going to be unusually low, then some or all of those who do not have immediate needs demand redemption of their demand deposits.

An important issue in the context of bank runs is efficiency. With regard to the issue of efficiency or inefficiency of runs, the two views of runs differ dramatically. Although a run is rational on the part of a depositor in both the views, the sunspots view shows a run to be inefficient and the business cycle view shows a run to be efficient, under some conditions.

There are many policies that have been discussed in the literature to eliminate the inefficiency (if any) due to runs. These are lender of the last resort, deposit insurance, suspension of convertibility and capital adequacy. There are two questions in this context. Is there a market failure in the context of bank runs? If yes, then what form should the policy take?

Allen and Gale (1998) showed that bank runs can lead to first best allocation, under some conditions. In other situations, bank runs are shown to result in inefficient allocation. In such cases, AG shows that by acting as lender of the last resort, central bank can restore efficient allocation. We will show that the results of AG critically depend on their use of demandable debt. If equity is used instead, then market outcome is efficient in most of the cases covered by AG. In particular, there are no bank runs and there is no need for lender of the last resort. Moreover, the issue of demandable debt by commercial banks that are supported by lender of the last resort is equivalent to the issue of equity followed by a partial repurchase of equity.

This dissertation is an attempt to see the role of equity vis-à-vis demand deposits in the context of AG. Central bank has played a very important role historically. In particular, the lender of the last resort policy has been extremely useful on many occasions. This dissertation is not a critique of central banking or of lender of the last resort policy. The purpose is however, to see whether the lender of the last resort policy is required in the context of the model as given by AG.

Plan of the Dissertation

In the second chapter we will give a review of the literature on bank runs

The third chapter will give the model by AG and its results. In the fourth chapter, we will discuss the implementation of the first best solution of AG by relying on market mechanism rather than central bank intervention. The market mechanism that we will use in this chapter is equity contract with repurchase. In this chapter, we will also explore combined risk sharing by risk neutral and risk averse agents. In fifth chapter, we will show that the first best of AG can be implemented by equity contract with dividend policy also. In sixth chapter, conclusions of the dissertation will be given.

Chapter II

Review of the Literature

In this chapter, we will discuss briefly some of the papers that are relevant to this dissertation.

Diamond and Dybvig (1983)

The sunspots view was formally modelled by Diamond and Dybvig (1983), henceforth DD. This is a three time period model (0, 1 and 2). They consider an economy with production technology which gives unit return after one period lag and more than unit return after two period lags if the initial investment remains uninterrupted. There are two types of consumers. These are type one consumers who only care to consume at date 1 only and type two consumers who want to consume at date 2 only. At date 0 all agents are identical with an endowment of one unit of consumption good each and none at subsequent dates. At date 0, the consumers do not know of their types. They only know the probability of their being in either of the types and this probability is same for each consumer. At date 1 each consumer realizes his type.

DD first considers the competitive solution where the consumers left to themselves without bank and financial institutions would invest directly in the production process. However if the types are publicly observable and there exist a social planner to maximise the expected utility of the consumers then the resulting allocation is a superior allocation than that would have been achieved by the consumers themselves. DD then consider a bank, which accepts a deposit from each agent at date 0, invests these goods in the production process, and gives depositors the option of withdrawing funds at date 1 or date 2. The bank pays a fixed amount for deposit redeemed at the option of the depositor at date 1 and then distributes its remaining resources in a pro rata basis to its remaining depositor at date 2. DD makes the assumption that this bank is subject to a "sequential servicing constraint". This implies that depositor payoff at date 1 depends upon their place in the queue.

DD shows that if the fixed amount promised by the bank at date 1 is equal to the consumption in first best allocation to the type 1 consumers, then there exist multiple Nash equilibria. In one Nash equilibrium, only type 1 consumers queue up to withdraw at date 1 and the first best is implemented. There also exist an another Nash equilibrium where all consumers queue up to withdraw at date 1 and the resulting equilibrium is even worse than the allocation achieved by the consumers themselves without any bank or financial institutions. DD proposes to eliminate this problem with what they call as a tax backed government deposit insurance program. According to this, the bank promises a fixed amount at date 1 to whatever proportion of the depositors wish to withdraw at date 1. If the bank fails to live up to its promise, the government pays off the remaining withdrawal at date 1.

This insurance is financed with a tax on depositors withdrawing at date 1.

McCulloh and Yu (1998)

McCulloh and Yu (1998), henceforth MY points that DD tax based scheme is based on the implicit assumption that taxes must be collected entirely in the form of deposits after depositors have declared their intention to withdraw, but before they actually withdraw. This is equivalent to the condition that depositors have to register their names at date 1. MY shows that if we have a contract between the depositors and the bank specifying this condition, we do not need any government to achieve the first best allocation. The first best allocation can be achieved by what MY calls a "contingent bonus contract".

Jacklin (1987)

Jacklin (1987) explores the role of demand deposits vis-à-vis equity in risk sharing. He shows that dividend paying equity shares provide the same risk sharing opportunities as demand deposits but do not introduce the possibility of runs. This result only requires that a market for ex-dividend shares exist. He then shows that equity shares cannot be used to achieve the same allocation as demand deposits for a large class of economies. The fact that they can do this in DD model is due to the extreme nature of assumed preference structure. Jacklin shows that for fairly general preference structure, the demand deposit provide opportunities beyond those provided by equity shares whether the firms issuing the shares pay dividend or not.

Gangopadhyay and Singh (2000)

Gangopadhyay and Singh (2000), henceforth GS, point out that the DD model tries to ensure for the depositors an outcome as close as possible to the insurance market equilibrium. In insurance schemes, agents who are more risk averse transfer a part, or all, of their risks to those who are less risk averse. In the DD model, all agents have identical risk aversion. Gangopadhyay and Singh by explicitly introducing agents with two different attitudes towards risk, reformulate the analysis as a true insurance problem.

GS retains all the assumption of the DD model except the assumption about knowledge of proportion of type one and type two consumers, attitude towards risk and initial endowments. The paper assumes that the proportion of type one consumer is not known at date 0, but is only known at date 1. What everyone knows at date 0 is the distribution function of this value and the proportion of risk averse and risk neutral consumers.

A bank is defined by GS as an institution that can sell shares and demand deposits. These are issued at date 0. Deposits claims at any date are senior to claims by the equity holders. Trading of equity shares is restricted at date 1 by assumption. With these assumptions

GS shows that a run proof bank and efficient banking is possible if sufficient amount of risk neutral capital is available. However, if sufficient risk neutral capital is unavailable, then a partial suspension of convertibility is optimal.

Allen and Gale (1998)

Allen and Gale (1998), henceforth we call them AG, attributes bank runs to a different reason. To them, banking panics are related to the business cycles and are not simply the result of independent beliefs. Runs occur when depositors perceive that the return on bank assets is going to be unusually low. This is also a three period model. There are two types of assets. These are safe assets and risky assets. In the paper there are two different cases. In one case, a safe asset is a storage technology which can be liquidated at either of the dates. In the other case it can be liquidated in either of the periods but if it is liquidated at date 2, then it yields higher return. The risky asset can be thought of as a production technology whose return is realized at date 2 but at date 0, the exact value of the return that will be realized at date 2 is not known. At date 0, every agent only knows the distribution function of this return. However, the expected value of the return on risky asset is more than the return on the safe asset in any case. At date 1, there is a real economic indicator, which tells to every agent the return on the risky asset that will be realized at date 2. AG assumes that only bank can distinguish the genuine risky assets from assets that have no value. This implies that only the bank can form a portfolio of both safe and risky assets. Assumption of free entry into the banking sector leads to maximization of the expected utility of the consumers by the bank. The first best is calculated by the expected utility maximization of the consumers.

According to the contract used by AG (given a two period economy), the bank promises a fixed amount to all withdrawers after one period if feasible, otherwise the entire fund available with the bank at that time is to be distributed equally among all withdrawers. After the end of second period, the residual with the bank at that time is to be distributed equally among all depositors who have not withdrawn at date 1. So although the pay-offs are supposed to be non contingent in the first period, the feasibility constraint makes the pay-offs contingent.

AG considers two cases. First if the return on safe asset is 1 at date 2, then the first best is implemented by the above mentioned demand deposit contract. AG explicitly shows in this case that if runs are prevented in equilibrium, then the first best allocation is not achieved. Hence according to AG runs are a way of risk sharing in this case. Second when the return on safe asset at date 2 is more than 1 then the demand deposit contract fails to achieve the first best allocation.

AG then proposes a simple monetary intervention by the central bank to eliminate this inefficiency. It consists of giving to depositors the money provided by the central bank instead of goods. In the event of run at date 1, the central bank gives the representative bank a loan in nominal terms, which is money. The bank gives depositors a combination of money and consumption goods whose nominal value equals the fixed amount promised in the contract. Type 2 consumers are of two kinds that are early withdrawing type 2 consumers and late withdrawing type 2 consumers. Since early consumers want to consume their entire wealth at date 1, they exchange the money for consumption goods with early withdrawing late consumers. It is shown in AG that the price level at date 1 adjusts so that the early consumers end up with the first best consumption level and the early withdrawing late consumers end up holding all the money. The money now held by the early withdrawing late consumers is just enough to allow the bank to repay its loan to the central bank and the bank has just enough consumption goods from its remaining investment to give the early withdrawing late consumers the corresponding first best consumption. It is shown that the price level at date 2 adjusts so that the bank and the early withdrawing late consumers can exchange money in the correct ratio and the bank ends up with the amount of money it needs to repay the loan and the consumers end up with the first best consumption goods.

AG then shows that if there exists a market for risky assets at date 1, then, even in the case when the return on safe asset has a unit return at date 2, the resulting allocation, given the demand deposit contract, is an inferior solution to the first best allocation corresponding the case when there does not exist such a market for risky assets. To introduce a market for risky assets, AG introduces a group of risk neutral speculators, who make direct investment in safe and risky assets at date 0. They consume only in the last period and their objective is to maximise the expected value of their portfolio at date 2. However, AG do not explore the possibility of combined risk sharing between risk neutral and risk averse agents in this case.

Besides beliefs and business cycles view, the literature on bank runs includes other issues also. Calomiris, Charles and Kahn (1991) models runs as a disciplining device. According to them, fear of runs on the part of investors prevents the bank to go for excessive risky projects. According to Diamond and Rajan (2000), banks can create liquidity because their deposits are fragile and prone to runs. In case of excessive fragility, there is role for outside bank capital which reduces liquidity creation by the bank but enables bank to survive more often and avoid crisis. Also banks with different amount of capital extract different amount of repayment from borrowers. The model points to overlooked side effects of policies such as regulatory capital requirements and deposit insurance. Diamond and Rajan (2002) shows that if the bank is having a capital structure fragile to runs, then it helps bank in creating liquidity. Stabilization policies, such as capital requirements, narrow banking and suspension of convertibility, may reduce liquidity. The literature also suggests that they serve as disciplining device. These two issues are discussed in the paper by Calomiris and Kahn (1991). In their format bank runs are an instrument of disciplining the firms rather than a way of risk sharing. Diamond and Rajan has given another advantage of runs. They show that the costs of illiquidity are avoided if the bank is having a capital structure fragile to runs.

However in this dissertation, we would only be concerned with the issues raised by AG.

Chapter III

Allen and Gale Model

1. Introduction

In this chapter, we will give describe the model as in Allen and Gale (1998). In section 2, we will give the specification and the assumptions of the model. In section 3, we will give the first best solution of the AG model. In section 4, we will give the result of the AG that competitive banking can implement the first best solution by a demand deposit contract in case of costless runs but fails to do so in the case of costly runs. In section 5, we will give the AG solution (central bank intervention in the form of lender of the last resort policy) to eliminate the inefficiency due to costly runs. In section 6, we will give a brief review of the AG model in case of presence of market for assets.

2. Model

The AG model has three dates (t= 0, 1 and 2) and a single consumption good. There are two types of assets, a safe asset and a risky asset. The safe asset can be thought of as a production technology that transforms one unit of the consumption good at date 0 into one unit of the consumption good at date 1 or r units of the consumption good at date 2, if left uninterrupted at date 1. Hence for r=1, the safe asset can be interpreted as a storage technology. The risky asset can be thought of as a production technology that transforms one unit of the consumption good at date 0 into R units of the consumption goods at date 2. If the project is liquidated at date 1, the output is 0. R is a non negative random variable with a density function f(R). AG assumes f(R) to satisfy the following assumption:

A.1: $E[R]>r$

This assumption guarantees that the choice of risky asset in equilibrium is strictly positive.

At date 0, all consumers are identical. AG assumes that the measure of consumers is 2. An individual consumer is having an endowment of E/2 at date 0 but none at other dates. Hence the total endowment in the economy at date 2 is equal to E. At date 1 the consumers are not identical. They can be type 1 (who only consume at date 1) or type 2 (who only consume at date 2). At date 1, they realize their types. But this realization is only private information. However, at date 0 they know the probability of their being of either type. This probability is same for each consumer and this is common knowledge. For simplicity assume that the probability of being type 1 is ½. Then both type 1 and type 2 consumers have a measure of 1 each. Consumers are risk averse. AG has also considered the case where the probability of the risk averse consumers of being of either of the types is not equal. The results are same for both the cases. The utility function u(.) of a risk averse consumer is twice continuously differentiable, increasing and strictly concave and is assumed to satisfy

A.2: $u'(0) > E[u'(RE)R]$

This assumption guarantees that the choice of safe asset in equilibrium is strictly positive.

At date 1, there is an economic indicator, which tells every agent with perfect accuracy the value of R that will be realized at date 2.

Assume that there exist banks in this particular economy, which make investment on behalf of these consumers. Only banks can distinguish the genuine risky assets from assets that have no value. Any consumer who tries to purchase the risky asset faces an extreme adverse selection problem. The banks differ from consumers in the sense that they can make a portfolio of safe and risky assets which consumer cannot. So a bank can hold a portfolio consisting of both types of assets but the consumers cannot do so. A portfolio consisting of both assets will be preferred to a portfolio consisting of safe asset alone by the consumers. So there is no loss of generality in assuming that consumers deposit their entire endowment with the bank at date 0. Hence at date 0, the bank has funds equal to E. Free entry into the banking industry forces bank to compete by offering a contract that maximizes the expected utility of the consumers. Before we discuss formally how banks operate, let us first consider the first best allocation in the next section.

3. The First Best Allocation

The optimal risk sharing, incentive compatible allocation can be found by solving the following problem

(P1)

Maximize $E[u(c_1(R))+u(c_2(R)]$

with respect to $L, X, c_1(R), c_2(R)$ subject to the following constraints

$$L + X \le E \tag{1}$$

$$c_1(R) \le L \tag{2}$$

$$c_2(R) \le r(L - c_1(R)) + RX \tag{3}$$

$$c_1(R) \le c_2(R) \tag{4}$$

where $c_1(R)$ and $c_2(R)$ are the consumption of type 1 and type 2 consumers conditional on return R. L and X denote the amount invested by the bank in safe and risky assets respectively. The first constraint says that the sum of investment made by the bank in safe and risky asset should be less than or equal to the amount invested. The second constraint says that the consumption of type 1 consumer cannot exceed the goods available at date 1, that is L. The third constraint says that the consumption of type 2 consumers cannot exceed the returns from risky assets (RX) plus the returns on the amount of the safe asset, if any, left over after the early consumers are paid off, i.e., $r(L-c_1(R))$ given the technology. The final constraint is the incentive compatibility constraint. It says that for every value of R, the late consumer must be at least as well as off as the early consumers.

Proposition 1: *Let A.1 and A.2 hold. The solution $\{L^*, X^*, c_1^*(R), c_2^*(R)\}$ to the optimal risk sharing problem is uniquely characterized by the following five conditions*

$$L^* > 0, X^* > 0 \qquad (1)$$

$$L^* + X^* = E \qquad (2)$$

$$E\left[u'(c_1^*(R))\right] = E\left[u'(c_2^*(R)R\right] \qquad (3)$$

$$u'(c_1^*(R)) = ru'(c_2^*(R)) \forall R < \overline{R} \qquad (4)$$

$$c_1^*(R) = L^*, c_2^*(R) = RX^* \forall R \geq \overline{R} \qquad (5)$$

Where $\overline{R}$ can be chosen to satisfy

$$u'(L^*) = ru'(\overline{R}X^*)$$

Proof The proof is given in the appendix of AG and is not repeated here.

Let us try to see some of the properties of the solution given by proposition 1. These properties will be used in the later part of the dissertation

$$c_1^*(R) \le L^* \tag{3.1}$$

From the definition of $\overline{R}$, the following two properties hold

$$r = 1 \Rightarrow \overline{R} = \frac{L^*}{X^*} \text{ and } r > 1 \Rightarrow \overline{R} > \frac{L^*}{X^*} \tag{3.2}$$

given the strict concavity of the utility function

For r=1, we have

$$L^* > RX^* \forall R < \overline{R}, \; L^* = RX^* \text{ if } R = \overline{R} \text{ and } L^* < RX^* \forall R > \overline{R} \tag{3.3}$$

Note that from condition (4) of the theorem, in case r=1,

$$c_1^*(R) = c_2^*(R) \forall R < \overline{R}. \tag{3.4}$$

However, in case r>1,

$$c_1^*(R) < c_2^*(R) \forall R < \overline{R} \tag{3.5}$$

The third constraint of the maximization problem is binding in equilibrium. This is because if $c_2^*(R) < r(L^* - c_1^*(R)) + RX^*$, then it is still feasible to increase the value of $c_2(R)$ and thereby to increase the expected utility function. Hence in equilibrium,

$$c_2^*(R) = r(L^* - c_1^*(R)) + RX^* \tag{3.6}$$

Now we show the first best allocation graphically by figure 1 (r=1) and by figure 2 (r>1).

P.T.O.

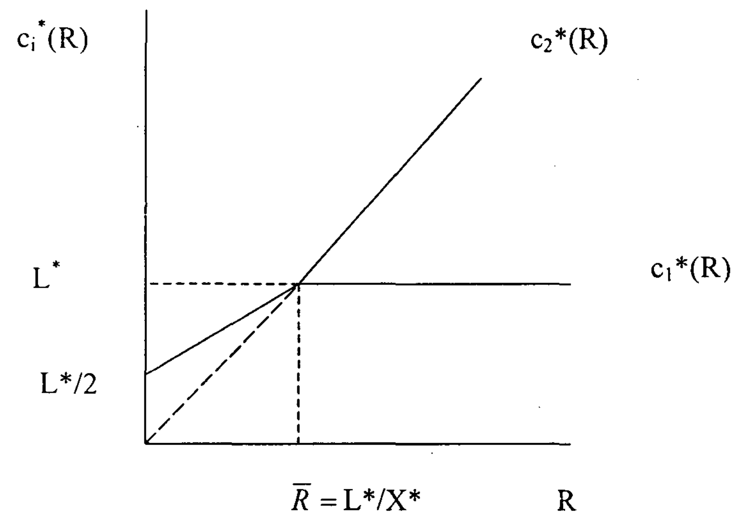

Figure 1

The optimal risk sharing allocation and deposit contract with costless runs (page 1255, figure 1, AG)

The figure plots the optimal consumption for early consumers at date 1, $c_1^*(R)$, and the late consumers at date 2, $c_2^*(R)$ in case of r=1. Given that r=1,

$$\overline{R} = \frac{L^*}{X^*}$$

from (3.2). For $R < \overline{R}$, we have

$$c_1^*(R) = \frac{L^* + RX^*}{2}, c_2^*(R) = \frac{L^* + RX^*}{2}$$

using condition (4) from proposition 1 and (3.6).

Hence for R=0, we have

$$c_1^*(R) = \frac{L^*}{2} = c_2^*(R)$$

for $R \geq \overline{R}$, we have

$$c_1^*(R) = L^*, c_2^*(R) = RX^*$$

from condition (5) of the proposition 1.

P.T.O.

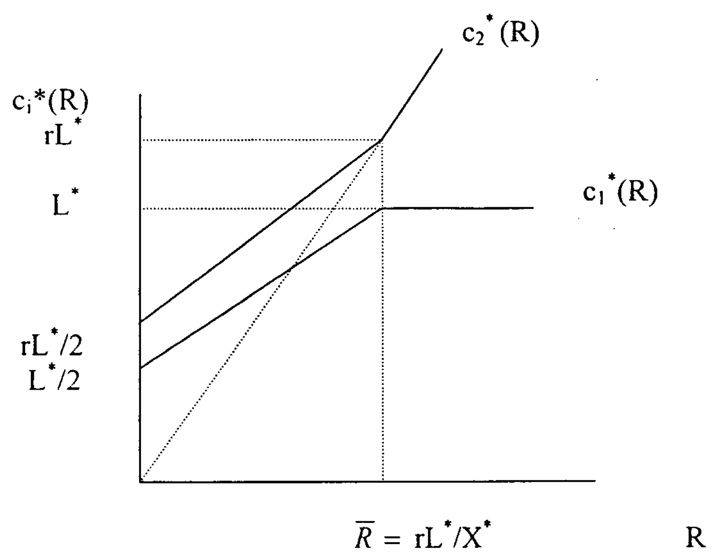

Figure 2

The optimal risk sharing allocation with costly liquidation (page-1264, figure 3,AG).

The figure plots the optimal consumption for early consumers at date 1, $c_1^*(R)$, and the late consumers at date 2, $c_2^*(R)$ in case of r>1. A specific utility function, that is

$u(c_i(.)) = \log(c_i(.)), i = 1,2$ is assumed to draw the above figure. Given this utility function, we have $\overline{R} = \frac{rL^*}{X^*}$ from the definition of $\overline{R}$.

For $R < \overline{R}$, we have

$$c_1^*(R) = \frac{rL^* + RX^*}{2r}, c_2^*(R) = \frac{rL^* + RX^*}{2}$$

from condition (4) of the proposition 1 and (3.6).

Hence for R=0, we have

$$c_1^*(R) = \frac{L^*}{2}, c_2^*(R) = r\frac{L^*}{2}.$$

For $R \geq \overline{R}$, we have $c_1^*(R) = L^*, c_2^*(R) = RX^*$ from condition (5) of the proposition 1.

So in this section we have seen that what the first best allocation is. In the next section we will see the key results of the AG that the first best allocation in case of r=1, can be implemented by relying on market mechanism. However, in the case of r>1, the market fails to do so.

P.T.O.

3. Demand Deposits, Bank runs and Efficiency

AG assumes a contract between the depositors and the bank. According to this contract the bank promises to pay a fixed amount $\bar{c}$ at date 1 to all those who withdraw if feasible, otherwise the entire amount with the bank at date 1 is to be divided equally among all withdrawers. At date 2, the residual with the bank is to be divided equally among all depositors who have not withdrawn at date 1. So given the definition of type 1 and type 2 depositors, all type 1 depositors will always withdraw at date 1 for any non negative value of $\bar{c}$. For lower values of R, there exists a possibility of some type 2 depositors also withdrawing at date 2. Let α be the proportion of type 2 depositors withdrawing at date 2. α can be interpreted as a measure of runs in AG.

Proposition 2: *If r=1, then the first best allocation can be attained by the issue of demand deposits by competitive banks that are subjects to bank runs. If r>1, then the first best allocation cannot be attained by the issue of demand deposits by commercial banks that are subject to runs.*

Proof: The rigorous proof is given in AG. See p.1258 and 1267

We now briefly discuss what is involved in the case of r=1. The logic is simple. The firm choose $\{L,X\}=\{L^*,X^*\},\bar{c}=L^*$. If there is run, that is if $\alpha>0$, then we have

$$c_{11}(R)=c_{21}(R)=\frac{L^*}{1+\alpha},c_{22}(R)=\frac{RX^*}{1-\alpha},\alpha>0$$

. . . .(3.7).

where c_{ij} is the consumption of type i consumer at date j.

In equilibrium, we have

$$c_{21}(R)=c_{22}(R)$$

.. ...(3.8).

Using (3.7) in (3.8), we get

$$\alpha=\frac{L^*-RX^*}{L^*+RX^*}$$

.. .. ..(3.9).

Using ((3.9) in (3.7), we get

$$c_{11}(R)=c_{21}(R)=c_{22}(R)=\frac{L^*+RX^*}{2}$$

...(3.10).

On the other hand if $\alpha=0$, all type one depositors get L^* units of goods and all type 2 depositors get RX^* units of goods.

It can be easily shown that given $\bar{c}=L^*$, there is run for $R<\bar{R}$, no run for $R>\bar{R}$, and consumers are indifferent between withdrawing at either of the dates (1 and 2) for $R = \overline{R}$. However, for $R = \overline{R}$, the outcome is same whether there is run or no run.

AG explicitly shows that in the case of r=1, if we choose such a value of $\bar{c}$ such that there are no runs for any value of R, then the resulting allocation is not the first best solution. Hence in case of r=1, runs are a way of risk sharing.

In case of $r > 1$, liquidation at date 1 is costly and hence runs are inefficient in this case.

So in this section, we have seen that demand deposit contract can implement the first best allocation in case of costless runs but fails to do so in case of costly runs.

In the next section, we give a description of the demand deposits supported by lender of the last resort that will implement the first best allocation even in the case of r>1.

5. Optimal Monetary Policy

If r=1, there is no market failure and hence no need for central bank intervention. Therefore this section deals with the case of r>1. AG shows that the inefficiency in the case of r>1 can be eliminated by a simple monetary intervention by the central bank. In this case, the central bank makes available to the representative bank a free line of credit of M units of money, which must be repaid at date 2. For simplicity AG assumes that this credit bears 0 rate of interest. It is also assumed that the bank will make use of the full line of credit or none of it. As in the previous section, consider a demand deposit contract, except that there is a little change in the form of the demandable debt contract between the bank and the depositors. Now the contract is specified in nominal terms. According to this contract, a depositor is promised the equivalent of a fixed amount of money D whether she withdraws at date 1 or at date 2. In real terms, the quantity of goods would depend on the prices of goods at date 1 and at date 2.

The details of the working of the contract are not fully worked out in AG. For sake of comparison in the later part of this dissertation, we give here a brief review of the working of the model. According to the contract, given $L, X, c_1(R), c_2(R)$, in states in which the consumption of the early consumers is L, there is nothing that the representative bank needs to do to prevent runs. In states where it is less than L, bank runs are valuable because they make the value of the deposit contingent on R, but here they operate through the price level (p.170, third paragraph of AG). Given that α proportion of type 2 depositor withdraw at date1, consider the transaction at date 1. The bank gives $\frac{c_1(R)}{1+\alpha}$ units of goods and $\frac{M}{1+\alpha}$ amount of money to each depositor who withdraws at date 1. Now for the contract to be satisfied,

$$p_1(R)\frac{c_1(R)}{(1+\alpha)}+\frac{M}{1+\alpha}=D \qquad (3.11)$$

Where $p_1(R)$ is the price of goods in terms of money at date 1. Given the definition of type 1 consumers, the type 1 consumers would like to exchange the money they get at date 1 with type 2 consumers in return for goods. If their happens to be a trade between type 1 consumers and type 2 consumers at date 1 (depending upon the fact whether type 2 consumer is willing to take part in this trade), a type 1 consumer purchases $\frac{M/(1+\alpha)}{p_1(R)}$ units of goods from type two consumers. On the other hand, a type 2 consumer, who withdraws from the bank at date 1, gets $\frac{c_1(R)}{1+\alpha}p_1(R)$ amount of money from type 1 consumers by selling $\frac{c_1(R)}{1+\alpha}$ units of goods. Hence the demand for goods by type 1 agents at date 1 is given by

$$\frac{M/(1+\alpha)}{p_1(R)}$$

and the supply of goods by early withdrawing type 2 agents at date 1 is given by

$$\alpha\frac{c_1(R)}{1+\alpha}$$

Assuming perfect competition, it follows that in equilibrium

$$p_1(R)=\left(\frac{1}{\alpha}\right)\frac{M}{c_1(R)} \qquad (3.12)$$

So as a result of withdrawing from the bank at date 1 and then purchasing goods from type 2 consumers at the price $p_1(R)$, the consumption of a type one consumer is

$$c_{11}(R) = \frac{c_1(R)}{1+\alpha} + \frac{M}{1+\alpha}\left(\frac{1}{p_1(R)}\right) \tag{3.13}$$

On the other hand, as a result of this trade, the total amount of money a type 2 depositor gets is given by

$$\frac{M}{1+\alpha} + \frac{c_1(R)}{1+\alpha} p_1(R)$$

At date 2, the residual Z with the bank is given by

$$Z = r\left[L - (1+\alpha)\frac{c_1(R)}{1+\alpha}\right] + RX \tag{3.14}$$

Note that at date 2, α proportion of type 2 depositors are having $\frac{M}{1+\alpha} + \frac{c_1(R)}{1+\alpha} p_1(R)$ amount of money and the rest of the type 2 depositors are having a deposit contract worth D amount of money. According to the contract, the residual is supposed to be divided on pro rata basis. Hence, we have

$$c_{21} = \left[\frac{\frac{c_1(R)}{1+\alpha} p_1(R + \frac{M}{1+\alpha}}{\alpha\left[\frac{c_1(R)}{1+\alpha} p_1(R) + \frac{M}{1+\alpha}\right] + (1-\alpha)D}\right] Z \tag{3.15}$$

$$c_{22} = \left[\frac{D}{\alpha\left[\frac{c_1(R)}{1+\alpha} p_1(R) + \frac{M}{1+\alpha}\right] + (1-\alpha)D}\right] Z \tag{3.16}$$

where Z is given by (3.14).

In equilibrium

$$c_{21}(R) = c_{22}(R) \tag{3.17}$$

Using (3.15) and (3.16) in (3.17) and thereafter using (3.12), we get

$$\alpha = \frac{M}{D} \tag{3.18}$$

Note that at date 2, those type 2 depositors who have withdrawn from the bank at date 1 gets goods from the bank at date 2 in return of money. The rest of the type 2 depositor gets goods from the bank at date 2 because they have not withdrawn from the bank at date 2 and hence they still have the withdrawal right which is worth D in nominal terms. Hence we can say that at date 2 the bank gives goods to depositors in return of money at a price $p_2(R)$ which is given by

$$p_2(R) = \frac{\alpha\left[\frac{c_1(R)}{1+\alpha}p_1(R) + \frac{M}{1+\alpha}\right] + (1-\alpha)D}{Z} \tag{3.19}$$

where Z is given by (3.14).

For the working of the model, the following constraints have to be satisfied. These are

$p_1(R) \geq p_2(R)$ and $0 < M \leq D$

The first constraint implies that it pays for type 2 depositors to sell their goods to type 1 depositors at date 1 rather than storing them till date 2. The second constraint implies that α is not more than 1.

Having discussed the working of the model, we now state the theorem given by AG. This is the theorem 4 of AG, but given the sequence of he dissertation, it is stated here as proposition 3.

Proposition 3: *Suppose that the central bank makes available to the representative bank an interest free line of credit of M units of money at date 1 which must be repaid at date 2. Then there exist equilibrium price level* $p_1(R)$ *and* $p_2(R)$ *and an equilibrium fraction of early withdrawers* $\alpha(R)$ *for every value of R, which will implement the incentive –efficient allocation* $\{L^*, X^*, c_1^*(R), c_2^*(R)\}$.

Proof: Given $\{L^*, X^*, c_1^*(R), c_2^*(R)\}$,

we have

$$p_1(R) = \left(\frac{1}{\alpha}\right)\frac{M}{c_1^*(R)} \qquad (3.12')$$

from (3.12).

Substituting $c_1^*(R)$ in place of $c_1(R)$ in (3.13), and thereafter using (3.12′), we get

$$c_{11}(R) = c_1^*(R) \qquad (3.13')$$

Similarly substituting $c_1^*(R)$ in place of $c_1(R)$, L^* in place of L and X^* in place of X in (3.15) and (3.16) and thereafter using (3.18), we get

$$c_{21}(R) = c_2^*(R) \qquad (3.15')$$

and

$$c_{22}(R) = c_2^*(R) \qquad (3.16')$$

from (3.6).

Similarly substituting $c_1^*(R)$ in place of $c_1(R)$, L^* in place of L and X^* in place of X in (3.19) and thereafter using (3.12′) and (3.18), we get

$$p_2(R) = \frac{D}{c_2^*(R)} \qquad (3.19')$$

after using (3.6).

Note that the constraints that $p_1(R) \geq p_2(R)$ is satisfied because $c_1^*(R) \leq c_2^*(R) \forall R$ from proposition 1.

To satisfy the other constraint, that is $M \leq D$, we assume that

$$0 < M < D^1 \qquad (3.20)$$

We can assume so because the choice of M and D do not affects the outcome.

Hence proved.

With the above mentioned optimal monetary policy by the central bank, the deadweight costs of bank runs in case of r>1 can be removed.

So in this section we have shown how demand deposit contract supported by lender of the last resort policy by the central bank can implement the first best solution for all values of r.

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[1] We could also assume that $0 < M \leq D$ but we do not do so because we will need (3.20) in proposition (6). So we rule out the possibility of $\alpha = 1$ by the assumption (3.20)

So far we have considered the case where there does not exist a market for risky assets at date 1. In the next section, we study the outcome in case where such a market exists.

6. Market for Risky Assets

Although demand deposit contract can achieve the first best allocation in case of r=1, without any central bank intervention, AG shows that if there exist market for risky assets at date 1, then, even in the case of r=1, given the demand deposit contract, runs lead to inefficient allocation. The resulting allocation which is obtained by the demand deposit contract is shown to be inferior to the allocation given by (P1).

To create a market for risky assets, AG introduces a group of risk neutral speculators, who make direct investment in safe and risky assets at date 0. They consume only in the last period and their objective is to maximize the expected value of their portfolio at date 2. The risk neutral agents are all identical, so they can be replaced by a representative individual who has an initial wealth W_s and chooses a portfolio (L_s, X_s) subject to the budget constraint

$$L_s + X_s = W_s$$

At date 1, the bank in case of not being able to give the promised units of goods to each depositor withdrawing at that date, sells its risky assets to the speculators in return of their safe assets.

If R is 'low', then it is in the interest of type 2 consumers to withdraw at date 1. However there cannot be a partial run. The terms of the deposit contract require the bank to liquidate all its assets if it cannot pay $\bar{c}$ to every depositor who demands. Given this, if there is a run, it will be complete, because a depositor, who waits till date 2, will get zero units of consumption good. Recall that in the earlier case when there was no market for risky assets at date 1, if a bank run occurred, it was a partial run, except for R=0. But in the case when there does exists a market for risky assets, a bank run is a complete run for all values of R. If the market for risky asset is illiquid, the sale of the representative bank holding of the risky asset will drive down the price. So, in this case even for r=1, when the representative bank is forced to liquidate the risky asset, it sells the asset at a low price. So even in the case of r=1, there is a deadweight cost of run, if their exist a market for risky assets at date 1.

The solution to (P1) is shown to be implemented by an intervention by the central bank (see theorem 5 and corollary 5.1 in AG). The details of this intervention are not needed for our analysis. AG shows that the laissez-faire outcome of the model with the type of asset market mentioned above is Pareto inferior to the solution given by (P1).

Proposition 4: *The central bank can implement the solution to problem (P1) by entering into a repurchase agreement with the representative bank at date 1. Given the allocation* $\{L^*, X^*, c_1^*(R), c_2^*(R)\}$, *corresponding to the solution of (P1), the equilibrium values of prices are given by the conditions*

$p_1(R)c_1^*(R) = D < p_2(R)c_2^*(R)$ *for* $R > \overline{R}$ *and*

$p_1 c_1^*(R) = D = p_2 c_2^*(R)$ *for* $R < \overline{R}$.

There is a fixed amount of money M injected into the economy in the event of a run and the fraction of late withdrawers who run satisfies $\alpha(R)D = M$. *The price of the risky asset (P(R)) at date 1 satisfies* $p_1(R)R = P(R)$ *and the optimal portfolio of the speculators is* $\{L_s, X_s\} = \{0, W_s\}$

Proof: The proof is given in AG. See p.1278-1279.

Corollary 4.1: *The solution to (P1), implemented by the policy described in proposition 4, is Pareto-preferred to the laissez faire equilibrium outcome of the model with asset markets.*

Proof: The proof is given in AG. See the appendix

However, given the introduction of risk neutral agents, AG does not explore the possibility of a combined risk sharing by risk neutral and risk averse agents.

To summarize, AG shows that in absence of speculators at date 1, demand deposit contract can achieve the first best solution in case of costless run. However in case of costly runs, demand deposit contract fails to achieve the first best solution. Even in case of costless runs, if there exist a market for risky assets at date 1, demand deposit contract fails to achieve the first best solution. The inefficiency is shown to be removed in both cases by central bank intervention.

In the next chapter, we will show that within the model specification given by AG, in the absence of speculators at date 1, the first best solution can be achieved without any central bank intervention if equity contract is used instead of demand deposit contract.

We will also show that in the absence of speculators at date 1, there also exists an equity contract which can achieve the first best contract in exactly the same manner as demand deposit supported by lender of the last resort policy does.

In the case of risk neutral agents, we will show that given combined risk sharing by risk neutral agents and risk averse agents, there exists a superior allocation than given by (p1).

Chapter IV

Equity, Repurchase and the First Best Solution

1. Introduction

In this chapter, we will show that in the absence of speculators at date1, the first best solution of AG can be implemented by relying on market mechanism, without any role of the central bank. In section 2, we will show that with an equity contract with repurchase of shares at date 1, the first best solution is achieved and that too without any run. In section 3, we will show that there also exists an equity contract with repurchase of shares at date 1, that can achieve the first best solution and it has runs in equilibrium. In section 4, we will show that equity contract discussed in section 3 is equivalent to the demand deposit contract supported by lender of the last resort policy (chapter 3, section 5) which is used by AG to implement the first best allocation in case of costly runs. In section 5, we will show that with combined risk sharing by risk neutral and risk averse agents, there exists an allocation superior to that given by (P1). However, we will not discuss the issue of implementation of this solution.

2. Repurchase of Equity and the First Best Solution

In this section we now show that the first best solution of AG (in case of risk averse agents only) is implemented in a very simple manner by an equity contract, which follows the policy of repurchase of equity.

Consider a firm instead of a bank. The only difference between the firm and the bank is that the bank issues demand deposits whereas the firm issue equity only. In what follows, we will use goods as the numeraire. Let the firm issues E number of equity shares at date 0 at a price 1. It is assumed that only the firm can form a portfolio of safe and risky asset but not the consumers and since the policy of the firm is to maximize the expected utility of consumers, it pays for the consumers to use their entire endowments to purchase equity shares from the firm at date 0. Hence funds with the firm at date 0 is equal to E. This is invested by the firm in safe (L) and risky asset (X) at date 0. At date 0 the firm announces the price of equity shares in terms of the consumption good at which it will repurchase equity shares from the investors at date 1. The residual with the bank at date 2 is to be divided among the investors on the basis of equity shares they have at that time. Since each consumer is having an endowment equal to E/2, each consumer purchases E/2 number of equity shares at date 0.

Let us consider an equity contract of the following kind between the firm and the investor. The firm repurchases equity shares from the investors at date 1. Let us have the simplifying assumption that the investors are allowed to sell either all of their equity shares (i.e. E/2 number of equity shares) or none of them if they queue up to sell their equity shares at date 1. The residual with the firm at date 2 is divided among the investors on the basis of equity shares they have at that date.

Let δ be the proportion of type 2 investors who sell their equity shares to the firm at date 1. Hence δ can be interpreted as a measure of runs in the context of equity contract with repurchase.

Proposition 5: *An equity contract which follows the policy of repurchase can implement the first best allocation without runs if the firm repurchases equity shares from the investors at date 1 with the condition that if an investor queues up to sell her equity shares to the firm at date 1, then she has to sell all of her shares, i.e.* $\frac{E}{2}$ *number of equity shares, at a price given by* $P(R) = \frac{c_1^*(R)/(1+\delta)}{E/2}$ *and portfolio choice*

$\{L, X\} = \{L^*, X^*\}$.

Proof: First we show that $\delta = 0$ is a unique equilibrium for all values of R.

Suppose $\delta = 0$. So only type one investors are selling their equity shares to the firms at date 1. Given the price, each type one investor gets $c_1^*(R)$ units of consumption good by selling her $E/2$ number of equity shares. The residual with the firm at date 2 is equal to

$$r(L^* - c_1^*(R)) + RX^* = c_2^*(R)$$

from (3.6).

Since the measure of type 2 investors is one and each type 2 investor is having equal number of equity shares, that is $E/2$ number of equity shares at date 2, each one of them gets $c_2^*(R)$ units of consumption good. Since $c_1^*(R) \le c_2^*(R) \forall R$, $\delta = 0$ is an equilibrium value for all value of R and the first best is implemented.

Next consider the case when $\delta > 0$. In this case, an investor who sells equity shares to the firm at date 1 gets $\frac{c_1^*(R)}{1+\delta}$ units of consumption good by selling her $E/2$ number of equity shares to the firm at date 1. Since $\frac{c_1^*(R)}{1+\delta} < c_1^*(R) \le c_2^*(R)$ for $\delta > 0$, a type two investor who sells her shares to the firm at date 1 is at a loss. So $\delta > 0$ is not an equilibrium.

Hence $\delta = 0$ is a unique equilibrium.

Given that $\delta = 0$ is a unique equilibrium, each type 1 investor gets $c_1^*(R)$ units of consumption good and each type 2 investor gets $c_2^*(R)$ units of consumption good in equilibrium. Hence the first best is implemented. This completes the proof.

So we see that the first best allocation can be implemented by an equity contract with repurchase of shares with the kind of contract, we have considered. In equilibrium there is no run. Next, we show that by reformulating the contract given in this section, we can make it vulnerable to runs and it achieves the first best allocation in the same manner as the demand deposit contact supported by lender of the last resort policy does.

3. Repurchase of Equity with Runs

The previous equity contract implements the first best allocation and there is no run in the equilibrium. Now we plan to look for an equity contract which has runs in equilibrium.

We consider a contract of the following kind between the firm and the equity holders. The firm puts a ceiling on the maximum number of equity shares it will accept per investor at date 1. Let the maximum number of equity shares an individual investor can sell to the firm at date 1 is equal to

$$\left[\frac{E}{2} - \frac{K}{1+\delta}\right]$$

Hence for a given δ, an individual investor who queues up for redemption at date 1, has to retain at least $\frac{K}{1+\delta}$ number of equity shares. Hence if each individual investor who sells her equity shares to the firm at date 1 retains $\frac{K}{1+\delta}$ number of equity shares, then K is the total number of equity shares retained by the investors who sell their equity shares to the firm at date 1.

Let P(R) be the price of equity shares at which the firm repurchases equity shares from the investors who queue up to sell their equity shares to the firm at date 1. Suppose all investors who queue up to sell their shares to the firm at date I sell $\left[\frac{E}{2}-\frac{K}{1+\delta}\right]$ number of equity shares to the firm at date 1. Given P(R), the units of consumption good each such investor gets from the firm is equal to

$$\left[\frac{E}{2}-\frac{K}{1+\delta}\right]P(R)$$

Note that for type one investors, the utility from consumption at date 2 is zero. Hence they would like to sell all of their retained equity shares to type 2 investors (who also have sold their shares to the firm at date 1) in return for consumption goods. Suppose the type 2 investors who have also sold their equity shares to the firm at date 1 are also willing to sell their goods to type 1 investors in return for equity shares. The total demand for goods in this case at date 1 is given by

$$\frac{K/(1+\delta)}{s_1(R)}$$

where $s_1(R)$ is the price of goods at date 1 in terms of equity shares in the secondary market. Similarly the total supply of goods at date 1 is given by

$$\delta\left[\frac{E}{2}-\frac{K}{1+\delta}\right]P(R)$$

Assuming perfect competition, $s_1(R)$ is given by the condition that the total demand of goods at date 1 is equal to the total supply of the goods at date 1. This implies that

$$s_1(R) = \frac{K/(1+\delta)}{\delta\left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)} \tag{4.1}$$

An individual type 1 investor gets $\left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)$ units of consumption good by selling $\left[\frac{E}{2} - \frac{K}{1+\delta}\right]$ number of equity shares to the firm at date 1. By selling the rest of the equity shares, that is $\frac{K}{1+\delta}$ number of equity shares to type 2 investors at price $s_1(R)$, she gets $\left[\frac{K}{1+\delta}\right]\frac{1}{s_1(R)}$ units of consumption good. Therefore we have

$$c_{11}(R) = \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R) + \left[\frac{K}{1+\delta}\right]\frac{1}{s_1(R)} \tag{4.2}$$

Similarly a type 2 investor has E/2 number of equity shares. She retains $\frac{K}{1+\delta}$ number of equity shares and sells the remaining of them to the firm. In return she gets goods which she sells to type 1 investors at price $s_1(R)$. Hence, she finally has the following number of equity shares

$$\frac{K}{1+\delta} + \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)s_1(R)$$

At date 2, the residual Z with the firm is equal to

$$Z = r\left[L - P(R)\left(\frac{E}{2} - \frac{K}{1+\delta}\right)(1+\delta)\right] + RX. \qquad (4.3)$$

At date 2, δ proportion of the type 2 investors are having $\frac{K}{1+\delta} + \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)s_1(R)$ number of equity shares. Rest of them are having $\frac{E}{2}$ number of equity shares. Denoting c_{ij} as the consumption of i type consumers at date j, we have according to the contract

$$c_{21}(R) = \left[\frac{\frac{K}{1+\delta} + \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)s_1(R)}{\delta\left[\frac{K}{1+\delta} + \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)s_1(R)\right] + (1-\delta)\frac{E}{2}}\right]Z \qquad (4.4)$$

where Z is given by (4.3). The numerator term in the square bracket is the number of equity shares a type 2 investor (who sells her equity shares to the firm at date 1) has at date 2 and the denominator term is the total number of equity shares with the investors at date 2. Similarly we have

$$c_{22}(R) = \left[\frac{\frac{E}{2}}{\delta\left[\frac{K}{1+\delta} + \left[\frac{E}{2} - \frac{K}{1+\delta}\right]P(R)s_1(R)\right] + (1-\delta)\frac{E}{2}}\right]Z. \qquad (4.5)$$

where the numerator term in the square bracket is the number of equity shares a type 2 investor (who have not sold her equity shares to the firm at date 1) has at date 2.

δ is determined by the condition

$c_{21}(R)=c_{22}(R)$ (4.6)

Since equity shares expire at date 2, it is as if the firm is selling the goods to investors in return for equity shares at a price $s_2(R)$, which is given by

$$s_2(R)=\frac{\delta\left[\frac{K}{1+\delta}+\left[\frac{E}{2}-\frac{K}{1+\delta}\right]P(R)s_1(R)\right]+(1-\delta)\frac{E}{2}}{r\left[L-P(R)\left(\frac{E}{2}-\frac{K}{1+\delta}\right)(1+\delta)\right]+RX}$$

(4.7)

The following conditions should be satisfied for the working of the model.

$$P(R)\left[\frac{E}{2}-\frac{K}{1+\delta}\right](1+\delta)\leq L \qquad \text{(A)}$$

$$s_1(R)\geq s_2(R) \qquad \text{(B)}$$

$$P(R)\geq\frac{1}{s_1(R)} \qquad \text{(C)}$$

$$0<K<\frac{E}{2} \qquad \text{(D)}$$

Condition (A) says that the total leakage of consumption goods from the firm at date 1 is less than equal to what the firm is having at date1 that is L. Condition (B) guarantees that there is trade between type 1 and type 2 investors. It is obvious for type 1 investor to sell her equity shares to type 2 investors in return of goods. Condition (B) implies that it pays for type 2 investors to sell her consumption goods to type one investors rather than carrying it to date 2. Condition (C)[2] guarantees that an investor who sells her equity shares to the firm at date 1 do not sell less than $\left[\frac{E}{2}-\frac{K}{1+\delta}\right]$[3] number of equity shares to the firm. Constraint (D) is obvious. We are considering partial repurchase of equity shares. Therefore, some equity shares are retained by consumers. Hence K>0. The firm is buying some positive number of equity shares at date 1.Hence

$\frac{E}{2}-K>0$, that is $\frac{E}{2}>K$.

Proposition 6: *An equity contract which repurchases equity shares from investors at date 1 with the rule that an investor is allowed to sell not more than* $\left[\frac{E}{2}-\frac{K}{1+\delta}\right]$ *number of equity shares at date 1 to the firm can implement the first best allocation. The required price and the portfolio choice is given by*

$$P(R)=\frac{2c_1^*(R)}{E(1+\delta)-2K} \qquad (4.8)$$

and

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[2] We have taken the reciprocal of $s_1(R)$ because $s_1(R)$ is the price of goods in terms of equity shares and P(R) is the price of equity shares in terms of goods.

[3] Note that selling more than $\left[\frac{E}{2}-\frac{K}{1+\delta}\right]$ number of equity shares to the firm a date 1 is not allowed by the firm. Hence this is the maximum number of equity shares an investor can sell to the firm at date 1.

$$\{L, X\} = \{L^*, X^*\}$$

Proof: Using (4.8) in (4.1), we get

$$s_1(R) = \frac{K\,\delta}{c_1^*(R)} \qquad (4.1')$$

Using (4.8) and (4.1′) in (4.2), we get

$$c_{11}(R) = c_1^*(R)$$

Using (4.8) and (4.1′) in (4.4) and (4.5) and substituting the value of Z from (4.3)), we get

$$c_{21}(R) = \frac{K/\delta}{K + (1-\delta)\frac{E}{2}}\left[r\left(L - c_1^*(R)\right) + RX\right] \qquad (4.4')$$

$$c_{22}(R) = \frac{E/2}{K + (1-\delta)\frac{E}{2}}\left[r\left(L - c_1^*(R)\right) + RX\right] \qquad (4.5')$$

Using (4.4′) and (4.5′) in (4.6), we get

$$\delta = \frac{2K}{E} \qquad (4.6')$$

Using (4.8), (4.1′) and (4.6′) in (4.7), we get

$$s_2(R) = \frac{\frac{E}{2}}{r\left[L - c_1^*(R)\right] + RX} \qquad (4.7')$$

Using (4.6′) and $L = L^*$ and $X = X^*$ in (4.4′), we get

$c_{21} = r\left(L - c_1^*(R)\right) + RX^* = c_2^*(R)$ from (3.6).

Similarly using (4.6′) and $L = L^*$ and $X = X^*$ in (4.5′), we get

$c_{22} = r\left(L - c_1^*(R)\right) + RX^* = c_2^*(R)$ from (3.6).

It can be checked that with $P(R)=\frac{2c_1^*(R)}{E(1+\delta)-2K}$, $L=L^*, X=X^*$, conditions (A),(B) and (C) are satisfied. Condition (D) needs to be assumed.

Hence the first best is implemented. This completes the proof

So we see that with the above mentioned equity contract, the first best is implemented. However, it differs from the previous equity contract, because in the previous equity contract, $\delta=0$ is an unique equilibrium. In this equity contract $\delta>0$ is equilibrium. But the allocation is same in both cases.

In the next section, we show that the equity contract mentioned in this section is equivalent to demand deposit contract supported by lender of the last resort policy by central bank (chapter 3, section 5).

4. Demand Deposit vis-à-vis Equity

In the previous section, we showed that it is possible to attain the first best allocation with an equity contract. Recall that in the previous chapter (section5), it was shown that with demand deposits issued by commercial banks and supported by lender of the last resort policy of the central bank, the first best solution can be attained. In this section, it will be shown that for given values of K and E, demandable debt supported by lender of the last resort policy and the equity contract with repurchase of shares are equivalent.

Proposition 7: *AG lender of the last resort policy and the equity shares contract used in the previous section are equivalent for K=M and E=2D*

Proof: First we show that K=M and E=2D is a feasible choice, that is it satisfies the constraint

$$0 < K < \frac{E}{2}.$$

The above constraint is satisfied because for the working of the monetary policy, we have assumed that

$$0 < M < D.[4] \qquad (3.20)$$

Substituting K=M, E=2D in (4.1), (4.2), (4.3), (4.4) and (4.5) and thereafter using (4.8) and $\{L, X\} = \{L^*, X^*\}$ in all of these equations, we get the following equations

$$s_1(R) = \left(\frac{1}{\delta}\right)\frac{M}{c_1^*(R)} \qquad (4.1'')$$

$$c_{11}(R) = \frac{c_1^*(R)}{1+\delta} + \frac{M}{1+\delta}\left(\frac{1}{s_1(R)}\right) \qquad (4.2'')$$

$$Z = r\left[L^* - c_1^*(R)\right] + RX^* \qquad (4.3'')$$

[4] If instead of assuming $0 < M < D$, we would had assumed that $0 < M \leq D$, then K=M and E=2D would not have been a feasible choice.

$$c_{21}(R) = \left[\frac{\frac{M}{1+\delta} + \frac{c_1^*(R)}{1+\delta} s_1(R)}{\delta\left[\frac{M}{1+\delta} + \frac{c_1^*(R)}{1+\delta} s_1(R)\right] + (1-\delta)D} \right] Z \qquad (4.4'')$$

$$c_{22}(R) = \left[\frac{D}{\delta\left[\frac{M}{1+\delta} + \frac{c_1^*(R)}{1+\delta} s_1(R)\right] + (1-\delta)D} \right] Z \qquad (4.5'')$$

Substituting K=M and E-2D in (4.6), we get

$$\delta = \frac{M}{D} \qquad (4.6'')$$

Substituting E=2D in (4.7) and thereafter using (4.8) and $\{L, X\} = \{L^* . X^*\}$, we get

$$s_2(R) = \frac{D}{r\left[L^* - c_1^*(R)\right] + RX^*} = \frac{D}{c_2^*(R)} \qquad (4.7'')$$

from (3.6)

Substituting (4.6$''$) in (4.7$''$), we get

$$s_2(R) = \frac{M}{\delta}\left(\frac{1}{c_2^*(R)}\right) \qquad (4.7''')$$

From (4.6$''$) and (3.18), we have

$$\delta = \alpha \qquad (4.6''')$$

Substituting (4.6$'''$) in (4.1$''$) and (4.7$'''$) and thereafter comparing them to (3.12) and (3.19), with $c_1^*(R)$ substituted in place of $c_1(R)$, L^* substituted in place of L and X^* substituted in place of X in (3.12) and (3.19) accordingly, we have

$$s_1(R) = p_1(R) \tag{4.1'''}$$

and

$$s_2(R) = p_2(R) \tag{4.7'''}$$

Using (4.6'''), (4.1''') and (4.7''') in (4.2''), (4.4'') and (4.5''), we get

$$c_{11}(R) = \frac{c_1^*(R)}{1+\alpha} + \frac{M}{1+\alpha}\left(\frac{1}{p_1(R)}\right) \tag{4.2'''}$$

$$c_{21}(R) = \left[\frac{\dfrac{M}{1+\alpha} + \dfrac{c_1^*(R)}{1+\alpha} p_1(R)}{\alpha\left[\dfrac{M}{1+\alpha} + \dfrac{c_1^*(R)}{1+\alpha} p_1(R)\right] + (1-\alpha)D}\right] Z \tag{4.4'''}$$

$$c_{22}(R) = \left[\frac{D}{\alpha\left[\dfrac{M}{1+\alpha} + \dfrac{c_1^*(R)}{1+\alpha} p_1(R)\right] + (1-\alpha)D}\right] Z \tag{4.5'''}$$

Note that Z given by (4.3'') is same as that given by (3.14) if we substitute $c_1^*(R)$ in place of $c_1(R)$, L^* in place of L and X^* in place of X in (3.14).

Hence $c_{11}(R), c_{21}(R)$ and $c_{22}(R)$ given by (4.2'''), (4.4''') and (4.5''') respectively are same as that given by (3.13),(3.15),and (3.16) if we substitute $c_1^*(R)$ in place of $c_1(R)$, L^* in place of L and X^* in place of X in (3.13), (3.15) and (3.16) accordingly.

Hence the two contracts are equivalent. This completes the proof.

$$c_1(R) + c_2(R) + c_{2n}(R) \leq L + RX \tag{3}$$

$$E[c_{2n}(R)] \geq E[R]W_s \tag{4}$$

$$c_1(R) \leq c_2(R) \tag{5}$$

and

$$L \geq 0, X \geq 0, c_1(R) \geq 0, c_2(R) \geq 0, c_{2n}(R) \geq 0 \tag{6}$$

The rationale for constraints (1), (2), (3), (5) and (6) is obvious. However, constraint (4) needs to be explained. Recall from proposition 4 where the risk neutral agent's portfolio choice is $\{0, W_s\}$. Given this portfolio choice, her expected utility is $E[R]W_s$. We want to show that an allocation superior than that given in this proposition exist. $E[R]W_s$ is the reservation utility of risk neutral agent.

Consider the following solution to (T1)

$$L = L^* \tag{4.9}$$

$$X = X^* + W_s \tag{4.10}$$

$$c_1(R) = c_1^*(R) \forall R \tag{4.11}$$

$$c_2(R) = \begin{cases} c_2^*(R), 0 \leq R < \overline{R} \\ c_2^*(R) + \varepsilon, \overline{R} \leq R < \hat{R} \\ c_2^*(R) - \varepsilon, \hat{R} \leq R \leq R^h \end{cases} \tag{4.12}$$

$$c_{2n}(R) = \begin{cases} RW_s, 0 \leq R < \overline{R} \\ RW_s - \varepsilon, \overline{R} \leq R < \hat{R} \\ RW_s + \varepsilon, \hat{R} \leq R \leq R^h \end{cases} \tag{4.13}$$

where $L^*, X^*, c_1^*(R), c_2^*(R), \overline{R}$ are defined as in proposition 1. R^h is the highest value of R. ε is assumed to satisfy

$$0 < \varepsilon \le \min\{\overline{R}W_s, \hat{R}X^* - L^*\}, \qquad (4.14)$$

Note that ε given by (4.14) is a feasible choice because both $\overline{R}W_s$ and $\hat{R}X^* - L^*$ are strictly positive because $\overline{R} > 0, W_s > 0$ and $\hat{R}X^* - L^* > 0$ from (3.3)

$\hat{R}$ is defined by the following condition

$$\int_{\overline{R}}^{\hat{R}} f(R)dR = \int_{\hat{R}}^{R^h} f(R)dR \ . \qquad (4.15)$$

Now we show that the above given solution is a feasible solution, that is, it satisfies all the constraints of (T1)

$L + X = L^* + X^* + W_s = E + W_s$ from (4.9), (4.10) and condition 2 of the proposition 1. Hence the first constraint is satisfied.

$c_1(R) = c_1^*(R) \le L^* = L$ from (4.9),(4.11) and (3.1). Hence the second constraint of (T1) is satisfied.

Now let us check whether the third constraint is satisfied. First consider the case when $R < \overline{R}$. In this case, we have

$c_1(R) + c_2(R) + c_{2n}(R) = c_1^*(R) + c_2^*(R) + RW_s, R < \overline{R}$ from (4.11), (4.12) and (4.13). But $c_1^*(R) + c_2^*(R) = L^* + RX^*$ by substituting r=1 in (3.6).

Hence $c_1(R)+c_2(R)+c_{2n}(R)=L^*+RX^*+RW_s=L+RX$ from (4.9) and (4.10). Hence the third constraint is satisfied for $R<\overline{R}$. Next consider the case when $\overline{R}\le R<\hat{R}$. In this case

$c_1(R)+c_2(R)+c_{2n}(R)=c_1^*(R)+c_2^*(R)+\varepsilon+RW_s-\varepsilon=c_1^*(R)+c_2^*(R)+RW_s$ from (4.11), (4.12) and (4.13) and as we have shown in the previous case the third constraint is satisfied in this case.

Similarly, it is easy to see that for $\hat{R}\le R\le R^h$, the third constraint is satisfied.

Next consider the fourth constraint. We have

$$E[c_{2n}(R)]=\int_0^{\overline{R}}RW_s f(R)dR+\int_{\overline{R}}^{\hat{R}}(RW_s-\varepsilon)f(R)dR+\int_{\hat{R}}^{R^h}(RW_s+\varepsilon)f(R)dR=E[R]W_s$$

from (4.13) and (4.15).

Next consider the fifth constraint, that is whether $c_1(R)\le c_2(R)$ is satisfied or not. For $R<\hat{R}$, the proof is obvious. For $\hat{R}\le R\le R^h$, we have $c_1(R)=c_1^*(R)=L^*$ from (4.11) and condition 5 of the proposition 1 and $c_2(R)=c_2^*(R)-\varepsilon=RX^*-\varepsilon$ from (4.12) and condition 5 of the proposition 1. But $RX^*-\varepsilon\ge L^*$ from (4.14). Hence the fifth constraint is satisfied.

Now consider the non negativity constraints of (T1). It is easy to see that $L, X, c_1(R)$ are nonnegative from (4.9),(4.10), (4.11) and proposition 1. It is easy to see that $c_2(R)$ is nonnegative for $R<\hat{R}$. For $\hat{R}\le R\le R^h$, we have $c_2(R)=c_2^*(R)-\varepsilon=RX^*-\varepsilon$ from (4.12) and condition 5 of the proposition 1. But $RX^{*}-\varepsilon \geq L^{*}>0$ from (4.14) and condition 1 of the proposition 1. Hence it is proved that $c_{2}(R) \geq 0 \forall R$.

Let us check the non-negativity of $c_{2n}(R)$. It is obvious for $0 \leq R<\overline{R}$ and $\hat{R} \leq R \leq R^{h}$. We need to check it for $\overline{R} \leq R<\hat{R}$. In this case, we have $c_{2n}(R)=RW_{s}-\varepsilon$ from (4.12). But $RW_{s}-\varepsilon \geq RW_{s}-\overline{R}W_{s} \geq 0$ from (4.14) and using the fact that $\overline{R} \leq R<\hat{R}$. Hence $c_{2n}(R) \geq 0, \overline{R} \leq R<\hat{R}$.

Hence it is proved that all the constraints of (T1) are satisfied by the solution given by (4.9),(4.10),(4.11),(4.12),(4.13),(4.14) and (4.15).

We now give the following lemma.

Lemma 1:

$$\int_{\overline{R}}^{\hat{R}} u\left(c_{2}^{*}(R)+\varepsilon\right) f(R) dR+\int_{\hat{R}}^{R^{h}} u\left(c_{2}^{*}(R)-\varepsilon\right) f(R) dR>\int_{\overline{R}}^{R^{h}} u\left(c_{2}^{*}(R)\right) f(R) dR$$

Proof

Choose $R_{1} \in[\overline{R}, \hat{R})$ and $R_{2} \in[\hat{R}, R^{h}]$

such that the following holds

$$u\left(c_{2}^{*}(R_{1})+\varepsilon\right)-u\left(c_{2}^{*}(R_{1})\right) \leq u\left(c_{2}^{*}(R)+\varepsilon\right)-u\left(c_{2}^{*}(R)\right), \overline{R} \leq R<\hat{R}$$

and

$$u\left(c_{2}^{*}(R_{2})\right)-u\left(c_{2}^{*}(R_{2})-\varepsilon\right) \geq u\left(c_{2}^{*}(R)\right)-u\left(c_{2}^{*}(R)-\varepsilon\right), \hat{R} \leq R \leq R^{h}.$$

Hence

$$\int_{\underline{R}}^{\hat{R}}\left[u\left(c_2^*(R_1)+\varepsilon\right)-u\left(c_2^*(R_1)\right)\right]f(R)dR \le \int_{\underline{R}}^{\hat{R}}\left[u\left(c_2^*(R)+\varepsilon\right)-u\left(c_2^*(R)\right)\right]f(R)dR$$

(4.16)

and

$$\int_{\hat{R}}^{R^h}\left[u\left(c_2^*(R_2)\right)-u\left(c_2^*(R_2)-\varepsilon\right)\right]f(R)dR \ge \int_{\hat{R}}^{R^h}\left[u\left(c_2^*(R)\right)-u\left(c_2^*(R)-\varepsilon\right)\right]f(R)dR$$

(4.17)

But note that $\overline{R} \le R_1 < R_2$. Hence $c_2^*(R_1) = R_1X^*, c_2^*(R_2) = R_2X^*$ from condition 5 of proposition 1. But $c_2^*(R_1) < c_2^*(R_2)$, because $R_1 < R_2$.

Therefore

$$u\left(c_2^*(R_1)+\varepsilon\right)-u\left(c_2^*(R_1)\right) > u(c_2^*(R_2))-u(c_2^*(R_2)-\varepsilon) \qquad (4.18)$$

because $u' > 0, u'' < 0$.

Hence

$$\int_{\underline{R}}^{\hat{R}}\left[u\left(c_2^*(R_1)+\varepsilon\right)-u\left(c_2^*(R_1)\right)\right]f(R)dR > \int_{\hat{R}}^{R^h}\left[u\left(c_2^*(R_2)\right)-u\left(c_2^*(R_2)-\varepsilon\right)\right]f(R)dR$$

(4.19)

after using (4.15).

From (4.16),(4.17) and (4.19), we have

$$\int_{\underline{R}}^{\hat{R}}\left[u(c_2^*(R)+\varepsilon)-u(c_2^*(R))\right]f(R)dR > \int_{\hat{R}}^{R^h}\left[u(c_2^*(R))-u(c_2^*(R)-\varepsilon)\right]f(R)dR.$$

(4.20)

Rearranging (4.20), we have

$$\int_{\underline{R}}^{\hat{R}} u\left(c_2^*(R)+\varepsilon\right) f(R)dR + \int_{\hat{R}}^{R^h} u\left(c_2^*(R)-\varepsilon\right) f(R)dR$$

$$> \int_{\underline{R}}^{\hat{R}} u\left(c_2^*(R)\right) f(R)dR + \int_{\hat{R}}^{R^h} u\left(c_2^*(R)\right) f(R)dR$$

$$= \int_{\underline{R}}^{R^h} u\left(c_2^*(R) f(R)dR\right)$$

Hence the lemma is proved.

Having proved this lemma, we can prove the following theorem

Proposition 8: *If risk averse and risk neutral agents share risk then a allocation superior than that given by (P1) can be shown to exist.*

Proof: Let $c_1^{**}(R)$ and $c_2^{**}(R)$ be the solution of (T1).

Hence $E\left[u\left(c_1^{**}(R)\right)+u\left(c_2^{**}(R)\right)\right] \geq E\left[u(c_1(R))+u(c_2(R))\right]$ (4.21)

where $c_1(R)$ and $c_2(R)$ are as given from (4.11) and (4.12).

But

$$E\left[u(c_1(R))+u(c_2(R))\right] = \int_0^{R^h} u\left(c_1^*(R)\right) f(R)dR + \int_0^{\overline{R}} u\left(c_2^*(R)\right) f(R)dR$$
$$+ \int_{\underline{R}}^{\hat{R}} u(c_2^*(R)+\varepsilon) f(R)dR + \int_{\hat{R}}^{R^h} u(c_2^*(R)-\varepsilon) f(R)dR$$

(4.22)

Using the previous lemma in (4.22), we have

$$E[u(c_1(R)) + u(c_2(R))] > E\left[u(c_1^*(R)) + u(c_2^*(R))\right] \tag{4.23}$$

From (4.21) and (4.23), we have

$$E\left[u(c_1^{**}(R)) + u(c_2^{**}(R))\right] > E\left[u(c_1^*(R)) + u(c_2^*(R))\right]$$

Hence the proposition is proved. This completes the proof.

So in this section, we have shown that given combined risk sharing by risk neutral and risk averse agents, there yields an allocation superior to (P1). However, we have not discussed the issue of implementation of this solution

In the next chapter, we show that in the absence of speculators at date 1, the first best solution can be implemented by an equity contract which follows dividend policy rather than repurchase of equity shares at date 1.

Chapter V

Equity, Dividend and the First Best Solution

1. Introduction

In the previous chapter we have shown that the first best allocation can be implemented without any role of the central bank if equity is used instead of demand deposits. This was done with repurchase of equity shares. In this chapter we show that in the absence of speculators at date 1, the first best allocation can also be implemented with an equity contract which follows dividend policy instead of repurchase of equity shares.

2. Equity Contract with Dividend Policy

So consider a firm instead of bank. The only difference between the two is that the bank issues demand deposits whereas the firm issue equity only. The firm issue one instrument only, which is equity, at date 0. At date 1, the firm pays out a dividend V(R) per equity share to all equity holders. At date 2, the residual with the firm is divided among investors on the basis of equity they have at that time. We plan to follow a treatment similar to Jacklin (1987).

Suppose at date 0, the firm issue E number of equity shares (there is no loss of generality in assuming so). The consumer would like to purchase these equity shares from the firm at date 0. This is because the firm's policy is to maximize the expected utility of its investors and only the firm is able to form a portfolio of safe and risky asset but the consumers are unable to do so. Given that the measure of the consumers is 2 and each individual consumer is having an endowment of $E/2$ number of consumption goods at date 0, an individual consumer purchases $E/2$ number of equity shares at date 0.

The proceeds from the sale of equity, that is E, is invested in safe (L) and risky asset (X).

At date 1, each investor gets V(R).$\frac{E}{2}$ units of consumption good. Hence the total dividend payout by the firm at date1 is equal to V(R) E. Hence V(R) should satisfy

$$V(R)E \leq L \qquad (5.1)$$

Observe that investors cannot redeem their holdings of equity at date 1 (because equity is not redeemable) if they so want depending on their types. Thus type 1 investors will be left with an asset, which will be redeemable only at date 2. The utility value to them from the ensuing consumption in period two is zero. Ideally, they would like to trade their equity shares with type 2 investors. On the other hand, the type 2 investors have received a dividend at date 1, which they would only consume at date 2. So if buying the equity shares from type 1 investors at date 1 pays to them, then they would like to buy them at date 1. For the moment if we take it for granted that it pays for type 2 investors to participate in this trade, the total demand for goods at date 1 is given by

$$\frac{E/2}{q_1(R)}$$

where $q_1(R)$ is the price of goods in terms of equity shares at date 1. On the other hand the supply of consumption goods is given by

$$V(R)\frac{E}{2}$$

Assuming perfect competition, $q_1(R)$ is determined by the condition that the total demand for consumption good at date 1 is equal to the total supply of consumption good at date 1, that is

$$\frac{E/2}{q_1(R)} = V(R)\frac{E}{2}.$$

This implies that

$$q_1(R) = \frac{1}{V(R)} \qquad (5.2)$$

As a result of dividend payout by the firm at date 1, followed by a trade between type one and type two shareholders where type one shareholders sell equity shares to type two shareholders in return for goods, the consumption of a type one investor, that is $c_1(R)$ and the consumption of a type two investor, that is $c_2(R)$ are given by

$$c_1(R) = V(R)\frac{E}{2} + \frac{E}{2}\left(\frac{1}{q_1(R)}\right) \qquad (5.3)$$

where the first term on the right hand side is the dividend payout by the firm and the second term on the right hand side is the amount of consumption goods a type 1 investor purchases from type 2 investors in return of her $E/2$ number of shares at price denoted by $q_1(R)$ and

$$c_2(R) = \frac{\frac{E}{2} + V(R)\frac{E}{2}q_1(R)}{E}\left[r\left(L - V(R)E\right) + RX\right] \qquad (5.4)$$

where $\frac{\frac{E}{2} + V(R)\frac{E}{2}q_1(R)}{E}$ is the proportion of total shares a type 2 investor is having at date 2 and [r(L-V(R)E)+RX] is the residual with the firm at date 2. Using (5.2) in (5.3) and (5.4) respectively, we get

$$c_1(R) = V(R)E \qquad (5.5)$$

and

$$c_2(R) = r\left(L - V(R)E\right) + RX \qquad (5.6)$$

The intuition is simple. At date 1, the dividend payout is V(R)E and the measure of type one investors is 1 So each type 1 equity holder consumes V(R)E. At date 2, the residual payout is [r(L-V(R)E)+RX] on E shares (which is the total number of shares). An individual type two investor is having E number of shares and the measure of type two investors is 1. Hence an individual type two investor consumes r(L-V(R)E)+RX.

Since equity shares expire at the end of date 2, it is as if the firm is selling the residual to the investors for E number of equity shares. Denoting $q_2(R)$ as the price of goods in terms of equity shares at date 2, we have

$$q_2(R) = \frac{E}{r(L - V(R)E) + RX} \tag{5.7}$$

Recall that we have taken it for granted it pays for type 2 investor to sell her goods to type one investors rather than to carry it to date 2. This requires the following condition to be satisfied

$$q_1(R) \geq q_2(R) \tag{5.8}$$

Given this analysis, we can state the following theorem:

Proposition 9: *An equity contract with dividend payout by the firm followed by trade in ex-dividend equity shares between type 1 and type 2 investors can implement the first best allocation given by proposition 1, that is* $\{c_1^*(R), c_2^*(R)$. *The following portfolio choice and the dividend payout by the firm can implement the first best:*

$\{L, X\} = \{L^*, X^*\}$ *and* $V(R) = \frac{c_1^*(R)}{E}$

Proof $V(R) = \frac{c_1^*(R)}{E}$ is a feasible value because

$V(R)E = c_1^*(R) \leq L^*$. from (3.1).

$q_1(R) = \frac{E}{c_1^*(R)}, q_2(R) = \frac{E}{c_2^*(R)}$ from (5.2) ,(5.7) and using the fact that

$V(R) = \frac{c_1^*(R)}{E}$.

Hence $q_1(R) \geq q_2(R)$ because $c_1^*(R) \leq c_2^*(R)$ (See the fourth constraint of (P1))

Therefore (5.8) is satisfied. Hence it pays for type 2 investors to sell their goods to type 1 investors at date 1. So given that there is trade at date 1, we have

$c_1(R) = c_1^*(R)$ from (5.5) and using the fact that $V(R) = \frac{c_1^*(R)}{E}$.

$c_2(R) = c_2^*(R)$ from (5.6) and using the fact that $V(R) = \frac{c_1^*(R)}{E}, \{L, X\} = \{L^*, X^*\}$.

Hence proved.

Hence an equity contract with dividend policy can also implement the first best allocation. Note that in the two equity contracts used in the previous chapter and the equity contract used in this chapter, the portfolio choice is the same. This is not just a coincidence but also the most basic tenants of the finance theory that dividend policy and repurchase of equity shares is equivalent under some given conditions. Note that this contract can implement the first best allocation for all values of r. Recall that the deposit contract used by AG can implement the first best allocation for all values of R only in case of r=1. Hence we see that the in the absence of speculators at date 1, the first best allocation can be implemented by an equity contract, which pays dividend to the investors at date 1 in the absence of speculators at date 1. Secondly there is no role of central bank in implementing the first best allocation.

Chapter VI

Conclusion

The issue of bank runs is discussed in this dissertation in the context of the model as in Allen and Gale (1998). AG has shown that bank runs occur due to business cycles and are not simply the results of self fulfilling beliefs. AG shows that runs are a way of risk sharing and are not something undesirable under some conditions. It is shown that market mechanism can lead to first best solution under some cases. In those cases, in which market mechanism does not lead to first best solution, AG shows that the central bank intervention of the right kind can lead to first best solution.

AG shows that market mechanism fails to achieve the first best solution in case of costly runs. The first best solution is shown to be achieved in this case by a demand deposit contract supported by lender of the last resort (LLR). It is shown that even in the case of costless runs, if there exist a market for risky assets, then the first best solution cannot be achieved. To introduce a market for risky asset, risk neutral agents are introduced by AG. AG shows that in this case there can be a Pareto improvement by an intervention by the central bank. However, In this case AG has not explored the possibility of risk neutral agents also investing their funds with the bank.

Working in the same format as AG, We have shown in this dissertation, that the first best solution can be achieved in all cases where AG has used the central bank intervention to do so. The first best is achieved in this cases by the use of equity contract. Since banks are known to issue demand deposit contract only, we have worked with firms which issue equity only in place of banks. Given an equity contract between the firm and the investors, the first best is shown to be implemented both by dividend policy and the repurchase policy. In case of equity contract with repurchase of equity shares, the first best is shown to be implemented in two different ways. In the first method, the first best is shown to be implemented without any run. In the second method, the repurchase policy is formulated in such a way that the issue of runs is also addressed. It is shown that the working of this contract bears resemblance to working of the demand deposit contract supported by lender of the last resort policy.

In case when risk neutral agents exist in the economy, it is shown in the dissertation that if these risk neutral agents also invest their funds with the bank, then there exists a superior allocation than what is the first best allocation in the case when only risk averse agents invest their funds with the bank.

We accept the fact that it is too simplistic to consider bank and firm on a single platform. In reality these two institutions differ in many respects and using firms instead of banks is far from reality. However, even the demand deposit contract used by AG and even in the case of central bank intervention, the contract used by AG is not fully non-contingent. What we have done is to model the contingent pay-offs explicitly. This not only clarifies some issues, but it also shows that central bank intervention is not required under some conditions.

Also while working with the equity contracts, we have assumed perfect competition at date 1, which is not very real.

Central bank has played a very important role historically. In particular, the lender of the last resort policy has been extremely useful on many occasions. This dissertation is not a critique of central banking or of lender of the last resort policy. The purpose is however, to see whether the lender of the last resort policy is required in the context of the model as given by AG.

However, we need to be clear theoretically about the conditions under which lender of the last resort policy is required. We hope this work throws some light on relevant issues.

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