# CHILDREN'S STRATEGIES TO SOLVE ADDITION AND SUBTRACTION PROBLEMS IN EARLY SCHOOL YEARS

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*In*

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*By*


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# CERTIFICATE

The research work embodied in the dissertation entitled, **CHILDREN'S STRATEGIES TO SOLVE ADDITION AND SUBTRACTION PROBLEMS IN EARLY SCHOOL YEARS** submitted by **ASHA VERMA** is in partial fulfillment for the degree of ***MASTER OF PHILOSOPHY***. The work is original and has not been submitted so far in part or full, for any other Degree or Diploma of any University.

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We recommend that this dissertation be placed before the examiners for evaluation.

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# ABSTRACT

The study of children's mathematical problem solving is a rapidly growing area of research within cognitive development. The solution strategies that children use for simple addition and subtraction problems provide a framework to understand how cognitive skills and strategies evolve and change across development. The present research entitled " children's strategies to solve addition and subtraction problems in early school years" focussed on multiple counting strategies, representations of word problems and solution processes for solving arithmetic operations at early age.

The study was designed with the following broad objectives that is (a) to study the counting strategies children used to solve addition and subtraction problems, (b) to examine children's ability to write numbers and number sentences to represent different addition and subtraction word problems, (c) to investigate children's understanding of place value from the solution strategies used for solving addition and subtraction problems, (d) to examine the differences in solution strategies used by second and third graders on addition and subtraction problems. The theoretical perspectives used for this study was Cognitive Development theory of Jean Piaget, and Socio-historical development of L.S. Vygotsky. Based on these objectives following research questions were formulated for this study. (a) Do children use different strategies for different problems? (b) Is there any pattern in the ways that children solve arithmetic problems?

An exploratory research, between group (2x2) design was chosen appropriate. The first two units of design resorted to class type (Grade II and Grade III) and the second two units were to gender (boys and girls). A random sampling technique was selected and the sample was drawn from New Delhi, MCD Primary Schools. Two different tests, one for class II and another for class III were developed to explore the strategies children used for solving the problems. The test was administered to forty (40) students selected from Primary School located in Basai Darapur, New Delhi. Proper care was taken to divide the sample equally in terms of gender and grade. These students were classified under four (4) groups having 10(ten) subjects in each group i.e. *class II boys,(10), class-II girls (10), class-III boys (10) and class-III girls (10).* The test items were self-administered and in-depth interview was conducted to

understand how did the children solve different addition and subtraction problems and what were the strategies they adopted. The identified strategies were statistically analyzed using frequency (f), percentage (%) and chi-square ($\chi^2$). The important findings of the present research were as follows: -

1. Grade II children preferred single strategy to solve both addition and subtraction where as, Grade III children used multiple strategies to solve particular problem. In other words, most of the second graders used conventional strategies i.e. finger counting and use of manipulatives but maximum third graders preferred to use recall number facts, and mental calculations.

2. In Grade II and Grade III, maximum number of students (both boys & girls) preferred counting–on strategies i.e. keeping the larger addend constant, add the smaller one, irrespective of single digit, double digit, multi digit numerical and word problems for addition.

3. There was a significant difference between boys and girls on strategies like mental calculations and putting tally marks to do arithmetic operations i.e. addition, subtraction, multiplication, and division. In other words maximum number of boys used covert counting (mental calculation) where as, more number of girls relied heavily on overt counting (putting tally marks, physical objects and fingers).

4. Girls from Grade II and Grade III committed more errors for both addition and subtraction as compared to boys. In case of subtraction the errors were more prominent than that of addition. The commitment of errors were dependent on problem types i.e. the problems where the first number was missing.

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# CHAPTER - 1

# INTRODUCTION

Learning is an active, continuous process in that learners take information from the environment and construct personal interpretation and meaning based on prior knowledge and exposure (Pitman, 1999). Science and mathematics education in the developing countries has been designated as highly abstract, authoritarian mode of transaction and dominance of recall type of course content (Pachaury, 1995). He states that adherents of constructivism emphasize on two major points. One being that the construction of meaning(s) by the learner takes place in a social context. Secondly, he creates his own meanings on the basis of his level of development by interacting with his peers. Cognitive theorists, developmental psychologists and educational researchers now endorse constructivistic approach to learning by the students so that they are able to display higher order cognitive functioning in their behavior. Development of cognitive thinking from a Piagetian perspective is a 'continuos and evolving process that includes four distinct invariant stages'. They are: sensory motor (0-2yrs); pre-operational (2-7yrs); concrete operational (7-11yrs); and formal operational (11-15yrs). His theory will provide a great help to the teachers and curriculum planners to understand how children perceive, construct and interpret their experiences at different stages of development (Pachaury, 1995).

Research over the past few decades confirms that children both in and out of school experience can construct methods for adding and subtracting multi digit numbers without explicit instructions. It is hypothesized that these invented strategies can play a central role in making problem solving a focus of learning arithmetic procedures and in helping students to develop number sense and understanding of multidigit operations (Carpenter et al., 1994). Children employ a number of strategies for solving multi digit problems; children construct many of these strategies independently or collectively, without direct instructions by the teacher. Review of related literature on addition and subtraction shows a meaningful picture on how children attain and solve simple addition and subtraction problems. The acquisition of addition and subtraction concepts describe the major stages in the development, the knowledge and the procedures underlying children's solution to simple word problems. Recent studies have shown that young children's solution of word problems reflect the semantic structure of the problem (Blume, 1981; Carpenter et al., 1981; Hiebert, 1982). In general, children tend to model the action or relationships described in a problem. Their solution to word problems is not limited to modeling and counting strategies. Children learn number facts both in and out of school and apply this knowledge to solve word problems, even though knowledge of the appropriate number facts does not insure that modeling or counting will not be used instead (Carpenter & Moser, 1984).

During the course of development, children transform historical form of number representation that are initially external to their cognitive repertoire in to symbolic vehicles which becomes an inherent part of their problem-solving activities. This means that the study of number development not only describes the child's acquisition of the numeration system of the culture but also provides an analysis of the changing relations over the course of the child's development between acquisition of numeration system and the process of problem solving. Children use number symbols to represent logico-mathematical operations such as addition, subtraction, multiplication and division. Numerical thought often entails operating with and upon a particular historical invention, the form of which may facilitate operations of certain types and limit others. Thus, the study of number development should provide an account of the way that differences in the socio historical construction of number which will lead to variations in the ways individual solve problems.

Informal knowledge of mathematics was to some extent present among children when they entered the school. This informal knowledge or invented strategies demonstrate a remarkable degree of insight in solving word problems (Carpenter 1983). Children can solve a variety of simple mathematics problems by counting, modeling, and using concrete objects or materials before they receive any formal instructions. Therefore, these cognitive socialization assist in constructing procedures or strategies to solve simple addition and subtraction problems without any formal instructions. Adding and subtracting are often having common place in any young child's life rather than the concrete activities. A child could understand that addition increases and subtraction decreases a quantity and that addition and subtraction cancel each other out without being able to work out actual sums in practice, we found that children's understanding of addition and subtraction comes from their performance in tasks in which they were asked to provide the answer to a particular sum in one way or other (Bryant, 1997)

Arithmetic word problems constitute an important part of the mathematics programme at elementary and primary school levels. Certain application functions of this are: to train children to apply formal mathematical knowledge and skills learned at schools to real world situations, thought of vehicle for developing student's general problem solving capacity, making math lessons more pleasant and motivating and finally a thorough understanding of basic arithmetic operations could be understood through arithmetic word problems (Decorte & Verschaffel, 1989). New ideas and methods from the information processing approach led to the emergence of new paradigm in addition and subtraction word problems. The representation of a problem solution is one of the fundamental problem solving processes. Many problems can be solved by representing directly the critical features of the problem situations or physical or pictorial representation. Modeling also turns out to be a relatively natural problem solving process for young children (Hiebert, 1988).

Much of the extensive body of research documents that children even before they receive formal instructions in arithmetic, they can solve a variety of addition and subtraction word problems by directly modeling with counters and different actions and relationships described in the problem (Carpenter, 1985; Fuson, 1992). Children's understanding of word problems has focused on the strategies that they use to solve different types of problems, especially addition and subtraction. Word problems in arithmetic do require representations that can be used to choose operations such as additions, subtraction and counting objects. A considerable body of research on solutions of simple word problems including the works of Carpenter & Moser (1982), De Corte & Verschaffel (1981), Nesher (1982) state that problems have been classified according to the semantic relations among quantities in the problems and data have been obtained that shows (a) how difficult different kinds of problems are for children of variant ages (b) what kind of solution processes are used for different problems (c) what kind of error occurs. These inferences aid in understanding the information structures formed in representing problems, inferences that are made and counting operations that are performed on set of objects, which are, represented (Kintsch & Greeno, 1985).

Children's solution of basic addition and subtraction problems has been thoroughly documented and there is some variability in their performance depending on the nature of actions and relationships in different problems. The relative difficulty of different kinds of problems emphasizes that more complex processes and structures are needed to solve the more difficult problems. Most of the first grade children can solve a variety of problem by directly modeling the relationships described in them. It involves the cognitive mechanism on these solution strategies.

Riley and Greeno (1988) propose that children's ability to solve simple addition and subtraction problems depend upon the availability of specific problem schemata for understanding various semantic relationship in the problems. Specific knowledge about additive and subtraction structures is also required to solve basic addition and subtraction problems. These cognitive structures are: -

(1) Number representation: - Individual can often do employ aspects of their environment as symbolic vehicles in order to increase the power of their problem solving.

(2) Individuals employ symbolic vehicles to represent logico-mathematical relations-relations that are not in the objects but are inherent aspects of subjects enumerative activities (such as addition, subtraction).

(3) Numerical thoughts often operating to facilitate operation of certain types and limit others.

The development of problem schemata, number representation and numerical thoughts to facilitate arithmetic operation could be better understood, if we critically see the theoretical perspectives proposed for cognitive development.

**Theoretical Perspectives: -**

Mathematics has traditionally been conceived as a highly abstract, formalized and theoretical system, which perfectly fits in to the mold of a decontextualised cognitive activity (Khan, 1994). Wittgenstein stated "our children are not only given practice in calculation but are trained to adopt a particular kind of attitude towards in calculating (Remarks on the foundation of mathematics Vol.40) From developmental psychological perspective we say that as age advances, children begin to give finer responses to the stimuli and their representational ability in terms of physical as well as symbolic becomes mature and judgmental

Mathematics is embedded in the context of practice and the type of involvement makes a difference to the child's knowledge. These relationships could be better understood in a theoretical perspective. This work uses the theories of Piaget and Vygotsky on cognitive development and Socio-historical analysis of knowledge development.

**Vygotsky's Approach: -**

L.S.Vygotsky (1962) understands cognition as a 'mediated' activity. By "mediated" it is meant that individuals do not interact with the world directly but with their personal representations of the world. The representations may include linguistic signs. discourses. orthography and numeration systems (Saxe, et al., 1983). According to Vygotsky the formative processes which influences the development of representational activities, general development of intelligence and establishes link between cognitive development of the individual with the collective practices of social group is language and its relation to the thought, (1962). Vygotsky argued that speech and thoughts are rooted in different kinds of activities and develop independently of one another. Vygotsky's approach to numerical cognition in particular emphasizes that how a representational system for number, which emerged in the social history of a cultural group is transformed by the individual in such that it becomes an intermediary in problem solving activities and a symbolic object with which an individual interacts (Saxe & Posner, 1983).

**The Acquisition of knowledge system: -**

Vygotsky's developmental approach emphasizes that there is a qualitative change in children's use of culturally organized knowledge systems for problem solving activities. Two types of learning experiences, those that occur from the "bottom up" is called "spontaneous concept" and those from "top-down" is called "scientific concept". Bottom up learning is resulted from the child's spontaneous attempt to understand aspects of social and physical reality without direct aid or peer tutoring. These experiences aid the child to acquire practical concepts and to find local solutions to a particular problem (Saxe & Posner, 1983).

Top down learning is described as resulting from interactions with adults or more capable peers. In these interactions, problems are posed for the child and he or she is presented with concepts of general applicability that are valued in the culture. Top down learning such as that encountered in school gives the child the opportunity to form general concepts that may be adapted to different problem types.

A fundamental aspect of adult-child interactions in problem solving contexts consist of a "scaffolding" process (Wood, Bruner & Ross, 1976) in which adults adjust their (top down)dialogue in such a way that the child can relate his or her (bottom up) experiences to novel problems (Gearhart & Newman, 1980).

Under the concept of 'Zone of Proximal Development' Vygotsky stated that there are two aspects of child's development; that are the actual development which at the outset are only operational in the social settings but gradually are "internalized" and they become part of the child's independent achievement (Vygotsky 1978). "The zone of proximal development is the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under guidance of adults, or in collaboration with more capable peers" (Vygotsky, 1978).

Early in the developmental process adults engage children in number related activities that children are not capable of doing on their own, but that which is within their grasp of understanding is called Zone of Proximal Development (ZPD). By guiding them through number tasks, adults introduce children the number symbols, general numeration strategies, which are specific to their culture.

**Piaget's Approach –:**

Piaget's theory is concerned with the origin of logical structures of thoughts and the characteristics of these structures. He rejects the nativist and empiricist formulations of the origins of logical structures. His theory, which focused on child's construction of reality and the intellectual development. He argues that the origins of logical structures are elaborated in sensori motor activities, that are transformed into mental operations in the course of development. Langer, (1980) according to him logic of thought is preceded by logic of action. The acquisition of knowledge is a constructive process that goes through an invariant sequence of stages, which he identified as sensori motor, the pre operational, the concrete operational and the formal operational stage. While studying the construction of reality Piaget has offered developmental analysis of several logico-mathematical concepts such as space, time, cardinal and ordinal number and the composition of numerical relations.

Piaget believed that "operations" were basic to thought. It is concerned with thinking that exemplifies the properties of a group in mathematics. These properties of formal structure exhibit the properties of identity, negation and inversion. There are three different ways in which an individual can conceive of such qualities as number, space and time and so on.

(1) One's actions can presuppose a certain concept of, say space.
Piaget calls this level of knowledge as sensori motor knowledge.

(2) Intuitive knowledge of number, cause, space and time were also distinguished by Piaget. This refers to a partial conceptual understanding of the notions in question.

(3) Operational knowledge of number, cause, space and time, these concepts are clearly defined. Piaget distinguishes concrete and formal operational knowledge.

According to Piaget, these three forms of knowledge emerge sequentially in development, in the order mentioned above for any particular concept. Each form is progressively "constructed". Piaget also makes the assumptions that the later developments build on the earlier development.

He considered the idea of conservation to be central to all rational thought and to the question of number in particular. The number is more directly concerned with topic, which include children's construction of one-to-one correspondence relations and their conception of the implications of these relations for the equivalence of two sets. Other additional topics include children's construction and understanding of serial order class inclusion and the additive composition of number. These topics build on one another. For example the studies of one-to-one correspondence focus on the question of whether children "Conserve" one-to-one correspondence or the equivalence that may be inferred from it.

Regarding the development of the concept of number Piaget gave certain arguments, which are: -

(i) The notion of conservation is central to all rational thought. It is central to the concept of number in particular.

(ii) The notion of conservation has to do specifically with the preservation of a thing or a quality across a possible set of changes related to that thing or quality. The set of changes to which Piaget refers the concept of number consists of changes in spatial arrangement or physical kind.

(iii) According to the developmental argument, there is an early period during which in judging the numerousity of observed arrays of objects (Specifically in judging whether the number of elements in two rows is the same), children will offer a judgement of what could describe as spatial extent. Even after having agreed that two rows contain the same number of objects, if one row is spread out, children will say that row has more. Children may be able to construct rows of equal number of objects during an intermediate period but they will still deny their equivalence when one of the rows is spread out. Later, they may be unsure of the answer and may count the rows to establish their equivalence.

(iv) Finally, they are able to draw a principled distinction between questions of numerousity and questions of spatial extent. They know that, if two rows of objects/elements contain the same number and if one row is spread out, then the two rows must continue to have the same numbers. They will argue spontaneously that (a) the altered ro[...] very same elements as it did before (identity); (b) if the row is longer b[...] having been spread out then there is correspondingly more space between [...] in it (Compensation); (c) One could simply 'undo' the spreading out o[...] moving the elements together (and without adding or subtracting anything) a[...] exactly the same display that one had before (reversibility).

Both the theoretical framework have stated the assumption about sem[...] knowledge required for representing the problem and the process of operating [...] numbers in the problems to find answers. Eventually children become more flexible [...] their choice of solution strategy as a result of changes in their conceptual knowledge, so that they can solve problems using variety of strategies (Briars et al., 1984). Riley and her colleagues propose that the flexibility is based on the development of an understanding of part-whole relationships which allows children to classify all addition and subtraction, word problems as either addition or subtraction. The major conceptual achievement of the early school years is the interpretation of numbers in terms of part-whole relationships. Part-whole schemata quality helps the child to think about numbers as composition of other numbers. This enrichment of number understanding permits the form of mathematical problem solving and interpretation that are not available to younger children (Resnick, 1983).

The schema specifies that any quantity (the whole) can be partitioned (into parts) as long as the combined parts neither exceeds nor fall short of the whole. The part-whole schema provides an interpretation of number that is similar to Piaget's (1965), definition of an operational number concept. Hence, if we see Vygotsky's and Piaget's approach as a whole then, we will find out this difference in their ideas of problem of cognitive development. Vygotsky's focus is on numeration as a mediated activity that has its roots in social interaction. Whereas Piaget's focus is on the emergence of logico-mathematical structures that underlie the use of numeration and have their roots in sensori motor activities.

**Classification of Addition and Subtraction word problems.**

Greeno et al., (1978) introduced a classification scheme of addition and subtraction problems for distinguishing between three basic categories of problem situation i.e. **change, combine** and **compare** problems. Change problems refer to the active or dynamic situations in which some event changes the value of an initial quantity. Combine problems relate to static situations involving two quantities that are considered either separately or in combination. Compare problems involves two amounts that are compared and difference between them are found out. Each of these three basic categories of problem situation can further be sub-divided into different problem types depending on the identity of the unknown quantity. And for change and compare problems further distinction can be made depending on the direction of the change (increase vs. decrease) or of cooperative relationship (more vs. less).

Large amount of investigations has analyzed the level of difficulty of different types of addition and subtraction word problems. It summarized that change problems in which the initial quantity is unknown are found to be more difficult than those unknown change set. Combine problems with an unknown sub set are more difficult than combine problems with unknown supper set. Compare problems with an unknown difference set are easier than problems with unknown compare set. Various researchers have stated that type of problem situation has a strong impact on the difficulty of addition and subtraction word problems. Besides this, task variable, the relative difficulty of a particular problem are affected by exact phrasing of the problem, the particular numbers used, the testing procedures the age and instructional background of the pupil (Bryant & Nunes 1997).

**The Development of Addition and Subtraction skills: -**

Counting strategies are more efficient and require more sophisticated counting skills than direct modeling. It suggests that young children use more concrete direct modeling strategies but older children use more abstract and efficient strategies (Carpenter & Moser, 1984). Riley et al., (1983) have developed computer simulation models of different levels of skilled performance in solving addition and subtraction problems. It identified three levels of skill for solving addition and subtraction word problem that is *level-1 (direct modeling only). Level-2 (modeling and counting) and Level-3 (Primary Counting).* For join and compare problems, children at Level-1 are limited to external representation of problem situation using physical objects. Level –1 children rely on counting all and separating strategy. They cannot use the adding on strategy, because they have no understanding of set-subset relationship. Therefore, they cannot solve missing addend problems. Level-2 includes the schema that enables the child to recognize set sub-set relationships. It allows children to solve missing addend problems. Level-2 children can also count on from the first number, but they cannot count from the larger. Level-3 is the most advanced level, where the child can use any strategy to solve a given problem.

Children at level 1 and level 2 are limited to direct representation of problem structure using counting or modeling. Level-3 includes a schema for representation of relationships among all pieces of information in the problem before solving it.

As for the combine and compare problems, children at level – 1 can solve compare problems, provided it is clear to them that matching is an appropriate strategy. Combine addition problems can be solved at level-1 but combine subtraction problems do require the combine schema that allows child to infer part- whole relations inherent in the problem (Briars and Larkin in Press)

The model projects that children at a given level consistently respond with specific strategies to a given type of problem. If a strategy is available, then child will use it. Briars and Larkin (1984) state that when alternative strategies are available, the child responds to the strategy that will result in the fewest counting steps or that avoids more difficult counting procedures. When children have several strategies available, they often use it interchangeably rather than exclusively using the most efficient one (Carpenter & Moser, 1984). On these three possible levels of skilled performance, variability in children's performance was marked. Again in order to avoid this difficulty, they devised a new model having five levels, which best suited. At the most primitive level (Level 0); the children were unable to solve any addition or subtraction problem. At the next level (Level 1) the children were limited to direct modeling strategies (counting all and adding on). Level 2 marks a transitional period. At this level, the children used both modeling and counting strategies. At level 3, the children relied primarily on counting strategies. At the highest level, level 4, the children solved addition and subtraction problems using number facts.

*Primary strategies used at each level of performance were as follows:*

| **Problem** | **Level –1** | **Level-2** | **Level-3** |
|---|---|---|---|
| Addition | | | |
| Join | Counting all | Counting all<br>Counting on from first<br>Counting on from larger | Counting on from larger |
| Subtraction | | | |
| Join missing addend given | | Adding on<br>Counting up from given | Adding on Counting up from given |
| Separate | Separating from | Separating from | Separating from<br>Counting down from<br>Counting up from given. |
| Combine | [most can't solve] | Separating from | Counting up from given |
| Compare | Matching | Matching<br>Counting up from given | Counting up from given |

**[Source: Carpenter and Moser, (1984). The acquisition of addition and subtraction concepts in grade one through grade three]**

**Strategies use by children for solving Addition & Subtraction Problems: -**

Review of literature on areas of addition and subtraction provide a coherent picture on major stages in the development of addition and subtraction. Children do construct strategies to solve simple problems on addition and subtraction word problems and numerical problems without any difficulty. In word problems a great deal of attention is required in identifying the strategies, since the problems are presented in a sentence form. Recent attention has been directed towards an identification of external variable that may hinder/pose problems in children's interpretation of verbal problems and affect the relative difficulty of various problem types (Carpenter, Hiebert and Moser, 1981). Hence, in this regard number of important factors such as semantic structure of the problem, number size, syntactic complexity and the position of unknown quantity in the associated number structure influence the interpretation part of the problem among children (Hiebert 1982). Many young children solve verbal addition and subtraction problems by representing the sets on the problems with objects and carrying out the prescribed actions on the object (Carpenter et al., 1981). They also stated that children solve simple addition and subtraction word problems by representing with physical objects. These counting strategies for simple problems are easy but becomes arduous for harder problems. Therefore, for the harder problems, mathematical representations are needed to conceptualize algorithm (mental heuristic for problem solving). Learning to represent problem situations with mathematical symbol is a major goal of mathematical curriculum. Writing number sentences to represent problem situations (addition/subtraction) may not be necessary to solve simple problems with small numbers but it is first step to represent problems mathematically.

**Addition Strategies**

Large body of research concerned with describing and understanding young children's solutions to simple addition and subtraction problems (Carpenter, 1985; Carpenter and Moser, 1984; De Corte and Verschaffel, 1987; Fuson, 1990; Nesher, 1982; Riley, Greeno and Heller, 1983; Siegler, 1987). Houlihan and Ginsburg (1981) suggested that techniques for dealing with addition develop before the onset of schooling and that these informal procedures have effects on what is learned in school. Russell (1977) reported that children tend to use procedures that are adaptive to the requirements of a problem. Thus, they may use Counting Procedures to deal with objects but written procedures with written numbers. Before formal instructions children can analyze and solve simple addition tasks, by directly representing the operation with physical objects or by using counting strategies. However, it has been suggested that these strategies are too cumbersome to be effective with more complex problems and larger number (Carpenter, Moser, and Bebout, 1988). There is evidence that at the time children are introduced to writing mathematical sentences they see no connection between them and their informal strategies (Carpenter, Hiebert and Moser, 1983). De Corte and Verschaffel (1987) compared their results, their results for strategies used by children in first grade to solve subtraction and addition problem, with those of Carpenter and Moser (1984). Carpenter and Moser's scheme for classifying children's solution strategies had two dimensions. First, strategies were identified as additive or subtractive and then ordered according to the level of internalization: material (using objects), verbal (using counting), and mental (using known number facts). It was also similar for De Corte and Verschaffel. Boulton Lewis (1993) found that, when children choose their own strategies and representation for subtraction and addition, the general developmental sequence from using objects, to using counting, to using known number facts, and that children tried to use recall of number facts as early as possible.

**Classification of Arithmetic Strategies: -**

Groen and Parkman (1972) summarized various strategies used by children while solving addition problems. The strategies are as follows: -

(a) **Counting all:** The Counting all strategy can be carried out using cubes or fingers as models or by counting mentally. If cubes are used both sets are represented and then union of the two sets is recounted beginning with one. If counting is done mentally or with fingers, the counting sequence begins with one and ends with the number representing the total of the two given quantities.

(b) **Counting on from smaller number :** In this strategy, the counting sequence begins either with the smaller (first) given number in the problem or the successor of that number. Counting may be done mentally or by using cubes or fingers to keep track of the number of steps in the counting sequence.

(c) **Counting on from large number :** In this strategy, the counting sequence begins with the larger (second) given number or with the successor of that number.

(d) **Number Fact:** The child gives an answer with the justification that it was the result of knowing some basic addition fact e.g. 5+5= 10.

(e) **Heuristic:** These strategies are employed to generate solutions from a small set of known basic fact. These are based on doubles or numbers, whose sum is 10,
e.g. - To solve a problem representing 5+7 =? It is responded as 5+5=10 and 5+7 is just 2 more than 10, therefore, the sum will be 12.

**Subtraction Strategies: -**

Carpenter Hiebert and Moser stated that on the whole, children were not quite as successful with the subtraction problem as they were with addition problems. Again they observed that as the age advances many children don't prefer to use written algorithm and analogs as an aid to perform the task but they rather prefer to use mental strategy including recall of fact and place value. They are very much successful while doing these operations.

**Classification of Subtraction Strategies: -**

(a) **Separating:** The large quantity (Minuend) is initially represented and the smaller quantity (Subtrahend) is then removed from it. When objects are used the child construct the larger given set and then takes away or separate one at a time, a number of cubes equal to the given number in the problem. Counting the set of remaining cubes yields the answer. In a more abstract representation of the same action, a child initiates a backward counting sequence beginning with the given larger number. The backward counting sequence contains as many counting number words as the given smaller number. The last number uttered in the counting sequence is the answer.

**(b)** **Separating to:** It is similar to the separating strategy described above, except that the separating continues until the smaller quantity is remaining rather than until it has been removed. When objects are used the larger set is counted out, and then the child removes the cubes one at a time until the remainder is equal to the second given number of the problem. Counting the number of cubes removed gives the answer. In the corresponding abstract representation, a child initiates a backward counting sequence beginning with the larger given number. The sequence ends with the smaller number.

By keeping track of the number of counting words uttered in this sequence the child determines the answer to be the number of counting words used in the sequence

(c) **Adding on:** The child starts with the smaller quantity and constructs the larger. With objects the child sets out a number of cubes equal to the smaller given number (an addend). Then cubes are added on to that set, one at a time until the new collection is equal to the larger given number. Counting the number of cubes added on gives the answer. Alternatively, a child initiates a forward counting sequence beginning with the smaller given number. The sequence ends with the larger given number. Again, by keeping track of the number of counting words uttered in the sequence, the child determines the answer.

(d) **Matching:** It is only feasible when object are available. The child puts out two sets of cubes, each set standing for one of the given numbers. Then the sets are matched one to one; counting the unmatched cubes gives the answer.

Again De Corte and Verschaffel, (1987) reported in their study classification of various addition and subtraction strategies in relation to (a) material (b) verbal and (c) mental representation.

These are grouped as: -

**(a) Material Strategies (addition)**

(1) Counting all with models using physical objects or fingers the child constructs two sets corresponding to the given addends and the union of addends is the answer.

(2) Reversed Matching: - The child constructs a set corresponding to the first number in the problem and a set corresponding to the first and second number, the answer is the number of object in the second set.

**Subtraction**

(1) Separating from

(2) Separating to

(3) Adding on

(4) Matching

**(b) Verbal Strategies (addition)**

(1) Counting all starting with first

(2) Counting all starting with larger

(3) Counting on from first

(4) Counting on from larger.

**Subtraction**

(1) Counting down from

(2) Counting down to

(3) Counting up from given

**(c) Mental strategies (addition)**

(1) Known facts starting with first – The child retrieves an addition number facts starting with first number in the problem immediately from long term memory (5+7 =12)

(2) Known facts starting with larger – The child retrieves an addition number facts starting with larger number immediately from long term memory (7+5 =12)

(3) Derived fact starting with first – Basing the answer on one or more recalled number fact , the child begins with the first number in the problem (e.g. 5+5=10 and 10+2=12)

(4) Derived fact starting with larger – Basing the answer on one or more recalled number fact, the child begins with the larger number in the problem (e.g. 7+3=10, 10+2=12)

**Subtraction**

(1) Direct Subtractive known fact – The child retrieves a direct subtractive number fact with the two numbers ( 5 and 12) immediately from long term memory ( 12-5=7)

(2) Indirect Subtractive know fact – The child retrieves an indirect subtractive number fact with the two numbers immediately from the long term memory (12-7=5)

(3) Indirect additive know fact – The child retrieves an indirect additive number fact with the two numbers ( 5 and 12) immediately from memory (5+7=12)

(4) Direct Subtractive derived fact – Relying on recalled number facts, the child finds the answer by subtracting the smaller number (5) from the larger (12) (e.g. 12 minus 2 minus 3 is equal to 7).

(5) Indirect subtractive derived fact – Relying on recalled number facts, the child finds the answer by determining what quantity should be subtracted from the larger number (12) to get the smaller (5). (e.g. 12-2=10 and 10-5=5 so the answer is 2+5=7)

(6) Indirect additive fact – Relying on recalled number facts, the child finds the answer by determining to what quantity the smaller number (5) should be added to obtain the larger (12) (e.g. 5+5=10 and 10+2=12 so the answer is 2+5=7)

**Kind of Representation for various problems-**

The transition to write number sentences for word problem is less clearly defined than the work with solution strategies. Number sentences may directly model a problem or may represent the arithmetic solution. For example 9-4= --- as 4 +---= 9 d directly models the action described in the problem and 9-4 = ---represents arithmetic solution.

Bebout (1990) reported that when children are familiar with alternative number sentences, then they naturally represent semantic structure of addition and subtraction word problems. But children initially have difficulty in writing number sentences for word problems like 4 + --- = 9 when they are limited to represents problems with standard number sentences, a + b= --- and a – b = --- that may not reflect the semantic structure of the problem situation (Hiebert, et al. 1983) over the time children become more flexible in their ability to write number sentences for word problems and are able to represent all addition and subtraction problems types with a + b= --- and a – b= --- As with the development of flexibility in the choice of strategies for solving these problems. this ability has been attributed to the development of an understanding of part whole relationship (Rathmell & Huisker, 1989). It suggests that children's ability to recognize appropriate alternative number sentence for addition and subtraction word problems may indicate a level of understanding about problems, similar to the way that children's solution strategies indicate different levels of problem solving ability. Riley, (1983) is of opinion that the knowledge required to solve certain problem type is not influenced by the numbers in the problem. Children's strategies and number sentence representation are determined by problem structure.

**Need and Significance of the Present Study: -**

The study of children's mathematical problem solving is a rapidly growing area of research within the framework of cognitive development. The solution strategies that children used for addition and subtraction provide a window to understand how cognitive skills and strategies evolve and change across development (Siegler, 1996). Traditionally many arithmetic models based on accuracy and latency data have been developed to infer the problem solving strategies. The present study completely relied on verbal reports ensued from the children to understand their mathematical cognition. Since, most of the previous researches have been undertaken to assess the solution strategies for mathematical problem solving, but few of them are well documented in Indian context. In India very few researches have been done to explore the solution strategies understanding encountering both numerical and word problems, type of errors and representation of number etc. through in depth interviews. Besides the solution to the given problems, the in-depth interview will be an effective tool for understanding how cognitive skills and strategies change and develop among the children with respect to grade.

From the present research the immediate need was felt to identify active or passive state of the situation. Also the order of the presentation of known and unknown quantities in the problem, which might influence children's solution strategies for addition and subtraction (Carpenter and Moser 1979). The need of this present research is to see (a) whether individual child have more than one model for addition and subtraction (b) whether children consistently employ similar model for different types of addition and subtraction problems (c) whether individual child displays different representation and strategies for problems requiring carry over sums and place value systems.

Children's concepts and processes related to addition and subtraction would be influenced by semantic classification of word problems and some understanding of why some problems are more difficult than others? What kind of counting strategies children adopt for addition and subtraction problems and the kind of representation they make to solve them.

Much of recent researches on young children's understanding of addition and subtraction has focused on the strategies that children used to solve different types of problems in different countries are well documented. The present study focused on the strategies used by children to solve both numerical and word problems in the Indian context. Although various researches relating to word problems have been documented, very few were reported regarding number problems relating to arithmetic addition, subtraction, multiplication and division. As a result of change in their conceptual knowledge, children become flexible in their approach to use various solution strategies (Carey, 1991). The standardized addition and subtraction form like a+b= --- and a-b=--- does not allow them to be flexible enough to solve word and numerical problems. They don't model directly the action within the problem (Briar and Larkin 1984).

Hence, the need of the present study was to determine how successful children are in solving different types of addition and subtraction problems, which kind of problem were difficult for them to solve, what are the strategies they used and how do they represent the word problems to open number sentences.

# CHAPTER – 2

## REVIEW OF RELATED LITERATURE

Education reformers currently suggest that in order to make mathematics learning meaningful, student should make conjectures, abstract mathematical properties, explain their reasoning capability, validate their assertions, discuss and question their own thinking and thinking of others. Some children view mathematics as a set of rules and procedures in which problems are solved through application of computational algorithm that have been taught explicitly by mathematics teachers. Hence, they perceive mathematics is a rule governed and abstract topic, which require formal instructions and well-developed strategies (Carey, & Franke, 1997). Before they receive any formal instruction, children can solve a variety of mathematics problems by counting and modeling with concrete materials. It means that without depending on formal instructions, children generally construct procedures or frame the cognitive map to solve simple addition and subtraction problems. Various researches have presented that children solve a variety of addition and subtraction problems by directly representing with physical objects, model the relationships described in the problem. Carpenter et al.. (1988) suggest that modeling and counting strategies that children use to solve simple problems with relatively small numbers are too cumbersome to be effective with more complex problems or problems with large numbers. Mathematical representations for problem situation are needed to solve arithmetic problems. Therefore, learning to represent problems mathematically, writing number sentences for addition and subtraction problems are supposed to be required.



### The development of counting strategies for addition and subtraction

Children use counting to solve problems involving addition, subtraction, multiplication, and division long before they come to school, as research has indicated (Suydam and Weaver 1981).

Counting nevertheless remains an integral aspect of children's beginning work with the operations. They need to know how to count forward, background, and by twos, threes and the other groups. They need to count as they compare and analyze sets and arrays and as they affirm their initial computational results. But they need more than counting to become proficient is computing.

Counting skills are started before, children's begin school but must be developed by careful and systematic instruction before written work is appropriate. Counting processes reflect various levels of sophistication, beginning with rote counting and eventually leading to rapid skip counting forward and backward. Although the four counting principles are established in the primary grades, counting skills are extended in the intermediate grades and often are further refined throughout our lives.

Pre-schoolers learn to use their internalized number sequence to solve addition problems at least for numbers up to single digit or so. Before they learn the "number facts", children solve (M+N) problems (where M and N range from 2 to 9) by relying on counting strategies that require the aid of countable objects. Developmentally, the most basic strategy is concrete counting-- fingers and other countable objects such as blocks are counted out one by one to represent an addend; the process is repeated for the other addend and then all the fingers put out are counted to determine the sum. Another labor saving short cuts for Concrete Counting All Short-cut (CCS) involves separate processes for representing the addends and determine the sum. During the sum count, the child saves effort by not counting from one (1) but starting with the cardinal designation of the first set (Baroody, 1987). Researches indicate that even before formal schooling, young children can solve addition sentences or word problem by direct modeling-by using concrete counting (CC) strategy (Resnick, 1983). Some researches (Ginsburg & Russell, 1981; Lindvall & Ibarra, 1979) suggest that the development of CC strategy cannot be taken for granted in all pre-schoolers- especially for disadvantaged children.

Carpenter and Moser (1984) carried out one study on first grade children. They found that about one-seventh of their entering first graders were unable to solve many addition world problems even when objects were available. In some cases children use finger pattern (simultaneous presentation of finger) and take a set of blocks equal to the addend to count the sum. This is termed as "counting finger strategy" which proves more efficient to solve addition problems. In this case, student creates finger patterns for each addend and then immediately recognize the sum either visually or kinesthetically (Siegler & Shrager, 1984)

Other counting strategies that is qualitatively superior to above-mentioned one, known as mental counting strategies. In these strategies, objects are not used to represent the second addend per se but to keep track of how far the sum count must be continued beyond the cardinal terms of the first addend (Carpenter & Moser, 1982). Counting all starting with the first addend (CAF) entails starting with "one" counting up to the cardinal value of the first addend, and then counting a number of steps equal to the cardinal value of the second. [e.g. 2+4: 1,2,3, (1 finger up ), 4 (2fingers up), 5 (3 fingers up), 6 (4 fingers up) =6]. That is the child counts on from the first to complete the sum count. The mental counting strategy utilizes concrete counting all starting with larger addend, which is a sophisticated one. It starts with "one" counting up to the cardinal value of the larger addend, and then counting on from there where the smaller terms is enumerated.

Robinson et al., (1982) observed that children resort to a counting finger strategy, when they could not determine the sums by using a counting strategy starting with first addend or fact retrieval. Baroody (1987) suggested that when children entering schools, we should not take for granted a concrete strategy for computing (M+N) sums. His data showed that only a few children immediately used concrete counting all strategy to calculate the sums of symbolically presented problems. This strategy especially needs repeated demonstrations for mastery. The difficulties in these strategies lie in direct modeling of a union of two set view of addition (Baroody & Ginsburg, 1986).

Cannon (1984) reported that, children should be given ample opportunities and time to practice to add with the concrete objects when entering schools. Other studies demonstrate that with computational practice or direct teaching, children will invent more advanced strategies (Groen & Resnick, 1977).

Briars & Larkin (1984) reported about development of mental strategies among children with respect to addition. They suggested that mental counting all strategies starting with larger addend were far more frequent than those starting with first addend. Kindergarten children develop a mental addition strategy to minimize the cognitively demanding keeping-track process by starting with the larger addend.

Again with respect to role of commutativity, Baroody and Cannon (1984) reported an appreciation of this commutative strategy which was not expected to be a necessary condition for the invention of addition strategy that disregard addend order. Apparently some children may add numbers in either order because they believe that they will get a correct answer. Riley et al., (1983) suggested that the strategy of counting on from the larger addend, which may not represent the problem directly in commutative one but children may invent strategies simply to save mental effort for conceptual advances..

Hebbler (1978) studied first and second grader's ability in solving addition and subtraction problems. He suggested that children of pre-school age can use concrete objects as aid for counting to solve addition problems. His study was also supported by Posner, 1978; and Ginsburg, 1978. Ginsburg (1977) studied the effects of informal procedures- what is learned in school to solve an arithmetic problem on pre school children and found that children assimilate school mathematics into their cognitive structures. This helps in developing counting strategies especially in the age of pre school years. He further reported that first grade's addition is usually accomplished by means of some forms of counting, even this is not the method taught in the school, which is formally called as an "invented strategy". It is basically the combination of remembered addition facts and counting procedures like 5+5= 10, 10+10= 20 etc.

Russell (1977) studied the invented strategies of older children (grade three) and found extensive use of invented strategies, some involving counting. Children tend to use procedures that are adaptive to the requirements of a problem. They use counting procedure to deal with objects but written procedures with written numbers. Inferences made about developmental sequence of children's addition and subtraction word problems were limited to the kind of conclusions that could be validly drawn from cross-sectional as well as cross-cultural research.

Since beginning a number of researchers have investigated how children solve addition and subtraction problems. Now various researchers have aggregated results in a different way and used different dimensions. Recent studies have shown that young children's solution of word problems and number problems reflects the semantic structure of problem. Three basic levels of addition and subtraction strategies were identified that is (a) strategies based on direct modeling with finger or objects, (b) strategies based on the use of counting sequences and (c) strategies based on recalled number facts. The most basic strategies children use are physical objects and fingers to represent each of the addends and then union of the two sets is counted starting with one. With respect to the development of addition and subtraction skills, counting strategies are expected to be more efficient and sophisticated than direct modeling. Young children use direct modeling whereas, older children use abstract and efficient counting strategies. The discrepancies in children's performances in addition and subtraction problems could be attributed to differences in cognitive mechanisms. For join and separate problems, young children are limited to external representation of problem situation using physical objects where as older children can count on from the first number by they cannot count on from the larger. As the child grows older, he/she develops a schema for representing relations that allows children to construct a representation of the relationship among all pieces of information in the problem before solving it. Children employ a number of strategies for solving varieties of problems at different levels. These strategies are developed individually or collectively without direct instruction of the teacher.

Besides the above-mentioned counting strategies, the child should look into the semantic aspect of the problem to understand it easily and decide which strategies to be used to make the task easier.

**Semantic structure of addition and subtraction problem**

Performances in addition and subtraction arithmetic problems of children depend on languages involved in it. Word problems basically cover the way problems are presented and language in which it was stated. It indicates that how words are used to state a problem may affect the comprehension and solution of the problem. Hudson (1983) stated that some words appear to make the semantic structure of the problem clearer and sometimes distorted. Researchers like Nesher and Teubal (1975) reported that, some words appeared as valid cues in the stated problems and as distractors in others. Carpenter, Hiebert & Moser (1981) and Lindvall & Ibarra (1980) supported this view.

Adetulla (1989) reported the significance of native language in understanding word problems. He found children faced difficulty in understanding instructions in math classes because the language was totally different from their native languages. Children may have difficulty with the language used in the mathematics classes in general and with the addition and subtraction problems in particular. Various related researches supported this hypothesis and reported, if mathematics problem presented to children in a simple and native language, then they would have solved the problem easily (Cuevas, 1984; Gaarder, 1975; Valverde, 1989;

Ehindero, 1980). Adettula (1989) commented that children's failure to understand the wordings or the language of a problem could create a stumbling block and eventually could decrease their level of mathematical attainment in skills and in use of advanced strategies.

Bebout et al., (1988) studied the representation of addition and subtraction word problems on first and second graders. They found that first graders represent word problems with open number sentences that directly modeled the actions described in the problem. They converted all problems with number sentences in standard form (a+b = --, a-b = --) and were limited in representing the problem. However, second graders could represent problems directly with open number sentences or transform them into number sentences in standard form.

Carpenter et al., (1988) suggest that writing number sentence to represent addition and subtraction situations is the first step in learning to represent problems mathematically. At the time children are introduced to writing mathematical sentences to represent word problems, first they see no connection between the informal modeling and counting strategies they use to solve problems and number sentences they are taught to write to represent them (Carpenter, Hiebert & Moser, 1988).

The differences in the semantics of different word problems are also reflected in children's ability to represent the problems with number sentences. In this regard, Carpenter et al., (1983) reported that children who are taught to represent word problems with number sentences in standard form (a+b= --, a - b = --) have no difficulty in representing problems describing simple joining and separating actions, but they do have difficulty in representing most other sentences. Carpenter and Moser (1984) stated that children might most naturally represent word problems with open number sentences that directly model the action in the problems. Children invent counting and modeling procedures to solve word problems without explicit instructions and symbolic representations. Again, they suggested that instructions might limit the range of symbolic representation that children recognize as acceptable. In order to investigate symbolic representations that children use to represent problems, they should be provided enough exposure to wide range of open number sentences.

Wilson (1967) investigated the relative effectiveness of instructions that included a complete range of open number sentences and instructions that focused on number sentences in standard form. This was primarily concerned with student's ability to find correct answers, not to understand the nature of their representations. The objective of the instructions was not to teach children to associate particular number sentences with specific types of problems; rather, it was to let the children know that different number sentences provided representations of word problems.

Carpenter and Moser's (1984) study relating to strategies based on addition and subtraction problems and they are arranged according to the level of internalization: **material strategy** based on direct modeling with fingers or physical objects, **verbal strategies** based on counting sequences and **mental strategies** based on recalled number facts. They found that children solved word problems using mainly material and verbal counting strategies and finally shifted to mental solution procedures based on knowing number facts. Again, children's strategies for solving addition and subtraction problems were significantly influenced by the problem structure.

There are several approaches that previous research has taken to characterize verbal problems. One approach is to classify the problems in terms of syntax, vocabulary level, and number of words in a problem (Jerman,1973; Suppes,Loftus&Jerman,1969). The second approach differentiates between problems in terms of the open sentences they represent (Grouws, 1972; Rosenthal & Resnick, 1974; Lindvall & Ibarra, 1980). Carpenter, Hiebert and Moser (1981) have chosen a third alternatives that consider the semantic characteristics of the problem. The focus of their study is on children's informal concepts of addition and subtraction as reflected in their ability to some certain verbal problems. They also identified the types of problems, which were most difficult for the children to solve and then they characterized the strategies children used to solve the problems. Four different classes of verbal problems that represent addition and subtraction were identified. Those were joining/ separating, Part-part whole, Comparing and equalizing. Various strategies for solving addition and subtraction were found, which were based on counting and there were few that were not based on counting. It was found that high level of success was achieved by the first grade children in solving verbal problems. Children were successful both in modeling action or relationships implied in problems and in using other appropriate model of addition and subtraction. This analysis is generally consistent with other analysis based on problem structure (Greeno, 1978; Nesher & Katriel, 1978). But certain distinctions that are made in this particular study were not included in previous analyses of problem types.

Fuson & Kuwon (1992) examined Korean second and third graders understanding of multidigit addition and subtraction and particularly their ability to explain the trading required when a column sum of the addends is 10 or more. The aim of the study was to explore Korean second and third grader's understanding of place value, multidigit addition and subtraction strategies used for solving the problems, the errors made by them on multidigit problems and also the conceptual structures used to explain the trading involved in such problems. It was found that Korean children's solutions were based on quantitative understanding of multi-digit number. They showed exceptional competence in multidigit addition and subtraction because they all explained the ones / tens / hundreds (place value) trading for both single digit addition and subtraction.

Carey (1991) investigated children's ability to write number sentences to represent different addition and subtraction word problems. Transition of time was taken into consideration in this study. Time in which children are limited to number sentences that directly model the action in problems to the time in which they are more flexible in their choice of number sentences and can represent any addition is subtraction problem with a standard number sentences. For this, 64 first grade children were taken which belonged to 3 schools representing different socio-economic groups. Children were free to generate any appropriate representation for a problem either a number sentence that represented the arithmetic solution of a word problem on a number sentence that represented the semantic structure of the problem. Children's strategies were assessed through both paper pencil assessment and interviews. Results of this study indicated that the relationships which children established between word problems and number sentences influenced children's flexibility in selecting number sentences. The explanations given by children during the interview indicated that number sentences served two purposes. First, to represent actions in the problems and secondly, to solve problems. It seems that young children can solve problems without using symbolic representation. Number sentences can be used as a window on the conceptual knowledge that children apply to the solution of problems. There may be developmental sequence in children's ability to deal with different representations, which is similar to the solution strategies for addition and subtraction word problems (Briar & Larkin, 1984: Carpenter & Moser 1984, Riley et al., 1983). There may be a relationship between solution strategy and the degree of flexibility in selecting appropriate representations. The context and language for studying children's thinking as well as a system for children to communicate their mathematical ideas can be provided by the use of alternative number sentences.

Kintsch & Greeno (1985) consider word problems in arithmetics, which require representations that can be used to choose operations such as addition, subtraction or counting of objects. A processing model is presented that deal explicitly with both the text comprehension and problem solving aspects of word arithmetic problems. General principle forms a theory of text processing (Dijk & Kintsch, 1983) that are combined with hypothesis about semantic knowledge of understanding problem texts (Riley, Greeno & Heller, 1983) in an integrated model of problem comprehension. The model simulates construction of cognitive representations, which includes information that is appropriate for problem solving procedures that children use. The outcome indicates that Van Dijk and Kintsch's model of text processing has sufficient generality to apply comfortably to the semi- technical subject matter of arithmetic problems. It also indicates that the structure hypothesized by Riley et al., (1983) can be formed with processes of language comprehension that can be assumed plausibly to be available for the task.

Frydman and Bryant (1988) claims that "if children have a full and explicit understanding of the quantitative significance of sharing then they should be able to infer the number of items in one shared set when they know the number in the other." Desforges and Desforges (1980) made a distinction between social sharing and mathematical sharing.. Davis and Hunting (1991) observed the systematic sharing in situation in which discrete items were to be distributed. It was found that in an informal situation without an adult presence sharing was totally absent.

There are theories explaining how children learn to count differ with respect to development of counting proccsses. Fuson and Hall (1983) & Fuson (1988) emphasized the role of language patterns as children acquire the conventional number word sequence. Gelman and Gallistel 1978. Gelman and Meck 1983, 1986, 1992 proposed a counting model in which the young children's counting skills are principle driven. Gelman and Gallistel (1978) proposed five counting principles. *(i.) One to one correspondence principle (ii) stable-order principle (iii) Cardinal principle (iv) Abstraction principle (v) An order-irrelevance principle.* They were concerned with that the counting abilities of many children were being underestimated because assessment of these abilities had been based on what children couldn't do rather than on the skills they exhibited.

Steffe & Cobb (1988) proposed that advance in counting competence is made through increasing independence of perceptual items and sensory cues to a stage in which abstract unit items can be created and counted. Later, they proposed that "computing types indicates what children's initial, informal numerical knowledge might be like and reflect our contention that children see numerical situation in a variety of qualitatively different ways."

Both counting and sharing require action on discrete elements entailing the logic of one to one correspondences. Pepper & Hunting (1998) conducted a study, in which the focus was on how counting and sharing related to one another. An experiment was conducted to examine strategies used by preschool children to subdivide items. Various tasks were designed in which application of counting skills, of visual cues such as subitizing and of informal measurement skills were made more difficult. It was found that dealing competence does not relate directly to counting skill. And there was no significant relationship between counting and sharing competence.

**Numerical competencies of primary grade children:-**

Recently educational and cognitive psychologists have given primary importance to children's computational skills and little attention is being paid to their problem solving and numerical understanding (Jordan & Hanich, 2000). Specifically this is talking about the conceptual understanding and procedural flexibility with special reference to performances. Children's performances in mathematics are the outcome of both procedural and conceptual understanding. It is not only guarded by conceptual understanding but influences the development of conceptual understanding (Klein & Burg, 2001). Hiebert and Wearne (1996) conducted a study on semantic aspect of the problem in relation to the conceptual understanding the problem. They concluded that those students who had better conceptual understanding were able to invent new solution strategies. They were able to know how to break the number down, recombine them, and calculate them correctly. This flexibility is expected to be the effect of the understanding the semantic aspect of the language used in it.

Cross-national differences among children with respect to numerical competencies and addition skills have been well documented. Studies like Stevenson, Chen and Lee (1993) stated the differences in addition skills were attributable to mathematical instruction at school, teacher's instructional behavior, parental support for learning, cultural emphasis on education and various social factors among Asian and American Children. Miura, Okamoto, Kim, Steere and Fayol (1993) have analyzed children's use of diverse solution strategy in specific arithmetic skills especially in addition and subtraction, and identified cognitive differences in their processing capacities. These cognitive differences were only because of specificity of number concepts and addition skills. They found that children's use of addition strategy rather then their solution accuracy, changed primarily as a function of schooling. Children's understanding of Base 10 number concepts improved with the amount of schooling as well as with other social and age-related factors. Therefore, they concluded that schooling is supposed to be dominant criteria for development of children's numerical competencies.

Practice counting skills including the conventional number name sequence. The cognitive skills of both counting and sharing seem to develop during the early childhood years. Carraber and Schilemann (1990) found that some children with no previous school instruction on the numeration system had an understanding of conceptual aspects of the numeration system despite an inability to generate the number name sequence systematically.

Competence with and understanding of the numbers zero through ten are essential for meaningful later development of larger numbers. The relation of the sets of objects, the number names, the written symbols, and the order between numbers must be well understood. This knowledge is the basis for the successful study of elementary mathematics and it prepares children for the necessary understanding of large number and place value. (Reys, Marilyn, Suydam and Linquist).

**Children's performances in different areas of mathematical cognition:-**

Hanich et al., (2001) conducted a study on performance across different areas of mathematical cognition in children studying at second grade. Their study found that various dimension of mathematics that is counting knowledge, arithmetic operations and strategy use were considered to be the best predictor of mathematical competence. Deficiencies in mathematical competence can seriously limit a students educational opportunities ( Rivera – Batiz , 1992).

Jordan & Hanich (2000) reported that different aspects of mathematics involved different cognitive abilities ( Carroll, 1996, Geary et al., 2000). It meant, some children might have relative weaknesses in fact retrieval, even though they understand counting principles, and mathematical concepts, where as others might have relatively strong computational skills despite a week understanding of concepts ( Jordan & Hanich, 2000; Jordan & Montani, 1997;. Russell & Ginsburg , 1984).

Hanich et al., (2001) assessed areas of mathematical cognition which directly related to teaching of mathematics, including basic calculation, approximate arithmetic's, problem solving, place value and written multidigit computation. They reported skilled performance in simple arithmetic developed gradually during early childhood ( Jordan & Huttenlocher, & Levine .1992; Jordan, Levine, and Huttenlocher,1994; Siegler, 1991) . In pre- school and kindergarten, many children solve arithmetic combinations by making rough estimates or by guessing. Gradually children learned to represent the problem with their fingers or other physical referents and to use these referents to count both addends, in case of addition (counting all), or to separate a subtrahend from the minuend , in case of subtraction (separating from). They again reported, by second grade children developed efficient counting strategies (e.g., for addition problem, they use a counting on procedures, which involves stating the larger addend and then counting upward the number of time equal to the value of the smaller addend.). Moreover, they (children second grade) began to use calculation shortcuts (Baroody, 1999; Dowker, 1998; Russell and Ginsburg , 1984) such as deriving answer from known number facts (doubles plus one pattern , 2+2=4 so 2+3=5). By the end of third grade, majority of children retrieve or construct answers by deriving answers from known arithmetic combinations with minimal cognitive efforts.

As per as problem solving is concerned, Riley, Greeno & Heller (1983) reported, in elementary school, children learned to solve mathematics story problem that involve basic arithmetic operation but vary in semantic complexity. These problems are referred to as *(a) change (b) equalize (c) combine and (d) compare.* Skill and solving these types of problem increases gradually in elementary school, with change and combine problem being the easiest and equalize and compare the hardest ( Riley and Greeno, 1988).

Ostad (1998) conducted a study on children having mental difficulties. He took three grade levels (second, fourth and sixth grades) of children. They were administered a set up change, combine , equalize and compare story problems. The results indicated that children with MD performed worse than normally achieving peers on all problem types. Using story problems involving change, Jordan & Montani (1997) found that third grade children with MD – only performed better than their peers with MD/RD (Mathematics difficulty, Reading difficulty) and as well as their normally achieving peers when the task was untimed but not when it was timed. Children with MD only had deficit with problem solving speed rather than with basic problem comprehension.

With respect to place value understanding, Hiebert & Wearne (1996) in a longitudinal study found a close connection between children's understanding of multidigit numbers and their computational skills. Children who developed the earliest understanding of place value and base-10 concept in first grade performed at the highest level in written computation in third grade.

Bisanz et al., 1995; Geary et al., 1996 conducted a study to differentiate influences of age from those of schooling on the development of children's addition skills. Their study reported children's strategies for solving addition problem did not vary with schooling. Only the efficiency (i.e. accuracy and solution time) of their problem solving improved during schooling periods. Their results suggest that schooling is not sufficient to produce within grade changes. Besides developmental changes in children's addition strategies, children's number concepts, number representations and place value understanding were assessed. Miura et al., (1993) reported children's Based 10 concepts which are acquired well before intensive math instructions, then their performance on tasks of number concepts would not change reliably across these early schooling periods but would be better at the start of schooling. Naito & Miura (2001) state that it has been found children's memory spans show age difference between pre school and elementary school years across cultures. Naito & Miura (2001) study on Japanese children's numerical competence, reported counting -on is the most advanced and efficient strategy than counts all, which just involves counting all the numbers indicated by both addends (e.g. Fuson, 1982). Therefore, having received pre school home training in numbers, the first grade students compared with kindergartners were already skillful enough to use counting on procedures

Baroody (1985) suggests that counting strategies used by younger children are gradually replaced by the means to answer questions more quickly, using a variety of strategies developed as they grow older and their experience increases. Carpenter & Moser(1983) suggest that there is not a sudden jump from counting to recall, but that a few facts are memorized by younger children, which are later used in derivation strategies, with other facts being memorized piecemeal over time. They also say that derivation strategies are not just a stage between counting and recall, but that a mix of all three is used for a long time. In relation to strategies, Baroody (1985) suggests, continues into adult life, and while adults rarely use fingers and they come to each answer quickly by a variety of methods rather than solely from memory, and may not even use the same method for the same fact on all occasions.

Siegler & Robinson (1982) suggest that children move from the exclusive use of a min counting strategy to using what is knows in any array of strategies. The selection between available strategies is made, they claim, on the basis of characteristics of the demand- the confidence which children feel they are required to have about the answer or the amount of effort it is appropriate to use. In other words children may use memory if speed is emphasized, but revert to slow. careful counting strategy if they feel they need to be sure of arriving at the correct answer.

Siegler (1987) suggests that only a mixed strategy can achieve the double ambition of fastest speed and maximum accuracy. Children seem to exercise a repeated judgement about which kind of method is most helpful for which kind of problems and on what occasions. As a result there can be inconsistency in the strategy used.

There is some variability in children's performance depending on the nature of the action or relationships described in different problems: It was found that by the first grade most children can solve a variety of problems by directly modeling the action or relationships described in the problems (Riley, Greeno and Heller 1983). In these solutions, two accounts of cognitive mechanisms are involved which differ in fundamental ways. Riley and Greeno, (1988) proposed that children's ability to solve simple addition and subtraction problems depends on the understanding the various semantic relationships in the problems. On the other hand, Briars and Larkin (1984) propose an analysis that, at the most basic level, it does not include separate schemata for representing different classes of problems. Problems are mapped directly in to the action schemata, which are required to solve the problem.

Carpenter, Ansell, Franke, Fennema & Weisbeck (1993) interviewed 70 kindergarten children individually, who had spent the year solving a variety of basic word problems including addition, subtraction, multiplication, division, multi-step & non routine word problems. The results of this study shows that a wide range of problems involving multiplication and division situations can be solved by children much earlier than generally has been presumed. With only a few exception children's strategies could be characterized as representing or modeling the action or relationships described in the problems as already mentioned above. The conception of problem solving as modeling could provide a unifying framework for thinking about problem solving in the primary grades. They concluded that modeling offered a parsimonious and coherent way of thinking about children's mathematical problem solving that is relatively straightforward.

Greer (1992) reported that problem structure and problem difficulty of multiplication and division is more difficult than addition and subtraction. The problem schemata for multiplication and division are more complex than those required for addition & subtraction. Kouba's (1989) study reported that the strategies that children used for multiplication and division word problems were consistent with the strategies that they use for addition and subtraction problems in that children directly modeled the explicit action and relationship described in the problem. However, they seemed to have more difficulty modeling multiplication and division situations.

**Representation and strategies for addition and subtraction problem**

In teaching mathematics the concrete representation can be thought of as the source and the concept to be taught the target. The value of a concrete representation is that it mirrors the structure of the concept and the child should be able to use the structure of the representation to construct a mental model on the concept, (Boulton-Lewis, 1993).

A child's capacity to process information and his or her knowledge base both develop with age (Carey 1985; Chi & Ceci, 1988; Halford, forthcoming). The difficulty that children (and often adults) experience in cognizing many concepts is due to the load imposed by the process of mapping the task into a mental model. Such difficulties have been shown for transitivity teachers except children to map complex tasks into mental models, and if the children do not have prior knowledge of the necessary components then the task can make a load on information processing capacity that will interfere with understanding.

Mapping from a concrete representation to a concept imposes a processing load and this can interfere with the understanding to a concept; if an analog is poor or not property understood it can generate incorrect information; and if it is not well mapped into the material to be remembered it can actually increase the processing or memory load of a task (Halford & Boulton-Lewis, 1992).

Use of analogs sometimes fails to produce the expected positive outcomes. Lesh, Behr & Post (1987) found that concrete problems often produce lower success rates than the word problems that become more difficult when additional information is given in the form of concrete material.

Mathematical problem solving task can be broken down into two major component processes: (a) problem comprehension, and (b) problem solution (Mayer, 1985, 1986, Mayer, Larkin, & Kadane, 1984). Problem comprehension involves translation of each sentence in the problem into an internal representation and integration of the literal information to form ac coherent structure. Problem solution, on the other hand, requires planning, monitoring, and executing the requisite mathematical operations. Gange (1983), however, suggests that there are three phases in the performance of a mathematical task: translation, execution (problem solution), and validation or monitoring of the problem solution. It has been found that students usually have trouble in the problem comprehension or translation phases (Mayer, 1985, 1986). However, most mathematics instruction focuses on the problem solution phase, particularly algorithmic computations (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Randhawa & Beamer, 1990). Also, students put little emphasis on the monitoring phase.

Kintsch and Greeno (1985) presented processing model that dealt with both the text comprehension and problem solving aspects of arithmetic word problems. The main features of this model include a set of knowledge structure and strategies for using them in building a representation and solving problem. The verbal input is transformed into a conceptual representation of its meaning that takes the form of a list of propositions. The knowledge structures of this model comprise (a) set of prepositional frames, (b) a set of schemata that represent properties and relations of sets in general form, and (c) set of schemata that represent counting and arithmetic operations in general form.

Kintsch and Greeno (1985) presented evidence to suggest that the general features of the comprehension process are alike in a variety of situations such as mathematics word problems, reading a story, but the content of the comprehension strategies, the nature of the task-and goal-specific. In their view, comprehension of a mathematical word problem is achieved by constructing a conceptual schema from the verbal form of the problem on which problem-solving processes can operate.

Sweller, Mawer, and Ward (1983) found that mathematics experts (graduates) and novices (9-12-years –olds) employed distinctive strategies while solving mathematics problems. Expert problem solvers preferred forward-chaining strategies, using a large formula, as opposed to various steps one at a time, whereas novices typically employed a means-ends approach working backward from the target solution.

Hall, Kibler. Wenger. and Truxaw (1989) analyzed the episodic structure of written protocols of 85 undergraduates who were asked to solve four story problems. Their results showed that comprehension and solution of the problems were complementary activities, rather than distinct phases of a problem solving task as Gagne (1983) and Mayer (1985), 1986) proposed. Also, Hall et al. (1989) found that the two complementary activities in the problem solving process resulted in a succession of episodes. Furthermore, competent problem solvers used various forms of model-based reasoning to identity, pursue, and verify quantitative constraints required for the solution. Russell and Ginsberg (1984) reported that fourth graders with mathematical difficulties or novices lacked sophisticated understanding and strategies, but they were not seriously deficient in basic mathematics concepts and non-algorithmic procedures.

Many young children solve verbal addition and subtraction problems by representing the problems with objects and carrying out the prescribed actions of the object (Carpenter et. al. 1981). For example. If we make counters available to children for representing the problem or situation then many children will be able to do some "missing addend" situation. It describes a joining action by adding on to the first set of numbers and counting the number of objects that must be needed on to yield the result. It seems that children rely on direct sequential representation to solve problems. So. on the basis of this finding the hypothesis can be made those problems, which are, not easy to represent sequentially will be more difficult to solve. Groen & Poll, 1973; Grouws. (1972) found that the problems in which the first quantity is missing would be more difficult to solve, interpret than the problems in which the first addend is given.

Hiebert (1982) extend this work by systematically manipulating the position of the unknown set on first grade children's representation and solution processes for verbally presented addition and subtraction problems. Two outcomes were taken in to considerations that are (a) children's modeling behavior on each type of problem and (b) their solution processes for each problem. The task used in this study were six verbal arithmetic problems of similar semantic structure i.e. three joining problems and three separating problems. These six problems were read to each student in an individual interview. In the findings of this study, several solution strategies were identified; most of them were based on counting. For addition, two counting strategies were found that are- count on and count all. In subtraction, four basic counting strategies were identified out of which two depends on modeling the problems with physical objects i.e. (a) Separate, (b) Separate to, (c) Add on and (d) Count down. Some common strategies were found which could be used in addition or subtraction problems, these are (a) Known fact, (b) Derived fact, (c) Uncodable. There were some inappropriate strategies, which were classified in to one of the following error categories. (a) Repeats given number (b) Wrong operation (c) Indeterminate. Hence, the results of this particular study indicates that the position of the unknown set in a verbal problem determines to a substantial degree whether or not the problem can be modeled successfully by first grade children.

Blume (1981) reported that the difficulties that children experience in writing number sentences to represent certain types of problems might occur because the representations that they have been taught (a+b and a-b) do not correspond to the interpretation of the problems. Children may naturally represent word problems with open number sentences directly model the actions in the problems. Carpenter & Moser (1984) stated that children invent counting and modeling procedures to solve word problem without explicit instructions. Instructions may limit the range of symbolic representations that children recognize as acceptable.

Geary (1994) states that counting strategies are typically the first type of strategy that children used and can include both finger counting and verbal counting strategies. The strategy chosen to solve a problem depends partly on problem difficulty. Basic strategies such as counting are used on problem that a more difficult for children, where as retrieval is used more often on problem that children consider to be easy.

From the above review of related literature we found that New ideas and methods from the information processing approach led to the emergence of new paradigm in addition and subtraction word problems. The representation of a problem solution is one of the fundamental problems solving processes. Many problems can be solved by representing directly the critical features of the problem situations or physical or pictorial representation.

# CHAPTER-3

# METHODOLOGY

The present study "children's strategies to solve addition and subtraction problems in early school years" intended to explore how children understood the addition and subtraction operations, the necessary symbols, represent them with analogs and what strategies they used to solve the problems.

Analogs are nothing but serve as memory aids which provide a means of verifying truth; increase flexibility in thinking; facilitate retrieval of information from memory and indirectly facilitate transition to higher level of transition (Boulton-Lewis, 1993). In order to explore the possible strategies and conceptualization of addition and subtraction problems, only Primary school students belonging to grade-II and III were approached for the study. Rationale of the present study were as follows:-

## RATIONALE OF THE STUDY

*A child's capacity to process information and his or her knowledge base both developed with age (Carey, 1985; Chi & Ceci, 1988). Boulton-Lewis (1993) interested to find out what strategies and representations children used for addition and subtraction operation in mathematics. He reported that the general developmental sequences are- use of objects, use of counting, mental calculations, using knowledge of number facts and place value.

* Large body of research which is concerned with young children's solution to simple addition and subtraction problems reported that, the knowledge of addition and subtraction develop before the onset of schooling and that these informal strategies do affect strategies learned at school (Houlihan & Ginsburg, 1981). The difficulty that children experience in Cognizing many concepts because of load imposed by the process of mapping the task into a mental model. The mental representation often becomes too difficult for children because they don't have prior understanding of processing of information at abstract level, but they become comfortable in using concrete materials and objects available to them for solving an addition and subtraction

problems. As the age advances the developmental patterns enable them to understand the operations, map the concepts and also relationships involved in addition and subtraction problems.

* Ample researches have shown that children of pre- school age can use counting methods to solve addition and subtraction problems involving concrete objects. First grade children solve addition and subtraction numerical problems through counting though it is not taught in school. Therefore, children assimilate school arithmetic into existing cognitive structure to develop their own invented strategies. Russell (1977) reported that children tend to use procedures that are adaptive to the requirements of a problem. Thus, they use counting procedures for dealing with objects but written procedures with written numbers.

* Data show that first graders add by counting methods involving concrete objects, fingers, blocks etc. Various studies report that most frequently used methods for counting are – counting on from the larger addend, counting from one and starting with the first addend. As the grade progresses, children tend to use both counting and non-counting methods to solve addition and subtraction place value problems (Groen, 1972 & Suppees, 1967).

* Children can analyze and solve simple addition and subtraction word problems by directly representing with physical objects before they receive formal instructions in addition and subtraction. At the time children are first introduced to writing mathematical sentences to represent word problems, they see no connection between the informal modeling and counting strategies they use, and the number sentences they are taught to write to represent them (Carpenter et al. 1983).

*They solve the problems by directly modeling with physical objects or the relationships described in the problem. The semantic structure of different word problems could be studied through the student's ability to represent with number sentences. Children who are taught to represent word problems with number sentences in standard form (a+b= ---- & a – b = ----) have no difficulty in representing simple joining and separating action but they do experience difficulty in other problems.

* Children pass through several stages in development of addition and subtraction concepts and skills. In the initial stages, children solve a variety of addition and subtraction problems by modeling the action in the problems using physical objects and counting strategies. At the higher grades, children's counting strategies become more flexible and are no longer limited to direct modeling the relationships. They can transform problems to solve them with a variety of different strategies and apply symbolic and physical representation of problems (Briar & Larkins, 1984). Higher Grade students are expected to be more successful in representing word problems to open number sentences and apply economical counting strategies. These open number sentences enable the children to provide symbolic representations and they can relate them to their informal counting and modeling strategies (Carpenter & Moser, 1984). With experience. children's single digit addition and subtraction become more complex abstract and interiorised. (Fenemma, et al., 1997)

From the above discussion, the following objectives for the present study were outlined. These are as follows:-

**OBJECTIVES**

(i) To study the counting strategies children use to solve addition and subtraction problems.

(ii) To examine children's ability to write numbers and number sentences to represent different addition and subtraction word problems.

(iii) To investigate children's understanding of place value from the solution strategies use for solving addition and subtraction problems.

(iv) To examine the differences in solution strategies use by second and third graders on addition and subtraction problems.

The following Research Questions were out lined for this study

## RESEARCH QUESTIONS

(i) Do children use different strategies for different problems?

(ii) Is there any pattern in the ways that children solve arithmetic problems?

## SAMPLE

In order to explore the possible strategies used to solve the arithmetic problems at early school years sample was collected from Municipal Corporation of Delhi Primary Schools located in the city. Only second and third grade school children were selected for the study. Two stage sampling techniques were followed: (a) the identification of school and (b) the identification of subjects for the study. The Government runs primary schools located in New Delhi were selected randomly for the present research. M.C.D. Primary School, Basai Dara Pur I and II of New Delhi were selected according to our convenience. This school comprised of two-session, morning Session (I) for girls and afternoon session (II) for boys.In this school all of our required sample size was met. Only grade-II and III boys and girls were included and approached for the present research, for which substantial research evidences has been mentioned before. From both the grades, the subjects were randomly drawn and care was taken to match the subjects with respect to socio-economic status, academic achievement and teacher-student interactions. Keeping all other background variables constant, care was also taken to make the sample equal in terms of gender. Therefore, 40 students from grade-II and III were randomly selected and again they were sub divided in terms of gender. So, finally sample for the present study comprised of four groups that is *Grade-II Boys, Grade-II Girls, Grade-III Boys, and Grade-III Girls* consisting of ten students each. The tabular representation of final sample is mentioned below: -

**Table A - Gender and Class Wise Distribution of Sample.**

| CLASS / GENDER | II | III | Total |
|---|---|---|---|
| **Boy** | n=10 | n=10 | n=20 |
| **Girl** | n=10 | n=10 | n=20 |
| **Total** | n=20 | n=20 | **Grand Total** N=40 |

## RESEARCH DESIGN

An exploratory research between group design was chosen to look for the possible strategies children use for writing numbers, number sentences, counting, understanding place value and representing the problems with concrete and abstract objects etc. to solve addition and subtraction problem in early school years. The strategies were identified through in-depth interview, proper interaction and systematic observation of the students during the time of data collection. Some explanations were ensued from the students for unique and extra ordinary responses.

## TOOLS USED

A number of problems were selected after a review of earlier research. The problem chosen for the study was based on the parameter of combine, change and compare types. These problems were selected because (a) they were representative of problems commonly included in elementary mathematics text books (b) the problems were appropriate because younger children would most likely be able to solve and (c) variety of basic problem types would elicit different solution strategies. In order to explore the possible strategies for addition and subtraction problems in early school years, two different sets of test were developed. One for grade II and another for grade III was prepared. The test items were selected from the categories of problems types stated in the mathematics text books published by Central Board of Secondary Education, New Delhi. Two separate tests were meant to understand student's developmental level in solving addition and subtraction numerical and word problems and to see their representational ability.

**TEST FOR GRADE –II**

Both numerical and word problems for addition and subtraction arithmetic operations were selected. Problems were included to understand students' ability to represent the word problems with open number sentences and the use of counting strategies. This test comprised sixteen problems, out of which eight are addition and rest are subtraction problems. Again addition and subtraction problems were decomposed in to five numerical and three word problems each. The nature of the problem is very simple and language pattern was according to the students' level of understanding. Care was taken to avoid ambiguity in word problems. Single digit vertical and horizontal problems along with missing addend in very simple forms were included in the test. The test items have been attached in appendix-I at the end.

**TEST FOR GRADE- III**

In like manner, test for the grade-III students was devised. The difficulty level of the items increased from simple to complex. Besides, addition and subtraction problems. some more problems on multiplication and division were added in order to understand their developmental pattern. Multidigit addition and subtraction with multi-step problems were included. Word problems of all arithmetic operations were presented to explore their capacity to represent them with open number sentences and derived number fact from recall. The test items have been attached in appendix-II at the end.

**PILOT STUDY**

After tool construction, a pilot study was conducted, on ten students (5 second-grade and 5 third grade) to evaluate the difficulty level of the items included in the test. Initially each test contained twenty items of different problem types. All children were grouped according to their school grade. The first test was administered to all the students in the school, after taking proper permission from both head master and head mistress. During the time of test administration all students were systematically observed. Student's reactions to the Problems, gestures they made were noted down. Some of the students did not understand the language of the word problems. Therefore, the ambiguities of the problems were reduced to maximum. All of them solved the problems personally. Because of huge number, individual interview was not made possible. All of them took more than one hour to complete the test. After taking suggestions from the participants and teachers, the items for the final study were reduced. On the very next day only third grade students were administered the second test. In the same way final tools were constructed. The addition problems, whose first addend was missing, created difficulties for students. For subtraction, students put wrong operation in solving carry over problems. Care was taken to reduce the number of items, nature of items and remove the ambiguity of the word problems. Based on participants' responses and suggestions only sixteen problems (eight for addition and eight for subtraction) were selected. Some more multiplication and division problems were included for grade-III students to understand their age and developmental level's influence on strategy use. The different problems for grade-II and III were selected so that they could be used to look at change in performance across the school year.

## ADMINISTRATION OF QUESTIONNAIRE

Headmasters and Headmistress of Municipal Corporation of Delhi Primary School, Basai Dara Pur were approached where both boys and girls studying in the different sessions simultaneously. After consultation with class teachers, students from both (morning and afternoon) sessions were randomly selected. In morning session, girls were administered the test where as in afternoon session, boys were administered the same test. For each group 10 students were selected randomly.

A small room in the school was allotted to conduct the study. In each session a single child was interviewed indepthly. First, the child was told to count the number from one to hundred and then some numbers were given for recognition. When the preliminary task became over, the test problems were administered randomly. For the addition and subtraction problems, the children were told that they would be asked to solve eight addition and eight subtraction problems. They were told to do their best and to use counters to solve the problem if they wanted. Counters in sufficient quantity to solve all of the problems were placed near the child. The researcher also told the children that they would be asked questions about how they solved the problems and why they solved the problem that way (strategies). The eight addition problems, printed on cards, were randomly presented first. Addition problems were presented first so that children could experience some success, particularly at the beginning (Carr & Jessup, 1997). Next eight subtraction problems were randomly presented. After one problem, the child was asked to explain how did he/she approach the problem and what led him/her to reach the solution. The strategies were studied by asking the children why and how they used different mathematical strategies. The children's responses to these questions were checked against the observed strategies. This technique is a commonly used procedure (e.g. Siegler, 1988; Siegler & Shrager, 1984) and has been shown to be a valid measure of actual strategy used (Siegler, 1989). In this way all problems in the test were administered. This process remained same for all problems and for all students belonging to grade-II and grade-III. When the students felt uneasy and were not able to tell the answer properly, rapport was established and proper care was taken to make him/her relaxed and he/she was made comfortable to elicit the answers. These interviews were designed to determine which strategies children use and what are the explanations they have for using these strategies.

## CODING

For representation of number and place value, children's responses were scored as correct, if they summed to the whole number using both the unit and ten blocks or either of the two. Children were shown one of the cards with written numerals and asked to read the number aloud. If children points first to the numeral in the ones position and then the numeral in tens position on the card, then it was considered to be correct. With respect to strategy use, children use both overt and covert strategies. Overt strategies were those techniques that were visible to the experimenter, e.g. finger counting or counters such as marbles, match sticks etc. Covert strategies such as mental calculation and retrieval were more difficult to assess because they were unobservable.

Mental calculation or retrieval strategies were determined by asking children about how they solved a problem. A strategy was considered to be retrieval if the child described pulling the information from memory or the information just crop up into his or her head.

## STATISTICAL ANALYSIS

All the possible responses, which were noted down on the note sheet, were written properly for each student. The content analysis was done on children's responses to the Questions about their strategies used were categorized. Strategies were categorized as (AS1-------AS9) for addition and (SS1-------SS8) for subtraction problems. Only frequency (f) and percentage (%) were calculated to see which strategies were more frequently used. The percentage of attempted strategy use by both boys and girls were calculated. The chi-square ($\chi^2$) was also calculated among boys and girls for preferred strategies used to solve the problem. Since, the present research based more on verbal protocol, quantitative techniques were least used for analysis. Qualitative analysis was suited best for proper analysis of the data.

## DESCRIPTION OF KEY WORDS

### Counting

It is a surprisingly intricate process by which children call number values by name. It is understood as reading of successive numerals or the extent of a sequence of numbers counted one after another by a child (either correctly or with incidence of error).

### Number

It is assumed to be the product of a set of rules applied to real word quantitative phenomenon.

**Representation**

The representation and the use of number refer to the numeral and digital codes that children designate. The digital and numeral number codes offer a set of elements to represent reality and understand the transformation, which it undergoes.

**Retrieval**

It referred to recall of answers directly from memory, as observed when children spoke the accurate answer quickly without using counting and did not specify how did they arrived at the answer.

**De-Composition**

It involved transforming the original problem into two or more simpler problems and was categorized when children described a stepwise process of calculation.

**Other strategy or guessing**

It refers to the situation when the children explained their answers by saying that they guessed or that they did not know the answer or when they choose not to give any answer.

# CHAPTER 4

# ANALYSIS OF RESULTS

*PRELIMINARY TASK*

**I- Counting**

**TABLE- 1 Percentage of Grade II Boys and Girls count correctly on different dimensions of Counting**

| Dimensions | Girls | Boys |
|---|---|---|
| Count correctly till 1000 | 00 | 70 |
| 100 onwards count 200,300,400 so on | 40 | 00 |
| Skip some numbers in between & knows till 100. | 60 | 30 |

From the preliminary task I in which children was asked to count loudly from 1 to 100, it seems that after this task has been completed with each and every child of Grade II, three dimensions of counting were found. First one deals with the children who could count fluently from 1 to 100 and knows further also till 1000. The second category is of children who can fluently count 1 to 100, but after 100 they start counting 100,200,300----900. Last category is of children who could count till 100 but they skipped some numbers in between. Now, we look at each of this category separately. It was found that 70% of the boys falling under the first category where as none of the girl could count fluently from 100 hundred onwards and nobody can count till 1000. In this category, basically when children were asked to count till 100, then those who could count fluently till 100 were told to count further. If they can count fluently then researcher told them to start counting from in between like count from 150 then stopped him/her at 180 and further asked them to count from 220 etc. After this, the researcher was came to know that how many of them can count fluently till 100, 500, or 1000. Now, comes to the second category of children in which they can count fluently from one to hundred but after reaching 100 they keeps on counting 200, 300, 400, 500-------10,00. Only girls (40%) fall under this category where as boys doesn't adopt this process of counting. This shows that girls who comes under this category doesn't have the conceptual understanding of place value system because they took Hundred (100) as a whole instead of dividing it in to 3 parts i.e. one's, ten's and hundred's. This is so, because after counting from 100,200-----when they reach 1000, they pronounce it as ten hundred (10+100 or 10,00). After completing this task, when the researcher asked them, what comes after 100, then some of them were able to count 101,102------ but they were unable to explain why 101 comes after 100 and previously why they count 100,200,300. The last and third category of students is of those who could count till hundred and further also, but while counting they skipped some numbers in between. Both boys (30%) and girls (60%) comes under this category, they start counting from one and till fifty they could count fluently but after fifty, they stuck at some numbers in between, both boys and girl stucked at the same number like 59,69,79,89,99 etc. Above hundred also, they stucked at the same number like 149,159 and so on. Whey they reach these above mentioned numbers either they skipped these numbers and jump on to the next or they followed wrong pronunciation for these numbers. All of them could count in Hindi but pronunciation of some Hindi numbers is a difficult one, that's why students faced difficulty in pronouncing them for e.g. for 59 they pronounce 69 and they were also confused in the pronunciation of 79 and 89. All these numbers are the numbers in which 9 is lying at the one's place. Hence, we can say that the children had problem in pronunciation of the numbers with nine. Thus. the error in this case produced by associative properties of mathematical terminologies in relation to digit numbers.

**TABLE 2 Chi-square value for Correct and Incorrect counting of numbers among second grade Boys and Girls**

| Dimensions | Count Correctly till one thousand | Hundred onwards Count 200 ,300--- | Skip numbers between one hundred |
|---|---|---|---|
| Chi-Square ($\chi^2$) df = 1 | ** 9.6 | ** 9.4 | 0.93(NS) |

**Note- ** = Value is significant at .01 level**
**NS = Value is not significant.**

Chi-square ($\chi^2$ ) value revealed a significant difference between boys and girls on different dimension of counting i.e. hundred onwards count 200,300-----, count correctly till one thousand and skipping some numbers in between one to one hundred. Boys and girls differed significantly ($\chi^2$ = 9.6/9.4, p < .01) on counting hundred onwards and correctly count till one thousand but they did not differ on skipping numbers ($\chi^2$ = .093, p > .05). It indicated that only boys could correctly counted numbers from one to hundred and above also where as girls could count fluently till one hundred only. Both boys and girls skipped some numbers between one to one hundred while counting. The results stated that boys were better performers in counting numbers till one thousand as compared to their female counter part.

**II- Recognition of Numbers**

To assess the recognition of written numerical, subjects were presented with cards with numbers ranging from 1 to 150 in a random order and asked to read them. A grading of the subjects was ordered on the basis of whether or not they could read correctly all the numbers. For analysis, these numbers are divided in to two parts

(1) Double-digit (2) Multidigit

From the results of the present study three dimensions were found for recognition of numbers.

(1) **Correct:** In this category, all the subjects were able to recognize the number correctly.

(2) **Wrong pronunciation:** In this, subject was able to recognize the presented number but the pronunciation made by them for a particular number is wrong.

(3) **Reverse Order:** Subjects recognize two numbers by changing their places in a reverse order e.g. they recognize 63 as 36, 107 as 170.

**TABLE 3 Percentage of Grade II Boys and Girls correctly recognized the given Numbers**

**TABLE 3A. Percentage of Grade II Boys and Girls correctly recognized Double digit Numbers**

| Dimensions | Girls | Boys |
|---|---|---|
| Correct | 10 | 70 |
| Difficulties as a function of pronunciation | 80 | 30 |
| Reverse Order | 10 | 00 |

Table 3A revealed that for the recognition of double-digit numbers, as compare to girls more number of boys i.e. 70% lies under the category of correct responses. They could recognize all the double-digit numbers correctly where as only 10% of the girls were successful in recognition of double-digit number. Those who were able to recognize the double digit-number correctly, they preferred some counting strategies for recognition. (a) To recognize a particular given number for e.g. 87 after reaching 87 while counting, they were able to read the given number.(b) some children adopted the decomposition strategy, which is of broken down the given number in to two parts. To recognize 84 they read it as 80 plus 4 are equal to 84. As far as category of wrong pronunciation is concerned, maximum number of girls i.e. 80% committed an error in the number pronunciation. In this category, the percentage of boys comes down to 30%. The errors were committed in the numbers like they read 49 as 39, 79 as 69 or 63 as 53, 23 as 33, 63 as 53, 87 as 77 etc. For recognition of each number, before recognizing it some of them start counting from some previous numbers for e.g. For recognizing 49 they start counting from 40,41,42---49 and finally after reaching 49 they pronounced it as 39.After seeing the number. they were not able to recognize it spontaneously.

One girl while recognition of numbers only preferred the third category, which is of Reverse Order. She recognizes the number by exchanging their positions. For e.g. To read out 63 in which 3 is at unit's place and 6 is at ten's place, she recognize it as 36 in which the position of both unit and ten's number was exchanged. Basically she read out 3 at place of ten's instead of unit and 6 at place of unit instead of reading it ten's place. Hence, she interchanged both the numbers given at unit and ten's place

**TABLE 3B. Percentage of Grade II Boys and Girls correctly recognized Multidigit Numbers**

| Dimensions | Girls | Boys |
|---|---|---|
| Correct. | 50 | 100 |
| Difficulties as a function of pronunciation. | 20 | 00 |
| Reverse | 30 | 00 |

Table 3B revealed that all the boys i.e. 100% could recognize the presented multi-digit number correctly, contrary to this only 50% of the girl (i.e. half as compare to boys) correctly recognize the multi-digit numbers. Other 50% of the girls comes under the other two categories

20% of them committed an error in the pronunciation e.g. they read out 122 as 112 or 121 other 30% did the same mistake as they did in double-digit number recognition. They interchange the position of numbers presented at unit and tens e.g. the number 107, they pronounce it as 170 because they exchange the position of 7 from unit to ten's place and bring 0 from ten's place to unit's place. Other e.g. is they read out 134 as 143, 145 as 154.

**TABLE 4 Chi-square value for Correct and Incorrect recognition of double and multidigit numbers among second grade Boys and Girls**

| Type of Nos. | Dimensions | Value of chi-square($\chi^2$) with df=1 |
|---|---|---|
| Double Digit | Correct recognition | 10.33 ** |
| | Wrong Pronunciation | 9.64 ** |
| | Reverse Order | 0.25 (NS) |
| Multi – Digit | CR. | 64 * |
| | WP | 2.25(NS) |
| | FR | 3.29 # |

**Note:-**
** = **Value significant at .01**
* = **Value significant at .05**
# = **Value significant at .10**
NS = **Not significant.**

The chi-square value showed a significant difference between boys and girls ($\chi^2$ = 10.33, $p < .01$) on correct recognition of double-digit number. It indicated that maximum number of boys correctly recognized the number, which were presented to them in the preliminary task. With respect to wrong pronounciation a significant difference ($\chi^2$ = 9.64, $p < .01$) was found among boys and girls. This result reported that girls were more likely than boys pronounced the number wrongly that is sixty-nine as seventy-nine and vice-versa. For multi digit numbers significant difference between boys and girls ($\chi^2$ = 6.4, $p < .05$) was marked. This stated that as compare to girls more number of boys was recognized the multi digit numbers correctly. With respect to recognition of number in a reverse order, significant difference was marked ($\chi^2$= 3.29, $p < .10$). This stated that more number of girls committed error while recognizing the multi digit numbers. They recognized 136 as 163 (one hundred thirty six as one hundred sixty three).

**III- Writing Numbers**

**TABLE 5 Percentage of Boys and Girls Write correctly the given Numbers**

**TABLE 5A Percentage of Boys and Girls Write correctly the Double Digit Numbers**

| Dimensions | Girls | Boys |
|---|---|---|
| Correct | 20 | 60 |
| Incorrect Numbers | 80 | 40 |

Children were told by the researcher to write some of the numerals on their papers. In this task also, these numerals divided to two parts i.e. double –digit and multidigit numerals. Results showed that 60% of the boys wrote double-digit numerals correctly and in case of girls this percentage comes down to 20% which is very less as compared to boys. Rest of the 80% girls and 40% boys under the category of incorrect numbers. This means that after hearing the right pronunciation of a particular number say 69, they interpreted it in a incorrect way and write it as 79 or 59. The error was committed by both boys and girls and the examples of this are as follows: - They wrote 57 as 67,95 as 85, 81 as 51, 89 as 69, 57 as 97, 41 as 81, 57 as 77, 81 as 21, 29 as 39 etc.

**TABLE 5B Percentage of Boys and Girls write correctly the Multidigit Numbers**

| Dimensions | Girls | Boys |
|---|---|---|
| Correct. | 80 | 50 |
| Incorrect Numbers | 20 | 20 |

In writing the multi digit number also, the number of girls and boys falls under the category of correct response varied. As compare to double-digit numerals the percentage of girls who makes a correct attempt to write the multidigit numerals has increased to 50% and in case of boys it increased to 80%. This shows that equal number of girls falls under both the categories (correct and incorrect). After hearing the number from researcher they immediately started writing. E.g.- they put 110 as 101,112 as 1012, 140 as 1040, and 143 as 133. In this basically, after hearing hundred's voice in the number they put extra zero for hundred place. Actually, they have a lack of conceptual understanding of place value. For writing 112 (One hundred and twelve), they interpreted it as 100 and 12, for clubbing it together they removed only one zero from it and the other zero denotes hundred for them where as one denotes 100 (10 in 1012 =100). Hence they wrote 1012 in which 10 denotes one hundred.

**TABLE 6 Chi-square value for Correct and Incorrect double and multidigit writing of numbers among second grade Boys and Girls**

| Type of Nos. | Dimensions | Value of chi-square($\chi^2$)with df=1 |
|---|---|---|
| **Double-Digit** | **Correct recognition** | **4.34*** |
| | **Wrong Presentation** | **15.4 **** |
| **Multidigit** | **Correct** | **15.1 **** |
| | **Wrong Presentation** | **15.1 **** |

**Note:-** ** = Value significant at .01
* = Value significant at .05

Table- 6 of chi-square revealed significant differences among boys and girls on different dimensions of writing numbers. With respect to double digit writing numbers, a significant difference ($\chi^2$ = 4.34, $P < .05$) was found between boys and girls on correct numbers. This stated that boys were correctly writing the double-digit numbers than girls. Again this difference was noticed ($\chi^2$ = 15.1, $P < .01$) incase of multidigit numbers. As per as wrong presentation of numbers are concerned both boys and girls differed significantly for writing double and multidigit numbers ($\chi^2$15.4 & 15.1, $P < .01$) respectively. This result reported that in writing double-digit numbers maximum girls wrote the numbers wrongly where as for writing multidigit numbers, the reverse was found.

Therefore, the over all findings indicated that boys were outperforming girls on counting numbers, recognition of double and multi digit numbers and writing double and multi digit numbers.

# RESULT ANALYSIS OF CLASS II

**TABLE 7- Percentage of Grade II Boys and Girls on preferred Strategies for Addition**

| PROBLEM TYPE ADDITION | SEX BOY/GIRL TOTAL | AS1 | AS2 | AS3 | AS4 | AS5 | AS6 | AS7 | AS8 | AS9 |
|---|---|---|---|---|---|---|---|---|---|---|
| P1 4 | G | 40 | 10 | 10 | 30 | - | 10 | - | - | - |
| + 7 | B | 10 | 30 | 10 | 10 | 20 | - | 20 | - | - |
| ------ | T | 25 | 20 | 10 | 20 | 10 | 05 | 10 | - | - |
| | G | 30 | 10 | 10 | - | 10 | 10 | - | 30 | - |
| P2 6+3 = --- | B | 80 | - | - | - | 20 | - | - | - | - |
| | T | 55 | 05 | 05 | - | 15 | 05 | - | 15 | - |
| P3 | G | 40 | - | 10 | - | - | - | 20 | 30 | - |
| -----+5 = 8 | B | 50 | - | - | - | - | - | 20 | - | 30 |
| | T | 45 | - | 05 | - | - | - | 20 | 15 | 15 |
| P4 15 | G | 40 | - | 20 | - | 20 | 20 | - | - | - |
| + 12 | B | 50 | - | - | - | 40 | - | 10 | - | - |
| ----- | T | 45 | - | 10 | - | 30 | 10 | 05 | - | - |
| P5 26 | G | 50 | - | 30 | - | - | 20 | - | - | - |
| + 16 | B | 40 | 20 | - | - | 40 | - | - | - | - |
| ---- | T | 45 | 10 | 15 | - | 20 | 10 | - | - | - |
| P6 26 | G | 60 | - | 10 | - | 10 | 20 | - | - | - |
| + 8 | B | 40 | - | 20 | - | 20 | - | 20 | - | - |
| ---- | T | 50 | - | 15 | - | 15 | 10 | 10 | - | - |
| P7 55 | G | 50 | 10 | 10 | - | - | 30 | - | - | - |
| + 28 | B | 30 | 10 | 30 | - | 10 | - | 20 | - | - |
| ---- | T | 40 | 10 | 20 | - | 05 | 15 | 10 | - | - |
| P8 47 | G | 40 | 10 | 10 | - | - | 30 | - | - | 10 |
| + 24 | B | 40 | - | 30 | - | - | - | 20 | - | 10 |
| + 34 | T | 40 | 05 | 20 | - | - | 15 | 10 | - | 10 |
| ----- | | | | | | | | | | |

Note: G= GIRLS, B=BOYS, T= TOTAL
AS : Addition Strategies
AS1: keeping the larger number constant count the smaller one with fingers
AS2: keeping the smaller number constant count the larger one with fingers
AS3: count the first addend then second and add all &vice- versa with fingers
AS4: count the smaller, add further larger until the answer comes with fingers
AS5: count the larger, add further smaller until the answer comes with fingers
AS6: putting tally marks on paper
AS7: mental calculation
AS8: errors
AS9: others

**Problem-1 (P1)-** Simple addition with single digit i.e. 4+7 given in a vertical form. It was revealed that all the students solved this problem correctly. Maximum number of student's i.e. 25% preferred AS1 whereas 20% adopted AS2 & 20% followed AS4. Only 10% followed AS3, AS4 & AS7 whereas rest of the 5% used AS6. It seems that AS1 was found to be the most convenient & effective strategy for maximum number of students.

**Problem-2 (P2)-** Simple addition with a single digit in a horizontal form i.e. 6+3 = ---. It was found that 85% of the students solved this problem correctly. Out of this 85% maximum number of students i.e. 55% preferred AS1 whereas only 55 followed AS2 as well as AS3. Only 15% followed AS5 & only 5% used AS6. None of the students followed strategies like AS4 and AS7.15% of the students who committed an error was confused in the operations and put the wrong operation. Interestingly all the students who committed an error were girls.

**Comparison of P1 & P2** – In P1 and P2 differences were found in terms of strategy, which were used, by both boys and girls. In P1 all the strategies have been used whereas in P2 maximum students used AS1 and no one preferred strategies like AS4 & AS7. The percentage of students followed AS1 was 25% in P1 and 55% in P2. As far as AS2 is concerned 20% students preferred it in P1 whereas only 5% followed it in P2.

Strategies preferred by boys varied in P1 and P2. Only 10% of the boys preferred AS1 in P1 whereas for P2 this was increased to 80% .In P1 30% followed AS2 whereas none of them used it in P2. No one used AS6 in both P1 and P2. Any boy in P2 did not use strategies like AS2, AS3. AS4 and AS7.

As far as girls are concerned, 40% of the girls preferred AS1 for P1 whereas 30% used it to solve P2. Only 10% of the girls used AS2 and AS3 to solve both P1 and P2.In P1 30% of the girls followed AS4 whereas no one used it in P2.Only 10% used AS6 in both P1 and P2. With reference to errors. none of the boy committed an error in any of the problem. Likewise, none of the girl committed an error in solving P1 but in P2 30% of the girls committed a mistake because the problem was given in a horizontal form. All the girls who committed an error put wrong operation, they did multiplication and subtraction instead of addition. This means that they were confused in arithmetic symbols.

**Problem-3 (P3)-** Single digit addition was presented in a horizontal form but instead of the total (answer), the first addend was missing (----+5 = 8). It was observed that most of the students solved this problem after some intervention of the researcher. Table-I revealed that 85% of the students solved this problem correctly. Maximum number of students i.e. 45% preferred AS1 whereas 20% used AS7 and only 5% used AS3. Rest of the 15% followed some other strategies like doing subtraction to find out the missing addend. Percentage of students who committed an error was 15% showing similar mistake. They added up the given total and second addend to find the first missing addend.

**Comparison of P2 & P3-** Differences were found in the strategies used by students to solve P2 and P3. In P2 all the strategies all the strategies have been used except AS4 and AS7 whereas in P3 only AS1, AS3 &AS7 were preferred. 85% of the students solved both the problems correctly. All the students solved P2 in first attempt whereas they solved P3 after some intervention of researcher. In both P2 & P3 majority of the students preferred AS1. In P2 the boys used only AS1 & AS5 whereas in P3 they preferred AS1, AS7 and some other strategies. In both. maximum number of boys i.e. 80% in P1 and 50% in P2 used AS1.30% of the boys followed some other strategy like doing subtraction to solve P3. It seems that girls to solve both the problems have used various strategies. Maximum number of girls in both P2 & P3 preferred AS1. In both P2 &P3 equal number of girls i.e. 30% had committed an error. As far as errors are concerned only girls had committed an error in both the problems.

**Problem-4 (P4)**-Problem4 showed double-digit simple addition in a vertical form without carryover (15 + 12). From the table, it was found that all the students solved this problem correctly. Majority of the students preferred AS1 whereas none of the student followed AS2.Only 10% followed AS3 whereas none of the student used AS4.10% adopted AS6 whereas only 5% used AS7 to solve the problem.

**Problem –5(P5) -** Like problem4, in problem5 where addition of double digits with carryover was presented (26 + 18), it was found that all the students solve this problem correctly. Majority of the students i.e. 45% preferred AS1 to solve the problem. Only 10% followed AS2 whereas 15% used AS3 to get the solution. None of the student used AS4 on the contrary 20% followed AS5 and only 10% adopted AS6. None of the students used AS7.

**Comparison of P4 & P5-**Strategies used by both girls and boys varied for both the problems. In both the problems maximum and equal number of students i.e. 45% preferred AS1.None of the students used AS2 in P4 whereas 10% used it to solve P5. No one followed AS4 in any of the problem.

All the boys solved both the problems correctly and maximum number of boys preferred AS1. In P4 none of the boys used AS2 whereas in case of P5 20% of the boys used this strategy. Equal number of boys i.e. 40% followed AS4 in both the problems.

Likewise boys, all the girls also solved both the problems correctly. Majority of girls preferred AS1 in both the problems whereas AS3 was used by 20% in P4 and 30% in P5. Equal number of girls followed AS6 in both P4 & P5 and 20% used AS5 in P4 whereas none of them used it in P5.

In both P4 & P5 neither boys nor girls committed an error in solving the problem.

**Problem-6 (P6)-** This was word problem in which single digit was supposed to be added in double digit and it is with carry over (26+8). It was found that all the students solved this problem correctly. Maximum number of students i.e. 50% preferred AS1 whereas 15% followed AS3 and AS5 each. Only 10% followed AS6 and AS7 each whereas none of the students used strategies like AS2 and AS4.

**Problem-7 (P7)-**Likewise problem 6 this was also a word problem of double digit with carry over (55+28). Table revealed that all the students solved this problem correctly. In this problem also, maximum number of students i.e. 40% preferred AS1 and only 10% used AS2.20% of the students followed AS3 whereas none of the students used AS4.Only 5% used AS5 whereas 15% followed AS6 and rest of the 10% used AS7.

**Problem-8 (P8)**-This represented the word problem which consisted of double digit with carry over in three columns (47+24+34). From the table, it seems that all the students solved this problem correctly. Majority of the students preferred AS1 whereas only 5% followed AS2 and 20% used AS3.None of them used AS4 and AS5 whereas 15% used AS6 and rest of the 10% followed AS7 to solve the problem.

**Comparison of P6, P7& P8**-In all three the problems majority of the students, preferred AS1.The percentage of students who used AS2 was 10% in P7, 5% in P8 whereas no one used it in P6.None of them followed AS4 in any of the problems and AS5 was used only in P6 and P7.Comparitively, AS6 was used by girls only whereas AS7 was preferred by boys. None of the students committed an error in any of the problem.

**TABLE- 8 Value of $\chi^2$ for Strategies preferred and not preferred among Grade II Boys and Girls for Addition**

| Problem Type Addition | Value of $\chi^2$ with df=1 | $AS_1$ | $AS_2$ | $AS_3$ | $AS_4$ | $AS_5$ | $AS_6$ | $AS_7$ | $AS_8$ | $AS_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| P1 4 + 7 = ? | $\chi^2$ | * 6.62 | ** 9.00 | .05 (NS) | ** 9.00 | ** 12.81 | ** 8.52 | ** 12.81 | 0.02 (NS) | .02 (NS) |
| P2 6 + 3 = ? | $\chi^2$ | ** 16.87 | ** 8.52 | ** 8.52 | .02 (NS) | # 3.17 | ** 8.52 | .02 (NS) | ** 10.77 | .02 (NS) |
| P3 ---+ 5 = 8 | $\chi^2$ | .43 (NS) | .02 (NS) | ** 8.52 | .02 (NS) | .02 (NS) | .02 (NS) | .05 (NS) | ** 10.77 | ** 10.77 |
| P4 15+12 =? | $\chi^2$ | .43 (NS) | .02 (NS) | ** 12.81 | .02 (NS) | * 4.16 | ** 12.81 | ** 8.52 | .02 (NS) | .02 (NS) |
| P5 26+18 =? | $\chi^2$ | .43 (NS) | ** 12.81 | ** 10.77 | .02 (NS) | ** 9.4 | ** 12.81 | .02 (NS) | .02 (NS) | .02 (NS) |
| P6 26+8 = ? | $\chi^2$ | 1.13 (NS) | .02 (NS) | ** 10.31 | .02 (NS) | ** 10.31 | ** 12.81 | ** 12.81 | .02 (NS) | .02 (NS) |
| P7 55+28 = ? | $\chi^2$ | 1.80 (NS) | .05 (NS) | ** 9.00 | .02 (NS) | ** 8.52 | ** 10.77 | ** 12.81 | .02 (NS) | .02 (NS) |
| P8 47+24+34=? | $\chi^2$ | .07 (NS) | ** 8.52 | ** 9.00 | .02 (NS) | .02 (NS) | ** 10.77 | ** 12.81 | .02 (NS) | .05 (NS) |

**Note -**

| | | | |
|---|---|---|---|
| | ** | = | **Value is significant at .01 level** |
| | * | = | **Value is significant at .05 level** |
| | # | = | **Value is significant at .10 level** |
| | NS | = | **Value is not significant.** |

**Problem-1** In Class-II, both boys and girls differed significantly ($\chi^2$ = 6.62, P < .01) on preferred strategy AS1 (Keeping the larger number constant count the small number) for solving single digit addition presented in a vertical form. More number of girls were found to prefer this strategy for solving the problem. sFor the same problem, significant difference was marked ($\chi^2$ = 9.00, P < .01) between boys and girls on preferred strategy AS2 (Keeping the smaller number constant adding the larger number).As compared to girls, more number of boys preferred this strategy. For AS3 no significant difference was found between boys and girls. A significant difference ($\chi^2$ = 9.00 P< .01) was marked for AS 4. Here maximum girls preferred this strategy as compared to their male counter part. With respect to AS5 both boys and girls differed significantly ($\chi^2$ = 12.81, p < .01) indicating that only boys used this strategy for solving the problems. There was a significant difference($\chi^2$ = 8.50,p < .01) between boys and girls for AS6. Girls were more frequently used tally marks while doing calculation. As per as mental calculation i.e. derived number facts and recall are concerned boys differed significantly ($\chi^2$ = 12.81, p < .01) from girls.

In brief, for problem-1, more number of girls preferred strategies like AS1 & AS 4 where as boys preferred AS2, AS5, and AS7. It was found that only girls used strategy of putting tally marks where as mental calculation was done by only boys.

**Problem-2** Chi-square ($\chi^2$) results for problem-2 i.e. single digit addition presented in a horizontal form showed a significant difference between boys and girls for preferred strategies such as AS1, AS2, AS3, AS5, AS6 and AS8. More number of boys preferred strategy of keeping the larger number constant and add the smaller number ($\chi^2$ = 16.87, p<.01) to get the answer, where as girls preferred to keep the the smaller number constant and add the larger number ($\chi^2$ = 8.57, p<.01) to do the operation for given problem. Again girls counted the first addend and then the second addend to get the results of the sum ($\chi^2$= 8.52, p<.01). More number of boys used to count the larger add further smaller addend until the answer comes as compared to girls ($\chi^2$ = 3.17, p< .10). For strategy like tally marks, girls used frequently ($\chi^2$ = 8.57, p< .01) and again maximum number of girls committed errors ($\chi^2$ = 10.77, p< .01) while doing single digit addition presented in horizontal form.

**Problem-3** In case of Problem-3, where the first addend was missing no significant difference was found between boys and girls on all the preferred strategies except AS3, AS8 and AS9. As compared to boys, more number of girls committed error ($\chi^2$= 10.77, p< .01) while doing the calculation. Boys were found to use other different strategies for solving this problem($\chi^2$ = 10.77, p<.01)

**Problem-4** Both boys and girls differed significantly on preferred strategies AS3, AS6, AS5 and AS7 for double digit addition ($\chi^2$= 12.81 p< .01, 12.81, p< .01, 4.16, p< .05 & 8.52, p< .01) respectively. It stated that girls preferred to count the first addend then second and add all to get the answers for double-digit sum. Most often they preferred to put lines for solving the sums ($\chi^2$ = 12.81, p<.01). Boys were found to count the larger, add further smaller until the answer comes for double-digit addition, which was revealed from the result ($\chi^2$ = 4.16, p<.05). With respect to mental calculation, boys differed significantly ($\chi^2$ = 8.52, p< .01) from girls for double-digit operation. It meant boys were more frequently using recall facts, decomposition to do the sum rather prefer to count with physical objects, finger and putting lines.

**Problem-5** Significant difference was observed between boys and girls in relation to strategy use for problem-5 i.e. double-digit addition with carry over,. Both boys and girls differed significantly ($\chi^2$ = 12.81, p < .01) on AS2. It indicated that boys preferred keeping the smaller addend constant and add the larger and to get the sum. Girls most often preferred AS3 ($\chi^2$ = 10.77, p< .01) i.e. count the first addend then second and add all to get the results. Boys preferred to count the larger, add further smaller until the answer comes ($\chi^2$ = 9.4, p< .01). As per as strategy for tally marks are concerned, girls more frequently use this strategy to count the numbers as compared to boys ($\chi^2$ = 12.81, p< .01).

**Problem – 6** No Significant difference was observed between boys and girls on as ($\chi^2$ = 1.13, p > .05) indicating that both boys and girls preferred the strategy that in to keep the larger number constant and add the smaller one for double digit addition. Boys differed significantly ($\chi^2$ = 10.31, p< .01) on AS3 which meant, more number of boys tried to count the first addend (whether smaller or larger) then second for getting answer. Girls differed ($\chi^2 = 10.31$, $p < .01$) significantly on the strategy that in to count from the larger and add further smaller addend to get the result. With respect to tally marks more girls adopted this strategy ($\chi^2 = 12.81$, $p < .01$) where as boys frequently used mental calculation ($\chi^2 = 12.81$, $p < .01$).

**Problem-7** A significant difference was observed between boys and girls on strategies preferred for double-digit addition with carry over. Boys differed significantly on AS3, AS5, and AS 7 ($\chi^2 = 9.00$, 8.52, & 12.81, $p < .01$). It indicated that boys preferred strategies that is count the first addend then second to get the sum, count the larger and add further the smaller one and the mental calculation. but girls differed significantly ($\chi^2 = 10.77$, $p < .01$) on AS6 i.e. putting the tally marks for counting.

**Problem-8** For multi step double-digit addition, significant difference was observed between boys and girls on AS2, AS3, AS6 and AS7. More number of girls used on the strategy that is keeping the smaller number constant adding the larger one and differed significantly ($\chi^2 = 8.52$, $p < .01$) from boys. Maximum girls than boys prefferd AS6 i.e. putting the tally marks for counting and differed significantly ($\chi^2 = 10.77$, $p < .01$). On the other hand more boys preferred mental calculation and count the first addend then second and add all to reach the sum. This is studied from the value of chi-square ($\chi^2 = 9.00$, 12.81, $p < .01$).

In nut shell. we found that more number of boys and girls used the strategy to keep the larger number constant and added the smaller one (AS1) irrespective of single digit and double digit addition with or without carry over. More number of girls used AS4 for single digit addition presented in vertical form. It was the least used strategy by both the girls and boys for other problems. Boys more frequently used AS5 (count larger, add further smaller until the answer comes) and AS7 (mental calculation) where as maximum girls were comfortable in AS7 (putting the tally marks) for doing addition.

**TABLE 9 Percentage of Grade II Boys and Girls on preferred Strategies for Subtraction**

| PROBLEM TYPE SUBTRACTION | SEX BOY/GIRL | SS1 | SS2 | SS3 | SS4 | SS5 | SS6 | SS7 | SS8 |
|---|---|---|---|---|---|---|---|---|---|
| P1 8 | G | 70 | - | 20 | - | 10 | - | - | - |
| -2 | B | 50 | - | - | 20 | 30 | - | - | - |
| ------ | T | 60 | - | 10 | 10 | 20 | - | - | - |
| P2 | G | 60 | - | 20 | - | 10 | 10 | - | - |
| 7 – 4 = ----- | B | 50 | - | - | 40 | 10 | - | - | - |
| | T | 55 | - | 10 | 20 | 10 | 05 | - | - |
| P4 25 | G | 40 | 50 | 10 | - | - | - | - | - |
| - 13 | B | 40 | 20 | - | 40 | - | - | - | - |
| ------- | T | 40 | 35 | 05 | 20 | - | - | - | - |
| P5 35 | G | 50 | 20 | 30 | - | - | - | - | - |
| - 18 | B | - | 70 | - | 10 | 10 | - | 10 | - |
| ------- | T | 25 | 45 | 15 | 05 | 05 | - | 05 | - |
| P6 76 | G | 40 | 20 | 30 | - | 10 | - | - | - |
| - 23 | B | - | 30 | - | 30 | 20 | 10 | 10 | - |
| ------- | T | 20 | 25 | 15 | 15 | 15 | 05 | 05 | - |
| P7 48 | G | - | 30 | 30 | 40 | - | - | - | - |
| - 27 | B | 10 | 10 | - | 60 | - | 10 | 10 | - |
| ------- | T | 05 | 20 | 15 | 50 | - | 05 | 05 | - |
| P8 600 | G | - | 30 | - | - | - | 60 | - | 10 |
| - 312 | B | - | - | - | 70 | - | 30 | - | - |
| ------- | T | - | 15 | - | 35 | - | 45 | - | 05 |

Note: G = GIRL, B=BOY, T= TOTAL
SS : Subtraction Strategies
SS1: Count the larger number, separated the smaller and count the remaining with fingers
SS2: Larger minus subtrahend with fingers
SS3: Putting tally marks on paper
SS4: Mental calculation
SS5: Larger number plus extra is equal to total minus smaller subtrahend with fingers
SS6: Errors
SS7: Reverse counting with fingers
SS8: Others

**Problem –1 (P1)-** Single digit subtraction was given in a vertical form (8 – 2=?). It was found that all the students solved this problem correctly. Maximum number of students i.e. 60% preferred SS1 whereas 20%followed SS5.Only 10% used SS3 and other 10% used SS4.Hence the SS1 was seemed to be the convenient and effective for large number of students

**Problem –2 (P2)-** Problem was given in the horizontal form that consisted of single digit number (7 - 4). It was found that 95% of the students solved this problem correctly. Maximum number of students' i.e.45% preferred SS1 whereas only 10% used SS3. Equal number of students i.e.20% followed SS4 & SS5 and rest of the 5% committed an error.

**Comparison of P1 & P2-**Maximum number of students preferred SS1 and equal number of students used SS3 in both the problems. Only 10% followed SS4 in P1 whereas in P2, 20% followed this strategy. The students in both the problems also used SS5.

It seems that maximum and equal number of boys i.e.50% preferred SS1 to solve both P1 & P2.In P1 20% of the boys used SS4 whereas in P2 40% boys used it. Only 10% followed SS5 in P1 but in case of P2 it was increased to 30%. None of the boys used strategies like SS2, SS3 & SS7.

As far as girls are concerned majority of the girls i.e. 70% in P1 & 60% in P2 preferred SS1.Equal number of girls followed SS3 and SS5 to solve both the problems. None of the girls used strategies like SS2, SS3 & SS7.

None of the boys committed any error in any of the problems. Interestingly, it was found that 10% of the girls committed an error in P2; she did multiplication instead of subtraction.

**Problem-3 (P3)-** All the strategies which are mentioned above and used for subtraction of single digit and double digit were not found totally true foe this problem, where larger number was missing and problem was given in a horizontal form (___- 5 = 4). Students were able to solve this problem only after some intervention of the researcher. So, this was a clue – dependent problem solving. For this problem some of the peculiar and unique strategies have been used by the students, which are as follows: -

1. 35% of the students preferred SS4 i.e. mental calculation to solve this problem. In this also, either they did 5+4=9 or 9-5=4. They used SS4 for both the ways.
2. 25% followed the strategy of first counting the total of 5 & 4 i.e.5+4=9 and then they separated 5 from 9 and count the remaining 4. Lastly, they put 9 in the box.
3. 20% of them solved this problem by using trial & error method. First they tried it with larger number like 10-5=how much? Then 8-5= how much? And lastly, they were able to solve it correctly by doing 9-5=4.
4. 20% of them committed an error; all of them solved this problem by separating 4 from 5 and put 1 in the box.

**Problem-4 (P4)-** Problem was consisted of double digit without carryover (25-13). It was revealed that all the students solved this problem correctly. Maximum number of students i.e.40% preferred Ss1 whereas 35% followed SS2. Only 5% used SS3 and rest of the 20% followed SS4.

**Problem-5 (P5)-** this problem was consisted of double digits with carry over given in a vertical form (35-18). It was found that maximum number of student's i.e. 45% preferred SS2 whereas 25% used SS1.15%adopted SS3 whereas only 5%followed SS4, SS5 & SS7.

**Comparison of P4 & P5-** The strategies used by both girls and boys varied in P4 & P5.Results showed that maximum number of students i.e. 40% in P4 preferred SS1 whereas in case of P5 majority of the students i.e. 45% preferred SS2. 35% followed SS2 in P4 whereas 25% used SS1 in P5 and only 5% used SS3 in P4 but in case of P5 it was followed by 15% of the students .20% adopted SS4 to solve P4 whereas only 5% used it for P5.None of the students followed SS5 & SS7 in P4 but 5% used these strategies in P5.

In P4 equal number of boys i.e. 40% preferred both SS1 & SS4 whereas in P5 majority of the students i.e. 70% preferred SS2.20% followed SS2 in P4 whereas only 10% followed strategies like SS4, SS5 &SS7 to solve P5.

In case of girls, same strategies were preferred for both P4 & P5. Maximum number of girls i.e. 50% preferred SS2 whereas in P5 majority of the girl's i.e. 50% preferred SS1.In P4 40% followed SS1 but in P5 20% used SS2. Only 10% used SS3 in P4 but this number was increased to 30% in P5.

None of the students committed an error in either P4 or P5.

**Problem-6 (P6)**-This was a word problem that consisted of double digits without carryover (76-23). It was found that 95% of the students solved the problem correctly. Maximum number of students i.e.25% preferred SS2 whereas 20% followed SS1 to solve the problem. 15% followed SS3, SS4 & SS5 each and only 5% followed SS7.Rest of the 5% committed an error.

**Problem-7 (P7)**- this was also a word problem that consisted of double digits with carryover (48-27). It was found that 95% solved the problem correctly. Maximum number of students i.e.50% preferred SS4 to solve the problem whereas 20% followed SS2.15% used SS3 to get the solution and equal number of students i.e. 5% followed the strategies like SS1 & SS7.Only 5% of the students committed an error, he did addition instead of subtraction.

**Problem-8 (P8)**- this problem consisted of three digits with two zeroes in it and with carry over (600-312). It was found that only 55% of the students solved this problem correctly. Majority of them i.e.35% preferred SS4 whereas 15% followed SS2 and only 5% used some other strategy. Rest of the 45% committed an error.

**Comparison of P6 & P8**- Both the problems were word problems and students were free to represent these problems either vertically or horizontally according to their own convenience. It was found that in P6 95% of the students solved the problem correctly whereas in P8 only 55% solved it correctly. Maximum number of students i.e. 25%

preferred SS2 whereas in P8 majority of the students i.e. 35% preferred SS4. In P6 the students also adopted other strategies like SS1, SS2, SS4, SS5 & SS7 whereas in P8 no one used these strategies.

Equal number of boys preferred both SS2 & SS4 to solve P6 whereas in P8 majority of the boys i.e. 70% preferred SS4.

In P6 except SS4 & SS7 girls have used all the strategies whereas in P8 only SS2 was preferred. In P6 maximum number of girls preferred SS1 whereas in P8 majority of them used SS2 and only 10% followed some other strategy.

It was found that in P6, only 10% boys committed an error, he did addition instead of subtraction. But in P8 45% of the students committed an error, out of which 30 % were girls and 15% were boys. Hence, in P8 girls committed maximum numbers of errors.

**TABLE 10 Value of $\chi^2$ for Strategies preferred and not preferred among Grade II Boys and Girls for Subtraction**

| Problem Type / Subtraction | Value of $\chi^2$ with df=1 | $SS_1$ | $SS_2$ | $SS_3$ | $SS_4$ | $SS_5$ | $SS_6$ | $SS_7$ | $SS_8$ |
|---|---|---|---|---|---|---|---|---|---|
| P1 8 – 2 = ? | $\chi^2$ | * 4.13 | .02 (NS) | ** 12.81 | ** 12.81 | ** 9.00 | .02 (NS) | .02 (NS) | .02 (NS) |
| P2 7 – 4 = ? | $\chi^2$ | .61 (NS) | .02 (NS) | ** 12.81 | ** 9.41 | .05 (NS) | ** 8.52 | .02 (NS) | .02 (NS) |
| P4 25 – 13 =? | $\chi^2$ | .75 (NS) | * 3.9 | ** 8.52 | ** 9.41 | .02 (NS) | .02 (NS) | .02 (NS) | .02 (NS) |
| P5 35 – 18 =? | $\chi^2$ | ** 9.7 | ** 7.88 | ** 10.77 | ** 8.52 | ** 8.52 | .02 (NS) | ** 8.52 | .02 (NS) |
| P6 76 – 23 =? | $\chi^2$ | ** 9.41 | * 5.53 | ** 10.77 | ** 10.77 | ** 10.03 | ** 8.52 | ** 8.52 | .02 (NS) |
| P7 48 – 27=? | $\chi^2$ | ** 8.52 | ** 8.7 | ** 10.77 | 0.93 (NS) | .02 (NS) | ** 8.52 | ** 8.52 | .02 (NS) |
| P8 600 – 312=? | $\chi^2$ | .02 (NS) | ** 10.77 | .02 (NS) | ** 13.10 | .02 (NS) | 2.31 (NS) | .02 (NS) | ** 8.52 |

**Note** - ** = **Value is significant at .01 level**
* = **Value is significant at .05 level**
NS = **Value is not significant.**

**Problem-1** Chi-square value ($\chi^2$) revealed a significant difference between boys and girls on subtraction strategy use for single digit subtraction presented in vertical form. Boys and girls differed significantly on SS1, SS3, SS4 and SS5 ($\chi^2 = 4.13, p < .05, \chi^2 = 12.81, 12.81$ & $9.00, p < .01$) respectively. It indicated that more number of girls used SS1 ( counted the larger, separated the smaller and count the remaining ) as compared to boys. Mostly girls were using tally marks, where as boys were doing mental calculation i.e. decomposition and recall number facts for solving the subtraction problems. As compared to girls, boys used SS5 (larger number plus extra is equal to total minus subtrahend ) which was revealed from the value of Chi - Square .

**Problem-2** When the problem was presented in horizontal form no significant difference ($\chi^2 = .61, p > .05$) was observed among boys and girls for SS1. Maximum number of girls ($\chi^2 = 12.81, p < .01$) used tally marks to do subtraction. More number of boys used mental calculation ($\chi^2 = 9.41, p < .01$) and a significant difference ($\chi^2 = 8.52, p < .01$) was found between boys and girls on commitment of errors. It stated that girls were committing error only when the problem was presented in horizontal form. Error was not found in vertical presentation of problem.

**Problem – 4** For double digit subtraction, no significant difference between boys and girls was marked ($\chi^2 = .75, p > .05$) for SS1, indicating that both the groups preferred to count the large number, separated the smaller and count the remaining. A significant difference ($\chi^2 = 3.9, p < .05$) was marked for SS2. It stated that maximum number of girls deducted the subtrahend from the larger number given in the problem than that of boys. Putting the tally marks was found to be a common strategy among girls ($\chi^2 = 8.52, p < .01$), where as boys used mental calculation for solving the problem ($\chi^2 = 9.4, p < .01$)

**Problem – 5** Boys and girls differed significantly ($\chi^2 = 9.7, p < .01$) on SS1 for double-digit subtraction with carry over. It indicated that most of the girls used this strategy i.e. counted the larger number, separate the smaller and count the remaining as answer to the sum. Maximum boys used SS2 ($\chi^2 = 7.88$, $p < .01$) to solve the subtraction problem. Boys directly deducted the subtrahend form the larger to get the result. In this problem a significant difference ($\chi^2 = 10.7$, $p < .01$) was marked between boys and girls for SS3 i.e. putting the lines. It was found that maximum number of girls were using this strategy to do the calculation or counting where as mental calculation was done by boys ($\chi^2 = 8.52$, $p < .01$). As per as reverse counting is concerned, both boys and girls differed significantly ($\chi^2 = 8.52$, $p < .01$) indicating that maximum boys used this strategy.

**Problem- 6** A significant difference was found between boys and girls ($\chi^2 = 9.4$, $p < .01$) on SS1 for double digit subtraction without carryover. It indicated that, more number of girls preferred to count the larger number and separated the smaller and finally counted the remaining. With respect to SS2 significant difference ($\chi^2 = 5.53$, $p < .05$) was marked between boys and girls, where more number of boys used the larger minus subtrahend strategy as compared to girls for this operation. As per as tally marks strategy is concerned only girls used this for counting ($\chi^2 = 10.77$, $p < .01$) but the reverse was found for boys ($\chi^2 = 10.77$, $p < .01$), i.e. they used mental calculation for doing the sum. With respect to error and reverse counting, boys differed significantly ($\chi^2 = 8.52$, $p < .01$) from girls. It indicated that boys used reverse counting method and committed errors while doing this operation.

**Problem:- 7** In this case though the problem was similar, but boys preferred SS1 strategy, where as girls used larger minus subtrahend strategy. The Chi – Square value revealed significant differences ($\chi^2 = 8.52$, $p < .01$ and $8.7$, $p < .01$) respectively. With respect to mental calculation and tally mark strategies, no difference ($\chi^2 = 1.13$, $P > .05$) was marked for mental calculation strategy but tally marks strategy was exclusively used by girls ($\chi^2 = 10.7$, $p < .01$). In this problem, boys did error while solving the operation and followed reverse counting method like the above problem (P6).

**Problem - 8** For multi digit carryover subtraction problem, both boys and girls differed significantly on SS2, SS4, and SS6. The Chi – Square value for these strategies were ($\chi^2 = 10.77$, $13.10$ and $2.31$, $P < .01$ & $P < .10$) respectively. It stated that only girls preferred larger minus subtrahend strategy to do calculation where as only boys used mental calculation. As per as errors are concerned maximum girls committed errors during the operation that of their male counter parts. With respect to other strategy such as guessing etc. girls used ($\chi^2 = 8.52$, $P < .01$) more than that of boys.

From the above analysis we may summaries that girls most frequently use the SS1 (count the larger number separated the smaller and count the remaining), SS3 (putting lines/tally marks) and committed errors during the problem solving period. Where as boys used SS1 for single digit subtraction presented in both horizontal and vertical form. Mental calculation found to be a common strategy among boys during the time of problem solving.

## CONCLUSIONS

On the basis of above results. following conclusions can be drawn for addition and subtraction problems: -

### Addition

- Maximum number of students irrespective of single digit, double-digit & word problems frequently preferred AS1.
- Larger number of students used AS3 only when problem consisted of double & triple digit with and without carryover irrespective of numerical / word problems.
- AS4 was used only to solve problem 1 where problem consisted of single digit, given in a vertical form.
- More number of boys followed AS5 as compared to girls in all the problems.
- Only girls preferred AS6 irrespective of single/double digit and word problems.
- Larger number of boys adopted AS7 irrespective of single/double digit and word problems.
- More number of girls committed an error only when the problem was presented in a horizontal form.

**Subtraction**

- Maximum number of students irrespective of single/double digit and word problems preferred SS1.
- SS1 was preferred by less number of students in double-digit problems as compared to single digit vertical/ horizontal problem.
- SS2 was followed by more number of students in double digit with and without carryover in both numerical and word problem.
- In single digit vertical and horizontal problem, SS2 was used by none of the students.
- Girls only in all types of problems preferred SS3.
- Larger number of boys followed SS4 in all the problems.
- More number of students used SS5 to solve single digit vertical and horizontal problem.
- SS7 was used by the same student (boy) in only double digit numerical and word problem with and without carryover.
- Maximum numbers of errors were committed in double / multi digit word problems.

## RESULT ANALYSIS OF CLASS III

**TABLE 11 Percentage of Grade III Boys and Girls on Correct and Incorrect responses for Addition**

| PROBLEM TYPE ADDITION | SEX BOY/GIRL | CORRECT | INCORRECT |
|---|---|---|---|
| P1 5437 +2209 +4388 +1879 --------- | G<br>B<br>T | 90<br>90<br>90 | 10<br>10<br>10 |
| P2 102 + ---- =155 | G<br>B<br>T | 70<br>50<br>60 | 30<br>50<br>40 |
| P3 2508 +2390 +3006 --------- | G<br>B<br>T | 90<br>90<br>90 | 10<br>10<br>10 |

G: GIRL, B: BOY, T: TOTAL

**Problem 1** This is a four steps four-digit addition numerical problem with carry over given in a vertical form (5437 + 2209 + 4388 + 1879 = ). Equal number of girls and boys i.e. 90% solved this problem correctly. The same student to solve the same problem has adopted various strategies. Some students first took two given equal numbers and added them in an abstract way. In this, actually multiplication was there because they added up 9 + 9 i.e. two times 9, which are 18 (9X2 =18). After adding 9 + 9 equal to 18 they start counting from the above given number of the same row i.e.18 plus 7 becomes 25 and plus 8 becomes 33. They count these numbers on fingers. Only girls were using the above-mentioned strategy. Some boys who used decomposition strategy for addition, they add 7 with 18 i.e. they add up 2 more first to 18 which becomes 20 then remaining 5 of 7 which makes it 25, then further add 5 first and 3 later which comes as 33. For doing this mental calculation some of the boys took the help of fingers to make it sure. For the second row, they put 3 as carry over on the top and start adding from upward to downward. Again, in this column they add the two equal numbers first i.e., 3 plus 3 which becomes 6 here also multiplication (3X2= 6), has been used. Further they start counting other numbers i.e. 8 & 9 either by fingers or through mental calculation. Though, one zero was there in this $2^{nd}$ row still it was four step problem because of 3 as a carry over. For the third row, they moves from upward to downward either it starts with smaller number or with the larger number. They used mental calculation for numbers like 4 plus 2 plus 3 and 8, they used (base 10) strategy which is of dividing the large number in to two parts like 4+ 2+ 3 becomes 9. There they took 1 from 8, which make the total 10 and 7 more becomes 17. There are some students who used fingers for calculating this sum. In $4^{th}$ row where the numbers are not too large most of them did mental calculation ( 5 + 2 + 4 + 1 = 12 + 1 carry over on the top). It was found that there are some girls who first put the tally marks aside and then count them all and wrote down the answer. It seems that the boys largely did mental calculation.

**Problem-2** This was a triple digit addition problem in which one addend was missing and the problem was given in a horizontal form (102 + ---- = 155.) To find out the missing addend only 80% students were made an attempt, and out of these 60% were able to solve it correctly. 20% of the girls who solved this problem correctly used the same strategy, which was of writing down the counting from 102 to 155 aside. Then after writing they start counting those written numerals from 1 to 53 and came up with the correct answer 53, which was the missing addend. Actually, this was a faulty strategy, girls were able to use this strategy here because the given numbers are small but when numbers are large like above 200, this strategy could not be possible. Other 40% students who solved this problem correctly, all of them preferred the same strategy i.e. by doing subtraction they were able to find out the missing addend. They separated the first addend from the given total and reach the solution that is 155 – 102 = 53 first they puted up in a verticle form then solve it. Because the given numbers were small and it consisted of zero, they (both boys and girls) solve the whole problem through mental calculation. Very few students followed counting strategies like count the above given number on fingers then separates the smaller (subtrahend) and count the remaining number. Rest of the 20% who committed an error, all of them added up the final number and the given total and came up with another total which they represented in place of missing addend that is 102 + 155 =257. Those who did addition and committed an error, they also used mental calculations but some girls added them up by putting the tally marks and count them up. There are other 20% who did not make even an attempt to solve this problem.

**Problem-3** This problem was a word problem of 3 levels multidigit with carryover i.e. 2508 + 2390 + 3006. Students were free to represent this problem either vertically or horizontally according to their own convenience. It was found that all the students represented this problem in a vertical form and except 10% all of them solved this problem correctly. One girl who committed an error missed one whole number while copying it down and did the addition with only 2 level multidigit where as the boy did some miscalculation. and came up with incorrect answer. 90% of the students who solved this problem correctly all of them used the same strategy of adding on from upward to downward by keeping the larger number constant at first stage. Later, they continued with mental calculation for other stage because the numbers are small and also consisted of zero. More number of boys preferred mental calculation to solve this problem where as some girls used strategy of putting the lines aside and count them all. For first row they started from upward by taking 8 and add another 6 in it. Some of them preferred (base 10) strategy like first they add 2 with 8 that becomes 10, then the remaining 4 of the 6 makes it 14. They took 4 of 14 and put 1 on the top as carry over, but for the same calculation some students preferred the counting strategy of keeping the larger number constant i.e. 8 and add on the 6 on fingers (8 [pause] 9,10,11,12,13,14). For second row, all of them did verbal mental calculations being it is a 1 carryover plus 9 more which makes it 10, it was just 2 numbers addition because the other two given numbers are zero. Then, they put 0 as a sum and take 1 carryover on the top of third row. Again some of them followed the strategy of keeping the larger constant adding the smaller i.e. 5 plus 3 makes it 8 plus 1 = 9. These children added the carryover at the end where as those who used math fact strategy they first added 5 + 1= 6 + 3 = 9. For the last row because there was no carry over, they simply add first the two equal numbers i.e. 2 + 2 = 4 + 3 more makes it 7. Boys followed this decomposition strategy. But girls who calculated the sum through manual strategy of counting on fingers though they knew that 2 + 2 = 4 + 3 = 7, but still for confirming the sum either they used counting or they put tally marks.

**TABLE 12 Value of $\chi^2$ for Correct - Incorrect responses of Grade III Boys & Girls for Addition**

| PROBLEM TYPE ADDITION | VALUE OF $\chi^2$ WITH df=1 | CORRECT – INCORRECT |
|---|---|---|
| $P_1$ 5437 +2209 +4388 +1879 | $\chi^2$ | .06 (NS) |
| $P_2$ 102 + ____ = 155 | $\chi^2$ | ** 7.52 |
| $P_3$ 2508 +2390 +3006 | $\chi^2$ | .06 (NS) |

Note:-

** = Value is significant at .01 level.
* = Value is significant at .05 level.
# = Value is significant at .10 level.
NS = Value is not significant.

There was no significant difference between boys and girls on multidigit addition. The chi-square ($\chi^2$) value ($\chi^2$= 0.6, $p > .05$) indicated that both boys and girls did the operation correctly. With respect to multidigit addition with second subtrahend missing a significant difference ($\chi^2$= 7.52, $p < .01$) was found. The chi-square results indicated that maximum number of girls did the operation correctly than that of boys.

**TABLE 13 Percentage of Grade III Boys and Girls on Correct and Incorrect responses for Subtraction**

| PROBLEM TYPE SUBTRACTION | SEX GIRL/BOY | CORRECT | INCORRECT |
|---|---|---|---|
| P1 9802 | G | 40 | 60 |
| - 7645 | B | 60 | 40 |
| --------- | T | 50 | 50 |
| P2 | G | 70 | 30 |
| 145 - ---- = 40 | B | 70 | 30 |
| | T | 70 | 30 |
| P3 4325 | G | 40 | 60 |
| - 2517 | B | 70 | 30 |
| --------- | T | 55 | 45 |

G: GIRL B: BOY T: TOTAL

**Problem 1:-** This was a four digit simple subtraction with carry over i.e. 9802 – 7645 given in the vertical form only 50% of the students were able to solve this problem correctly 20% of the students solved the problem by first counting the large number on fingers from upwards after taking carryover, then separated the smaller one and write down the remaining. Then at second step, at place of zero, which they have written as 9, they separated 4 through mental calculation and continued using this method only for further steps also because the difference between the two numbers is very less. 30% of the student used some other strategies like ---for them, in first step 4 fingers represents 12 (each finger consisted of 3 boxes) they directly took the fingers and separated the given smaller number i.e. 5 and count the remaining number. They also put the carryover on top and goes from upward to downward. Further also, they used this strategy when the difference was large like for 9 – 4 but for last two steps they also followed mental calculation and solved the sum.

Rest of the 50% students committed an error, out of which some always separated smaller from larger either from downward or upward for first step they did subtraction from down to up because large number was given downwards. But in place of zero they took carryover where as at other places where carryover was needed they did not take it at the end, i.e. at hundred and thousand's place, because the large number was given above, they were able to separate the smaller from above larger number. They did it mentally because the difference between two numbers was very less i.e. only two (8-6=2 and 9-7=2).

For the first row, because the above given number is smaller than the below given number and it cannot be separated. They cannot borrow one from the second row's but it is zero from then they borrow carryover from third row's 8 and makes it 7, now 0 becomes 10, they borrow one for the given number 2 which makes it 12 and at zero's place they put 9 after cutting zero. Then they separated 5 from 12 by counting on fingers and for second row also they separated 4 from 9 and get 5. Some of them preferred mental calculation for this where as some of them used counting strategy of first count 9 on fingers with both the hands i.e. 9 fingers up then they put 4 fingers down and count the remaining fingers which were 5. For the third row where seven is already there after borrowing one from it, all of them did mental calculation because the difference between the two gives numbers is very less i.e. 7 – 6 = 1. For the last row also students used both counting strategy and mental calculation, some count 9 on fingers and take out 7 from it, 2 remains where as some of them already knew that 9 – 2 = 7. Like other previous problems, in this problem also some of the girls solved the sum by putting the tally marks and cutting down the wanting ones from it and count the remaining ones.

Those who committed an error they move from downward to upward because the larger number was given below and we can't separates the larger number from the smaller one. They did it like 5 – 2 = 3, then 0 – 4 we can't separate therefore they put zero only and some of them put 4 as it is and for row third and four, they move from upward to downward because here, the above given numbers are larger than the below gives numbers and we can separate them ( 8 – 6 = 2 and 9 – 7 = 2) final answer they get 9802 – 7645 = 2243. The difference between the numbers is very less that is why most of them solve it through mental calculation. There were some that adopt the right process but they got confused with zero in between. For the first row, they think that they can't borrow one from the left side second row because it is zero, So, they borrow one directly from third row i.e. from 8 and did it like 12 – 5 which comes as 7, for second row where zero was there they borrow again one from left side 8 and makes it

10 - 4 = 6. But at 8 place they did not reduce 8 to 6 because they borrow two from it. They separated 6 from 8 and get 2 and at last they take out 7 from 9 and get 2. Hence, after miscalculating the sum, the final answer which they come up with was 2267.

**Problem-2** Multidigit subtraction problem was given in a horizontal form in which the subtrahend was missing and the answer was given (145 - ---- = 40 ) 70% of the students solved this problem correctly. First they put this problem in a vertical form, then they did subtraction from up to down (145-40=105). It was found that equal number of boys and girls preferred the same method that is of separating the given remaining answer from the first larger number, all of them did subtraction through mental calculation. Because, the numbers were small and consisted of zero, therefore they were able to solve it mentally. All the students, who committed an error i.e. 30%, they added up the first number and the given answer (145+40=185) and some of them repeated one of the given numbers i.e. either 145 or 40 and put it up in the box. Among these some of them adopted the right process i.e. of putting down two given numbers in a vertical form. But though, these two numbers were different in nature, one is of three digit and another is of double digit and it also contains one zero at units place therefore, they got confused and miscalculated the sum. Because they put 40 below 14 and separate it like put 5 of 145 as it is then 4minus 0 is 4 and 1 minus 4 is 3.

**Problem –3** This was a multidigit word problem of subtraction with carry over (4325 – 2517). It was found that all the students noted down this problem in a vertical form. 60% of the students solved this problem correctly. First, they borrow one for the first row from the second row's 2 and separated 7 from it by counting down on fingers. All of them used counting for double- digit number i.e.for number larger than 9 where as for single digit subtraction they preferred mental calculation. Some of the girls solved the problem by putting tally marks. Rest of the 40% students committed an error and come up with incorrect answers. Out of these 40% some of them followed the right process to solve the problem like they know how to borrow, when small number was presented above at first place. Though, they adopted the correct process but due to some miscalculation they came up with incorrect answer. They also took carry over for the numbers where there was no need for it for e.g. at ten's place for 1 – 1 they borrow one from left row and miscalculated the sum by doing 11-1. Some of them committed an error because they always separated the small number from the larger either it was from upward to downward or downward to upward. They solve the sum without borrowing therefore they came up with incorrect responses.

**TABLE 14 Value of $\chi^2$ for Correct - Incorrect responses of Grade III Boys and Girls for Subtraction**

| PROBLEM TYPE SUBTRACTION | VALUE OF $\chi^2$ WITH df=1 | CORRECT – INCORRECT |
|---|---|---|
| $P_1$<br>9802<br>- 7645 | $\chi^2$ | **<br>7.22 |
| $P_2$ 145 - ____ = 40 | $\chi^2$ | .02<br>(NS) |
| $P_3$<br>4325<br>- 2517 | $\chi^2$ | **<br>16.98 |

**Note:**

| | | |
|---|---|---|
| ** | = | Value is significant at .01 level. |
| * | = | Value is significant at .05 level. |
| # | = | Value is significant at .10 level. |
| NS | = | Value is not significant. |

A significant difference between boys and girls ($\chi^2$= 7.27, $p < .01$) was reported for multidigit subtraction. It stated that more number of boys did the subtraction correctly as compared to girls. No significant difference ($\chi^2$= .02, $p < .01$) was marked between boys and girls for subtraction problem presented in horizontal order with second missing subtrahend. Significant difference ($\chi^2$= 16.98, $p < .01$) was found for multidigit subtraction. The chi-square value stated that more number of boys did the subtraction operation correctly as compared to girls.

**TABLE 15 Percentage of Grade III Boys and Girls on Correct and Incorrect responses for Multiplication**

| PROBLEM TYPE MULTIPLICATION | SEX BOY/GIRL | CORRECT | INCORRECT |
|---|---|---|---|
| P1 ___ X 8 = 96 | G<br>B<br>T | 70<br>80<br>75 | 30<br>20<br>25 |
| P2 397<br>X 79<br>--------- | G<br>B<br>T | 50<br>60<br>55 | 50<br>40<br>45 |
| P3 23<br>X25<br>- --------- | G<br>B<br>T | 50<br>60<br>55 | 50<br>40<br>45 |
| P4 365<br>X 25<br>--------- | G<br>B<br>T | 80<br>40<br>60 | 20<br>60<br>40 |

G: GIRL B: BOY T: TOTAL

**Problem- 1** In this problem, the multiplication sum was given in a horizontal form in which students were supposed to find out what number, eight times becomes 96 ( ___X 8 = 96). 70% of the students solved this problem correctly because it was found that all of them know the table of 12. After seeing the given problem, first they got confused and think what to do? But later, with some help of the researcher they were able to understand what to do and frequently solving it by reciting the table of 12 from the starting till eighth place i.e.12X8=96. Out of 70%, some of them first recites the table of 9 & 11 then come to table of 12 this is a trial and error method. Others directly start reciting the table of 12 from the starting point. Rest of the 30% committed an error out of these there were some that did not make an attempt to solve the problem. Others did the same mistake of multiplying 96 in to 8 and wrote the attained answer in the box i.e. 96x8=768.

**Problem 2** This was a multiple digit multiplication problem given in a vertical form ( 397 X 79). It was found that maximum number of students' i.e. 50% committed an error.. While multiplying some of the girls first wrote down the tables of required number on side and then continued with multiplication. Students who solved this problem correctly followed different strategies for addition in this multiplication problem In this problem,where the small number was given above they moved from downward to upward by keeping the larger number constant, add on the smaller like in second row this start adding 9+7=16 on fingers and some did it with 10 base strategy i.e. 9+1=10+6=16. But, there are some boys who preferred mental calculation to solve this problem.

Among those who committed an error, there were some students who repeated the same numbers given in the problem like they put 7 and 9 thrice below the sum, and added them up. But, some adopted the right process and did the multiplication correctly but while doing addition they miscalculated the sum. In these 50% there were some students who did not make even an attempt to solve the problem and it was found that these were girls and maximum number of errors were committed by girls.

**Problem –3** This was a word problem, consisted of double-digit multiplication (23 X 25). It was found that only 55% students solved this problem correctly. All of them adopted the same strategy for solving it like first they did multiplication successfully because the numbers were small. Later, for addition they preferred mental calculation because the numbers, which they get after multiplication, were also small i.e. below 10 and without carry over. It was found that none of them used fingers to count numbers in this problem for addition and all of them did it through mental calculations.

Rest of the 45% of the students committed an error because they were not able to comprehend the language of the problem. They were failed to take out the meaning of the problem, exactly what operation was needed to solve it. Even after getting some clues from the researchers they were not able to solve it. By guessing, instead of multiplication, some of them did addition and some did subtraction.

**Problem –4** Likewise problem 3 this is also a word problem in which multidigit number was supposed to be multiplied by double digit number (365 X 25). It was found that maximum number of student i.e. 60% solved their problem correctly. 40% of the student committed an error. Out of these 40% some of them adopted the right process they did the multiplication correctly because they knew tables of 2 & 5. But later, in addition because it consisted of carry over, they miscalculated the sum and get the incorrect answer. Some of them were not able to understand the language of the problem and did not know what operation was supposed to apply. They applied either addition or subtraction without any understanding and they committed a mistake in addition & subtraction also. Because in the word problem the numbers were given in horizontal way & while solving it first they put it down in a vertical form. Though, the two given numbers were different, one was multidigit and another was double digit some of them put it down like **365**

**25**

And did addition and subtraction by putting zero in empty place i.e. below 5. Those who solve it correctly they adopted different process while adding it. Some of them count the numbers on fingers starting from up to down whereas some of them preferred mental calculations.

**TABLE 16 Value of $\chi^2$ for Correct - Incorrect responses of Grade III Boys and Girls for Multiplication**

| PROBLEM TYPE MULTIPLICATION | VALUE OF $\chi^2$ WITH df=1 | CORRECT – INCORRECT |
|---|---|---|
| $P_1$ _____ x 8 = 96 | $\chi^2$ | 2.16 (NS) |
| $P_2$ 397 X 79 | $\chi^2$ | 1.64 (NS) |
| $P_3$ 23 X 25 | $\chi^2$ | 1.64 (NS) |
| $P_4$ 365 X 25 | $\chi^2$ | ** 15.5 |

Note:-

** = Value is significant at .01 level.
* = Value is significant at .05 level.
# = Value is significant at .10 level.
NS = Value is not significant.

Chi-square value revealed no significant difference between boys and girls on multiplication problems of $P_1$, $P_2$ & $P_3$. The value of these problems were ($\chi^2 = 2.16$, 1.64, 1.64, $p > .05$) respectively. In terms of percentage, boys were out weighing girls for problems in which first number was missing, multiplication of multidigit with double digit and double-digit with double-digit sum. But for problem like multidigit with double-digit multiplication a significant difference was noticed ($\chi^2 = 15.5$, $P < .01$) indicating that maximum number of girls did the operation correctly as compared to boys.

**TABLE 17 Percentage of Grade III Boys and Girls on Correct and Incorrect responses for Division**

| PROBLEM TYPE DIVISION | SEX BOY/GIRL | CORRECT | INCORRECT |
|---|---|---|---|
| P1 | G | 50 | 50 |
| 946 ÷ 7 | B | 60 | 40 |
| | T | 55 | 45 |
| P2 | G | 50 | 50 |
| 808 ÷ 4 | B | 50 | 50 |
| | T | 50 | 50 |
| P3 | G | 60 | 40 |
| 801 ÷ 3 | B | 40 | 60 |
| | T | 50 | 50 |
| P4 | G | 70 | 30 |
| 648 ÷ 8 | B | 50 | 50 |
| | T | 60 | 40 |

G: GIRL B: BOY T: TOTAL

**Problem –1** Simple division with multiple digits and single digit was presented (947 ÷ 7). All the students first put this problem in a vertical form and it was found that only 55% were able to solve this problem correctly. Among those who committed an error, some were there who repeated the same given number below it where as others who did not know the operation of division they put any number in place of answer by guessing for e.g. 946 ÷ 7 = 11 or 946 ÷ 7 = 62 etc.

Those who committed an error they know table of 7 but they did not understand how to pursue with the division sum by reciting table.

**Problem –2** Likewise problem 1 ( 808 ÷ 4) numerical division problem. But in this problem multidigit consisted zero in it because of which 50% of the students committed an error. Though, some of them adopted the right process of division because the others numbers given were same i.e. 808 and it was the direct divisible of 4. But they did not know that to do with zero and they come up with 22 as an answer. Some adopted the incorrect process and wrote down the numbers by guessing only like at place of answer they put 222, 231 etc. those who solved it correctly they know that how to proceed with zero also and were able to reach the right answer i.e. 202. All of them preferred mental calculations to solve this sum.

**Problem –3** This was a word problem which was also consisted of zero in it ( 801 ÷ 3) likewise Problem 2 is this problem also only 50% of the students solved this problem correctly. Only after reading & comprehending the language of the given problem students were able to understand which operation was needed in this problem. Rest of 50% who committed an error they were not able to comprehend the problem. Even after getting some clues from the researcher they were failed to understand and applied the wrong operation to solve it. Some of them did addition, subtraction and some put multiplication sign and left the problem incomplete because of zero. None of them, adopted the right operation i.e. division. Those who solved it correctly they did not face any problem in the process of division because the given number was small i.e. 3. All of them has an understanding of operation and know how to proceed further with a digit zero. Some of them used mental calculation where as others used fingers to count, while subtracting because carry over was also there.

**Problem –4** A word problem consisted of multidigit and single digit (648 ÷ 8). Likewise problem 3 in this problem also those who committed an error were not able to comprehend the problem. Instead of division they did addition subtraction and multiplication just by guessing without any understanding. It was found that 60% of the students solve this problem correctly because the given number i.e. 648 is directed divisible of 8. None of them committed an error in the process of solving this problem. There is no addition and subtraction involved in this process because 8 in to 8 are equal to 64 and 8 in to 1 is 8. In both the steps the remaining answer is zero. There was no carryover or without carryover that is why nobody committed a mistake in the process.

**TABLE 18 Value of $\chi^2$ for Correct - Incorrect responses of Grade III Boys and Girls for Division**

| PROBLEM TYPE MULTIPLICATION | VALUE OF $\chi^2$ WITH df=1 | CORRECT - INCORRECT |
|---|---|---|
| $P_1$ 946 ÷ 7 | $\chi^2$ | 1.64 (NS) |
| $P_2$ 808 ÷ 4 | $\chi^2$ | .02 (NS) |
| $P_3$ 801 ÷ 3 | $\chi^2$ | ** 7.22 |
| $P_4$ 648 ÷ 8 | $\chi^2$ | ** 7.52 |

Note:-

** = Value is significant at .01 level.
* = Value is significant at .05 level.
# = Value is significant at .10 level.
NS = Value is not significant.

The chi-square ($\chi^2$) value revealed a significant difference between boys and girls for solving divisor operation correctly or incorrectly. No statistical difference was found ($\chi^2$= 1.64, $p > .05$) between boys and girls for solving multidigit sum with single division where remainder becomes one or more than one. Again in $P_2$ (the same type with remainder is zero) no statistical difference ($\chi^2$= .02) was noticed. It meant both boys and girls correctly solved this division operation. A significant difference was found for $P_3$ and P4, ($\chi^2$= 7.22, 7.52, $p < .01$) respectively, indicating that girls are better performed than boys in solving the given operation.

# CHAPTER - 5
# DISCUSSION

The present study explored different **counting strategies** used by children to solve different arithmetic problems i.e. Addition and Subtraction. Counting strategies for **addition,** which were found are as follows: -

**AS1:** keeping the larger number constant adding the smaller one
**AS2:** keeping the smaller number constant adding the larger one
**AS3:** count the first addend then second and add all &vice- versa
**AS4:** count the smaller. add further larger until the answer comes
**AS5:** count the larger. add further smaller until the answer comes
**AS6:** putting the tally marks
**AS7:** mental calculation
**AS8:** errors
**AS9:** others

The results of this present study indicate that children are not entirely consistent in their choice of strategies. They often used several strategies to solve different types of problems. They used them inter changeably rather than exclusively using the most efficient one.Even when a more efficient strategy like counting on from larger has been acquired, children often revert to a less efficient strategy like AS 4 which is of counting the first addend then add further the other one until the answer comes. The data showed that AS1 which was the strategy of keeping the larger addend constant count on the smaller one was preferred by a maximum number of students irrespective of single-digit, double-digit and word problems. This strategy seems to be the most convenient and effective to reach at the solution for the larger number of children is Grade- II. This strategy seems to be the easiest strategy for children of second grade because in this particular strategy they only need to count the one addend on fingers.

It was found in the present study that after AS1 the most frequently used strategies were AS3 which was of count-all and AS5 which means count the larger addend first then the smaller until the answer comes. Children irrespective of single/double digit and word problems frequently used these two strategies. As compare to these strategies the strategy of keeping the smaller number constant add the larger one (AS2) was preferred by less number of students mostly it was used in those problems in which the smaller no was presented above like 4 + 7, 55 + 28, 26 + 18. In these problems, children used this strategy because they want to solve the sum in a right process i.e. as it was given. Regarding the above mentioned strategies, results of the study of Carpenter and Moser (1984) clearly suggest that children initially solve the problem with counting-all strategy ( i.e. AS3 and AS 5 in the present study ) and this strategy gradually gives way to counting on and the use of number facts. The shift to counting on (AS1) was generally not initially complete and counting all ( AS3 and AS5) and counting on were often used concurrently for some time. The evidence regarding separate stages for counting-on from first (AS2 ) and counting on from larger (AS1) is less compelling. It seems that children who could count on from the larger number would choose to do so rather than use the less efficient counting-on-from first strategy. We have already observed however that, children do not consistently use the most efficient strategy available.

Furthermore, the data agree with Groen's (1972) and Suppes (1967) findings that the most frequently used first grade methods are counting on from the larger addend and counting from 1, starting with the first addend. Their bindings are based on response latency data, a very different data base than that acquired through the clinical interview techniques. Contradictory to these findings, Baroody (1987) found that mental counting all strategies starting with the larger addend were far more frequent than those starting with the first addend. Even, among kindergarten-age children just develop a mental addition strategy, there is a tendency to minimize the cognitively demanding keeping-track process by starting with the larger addend. Counting all starting with the larger addend may be an important transitional step, at least for some children. For children who invent that strategy, counting on from the first addend makes little sense as the next developmental step, because it does not minimize the number of steps in the cognitively demanding keeping-track process.

Various counting strategies, which were found for **subtraction** in Grade II are as follows: -

**SS1:** Count the larger number, separated the smaller and count the remaining

**SS2:** Larger minus subtrahend

**SS3:** Putting the tally marks

**SS4:** Mental calculation

**SS5:** Larger number plus extra is equal to total minus smaller subtrahend

**SS6:** Errors

**SS7:** Reverse counting

**SS8:** Others

Results of the present study showed that in subtraction problems, maximum number of students preferred to SS1 which denotes count the larger number first then separates the smaller from it and finally count the remaining ones. Another strategy i.e. SS2 (Total minus subtrahend ) which was preferred by the students only to solve double-digit and multi-digit both numerical and word problems. In all the above mentioned strategies children count manually and they used fingers to count.

As far as mental calculation is concerned, it was found that larger number of children used it to solve subtraction problems as compare to addition. This is basically a verbal counting strategy in which the response is based on math fact without counting on fingers children were able to count forward or backward to find out the answer. This whole process takes place in an abstract form.

According to Carpenter and Moser (1982), users of these informal solution strategies frequently are not aware of the interchangeability of these strategies and are unable to link them to one single formal arithmetic operation ( + or –).Decorte and Verschaffel (1987a) complemented these findings by using a more differentiated problem set as well as elaborated scheme for classifying pupil's solution strategies showing that : (1) Solution strategies for addition problems of children operating at the material and verbal counting levels are strongly influenced by the situational structure of the problem, (2) for addition and subtraction problems, the type of situation keeps a significant influence on children's mental solution processes based on known-fact or derived fact strategies.

In the join-missing addend i.e. ___ + 5 = 8 in this problem actually subtraction was there as 8 – 5 = 3 which gives the answer to missing addend. It was also found that children were able to solve this problem only after some intervention made by the researcher. The absence of a dominant strategy on the __ + /- b = c problems may reflect children's confusion about how to model or solve these problems the large number of inappropriate strategies for these problems provides further evidence that problems with unknown in the fist position not only are more difficult to model but also are more difficult to solve (Hiebert, 1982). Because the missing addend is the smaller number hence, maximum no of children use the strategy of keeping the larger addend constant on fingers and add the smaller until they get the answer. By using this strategy find out that how many more, add in 5 gives us 8.rest of the 30% used the other strategy of separating i.e. subtraction they find out the missing addend by doing 8-5=3,they solved this by counting on fingers. Basically this is the correct process of solving this problem. Some children solved this problem through mental calculation whereas some committed an error. All of them who committed an error they interpret __ + 5 =8 as 8 plus 5 equals something and wrote 13 in missing addend box, this shows an inability to read sentences correctly. Also, of course, this study's analysis of the relationship of correct reading capability to be an essential prerequisite to arriving at a correct answer.

(Parkman 1972) in his results indicated that adult answered addition and subtraction facts through retrieval from memory. Groen & Poll (1973) applied this some procedure of analysis of response times or latencies to a study of how children solve open addition sentences. Their results indicated that for problems of the form x + __ = y, performance is best explained by model that assumes the student either count up from addend or counts down from the sum whichever is quicker.

The results of study of the Carpenter & Moser (1984) indicated that the children almost exclusively used the modeling and counting strategies that reflect the additive action of the problem. As with addition, the children initially modeled the problem directly using adding-on strategy, which was later replaced by the more efficient counting-up-form-given strategy.

Both Briars and Larkin (in press) and Riley et al. (1983) hypothesize that children solve addition problems before they can solve join-missing addend problems. They propose that the ability to solve missing addend problems emerge at the some time as the ability to count- on.

The results of the present study is in line with the findings of (Briar & Larkins) study which indicates, however, that the ability to solve missing addend problems develops before children count-on from first or count-on from larger to solve addition problems. With the adding- on strategy, children physically represent the action in the missing addend problem and almost all the children in the study used this strategy to solve missing-addend problems before they counted-on to solve addition problems. In fact, the counting equivalent of adding on counting up from given was the strategies that parallel the use of counting-on. Children keeps the 6 constant and add 3 more by counting on fingers 6 pause 7, 8, 9 and they come up 9 as a answer for AS2 they did it like 3 pause 4,5,6...9.

In one other problem which was presented in a horizontal form i.e. 6 + 3 = ---. It was found that in this problem also, AS1 was used by maximum number of children of grade II . Children keeps the 6constant and add 3more by counting on fingers 6 [pause] 7, 8, 9 and they come up 9 as an answer. For AS2 they did it like 3[pause] 4,5,6...9.

Various strategies like AS2, AS3, AS5, AS6 were used by children to solve this problem but there were some children who committed an error, all of them applied operation of multiplication in place of addition and solve it as 6 in to 3 which gave 18 as an answer. This is because the symbol of addition and multiplication is similar to some extent. Hence, these children got confused between the signs of two operations i.e. addition (+) and multiplication. (x).

Ample researches have been done which concerned with determining the procedures used by students in solving open sentences. Grouws (1974) used an interview technique to identify how students went about solving addition and subtraction sentences. He identified such steps as (a) direct addition or subtraction (b) recall, (c) trial substitution with verification and correction as needed, and (d) counting.

Some research (Grouws, 1972; Weaver, 1971; Suppes, Note 1) has attempted to identify the characteristics of the sentences that are associated with differences in difficulty of solution. This work, has been concerned with differences in difficulty between addition and subtraction sentences and between sentences having the operation on the left (e.g. a + b = c) and those having the operation on the right (c= a + b). Some such studies (Groen & Parkman, 1972; Woods, Resnick, & Groen, 1975) have concluded that a simple counting mode best explains the results when students solve simple addition or subtraction problems.

Case (1978) using a neo-Piagetian analysis has pointed to the importance of knowing what incorrect procedure is being used so that the student can be convinced that his procedure does not produce the correct answer. Information on the strategies used by pupils who are unable to solve open sentences correctly should be helpful to teachers in alerting them to possible explanations for the performance of these pupils.

The findings of the current study suggests the importance of teaching students to comprehend number sentences in terms of all the meanings that are essential to their solution and their application to real problems.

Like addition for solving subtraction problems also it was found that in Grade-II multiple subtraction solution strategies were expected to be reported because children tend to use several strategies to solve even fairly simple subtraction (e.g. problems such as 8 – 2 and slightly more difficult problem 25 –13 presented in a vertical form). Counting strategies are typically the first type of strategy that children use and this includes both finger counting and verbal counting strategies.

It was observed that for solving the problem like 8 – 2, children use the SS1 i.e. strategy of first count the larger number then separates the smaller and count the remaining ones. On fingers firstly they start counting from 1 to 8 then they reduced 2 from 8, 1, 2 and further starts counting the remaining ones 1,2,3,4,5, and 6. Finally, they reach at 6 as an answer. But there were some children who preferred mental calculation to solve the same problem. After seeing 8 – 2 they immediately gave a response that 8 – 2 is 6. This response of children is based on their memory retrieval.

Contradictory to these findings, Geary (1994) postulated that decomposition strategies also referred to as derived facts or 'special tricks' in his study, are the strategies that children start to use. Although decomposition strategies are numerous they all involve breaking a problem in to smaller and easier parts. For example on a problem such as 14 – 6, a child might break the problem in to 14 – 4 = 10 and then subtract 2 more from 10 to get the answer the answer 8. Strategies such as these are more sophisticated than counting and require at least some understanding of the concepts involved in subtraction.

Siegler & Shipley, 1995, Siegler & Shrager, 1984 found that children who use retrieval often report that the answer to a problem such as 8 – 3 just post in to their heads. Retrieval involves accessing the answer to a problem directly from memory. It is a quick and automatic process that is usually accurate.

Hence, the strategy chosen to solve a problem depends partly on problem difficulty. Basic strategies such as counting are used on problems that are more difficult for children, where as retrieval is used more often on problems those children consider being easy.

In the present study some other subtraction strategies were also found like putting the tally marks (SS3) which was preferred by only girls of Grade-II for every given problem irrespective of single/double digit, with or without carry over and numerical/word problem they used this particular strategy. For Example word problem which denotes 48- 27, first they put 8 tally marks in their papers then they cut down 7 from it and count the remaining ones i.e. 1, for second place also, they put 4 tally marks and cut down 2 from it, then 2 remains.

One other strategy was found which is a complicated one and unique this is of first takes the larger quantity in the subtraction problem is initially represented in to two parts then smaller quantity is subsequently removed from it. For example in the problem 8 –2 children first count 8 as 4 + 4 or 5 + 3 (they represented this addition process on fingers) and comes up with 8 minus 2 is equal to 6.Hence, in this strategy first they did addition then subtraction.

Another subtraction solution strategy was found that is "ReverseCounting" (Counting Down) in which the separating action is represented by counting backward for example to solve the problem 76 – 23 =?, first the child would count 6,5,4,[pause], 3 the answer is 3. Then further for 7 – 2 child count 7,6,5,4,3[pause] 2, the answer is 5 because the child count these numbers on fingers and 7 to 3 represented 5 fingers so he came up with 5 as an answer. It was found that boys only used this strategy.

Subtraction of multidigit numbers requires only knowledge of the basic subtraction facts and of Place Value. Just as addition, without regrouping is comparatively easy for most children, so is subtraction without regrouping. Subtraction with regrouping (Sometimes called borrowing) is difficult for many children. It is wise to make sure that they are proficient with place value and basic facts (especially those for sums greater than 10) and plan on systematically developing the algorithm with materials and then matching the materials with the symbolic representation.

Arithmetic word problems constitute an important part of the mathematics program at elementary school. Initially, they were used to train children to apply the formal mathematical knowledge and skills learned at school to real-word situations, later on, word problems were thought of as a vehicle for developing student's general problem-solving capacity or for making the mathematics lessons more pleasant and motivating. At present word problems are also mobilized in the early stages of learning a particular concept or skill i.e. to promote a thorough understanding of these basic arithmetic operations ( De Corte & Verschaffle, 1989; Treffer 1987).

According to Nesher, 1980 by the end of elementary school many students do not see the applicability of their formal mathematical knowledge to real word situations; they do not have flexible access to heuristic and metacognitive strategies for attacking non-standard problems (Dc Corte, 1992; Van Essen, 1991) ; they have only a weak understanding of arithmetic operations as models of situations (Greer, 1992); finally they seem to dislike mathematics in general and word problems in particular (Mc Leod, 1992)

Word Problems have attracted the attention of researchers too. In the present study, different types of word problems were taken in to consideration involving either an addition, a subtraction , a multiplication or a division. The main aim of the researcher was to see the representation of the problems the selection and execution of a solution strategy and the interpretation and verification of the result. It was observed that all the students, except one girl first represented the problem in a vertical form and their adopted a required operation. They put up the numbers in the sequence as they were given in the problem. It was found that there was a difference in the strategies used for addition word problems and strategies used for subtraction word problems in grade two. It seems that for solving addition problems maximum number of students preferred as 1 of (keeping the larger number constant adding the smaller one) irrespective of single/double digit, with or without carry over. But in subtraction, larger number of students followed SS4, which was the mental calculation irrespective of double or multi digit problem with or without zero. As far as the representation of arithmetic word problems are concerned. It is generally accepted that a skillful solution process of a word problem starts with the construction of a network representation of the basic semantic representation of the basic semantic relationship between which emerges at the end of this first stage, is the result of a complex interaction of bottom up and top down processing. Throughout that constructive process or problem representation, different kinds of knowledge seem to play an important role. There are three types of such knowledge schemata of problem situations, linguistic knowledge and knowledge about the game of school word problems. First stage of a competent problem-solving process consists of constructing and appropriate representation of the problem situation in terms of sets and set relations. To arrive at such a problem representation can not be understood exclusively in terms of the interplay between the particular text and the person's knowledge about problem situations and linguistic terms. As several authors have argued De Corte and Vershcaffel, 1985 a; Nesher, 1980; Schoenfeld 1991, the relationship of s word problem in terms of the semantic relations between the constituting elements is also seriously affected by the solvers knowledge of the peculiar type of text that a word problem is students seem to develop this knowledge as they participate in the culture of solving traditional school word problems (De Corte and Verschaffel, 1985 a; Schoenfeld, 1991). But failure to acquire this knowledge may lead to " bizarre" errors and reactions.

**Gender specific results** showed that for solving both addition and subtraction problems irrespective of single/double digit, numerical/word problem the strategy like putting the tally marks was more preferred by girls of both the grades i.e. Grade II and III.

For example: In case of grade II, for addition word problem carried 26+8 =? First they put this number sentence in a horizontal from then some of them put 26 tally marks aside and 8 tally marks separately, then they start counting all the tally marks & combined them. So they come up with 34 as an answer. But for the same problem there are some girls who first put 6 tally marks then 8 and count all of them as 14, then they take carryover and verbally count 2 plus 1 is equal to 3. Hence, the answer is 34.For subtraction problems also, they used the same procedure/strategy. To solve 35-18, first they put 15 lines, after borrowing 1 from 3 (given on left side) then cut down 8 lines from it & count the test as 7, lastly verbally they put 2 minus 1 = 1.

In Grade three for all addition, subtraction, multiplication and division problems, girls used either the strategy of putting the tally marks or manipulative like counting on fingers. It was found that for solving multi-digit subtraction problems irrespective of numerical or word problem more number of errors was committed by girls. Where as in addition this is not reported and the percentage of both boys and girls who solved the problem correctly was almost similar. In multiplication and division also more errors like apply the wrong operation, miscalculations etc. were committed by girls.

As compared to girls, the mental calculation strategy was preferred by more number of boys in which they solved the problem verbally i.e. without using the fingers they were able to count. In class II, even for the more complex word problem like 600-312 =? First they borrowed 1 from the left given number then they did 10 minus 2 is 8, at the place of other 0, 9 remains so 9 minus 1 is 8 and finally in thousand place 5 remains because they have already borrow 1 from it. So, 5 minus 3 are 2 hence, the final answer is 288.

The presence of zero in the sum demands special attention. If the zero is in the ones place, it causes little difficulty. Zero in the tens place is slightly more difficult, especially when regrouping in the one's place is also necessary. The biggest difficulty lies with numbers having more than one zero. One alternative is multiple renaming, from hundreds to tens, then from tens to ones. Place value experiences are important in preparing children to cope with these problems.

*For the same problem one girl named Jyoti of class II used the unique decomposition strategy. She put down this sum in a horizontal way 600-312=? All the calculations she did verbally with the help of fingers. First she separated 300 from 312, she divide*d *it in two parts like* 300 and 12, then she separated *250 from 600 it was like 250+300+12+? = 600. From 250 she takes out 200, then she start counting the* remaining tens *from 50 onwards i.e. 50, 60, 70, 80, 81, 82, -----------88. Basically she was trying to find out 100-12 = 88. Finally, she added these 88 to the previous 200 which she kept separately hence 200+88 is 288 i.e. the answer to 600-312.*

*For using this strategy, she gave the reason that she doesn't know how to borrow ones from ten place. This was the most complex word problem of subtraction. In addition problems, she preferred a different strategy like for solving 47+24+34word problem. She solved it verbally first, she added up two given equal numbers i.e. 4 plus 4 is equal to 8 then she added the remaining 7 to 8. For adding 8+7, she separated 1 from 8 hence she made 7 plus 7 equal to 14, then she added previous 1 with 14and got 14+1 = 15. She put 5 at unit place and took 1 as carryover at tens place again she added 1 with 4. After that she added 5 with 2 and made it 6, 7*

*& 3 with 7 made 8,9&10. Finally, the answer became 105.The stepwise presentation of the problem is presented here as follows:-*

***Problem:*** ***47+24+34***

*Ist Row-* *7+4+4 (Unit-Place)*

*$1^{st}$ step* *4+4=(4 X 2) = 8*

*$2^{nd}$ step* *8+7= 8 -1+7 = 7+7+1= 14+1 = 15*

*IInd Row-* *4+2+3 (Tens Place)*

*$3^{rd}$ step* *4+ 1 (carry over) = 5*

*$4^{th}$ step* *5+2 (6,7) +3 (8,9,10) on fingers.*

*In this whole process, actually multiplication was in hidden form, e.g. two times 4 ( 4X2 ) is 8, and 2 times 7 ( 7X 2) is 14.*

Fennena and her colleagues (Fennena, Carpenter, Jacobs, Franke and Levi, 1998) found gender differences in later grades and with a range of mathematics problem. They found that older boys were more likely than older girls to use decomposition strategies in which problems are broken down into smaller problems. Girls in contrast were found to keep on using manipulative to solve a series of mathematics problems in grade three. Contrary to above findings, the present study revealed an exceptional case in which the second grade girl used the decomposition strategy to exceptional case in which girls used the decomposition strategy to solve different addition and subtraction problem.

But in grade 3 our findings are in line with the findings of Fennena et al., (1998) which revealed that though in grade three, both girls and boys were older than grade two still some girls used the strategy of putting the tally marks to solve the complex problem of four step multidigit addition and subtraction whereas boys preferred decomposition strategies like first add on the two given equal numbers then, adopted the strategy of keeping the larger first add the smaller. In grade three most of the boys solved the problem through mental calculation whereas girls preferred to use manipulative such as counting with counters or fingers to solve the problems. One explanation for these gender differences in strategy use is that they reflect real differences in mathematics knowledge and skills. Another explanation for these gender differences is that girls and boys have different motivations for mathematics and that gender differences in strategy use are driven by girls' and boys' strategy preferences. By strategy preferences we mean that girls and boys may use different strategies either consciously or unconsciously, but that gender differences in strategy use do not reflect fundamental differences in skill.

It was found that more number of boys preferred mental calculation for subtraction problem as compared to addition. The addition, which consist of place value, they used mental calculation to solve them. For example: 55+28 =----, they first added up 5+8 as 8+5 (keeping the larger first) is 13, they put 3 and take 1 as a carryover above 5, then they join 5+1+2 which gives them 8 i.e. (5 plus 1 is 6 plus 2 is 8) For the same problems girls are more likely using the manipulative such as counting on fingers.

Fennema and Peterson (1985) suggested that one way girls differ from boys is that girls do mathematics in a rote fashion and boys are autonomous in their mathematics. We believe that children who reflect on their strategy use and mathematics knowledge will be autonomous in their mathematics. In contrast, children who see mathematics as the rote application of procedures may see no need to reflect on their mathematics. If this is true then these difference should be evident in the types of strategies and metacognitive knowledge girls and boys use in problem solving. Girls rote approach to mathematics would result in the use of strategies that are algorithmic and may result in the superior calculation documented by Armstrong (1981) and Marshall (1984), but girls may not reflect on their strategy use. As a result girls would not be able to move from rote procedures and superior calculation skills to good problem solving skill; they would not reflect on what, why, and how they solve mathematics problems. In contrast, boys are more independent in their approach to mathematics, they should reflect more on their mathematics and this reflection should result in the use of more complex strategies related to metacognitive knowledge about strategies, and subsequently better problem-solving skills (Carr & Jessup, 1997).

Gender differences in the development of mathematics skills and knowledge are also believed to emerge as a function of the different experiences of girls and boys in group settings and under peer influence in the classroom and neighbourhood (Kimball, 1989)

# CHAPTER - 6
# CONCLUSION, LIMITATIONS, IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

## Conclusion

Educational and cognitive psychologists are beginning to devote serious attention in the areas of counting (Briars & Seigler, 1984; Fuson, 1988; Yao, 1992; Gelman & Gallistel, 1978), arithmetic operations (Hutenlocher, Jordan, & Levine, 1994; Jordan, Levine, & Hutenlocher, 1995), problem solving (Riley & Greeno, 1988) and strategy use (Siegler & Jenkins, 1989). Researchers have analyzed children's use of diverse solution strategies appearing in specific arithmetic skills such as addition and subtraction. The present study found significant differences in strategy use and attributed it to gender and grade development. In this, we may conclude that irrespective of the nature of Problem whether simple or complex, both boys and girls went for most common strategy i.e. keeping the larger number constant adding the smaller number. The second most frequently used strategy is -count the first addend and then second addend and add all. From the discussion we knew that both the strategies reduce cognitive load during the time of processing information in problem solving situation. As we know different aspect of mathematics involve different cognitive abilities (Geary et al.1996). For example, some children might have relative weakness in fact retrieval, even though they understand counting principles and mathematical concepts better where as, others might have relatively strong computational skills despite a weak understanding of concepts (Jordan & Hanich, 2000). The absence of a dominant strategy on the (--- + b = c and --- - a = c) problems may reflect children's confusion about how to model or solve these problems. The large number of inappropriate strategies used on these problems provide further evidence that problems with the unknown in the first position not only are more difficult to model but also are more difficult to solve (Hiebert, 1982). In our study most of the students from grade-II committed error and did wrong operations for this operation. They reported that the wrong operations were found because of the mathematical symbol rather than the conceptual understanding of the problem.

Both second and third grade girls more likely used the traditional methods (putting the tally marks) for counting than that of boys. This strategy is considered to be the most easiest and efficient for girls because they directly model the actions involved in the problem. Significant gender differences were noticed for mental calculation strategy. Maximum boys irrespective of grades, used mental calculation strategy for counting. Girls were more likely than boys committed errors, when complex operations were presented to them, these errors were confusion of operation and lack of conceptual understanding of problems. Second grade boys put the biggest first than girls for counting forward for subtraction and addition operations. We would say that children's problem solving strategies are cropped up, when the quantitative relations in the problem become visible to them. When they analyze the problem with systematic conceptual understanding use of counters (fingers & concrete objects) found to be an important quality among children. The present study revealed that third grade childrens' improved performances in counting, recognition of number, complex numerical & word problems and advanced strategies were used as a function of both age and grade level. Children's grade was related to the type of solution strategies they used in addition and subtraction problems. Base 10 concept and place value understanding help the children to use mental calculation and advanced strategies for number understanding and arithmetic reasoning. Therefore, we predicted that some abilities such as using counting to solve simple addition and subtraction would manifest themselves in second grade but others strategies such as understanding of the base 10 concept, decomposition etc. would emerge as the children moved to the higher grade. This study clearly indicated the developmental pattern in strategies used for subtraction and addition operation among second and third grade students, whereas gender differences were prominent for some basic strategies but not for all. These gender differences cannot be generalized and explained to skills and performances for addition and subtraction strategies.

**Limitations of the present study**

Some limitations were mentioned regarding the study with respect to test items, sample size and nature of the sample. Though all the test items were lifted from the math textbook, but they were not enough to understand children's processing capacity for solving the problem. If we had kept more items covering all the parameters mentioned in the test, then that could have helped us to determine students' conceptual understanding of the problem, representation of number, writing number sentences, knowledge of place value and mental calculations, and semantic aspects of word problems etc. Since we were observing different strategies for addition and subtraction word & numerical problems. so, that could have provided us a better insight about their meanings, and structures.

The participants for this research were primary school children and they were not well versed in stating what they felt, how did they solve the problems, and what strategies did they prefer. Sometimes, main themes of the problems remained disguised and hidden. Therefore it became difficult to explore their affective component. Results for the present study would have been rich, if we had talked with the teachers about math instructions in the schools and their own methods of teaching. This could have pave the way for discriminating between formal and informal ways of teaching mathematics.

The sample size was less and collected from urban area of Delhi. Therefore, the study may be generalized with caution. If the same test had been administered to all possible groups, then statistical comparisons would have been possible between the groups and influences of age and grade levels could have been assessed. A clear examination of language content, semantic aspect of word problems needs further probing, for better understanding of the language pattern.

**Implications and Suggestions for future research**

Following implications were drawn from the present study. Mathematics teachers in primary and secondary schools generally teach through conventional teaching method. This approach may be effective for high ability students but low ability students profit less from it. Therefore the present study stated that: -

All students irrespective of high and low ability should be taught through appropriate methods and teachers should look at their possible strategies for problem solving.

As we know students' mental processes are largely influenced by their ability to understand the words present in the problem, recognize the nature and type of problems, monitor solution processes, and carry out calculation. Therefore, the teachers are needed to look at these aspects minutely by developing abilities to solve problems, create confidence in students' problem solving ability and explain them the right way of thinking to solve problems.

An effort will be made to understand children's cultural background where they accommodate and assimilate relevant information for doing mathematics. Their informal strategies should not be blocked/suppressed by imposing formal methods of instruction taught at the school settings.

Besides teaching methods, further look is needed at curriculum level, scanning of textbooks and pedagogy of mathematics so that underlying procedures for problem solving could be tapped easily.

If each of the suggestion implemented successfully these could provide us better platform to vividly understand children's strategies for solving addition and subtraction problems in early school years.

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# APPENDIX –I

## Test items Used for Class II Children

*Preliminary Tasks*

(I) **Counting :-** Count (1 to 100) loudly

(II) **Writing Numbers :-**
18, 24, 29, 33, 39, 57, 60, 64, 70, 79, 81, 86, 89, 95, 99, 110, 112, 140, 143

(III) **Recognition of Numbers :-**
(a) 13, 27, 32, 35 (b) 63, 46, 58, 49
(c) 75, 87, 79, 91, 84 (d) 107, 122, 134, 145

***Numerical Problems***
**Parameter :-**

Single-Digit Addition presented in a vertical form

(1)
$$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$$

Single-Digit Addition presented in a horizontal form

(2) $6 + 3 = \square$

Missing-addend problem

(3) $\square + 5 = 8$

Double-Digit addition without carry over

(4)
$$\begin{array}{r} 15 \\ +12 \\ \hline \end{array}$$

Double-Digit addition with carry over

(5)
$$\begin{array}{r} 26 \\ +18 \\ \hline \end{array}$$

Single-Digit subtraction given in a vertical form

(6)
$$\begin{array}{r} 8 \\ -2 \\ \hline \end{array}$$

Single-Digit subtraction given in a horizontal form

(7) $7 - 4 = \boxed{\phantom{0}}$

Missing subtracted problem

(8) $\boxed{\phantom{0}} - 5 = 4$

Double-Digit subtrahend without carry over

(9)
$$\begin{array}{r} 25 \\ -13 \\ \hline \end{array}$$

Double-Digit subtraction with carry over

(10)
$$\begin{array}{r} 25 \\ -18 \\ \hline \end{array}$$

**Word Problems**

**Parameter**

Combine Double-Digit with single digit

(11) बाबू लाल के पास कहानी की 26 पुस्तकें हैं। उसने 8 पुस्तकें और खरीद लीं है। अब उसके पास कुल कितनी पुस्तकें हो गयीं ?

Combine doubled-digit addition

(12) एक पार्क में 55 बच्चे हैं। वहाँ 28 बच्चे और आ गए। अब बताओ, पार्क में कुल कितने बच्चे हो गए।?

Multi-step double-digit addition

(13) एक गाँव में 47 गायें, 24 भैंसे और 34 बकरियाँ हैं। उस गाँव में कुल कितने जानवर हैं?

Compare

(14) एक दुकानदार के पास 76 अण्डे थे। उस दुकानदार ने 23 अण्डे बेच दिये। उसके पास अब कितने अण्डे बच गए ?

Compare

(15) बच्चों के एक समूह में 48 लड़के और 27 लड़कियाँ हैं। लड़कियों से लड़के कितने अधिक हैं ?

Multi-digit subtraction with carry over

(16) एक स्कूल में 600 विद्यार्थी हैं। उनमें से 312 लड़कें हैं, और बाकी लड़िकयाँ हैं। बताओ स्कूल में कुल कितनी लड़कियाँ हैं ?

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## APPENDIX – II
## Test Items Used for Class III Children

***Preliminary Tasks***

**Counting :-** 1 to 100

**Writing Numbers :-**

64, 86, 70, 79, 95, 105, 89, 110, 114, 107, 1004, 1004,

1008, 1000, 1015, 1015, 1040, 504, 400, 33, 81, 500.

**Recognition of Numbers :-**

(1) 13, 27, 32, 35 (2) 63, 46, 58, 49

(3) 75, 87, 79, 91, 84 (4) 107, 122, 134, 145

***Numerical Problems***

**Parameter :-**

Multi-step multi-digit with carry over

(1)
$$\begin{array}{r} 5437 \\ +\ 2209 \\ +\ 4388 \\ +\ 1879 \\ \hline \end{array}$$

Multi-digit subtraction with carry over

(2)
$$\begin{array}{r} 9802 \\ -\ 7645 \\ \hline \end{array}$$

Missing subtrahend

(3) $145 - \square = 40$

Missing addend

(4) $102 + \square = 155$

Multiplication of missing with single-digit

(5) $\square \times 8 = 96$

Multiplication of multi-digit with double-digit

(6) $$\begin{array}{r} 397 \\ \times\ 79 \\ \hline \end{array}$$

Simple Division without remainder

(7) $808 \div 4 = \square$

Division with remainder

(8) $946 \div 7 = \square$

***Word Problems***

Multi-digit subtraction with carry over

(9) एक गांव में 4325 बच्चे हैं । इनमें से 2517 बच्चे स्कूल जाते हैं । बताओं कितने बच्चे स्कूल नहीं जाते हैं?

Multi-step multi-digit with carry over addition

(10) एक गांव में 2508 पुरुष, 2390 महिलाएँ तथा 3006 बच्चे हैं । गाँव की कुल जनसंख्या कितनी हैं?

Double digit multiplication word problem .

(11) एक बाग में फूलों की 23 पंक्तियाँ हैं। प्रत्येक पंक्ति में 25 फूलों की पौधे हैं। बताओ, उस बाग में कुल कितने फूल के पौधे हैं?

Multiplication of multi-digit with double-digit .

(12) एक टोकरी में 365 आम हैं । बताओ, ऐसी 25 टोकरियों में कितने आम होंगे ?

Multi-digit division without remainder.

(13) एक व्यक्ति ने तीन बच्चों में 801 रुपये बराबर-बराबर बाँटे । बताओ प्रत्येक बच्चे को कितने रुपये मिलें?

Multi-digit division without remainder.

(14) यदि 8 ट्रांजिस्टरों का मूल्य 648 रुपये है, तो एक ट्रांजिस्टर का मूल्य क्या होगा?

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