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INTERCELL INTERFERENCE STATISTICS UNDER DIFFERENT FADING DISTRIBUTIONS IN MOBILE COMMUNICATION

Dissertation submitted to
Jawaharlal Nehru University
In partial fulfillment of requirements
for the award of the degree of
Master of Technology
In
Computer Science and Technology

By

ANUPARTHA HOWLADER

Under Guidance of

PROFESSOR KARMESHU

SCHOOL OF COMPUTER & SYSTEMS SCIENCES
Jawaharlal Nehru University
New Delhi -110067
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SCHOOL OF COMPUTER AND SYSTEMS SCIENCES

CERTIFICATE

This is to certify that the dissertation entitled ***"Intercell Interference Statistics Under Different Fading Distributions in Mobile Communication"*** which is being submitted by **Mr. Anupartha Howlader** to the School of Computer and Systems Sciences, Jawaharlal Nehru University, for the award of ***Master of Technology in Computer Science and Technology***, is a record of bonafide work carried out by him.

This work is original and has not been submitted in part or full to any university or institution for the award of any degree.

Date:
Place: 5\11\04

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New Delhi-110067,
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DECLARATION

This is to certify that the dissertation entitled "***Intercell Interference Statistics Under Different Fading Distributions in Mobile Communication***" which is being submitted to the School of Computer and System Sciences, Jawaharlal Nehru University, for the award of ***Master of Technology in Computer Science and Technology***, is a record of bonafide work carried out by me.

This work is original and has not been submitted in part or full to any university or institution for the award of any degree.

Anupartha Howlader
9/01/04

Anupartha Howlader

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I shall always be grateful to my parents for providing me with the best educational opportunities. They have been a source of incessant support and encouragement, which have been vital to all of my accomplishments.

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Anupartha Howlader

New Delhi,
5th January, 2004.

...dedicated to my parents

Abstract

This work proposes an analytical model for estimating Quality of service parameters i.e. intercell interference statistics, in mobile communication. In cellular communication system signal gets scattered from various structures coming along in its path. Due to scattering from a number of obstruction, randomness arises in the received signal which gets faded. Along with the fading and shadowing signal is also affected by intercell interference which keeps on varying depending upon the receivers position. Now a days wireless mobile communication systems support a wide range of traffic. At the same time, the quality of the transmitted information must be guaranteed. QoS provides consistent and predictable data delivery service. In wireless communication, complexities due to fading and shadowing are the major issues that degrades QoS parameters to be explored. In the previous work, the estimation of QoS parameter is done considering fading aspects. A more realistic model is used incorporating fading and shadowing. This leads to a form which is intractable and difficult to work with. An approximation using three points distribution is employed to get the expression in analytic form. The proposed work gives a simple though approximate form to calculate QoS parameter in wireless system. After numerical simulation it is observed that the proposed three point distribution can approximate Rayleigh-Lognormal distribution quite accurately for lower values of ratio of mean and standard deviation of the distribution, that is for $\eta/\mu << 1$.

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Chapter 1

1. Introduction:

In cellular communication system signal gets scattered from various structures coming along in its path. Due to scattering from a number of obstruction, randomness arises in the received signal which gets faded. Along with the fading and shadowing signal is also affected by intercell interference which keeps on varying depending upon the receivers position. For efficient reception the Signal-to-Interference Ratio (SIR) should be greater than or equal to the threshold limit of the system. We will briefly discuss about mobile communication systems before going to the main topic.

Ability of communicating with mobile persons has led to wide spread adoption of wireless communication [11]. The developmental achievements in wireless communication over the past decade have been motivated by huge increase in the demand of wireless connectivity. It has taken quite long to emerge from the wireless telegraphy to third generation mobile communication, and the growth in last decade has seen the most rapid progress without any indication of decrease in the pace in near future. Accordingly, considerable amount of growth in very large scale integration (VLSI) that has increased the ratio between computational capabilities versus cost is making the implementation of novel technologies more feasible.

1.1 What is Mobile Communication?

Mobile communication helps the users to communicate with each other without being at a fixed location. Since the users are not at a fixed location, so the obvious means to enable mobile communications is radio communications. Radio communications allow the highest possible degree of mobility.

The main idea behind the emerging mobile and personal communication services and systems is to enable communication with a person, at anytime, at any place, and in any form [12]. Additional to unlimited reachability and accessibility, this concept for personal communications also underlines the increasing need for users of communications services to be able to manage their individual calls and services according to their real-time need. Suppose, during certain period of the day a user may wish to divert his or her calls to message center or to have the caller number flashing on the display of the handset so that the incoming calls can be treated according to the user's instructions.

1.2 Various Terms and Systems in Mobile Communication:

In everyday use, we are familiar with different kind of mobile radio communication systems, i.e. remote controllers for personal entertainment systems, pagers, cordless telephones, hand held walkie-talkies and cellular telephone fitted in vehicle etc. These systems differ in terms of cost, complexities, performance and types of service offered.

1.2.1 Terms Used In Mobile Communications:

For better under standing about various concepts we will discuss about the following terms as defined in [11], which are widely used for mobile communication systems,

Base Station (BS): This is a fixed station to provide radio communication with mobile terminals (MT). BSs are positioned in the center or on the edge of a coverage area and consist of a set of radio channels for transmitter and receiver antennas mounted on a tower.

Control Channel (CC): Radio channel that is used for different services like call request, call initiation, call setup and other beacon or control services.

Forward or Up-Link Channel (UL): Radio channel used for transmitting information from the base station to the mobile terminal.

Full Duplex Systems: Communication systems that allow simultaneous two-way communication, generally through two different channels.

Half Duplex Systems: Communication systems that allow both way communications, but only one-way communication at a time using the same radio channel for transmitting and receiving.

Handoff: Process involves in transferring of mobile terminal from one channel or base stations to another while call is in progress.

Mobile Terminal (MT): A radio device fitted with transmitter and receiver, which is not fixed at any geographic location and intended to use while in motion. Mobile terminals can be portable handheld device or can be installed in vehicles.

Mobile Switching Center (MSC): It involves with coordinating the routing of calls in a considerably large service area. In cellular systems the MSC connects the base stations and the mobile terminals to the PSTN network. MSC is also called Mobile Telephone Switching Office (MTSO).

Page: A brief message that is usually broadcast in simulcast fashion by many base stations at the same time over the entire service area.

Reverse Link Channel (RL): Radio channel that is used for receiving information by base station from the mobile terminal.

Roamer: A mobile station, which is using services from a network other than its home or subscribed network.

Simplex systems: Communication system that allows only one-way communication. Here transmission of information by the receiving is not possible.

Subscriber: A user who is registered for using the communication service on payment basis.

Transceiver: A device that is capable of transmitting and receiving radio signal simultaneously.

1.2.2 Basic Systems in Mobile Communications:

Systems in mobile communication vary depending upon communication types (like full duplex, half-duplex and simplex), complexities, costs and mobility types and range or distance. According to these parameters followings are the basic systems currently available in mobile communication:

1.2.2.1 Paging Systems:

In paging systems a brief message is sent to the subscriber, which can be numeric or alphanumeric or voice message depending upon the type of service. The main intention here is to notify the subscriber about some instructions. There are wide varieties in paging systems depending upon the complexity and coverage area [11]. Simple paging systems have coverage area of 2 to 5 kilometers, or can even be confined to within individual buildings; while wide area paging systems can give up to world wide coverage. Paging receivers are simple and inexpensive, but its transmission system is quite sophisticated. A wide area paging system consists network of telephone lines (PSTN), number of base station transmitters, and large radio towers used for simultaneous broadcasting a page from each base stations. The architectural aspect of paging systems is illustrated by the figure-1.1,

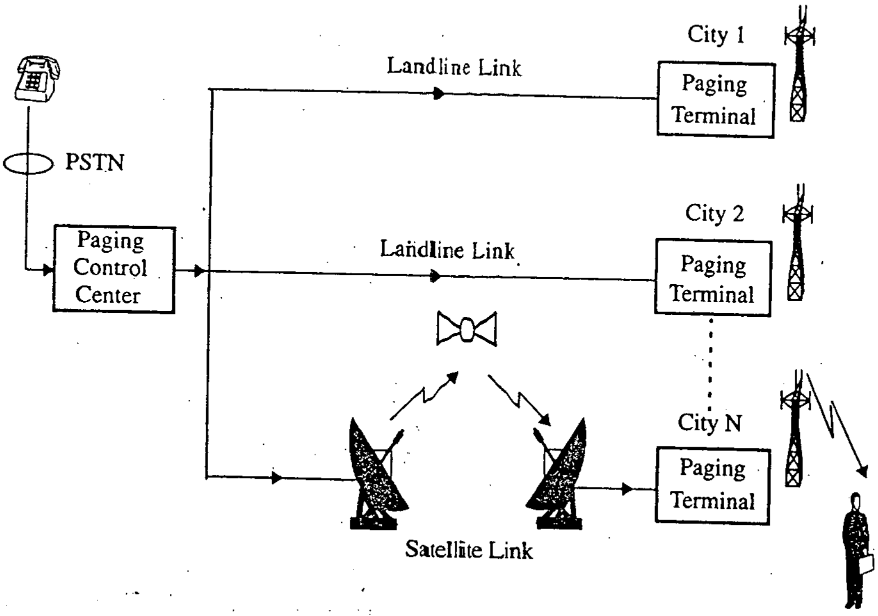

Figure - 1.1: Architectural layout of a wide area paging system.[11]

1.2.2.2 Cordless Telephone Systems:

This is a full duplex communication system, which has two units; one is dedicated base unit that is connected to public switched telephone network (PSTN) and another is portable handset that is connected to the dedicated base unit through radio link. This portable unit can communicate with it's dedicated base unit over distances of a few tens of meteres.

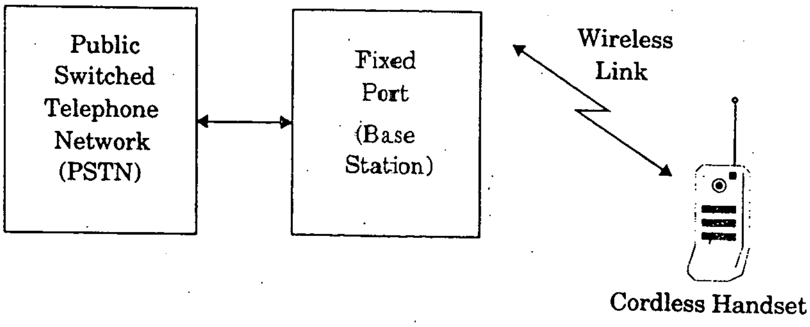

Figure - 1.2: A cordless telephone system.[11]

1.2.2.3 Cellular Telephone Systems:

A wireless connection to the PSTN is provided by cellular telephone system for any user location within the radio range of the system. Within a limited frequency spectrum this system accommodate large number of user over a geographic region. Quality of service in cellular systems is comparatively higher than that of the PSTN telephone. Since the main interest of this work is concerned with cellular systems, we are going to give the details description of cellular systems.

1.3 Cellular Concept:

The cellular concept was developed and introduced in the early 1970s by the Bell Laboratories. Advanced mobile phone system (AMPS) which has been available in US since 1983, was one of the most successful cellular concepts. According to Raj Pandya[12] A cellular system can be defined as

> "A high capacity land mobile system in which available frequency spectrum is portioned into discrete channels which are assigned in groups to geographic cells covering a cellular Geographic Service Area (GSA). The discrete channels are capable of being reused in different cells within the service area."[12]

Cellular systems usually consist of number MTs, BSs and MSCs those are connected with PSTN service. Each MT communicates with one of the BSs through radio channel and can change the BS or the radio channel while the call is in progress. A typical cellular system divides a large geographic area into small hexagonal cells with diameter 2km to 50km and allocates each small cells a number of radio frequency (RF) channels. In order to avoid interference, each cell's transmitter operates on different frequencies than its adjacent cell. In this system antenna height is moderately low and the cells considerably far apart can use same set of frequency without having co-channel interference (CCI). Thus the coverage range and capacity of cellular systems become theoretically unlimited which provides scalability for future expansion.

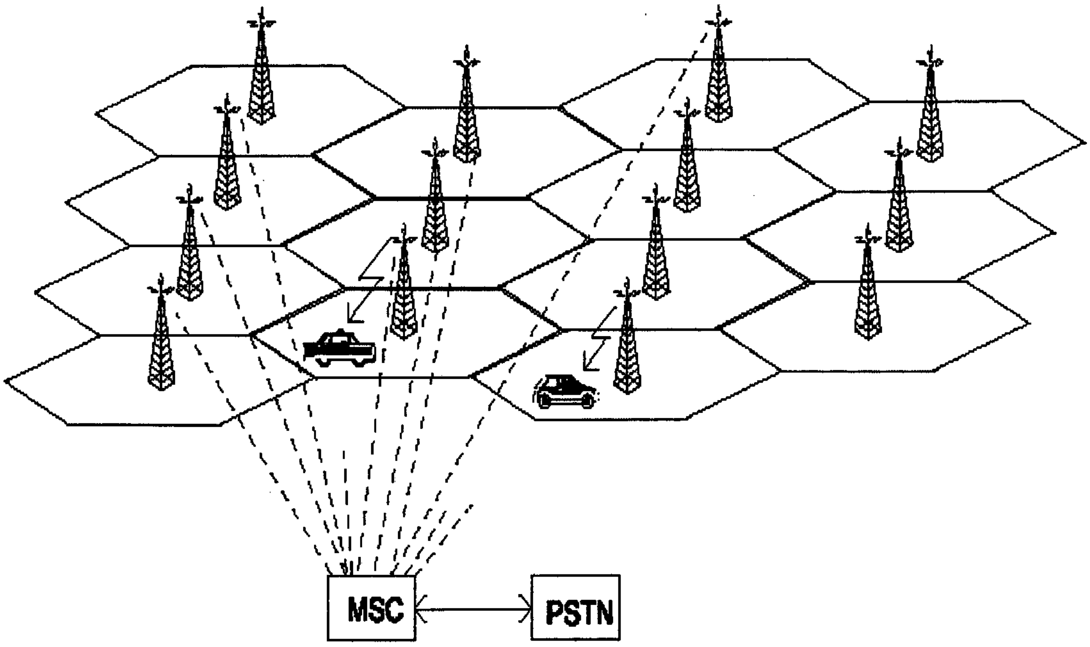

Figure – 1.3: Cellular systems

1.3.1 Size and shape of Cells:

The size of a cell is usually irregular, that is roughly circular. It is conceptually assumed hexagonal shape that is a simplistic model of the radio coverage area for each base station for easy and manageable analysis of a cellular system. Consideration of the hexagonal shape leaves a geographic area without any gap or uncovered region and overlapping region. The actual radio coverage of a cell is known as the *footprint* and is determined from field measurements.

Among the other geometric shapes hexagonal shape covers the largest area using the fewest number of cells, and the hexagon closely approximates a circular radiation pattern, which could occur for an *omni-directional* BS antenna and free space propagation. BS are generally located in the center of the hexagonal cell.

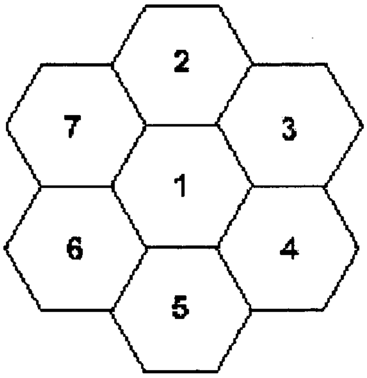

Figure - 1.4: Cell organization for a coverage area.

1.3.2 Frequency Reuse:

Cellular systems employ efficient allocation and reuse of channels for an entire coverage area. Each *cell* uses a group of radio channels for it's *BS* which is completely different from that of it's neighboring *cells' BSs*. Base stations those are considerably far apart from each other can use same set of radio channel. For understanding about the frequency reuse concept,

Let, S = total duplex channels available for use

N = number of cells using distinct channel within S.

k = number ofchannels allocated for each cell

So we can get the expression $S = kN$.

A new term we can introduce that is *Cluster* - the set of N cells, which use the complete set of available frequencies (S) altogether. Throughout any geographic area a cluster can be replicated M times. So the total number of duplex channels C can be used as a measure of capacity of the system and can be expressed as: $C = MkN = MS$. This equation shows that the capacity of a cellular system is directly proportional to the number of times a cluster is replicated in a fixed service area. The factor N is called the *cluster size* and is typically equal to 4, 7, or 12. If N is reduced and the cell size is unchanged, then more number of clusters will be required to cover an area. This will increase the capacity of the cellular system.

For lager cluster size ratio between the cell radius and the distance between co-channel cells is large, and smaller cluster size the ratio is small that is co-channel cells are located much closer. While maintaining the sufficient quality of communications, the value of N can be expressed as a function of the magnitude of interference that a MT of BS can tolerate. Here, the lowest possible value of N maximizes the capacity for a given coverage area.

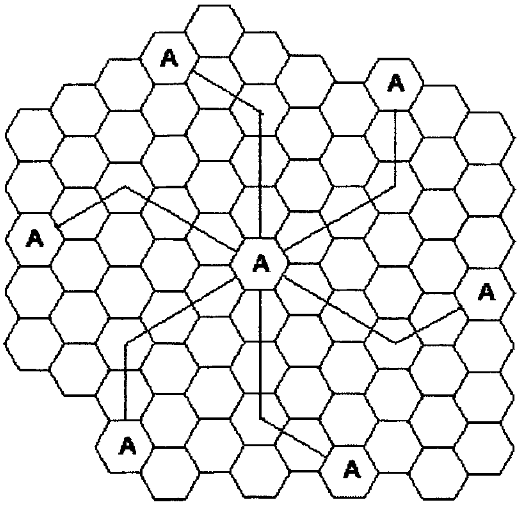

Figure1.5: Co-channel cells in a cellular systems[11]

1.4 Operational issues in mobile systems:

We are going to discuss various operational issues in mobile communication, like noise limited and interference limited systems, system load and quality of service(QoS).

1.4.1 Noise limited and interference limited systems:

The performance of mobile radio systems is hampered by noise and interference. Noise can be thermal, atmospheric or man made. This noise degrades the *quality of service* (QoS) parameters like signal to noise ratio (SNR). Interference, that is signals from other users of the same mobile radio system, also degrades QoS parameters like signal to interference ratio (SIR). Usually, cellular mobile radio systems are interference limited due to sufficiently large transmission powers. However, for MTs operating at the cell border, which is far from their BS, noise limitation may be the case.

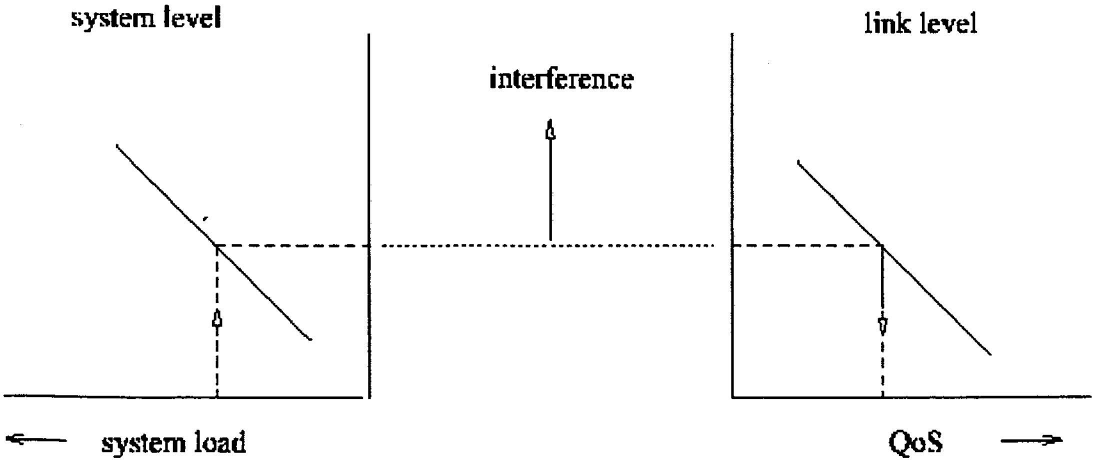

Figure 1.6: Relation between system load, interference and Qos.

1.4.2 System load versus Quality of Service:

In cellular systems each user expects a certain Quality of Service (QoS). QoS is defined by the probability that a user gets the desired connection, and that the speech quality or error probability (data transmission) fulfills given requirements. System load and Quality of Service is related to each other. With the increase in number of supported users that is with increasing system load interference increases, and QoS decreases with increasing interference, Therefore, QoS decreases with increasing system load as shown in Fig.(**):

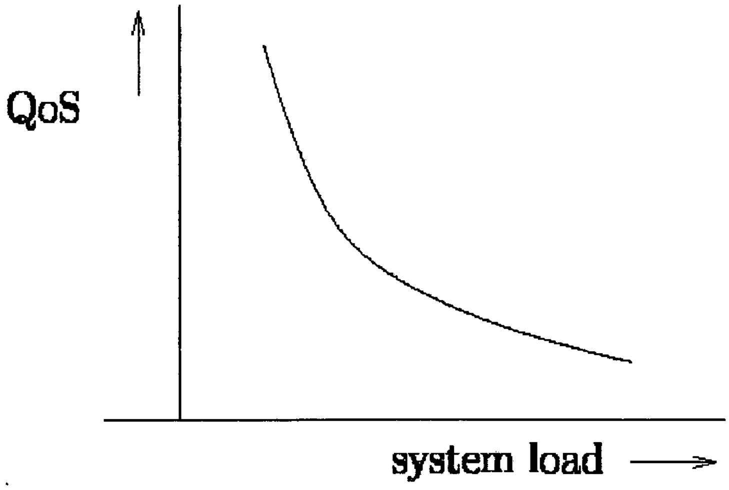

Figure 1.7: Relation between system load and QoS.

1.5 Organization of Dissertation:

In this chapter we have discussed briefly about the basic terminologies and systems in mobile communication. Chapter 2 describes about radio wave propagation model and different kinds of impairments like fading and shadowing aspects which is concerned to our work. After having understanding about fading and shadowing aspects and the problem and previous work done related to the problem is discussed in chapter 3. Then, a detail about proposed work and comparison and analysis is explained in chapter 4 and chapter 5 respectively. Finally, suggestion for future development and conclusion is drawn in chapter 6.

Chapter 2

Wave Propagation in Mobile Communication

2.1 Overview:

Mobile communication has particular propagation complications compared to the channel characteristics in radio systems with fixed and carefully positioned antennas. The antenna height at a mobile terminal is usually very small, typically less than a few meters. For this the antenna has very little clearance, so obstacles and reflecting surfaces in the vicinity of the antenna have a substantial influence on the characteristics of the propagation path. Moreover, the propagation characteristics change from place to place and, if the mobile unit moves, from time to time. Thus, the transmission path between the transmitter and the receiver can vary from simple direct line of sight to one that is severely obstructed by buildings, foliage and the terrain.

The study of mobile communication channel is evaluated from statistical propagation models - no specific terrain data is considered, and channel parameters are modeled as stochastic variables. The mean signal strength for an arbitrary transmitter-receiver (T-R) separation is useful in estimating the radio coverage of a given transmitter whereas measures of signal variability are key determinants in system design issues such as antenna diversity and signal coding.

2.2 Propagation Mechanisms:

In mobile radio propagation environment there are three basic propagation mechanisms. These are

- Reflection
- Diffraction
- Scattering.

Reflection:

When a propagating electromagnetic wave impinges upon an object which has very large dimensions as compared to the wavelength of propagating wave then the propagating wave gets reflected from that object. Reflection occurs from the surface of the earth and from buildings and walls and depends on the geometry of the objects.

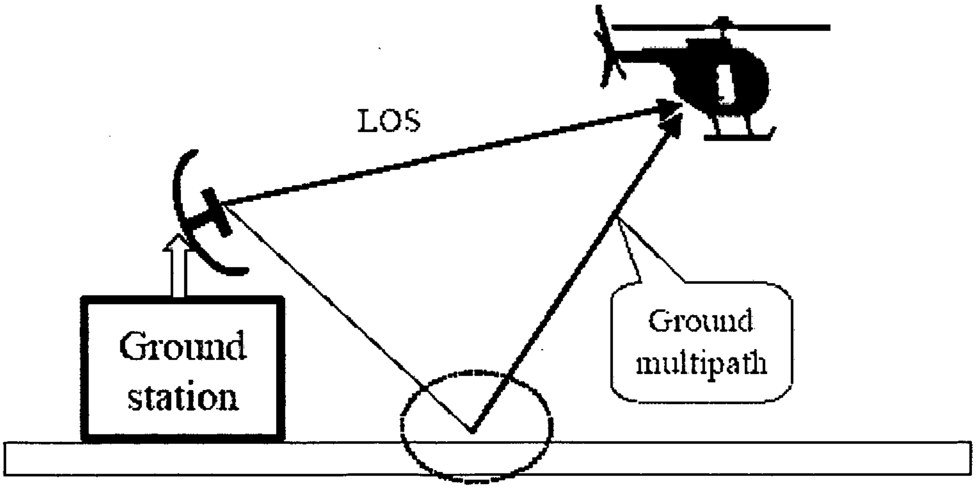

Figure 2.1 - Reflection of radio wave.

Diffraction:

Diffraction occurs when there are some obstructions with surfaces having sharp irregularities, between the path of the transmitter and receiver. The secondary waves resulting from the obstructing surface are present throughout the space and even behind the obstacle, giving rise to a bending of waves around the obstacle, even when a line of sight (LOS) path does not exist between transmitter and receiver. Thus diffraction allows radio signals propagate along curved path. As the receiver moves deeper into the obstructed or shadowed region received field strength decreases rapidly, but still diffraction field can have sufficientstrenght to produce signal [Rapapport]. At high frequencies, diffraction, like reflection, depends on the geometry of the object, as well as the amplitude, phase, and polarization of the incident wave at the point of diffraction.

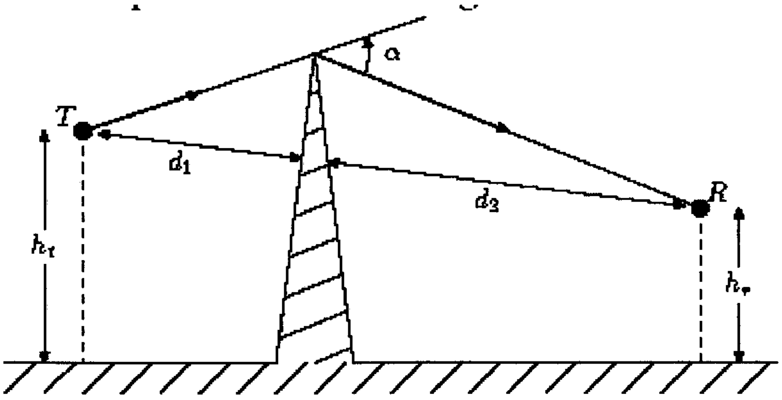

Figure 2.2 - Typical Diffraction (Knife edge model).

Scattering:

During propagation, if there are any objects with dimensions smaller than the wavelength and the number of those is comparatively large per unit volume, then the radio wave gets scattered. Rough surfaces, small objects or other irregularities in channel causes scattering. In practical, foliage, street signs and lamppost induce scattering in mobile communication system.

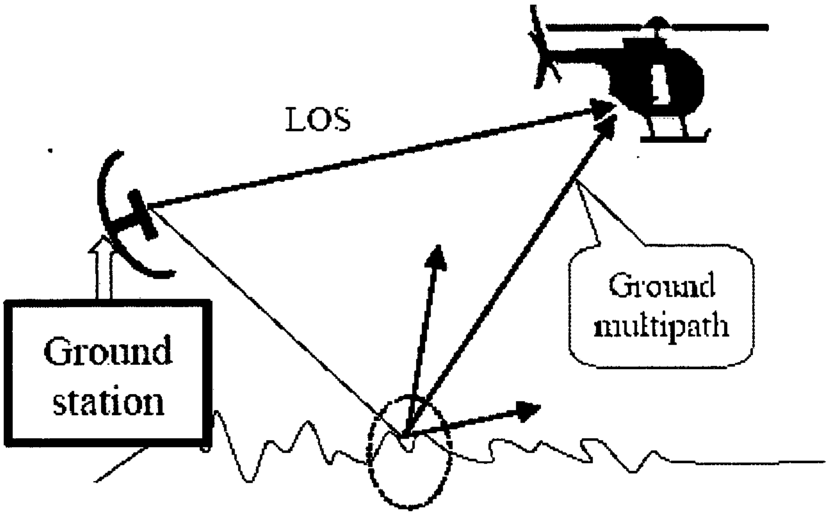

Figure 2.3 - Scattering causing due to uneven surface.

2.3 Various Phenomena due to Propagation Mechanisms:

Due to these above mechanisms the radio wave propagation can be characterized by three mutually independent, multiplicative propagation phenomena.

- Large-scale path loss.
- Fading
- Shadowing

2.3.1 Large-Scale Path Loss:

The large-scale effects of path losses cause the received power to vary gradually due to signal attenuation, determined by the geometry of the path profile in its entirety. This is in contrast to the local propagation mechanisms, which are determined by building and terrain features in the immediate vicinity of the antennas. The average received signal power decreases logarithmically with distance. The average large-scale path loss is expressed as the function of distance between transmitter and receiver with path loss exponent n.[11]

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$$

$$or \qquad \overline{PL}(dB) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) \qquad (2.1)$$

where the path loss exponent n indicates the rate at which the path loss increasing with distance, d_0 is the close reference distance and d is the transmitter and receiver separation distance. Here the value n depends of the specific propagation environment. A table is given for different values for different propagation environment.

Propagation Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line of sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Table 2.1 - Path Loss Exponents for Different Environments [11]

2.3.2 Fading:

Multipath propagation leads to rapid fluctuations of the phase and amplitude of the signal if the mobile terminal (MT) moves over a distance in the order of a wave length or more. *Multipath fading* thus has a small-scale effect. Multipath is usually described by

- **Line-of-sight (LOS)**: the direct connection between the transmitter (TX) and the receiver (RX).
- **Non-line-of-sight (NLOS)**: the path arriving after reflection from reflectors.

The illustration of LOS and NLOS is shown in figure 2.4.

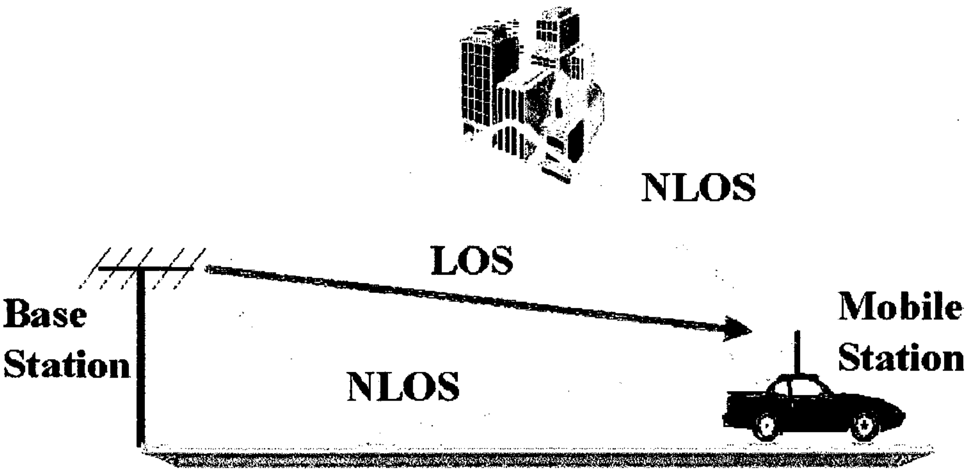

Figure 2.4 - Multipath Reception.

In *multipath reception* the signal offered to the receiver contains not only a direct line-of- sight radio wave, but also a large number of reflected radio waves. These reflected waves interfere with the direct wave, which causes significant degradation of the performance of the network. A mobile communication network has to be designed in such way that the adverse effect of these reflections is minimized.

The effect of multipath reception-

- for a fast moving user: rapid fluctuations of the signal amplitude and phase.
- for a multi-carrier signal: different attenuation at different sub-carriers and at different locations

For multipath reception, Narrowband Rayleigh or Rician models mostly address the channel behavior at one frequency only and Dispersion is modeled by the delay spread.

2.3.2.1 Rayleigh Fading:

Rayleigh fading is caused by multipath reception. The mobile antenna receives a large number, say *N*, reflected and scattered waves. Because of wave cancellation effects, the instantaneous received power seen by a moving antenna becomes a random variable, dependent on the location of the antenna. Density function for Raleigh fading distribution is,

$$p(x) = \begin{cases} \dfrac{x}{\sigma^2}\exp\left(-\dfrac{x^2}{2\sigma^2}\right) & (0 \le x \le \infty) \\ 0 & (r < 0) \end{cases}$$

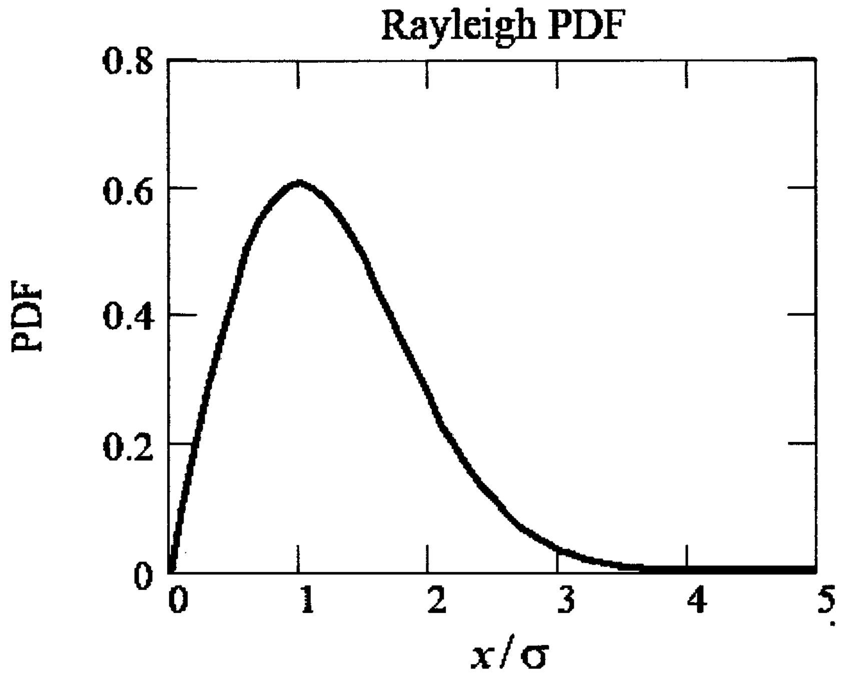

Figure 2.5 – Plotted probability density function of Rayleigh distribution.

2.3.2.2 Rician Fading:

The model behind Rician fading is similar to that for Rayleigh fading, except that in Rician fading a strong dominant component is present. This dominant component can for instance be the line-of-sight wave. Probability density function of signal amplitude

$$f_{\mathrm{P},\Phi}(\rho,\phi)=\frac{\rho}{2\pi\sigma^2}\exp\left\{-\frac{\rho^2+C^2-2\rho\cos\phi}{2\sigma^2}\right\} \qquad (2.2)$$

Here, p is the local-mean scattered power and $C^2/2$ is the power of the dominant component. The pdf of the amplitude is found from the integral

$$\begin{aligned} f_{\mathrm{P}}(\rho) &= \int_{-\pi}^{\pi} f_{\mathrm{P},\Phi}(\rho,\phi)\,d\phi \\ &= \frac{\rho}{\sigma^2}\exp\left\{-\frac{\rho^2+C^2}{2\sigma^2}\right\} I_0\left(\frac{\rho C}{\sigma^2}\right) \end{aligned} \qquad (2.4)$$

where $I_0(..)$ is the modified Bessel function of the first kind and zero order, defined as

$$I_0(x) \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\{-x\cos\phi\}\,d\phi \tag{2.5}$$

Rician factor - The Rician *K*-factor is defined as the ratio of signal power in dominant component over the (local-mean) scattered power. Thus

$$K \equiv \frac{C^2/2}{\sigma^2} = \frac{\textit{direct power}}{\textit{scattered power}} \tag{2.6}$$

Expressed in terms of the local-mean power $\overline{p}$ and the Rician *K*-factor, the pdf of the signal amplitude becomes

$$f_\rho(\rho) = (1+K)e^{-K}\frac{\rho}{\overline{p}}\exp\left(-\frac{1+K}{2\overline{p}}\rho^2\right) I_u\left(\sqrt{\frac{2K(1+K)}{\overline{p}}}\,\rho\right) \tag{2.7}$$

2.3.3 Shadowing:

Shadowing is a medium-scale effect. Field strength variations occur if the distance between transmitter and receiver (T-R separation) is larger than a few tens or hundreds of meters. From previous measurement it is shown that for any value of T-R separation d, the path loss *PL(d)* is random and log-normally distributed, that is

$$\begin{aligned} \overline{PL}(d)[dB] &= \overline{PL}(d) + X_\sigma \\ &= \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma \end{aligned} \tag{2.8}$$

where X_σ is zero mean Gaussian distributed random variable (in dB) with standard deviation σ (in dB).

Chapter 3

Problem Definition and Previous Work

3.1 Problem definition:

Fading and shadowing degrades the signal strength in stochastic manner, accordingly a probabilistic model becomes inevitable [11]. Extensive research has been done on emerging statistical distributions for capturing fading and shadowing aspects in mobile communication. Statistical distribution of fading effect caused by multipath reception is captured by Rayleigh distribution and shadowing by lognormal distribution [7, 11]. The combined effect of Fading and Shadowing will be a mixture of Rayleigh-Lognormal distribution, which is not available in closed form. In the previous works [2, 3] estimation of Quality of Service (QoS) parameters like intercell interference statistics is done considering fading contribution to Rayleigh distribution and combined effect of Fading and Shadowing represented by mixture of Rayleigh-Lognormal distribution is not considered. So it is necessary to use a closed form of this distribution or an approximation of this distribution as obtained by Karmeshu & Agrawal. It is also important to study the intercell interference statistics under the hypothesis of perfect load control [2].

3.2 Previous works:

In order to get idea about the real environment let us discuss briefly the environment, assumptions and work done in [2]. Abrardo et. al. [2] considered typical urban path loss, which depends on the αth power of the distance. The shadowing attenuation is log-normally distributed, i.e., of the form $e^{\varepsilon(t)}$, where $\varepsilon(t)$ is a correlated zero-mean Gaussian process with standard deviation σ [9]. Each MT is assumed to be connected to the BS from which it receives the strongest pilot power. Here total received power P_{tot} is expressed as,

$$\begin{aligned} P_{tot} &= I + N \\ &= I_{\text{intra}} + I_{extra} + N \end{aligned}$$

where, I_{intra} is power received from MTs connected to the same cell, I_{extra} is power received from MTs connected to other cells, i.e. the external interference or intercell interference, and N is noise. So, here I_{extra} is calculated as,

$$I_{extra} = P_{tot} - I_{\text{intra}} - N$$

Here all the other assumptions made in [2] are beyond the scope of our work. Intercell interference, I_{extra} is received from equivalent MT (EMT, hypothetical MTs assumed to be connected with surrounding interfering cells), and P_E is power transmitted by each EMT. If all power received at BS was transmitted by EMTs, then number of total EMTs N_E, that would contribute to the total received power, is expressed as,

$$N_E = I/P_E$$

we know that,

$$N_E = M_E + K_E$$

where

$$M_E = I_{\text{intra}}/P_E \qquad \text{and} \qquad K_E = I_{extra}/P_E$$

Here I_{extra} can be characterized by deriving K_E. In [10] I_{extra} is modeled as log-normally distributed. $K_E^{(h)}$ denotes the normalized I_{extra}, contribution at the tagged BS(BS_0), from an interfering MT connected to $BS_{(h)}$. Assuming a simple case the statistical evaluation of $K_E^{(h)}$ is done in [3]. A more realistic case is taken into consideration in [2], here the results of statistics of the intercell interference derived in [9] are used to characterize the $K_0^{(h)}$. So as shown in [3] the terms can be expressed as,

$$\begin{aligned} K_E^{(h)} &= \frac{\lambda_0}{\lambda_h}\left\{\left(\frac{r_h}{r_0}\right)^{\alpha} e^{\varepsilon_0-\varepsilon_h} \,\middle|\, r_h^{-\alpha}e^{\varepsilon_h} \ge r_w^{-\alpha}e^{\varepsilon_w}, \forall w\right\} \\ &= \frac{\lambda_0}{\lambda_h}\, K_o^{(h)} \end{aligned} \qquad (3.1)$$

where

$$K_o^{(h)} = \left(\frac{r_h}{r_0}\right)^{\alpha} e^{\varepsilon_0-\varepsilon_h} \,\Big|\, r_h^{-\alpha}e^{\varepsilon_h} \ge r_w^{-\alpha}e^{\varepsilon_w}, \qquad \forall w \qquad (3.2')$$

Following Hasem et. al.[3], we give below the meaning of notations used by them;

- subscript l, $l \in \{0, 1, \dots, C\}$ denotes the quantities related to BS_l, where C is the number of interfering cells;
- r_h and r_0 are the distances of MT from BS_h and BS_0, respectively;
- ε_h and ε_0 are shadowing contributions towards BS_h and BS_0, respectively;
- λ_h and λ_0 are fading contributions towards BS_h and BS_0, respectively;

- cell $w \in \{0, 1, \ldots, C\}$ is a generic-interfering cell.

We find it is convenient to introduce random variables U_1 and U_0 which represent λ_h and λ_0, and W such that eq.(3.1) becomes,

$$K_E^{(h)} = \frac{U_0}{U_1}[W] \tag{3.3}$$

Noting that U_1, U_0 and W are independent, one can write,

$$\begin{aligned} E\left[K_E^{(h)}\right] &= E\left[\frac{U_0}{U_1}W\right] \\ &= \frac{E[U_0]}{E[U_1]}E[W] \end{aligned} \tag{3.4}$$

In [2], it is assumed that U_0 and U_1 are distributed each as chi-squared variate with m = L degree of freedom. Using the results, their ratio of two individual chi-squared variate is a Fisher distribution, Abrardo et.al. [2] evaluated the fading. In next chapter we are going to discuss more realistic model for the random variables U_0 and U_1.

Chapter 4

Proposed Work

4.1 Fading Channel Model:

In [2] fading contributions U_1 and U_0 are assumed to follow chi-squared distribution. However, in reality there will be a combined effect of fading and shadowing, which is a mixture of Rayleigh-Lognormal distribution [5]. Accordingly we consider U_1 and U_0 to be Rayleigh-Lognormally distributed, which is expressed in integral form. This form is analytically intractable and accordingly not much progress could be made. The density function of the mixture of Rayleigh-Lognormal distribution can be expressed as:

$$f_{RL}(u) \equiv f_U(u) = \int_0^\infty f_{U|X}(u \mid X = x)\,\phi_X(x)dx; \qquad x \geq 0 \qquad (4.1)$$

where U corresponds to Rayleigh-Lognormal variate, the conditional density for $f_{U|X}(u \mid X = x)$ denotes the Rayleigh distribution given as:

$$f_{U|X}(u \mid X = x) = \frac{u}{x}\, e^{-\frac{u^2}{2x}}; \qquad u \geq 0 \qquad (4.2)$$

In eq.-(4.1) random variable X is assumed to follow a Lognormal distribution with parameters μ and η and the pdf lognormal distribution with modal value x is given as,

$$\phi_X(x) = \frac{1}{\sqrt{2\pi\eta^2}\, x} e^{\frac{-(\ln x-\mu)^2}{2\eta^2}} ; \qquad x \geq 0 \qquad (4.3)$$

So the explicit pdf of Rayleigh-Lognormal becomes,

$$f_U(u) = \int_0^\infty \frac{u}{x} e^{-\frac{u^2}{2x}} \frac{1}{\sqrt{2\pi\eta^2}\, x} e^{\frac{-(\ln x-\mu)^2}{2\eta^2}} dx \qquad x \geq 0 \qquad (4.4)$$

Eq.(4.4) represents the pdf of Rayleigh-Lognormal distribution. This expression is not available in analytical form and this aspect makes the distribution difficult to work with. Any closed form of this distribution will be useful in providing quality of service parameters of fading-shadowing in mobile communication. We shall follow a new approximation [6] of Rayleigh-Lognormal distribution in analytical form. The next section gives the approximation of normal distribution using three point approximation for Gaussian density. The advantage results from this is that we get an analytical expression, though approximation.

4.2 Approximating Normal Distribution with Point Distribution:

Now we will discuss about an approximation method of Normal distribution by Three Point distribution. Before discussing about three point distribution, for the sake of completeness we will discuss about the two point distribution proposed by Rosenblueth[4]:

4.2.1 Two Point Distribution:

It is an approximation method to compute expectation of first few moments of function of a random variable when the first two moments of random variable X are known. Let X and Y be real random variables and Y be the function of X, that is,

$$Y = Y(X)$$

Here we have to find the expectation of Y, based as available information about two moments of X. Rosenblueth [4] suggested the following pdf,

$$p(x) = P_{+}\delta(\overline{X} - x_{+}) + P_{+}\delta(\overline{X} - x_{-}) \qquad (4.5)$$

where P_{+} and P_{-} denotes probability weights, $\delta(\ )$ is Dirac's delta function, x_{+} and x_{-} are specific values for X to be chosen.

If the three moments like mean ($\overline{X}$), standard deviation (η_X) and skewness (υ_X) of the random variable X are specified, the following simultaneous equations are to be satisfied,

$$\begin{aligned} &P_{+} + P_{-} = 1 \\ &P_{+}x_{+} + P_{-}x_{-} = \overline{X} \\ &P_{+}(x_{+} - \overline{X})^2 + P_{-}(x_{-} - \overline{X})^2 = \eta_X \qquad (4.6) \\ &P_{+}(x_{+} - \overline{X})^3 + P_{-}(x_{-} - \overline{X})^3 = {\eta_X}^3 \upsilon_X \end{aligned}$$

whose solution becomes,

$$P_+ = \frac{1}{2}\left[1 \mp \sqrt{1 - \frac{1}{1-(\upsilon_X/2)^2}}\right], \qquad P_- = 1 - P_+$$

and $$x = \overline{X} \mp \eta_X \sqrt{P_\mp / P_\pm}$$

So for two point distribution the expectation of Y is such that,

$$E(Y) = \int_{-\infty}^{\infty} Y(x)p(x)dx$$

$$E(Y) = P_+ Y(x_+) + P_- Y(x_-) \qquad 4.7$$

The advantage of two-point distribution is that moments of random variable Y can be obtained in terms of function value at two points. When the skewness is zero, the density function of two point distribution becomes,

$$p(x) = \frac{1}{2}\delta\left[x - \left(\overline{X} - \eta_X\right)\right] + \frac{1}{2}\delta\left[x - \left(\overline{X} + \eta_X\right)\right] \qquad 4.8$$

This is a symmetrical distribution and the moments can be obtained as follows,

$$\overline{X} = \int_{-\infty}^{\infty} xf(x)dx = \overline{X}$$

$$Var(X) = \int_{-\infty}^{\infty} \left(x - \overline{X}\right)^2 f(x)dx = \eta_X^2$$

$$E\left[X - \left(\overline{X}\right)^3\right] = \int_{-\infty}^{\infty} \left(x - \overline{X}\right)^3 f(x)dx = 0$$

$$E\left[X - \left(\overline{X}\right)^4\right] = \int_{-\infty}^{\infty} \left(x - \overline{X}\right)^4 f(x)dx = \eta_X^4$$

It may be noted that this Two Point approximation will not work for normal density as the fourth moment differs from that of a normal variate which is $3\eta^4$. It is important to point out two point distribution yields moments, which can match only the first three moments of a normally distributed random variable. Extending this, we find for three point as discussed in [8], will give a better approximation of normal distribution.

4.2.2 Three point distribution: for Gaussian Variate $T(\mu, \eta^2)$:

For a normally distributed random variable T ~ $N(\mu, \eta^2)$, it is shown in literature that a pdf of a normal variate can be replaced by three point distribution such that first four moments are same as that for normal variate. This idea of replacing normal distribution by three point distribution is quite elegant as it yields further possibility of obtaining analytical results. So, after replacing normal distribution $N(\mu, \eta^2)$ by three Point distribution, we can

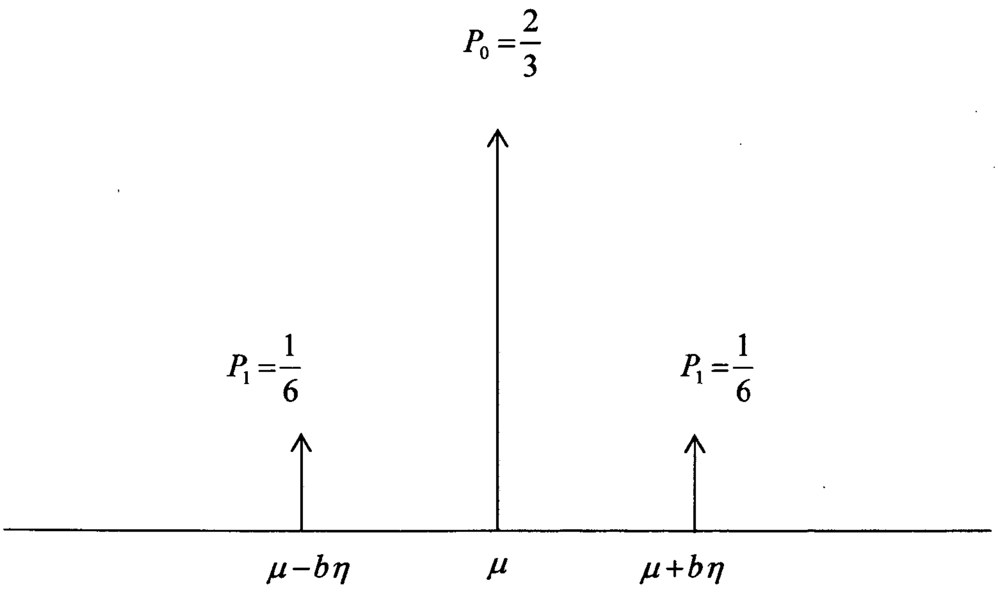

Figure - 4.1: **Three Point Distribution** have the following three point density function, which approximates Gaussian density

$$\phi(T)=P_1\,\delta[T-(\mu-b\eta)]+P_0\,\delta[T-\mu]+P_1\,\delta[T-(\mu+b\eta)] \qquad (4.9)$$

Here, P_0 and P_1 denotes probability weights given at points $T = \mu$ and $T = (\mu \pm b\eta)$ respectively and b is a positive real number. And $\delta(\ )$ is Dirac's delta function, which is a normal function that follows the equation,

$$\int_{-\infty}^{\infty}\delta(x-a)\,f(x)\,dx=f(a) \qquad (4.10)$$

HereP_1, P_0 and b must satisfy the following simultaneous equations,

$$\begin{aligned}&2P_1+P_0=1\\&2P_1(b^2\eta^2)+0=\eta\\&2P_1(b^4\eta^4)+0=3\eta^4\end{aligned} \qquad (4.11)$$

solving the above equations we get

$$b=\sqrt{3}, \qquad P_1=\frac{1}{6} \qquad \text{and} \qquad P_0=\frac{2}{3}$$

So the equivalent probability density function will be,

$$\phi(T)=\frac{1}{6}\delta\left[T-\left(\mu-\sqrt{3}\eta\right)\right]+\frac{2}{3}\delta[T-\mu]+\frac{1}{6}\delta\left[T-\left(\mu+\sqrt{3}\eta\right)\right] \qquad (4.12)$$

It can be easily verified that the first five moments of three point distribution are same as that of normal distribution. For random variable T, we get,

$$E[T]=\int T\phi(T)\,dt=\frac{1}{6}\left(\mu-\sqrt{3}\eta\right)+\frac{2}{3}\mu+\frac{1}{6}\left(\mu+\sqrt{3}\eta\right)=\mu$$

$$E[(T-\mu)] = \int (T-\mu)\,\phi(T)dt = \frac{1}{6}\sqrt{3}\eta - \frac{1}{6}\sqrt{3}\eta = 0$$
$$E[(T-\mu)^2] = \int (T-\mu)^2\,\phi(T)dt = \frac{1}{6}3\eta^2 + \frac{1}{6}3\eta^2 = \eta^2$$
$$E[(T-\mu)^3] = \int (T-\mu)^3\phi(T)\,dt = \frac{1}{6}\left(\sqrt{3}\eta\right)^3 - \frac{1}{6}\left(\sqrt{3}\eta\right)^3 = 0$$
$$E[(T-\mu)^4] = \int (T-\mu)^4\phi(T)\,dt = 3\eta^4$$

$$E[(T-\mu)^5] = \int (T-\mu)^5\phi(T)\,dt = 0$$
$$E[(T-\mu)^6] = \int (T-\mu)^6\phi(T)\,dt = 9\eta^6$$

Here we note that, the first five moments of three point distribution are same as that of the normal variate $T(\mu,\ \eta^2)$. However, t starts departing when we compute the sixth moment for a normal variate T,

$$E[(T-\mu)^6] = 15\eta^6$$

It may be worth noting that for a small standard deviation η, i.e. $\eta/\mu >> 1$, the approximation will perform well.

4.2.3 Approximation of Rayleigh-Lognormal Distribution by Three Point Distribution:

Now using three point distribution we can approximate the Normal Distribution. Recall the pdf of Rayleigh-Lognormal distribution shown in eq.-(4.4), we can reduce it in a mixture distribution from with one density being normal.

$$f_U(u) = \int_0^\infty \frac{u}{x} e^{-\frac{u^2}{2x}} \frac{1}{\sqrt{2\pi\eta^2}\,x} e^{\frac{-(\ln x-\mu)^2}{2\eta^2}}\,dx \qquad x \geq 0$$

defining, $\ln x = t$

$$f_U(u) = \int_{-\infty}^{\infty} \left[\frac{u\, e^{-\frac{u^2}{2e^t}}}{e^t} \right] \left[\frac{1}{\sqrt{2\pi\eta^2}} e^{\frac{-(t-\mu)^2}{2\eta^2}} \right] dt$$

$$= E_\Phi \left[\frac{u\, e^{-\frac{u^2}{2e^t}}}{e^t} \right] \tag{4.13}$$

where $E_\Phi\left[\,.\,\right]$ denotes expectation w.r.t. normal variate.

Following [6], we approximate $f_U(u)$ by three point distribution (matching values of t with $\mu - \sqrt{3}\eta$, μ and $\mu + \sqrt{3}\eta$ from eq.-4.10 dirac's delta) we can have,

$$f_U(u) = \int_{-\infty}^{\infty} \left[\frac{u\, e^{-\frac{u^2}{2e^t}}}{e^t} \right] \left[\frac{1}{6}\delta\left[t - \left(\mu - \sqrt{3}\eta\right)\right] + \frac{2}{3}\delta\left[t - \mu\right] + \frac{1}{6}\delta\left[t - \left(\mu + \sqrt{3}\eta\right)\right] \right] dt$$

$$= \frac{1}{6}\left[\frac{u e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}}}{e^{(\mu-\sqrt{3}\eta)}} \right] + \frac{2}{3}\left[\frac{u e^{-\frac{u^2}{2e^{\mu}}}}{e^{\mu}} \right] + \frac{1}{6}\left[\frac{u e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}}}{e^{(\mu+\sqrt{3}\eta)}} \right] \quad u \geq 0 \tag{4.14}$$

As this form of $f_U(u)$ is quite convenient for analytical manipulations. We can now explicitly obtain various moments of its values U.

$$\begin{aligned} E[U] &= \int_0^\infty u\, f_U(u)\, du \\ &= \int_0^\infty u \left[\frac{1}{6} \left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}}}{e^{(\mu-\sqrt{3}\eta)}} \right\} + \frac{2}{3} \left\{ \frac{u e^{-\frac{u^2}{2e^{\mu}}}}{e^{\mu}} \right\} + \frac{1}{6} \left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}}}{e^{(\mu+\sqrt{3}\eta)}} \right\} \right] du \\ &= \frac{1}{6e^{(\mu-\sqrt{3}\eta)}} \int_0^\infty u^2 e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}} du + \frac{2}{3e^{\mu}} \int_0^\infty u^2 e^{-\frac{u^2}{2e^{\mu}}} du + \frac{1}{6e^{(\mu+\sqrt{3}\eta)}} \int_0^\infty u^2 e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}} du \\ &= \frac{1}{3a} \int_0^\infty u^2 e^{-\frac{u^2}{a}} du + \frac{4}{3b} \int_0^\infty u^2 e^{-\frac{u^2}{b}} du + \frac{1}{3c} \int_0^\infty u^2 e^{-\frac{u^2}{c}} du \end{aligned}$$

$$\textit{where,} \qquad a = 2e^{(\mu-\sqrt{3}\eta)}, \qquad b = 2e^{\mu} \textit{ and } c = 2e^{(\mu+\sqrt{3}\eta)}$$

Here there are three terms in the integral, we evaluate the integral of the first term. Noting the first term that,

$$\begin{aligned} I_1 &= \int_0^\infty u^2 e^{-\frac{u^2}{a}} du \\ &= \frac{a^{\frac{3}{2}}}{2} \int_0^\infty x^{\frac{1}{2}} e^{-x} dx \qquad \left(here, x = \frac{u^2}{2}\right) \qquad (4.15) \end{aligned}$$

is in a form of gamma function, which is defined as

$$\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx \qquad (4.16)$$

$$\textit{where} \qquad \Gamma(n+1) = n\Gamma(n), \qquad \Gamma(1) = 1 \qquad \textit{and} \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

So the eq.-(4.15) becomes,

$$\frac{a^{\frac{3}{2}}}{2} \int_0^\infty x^{\frac{1}{2}} e^{-x} dx = \frac{\sqrt{\pi}}{4} a^{\frac{3}{2}}$$

Similarly we can get the integrals of the other terms, we find,

$$E[U] = \frac{\sqrt{\pi}}{12} a^{\frac{1}{2}} + \frac{\sqrt{\pi}}{3} b^{\frac{1}{2}} + \frac{\sqrt{\pi}}{12} c^{\frac{1}{2}} \tag{4.17}$$

After substitution of a, b and c we get the final result,

$$E[U] = \frac{\sqrt{\pi}}{6} e^{\frac{\mu-\sqrt{3}\eta}{2}} + \frac{2\sqrt{\pi}}{3} e^{\frac{\mu}{2}} + \frac{\sqrt{\pi}}{6} e^{\frac{\mu+\sqrt{3}\eta}{2}} \tag{4.18}$$

Similarly we can find out the E[U²].

$$\begin{aligned}
E[U^2] &= \int_0^\infty u^2 f_U(u)\,du \\
&= \int_0^\infty u^2 \left[\frac{1}{6} \left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}}}{e^{(\mu-\sqrt{3}\eta)}} \right\} + \frac{2}{3} \left\{ \frac{u e^{-\frac{u^2}{2e^{\mu}}}}{e^{\mu}} \right\} + \frac{1}{6} \left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}}}{e^{(\mu+\sqrt{3}\eta)}} \right\} \right] du \\
&= \frac{1}{6 e^{(\mu-\sqrt{3}\eta)}} \int_0^\infty u^3 e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}} du + \frac{2}{3 e^{\mu}} \int_0^\infty u^3 e^{-\frac{u^2}{2e^{\mu}}} du \\
&\quad + \frac{1}{6 e^{(\mu+\sqrt{3}\eta)}} \int_0^\infty u^3 e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}} du \\
&= \frac{1}{3a} \int_0^\infty u^3\, e^{-\frac{u^2}{a}}\, du + \frac{4}{3b} \int_0^\infty u^3\, e^{-\frac{u^2}{b}}\, du + \frac{1}{3c} \int_0^\infty u^3\, e^{-\frac{u^2}{c}}\, du
\end{aligned}$$

where $a = 2e^{(\mu-\sqrt{3}\eta)}$, $b = 2e^{\mu}$ *and* $c = 2e^{(\mu+\sqrt{3}\eta)}$

There are three terms in the integral. We evaluate the integral of the first term,

$$\begin{aligned}
I_1' &= \int_0^\infty u^3 e^{-\frac{u^2}{a}} du \\
&= \frac{a^2}{2} \int_0^\infty x\, e^{-x} dx \qquad (here, x = \frac{u^2}{2})
\end{aligned} \tag{4.19}$$

which is again in a form of gamma function, so the eq.-(4.19) becomes,

$$\frac{a^2}{2}\int_0^\infty x\, e^{-x}dx = \frac{a^2}{2}\Gamma(1+1)$$

$$= \frac{a^2}{2}$$

In similar manner we can get the integrals of the other terms that make $E[U^2]$,

$$E[U^2] = \frac{a}{6} + \frac{2b}{3} + \frac{c}{6} \qquad (4.20)$$

Substituting the values of a, b and c

$$E[U^2] \qquad = \frac{e^{(\mu-\sqrt{3}\eta)}}{3} + \frac{4e^{\mu}}{3} + \frac{e^{(\mu+\sqrt{3}\eta)}}{3} \qquad (4.21)$$

4.2.4 Calculation of kth Moment:

It is easy to consider the general case of kth moment.

$$E[U^k] = \int_0^\infty u^k f_U(u)\,du$$

$$= \int_0^\infty u^k \left[\frac{1}{6}\left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}}}{e^{(\mu-\sqrt{3}\eta)}} \right\} + \frac{2}{3}\left\{ \frac{u e^{-\frac{u^2}{2e^{\mu}}}}{e^{\mu}} \right\} + \frac{1}{6}\left\{ \frac{u e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}}}{e^{(\mu+\sqrt{3}\eta)}} \right\} \right] du$$

$$= \frac{1}{6e^{(\mu-\sqrt{3}\eta)}} \int_0^\infty u^{k+1} e^{-\frac{u^2}{2e^{(\mu-\sqrt{3}\eta)}}} du + \frac{2}{3e^{\mu}} \int_0^\infty u^{k+1} e^{-\frac{u^2}{2e^{\mu}}} du$$

$$+ \frac{1}{6e^{(\mu+\sqrt{3}\eta)}} \int_0^\infty u^{k+1} e^{-\frac{u^2}{2e^{(\mu+\sqrt{3}\eta)}}} du$$

$$= \frac{1}{3a}\int_0^\infty u^{k+1} e^{-\frac{u^2}{a}} du + \frac{4}{3b}\int_0^\infty u^{k+1} e^{-\frac{u^2}{b}} du + \frac{1}{3c}\int_0^\infty u^{k+1} e^{-\frac{u^2}{c}} du \qquad (\;)$$

where

a, b and c have the values previously discussed

We evaluate the integral of the first term,

$$\int_0^\infty u^{k+1} e^{-\frac{u^2}{a}} du$$

$$= \frac{a^{\frac{k}{2}+1}}{2} \int_0^\infty x^{\frac{k}{2}} e^{-x} dx \qquad (here, x = \frac{u^2}{2})$$

$$= \frac{a^{\frac{k}{2}+1}}{2} \Gamma\left(\frac{k}{2}+1\right)$$

Similarly we can get the integrals of the other terms that make $E[U^k]$,

$$E[U^k] = \frac{a^{\frac{k}{2}}}{6}\Gamma\left(\frac{k}{2}+1\right) + \frac{2b^{\frac{k}{2}}}{3}\Gamma\left(\frac{k}{2}+1\right) + \frac{c^{\frac{k}{2}}}{6}\Gamma\left(\frac{k}{2}+1\right) \qquad (4.22)$$

Substituting the values of a, b and c

$$E[U^k] = \frac{a^{\frac{k}{2}}}{3}\Gamma\left(\frac{k}{2}+1\right) + \frac{4b^{\frac{k}{2}}}{3}\Gamma\left(\frac{k}{2}+1\right) + \frac{c^{\frac{k}{2}}}{3}\Gamma\left(\frac{k}{2}+1\right)$$

$$E[U^k] \quad = \frac{e^{\frac{k(\mu-\sqrt{3}\eta)}{2}}}{3} + \frac{4e^{\frac{k\mu}{2}}}{3} + \frac{e^{\frac{k(\mu+\sqrt{3}\eta)}{2}}}{3} \qquad (4.23)$$

4.3 Statistical evaluation of $\mathrm{K_E}^{(h)}$:

In eq.-4.18 and eq.-4.21 we have computed

$E[U_0]$, $E[U_1]$ and $E[U_0^{\,2}]$ & $E[U_1^{\,2}]$

For computation of $\mathrm{K_E}^{(h)}$,

$$E\left(K_E^{(h)}\right) = E\left(\frac{U_0}{U_h}\right)E\left[K_E^{(h)}\right] \tag{4.24}$$

$$\mathrm{Var}\left[K_E^{(h)}\right] = \mathrm{E}\left[\frac{U_0^{\,2}}{U_h^{\,2}}\right]\ E\left[\left(K_0^{(h)}\right)^2\right] - E\left[K_E^{(h)}\right]^2, \tag{4.25}$$

Under the assumption of independence of U_0 and U_1,

$$E\left(K_E^{(h)}\right) = \frac{E[U_0]}{E[U_h]}\left[E\left(K_0^{(h)}\right)\right] \text{ and}$$

$$\text{and } \mathrm{Var}\left[K_E^{(h)}\right] = \frac{E\left[U_0^{\,2}\right]}{E\left[U_1^{\,2}\right]}\left[E\left[W^2\right] - \left[E(W)\right]^2\right]$$

The computation of $K_E^{(h)}$ requires knowledge of moments of U_0, U_h and using moments of U_0 and U_h from equation,

$$E\left[K_E^{(h)}\right] = \frac{\int_0^\infty u\, f_{U_0}(u)du}{\int_0^\infty u\, f_{U_h}(u)du}\, E[W] \tag{4.26}$$

$$\mathrm{Var}\left[K_E^{(h)}\right] = \frac{\int_0^\infty u^2\, f_{U_0}(u)du}{\int_0^\infty u^2\, f_{U_h}(u)du}\, \mathrm{Var}[W] \tag{4.27}$$

Eq.-4.26 & eq.-4.27 will yield results for a more realistic situation. We have thus been able to extend the results of Abrardo et. al. [2]. When fading is derived by Rayleigh-Lognormal distribution.

Chapter 5

Comparison and Analysis:

After performing numerical simulation for both Rayleigh-Lognormal distribution (RLD) and Three Point Distribution, the output that is quite satisfactory. Computing the RLD with proposed three point approximation gives good results for the parameter values such that $\frac{\eta}{\mu} << 1$.

Here numerical simulation is done for different parameter values of μ and η.

Case 1:

For μ =2.0 and η = 1.0, three point distribution almost approximates the Rayleigh Lognormal distribution.

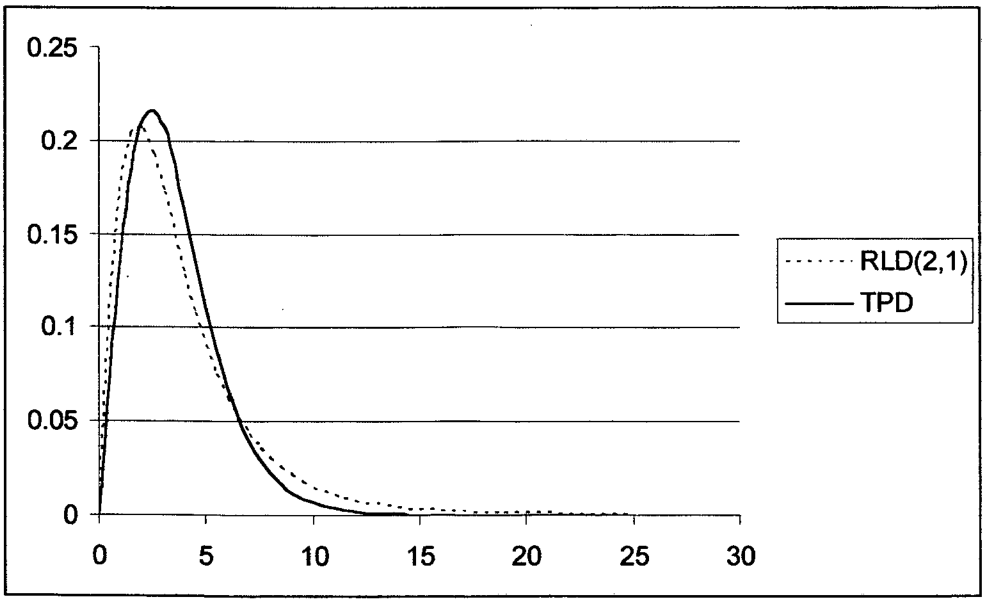

Figure 5.1 - Comparison of RLD (Rayleigh Lognormal distribution) and Three Point distribution for μ =2.0 and η = 1.0, $\eta/\mu = 0.5$

Case 2:

For μ =2.0 and η = 0.75, three point distribution approximated the RLD better than the previous case.

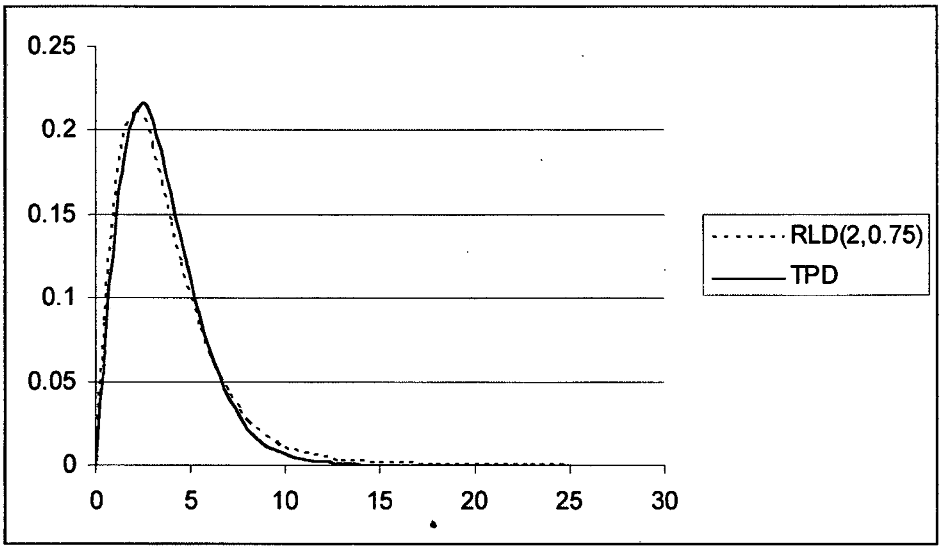

Figure 5.2 - Comparison of RLD (Rayleigh Lognormal distribution) and Three Point distribution for μ =2.0 and η = 0.75, $\eta/\mu = 0.375$

Case 3:

For μ =2.0 and η = 0.5, three point distribution approximated the RLD the best among the three cases.

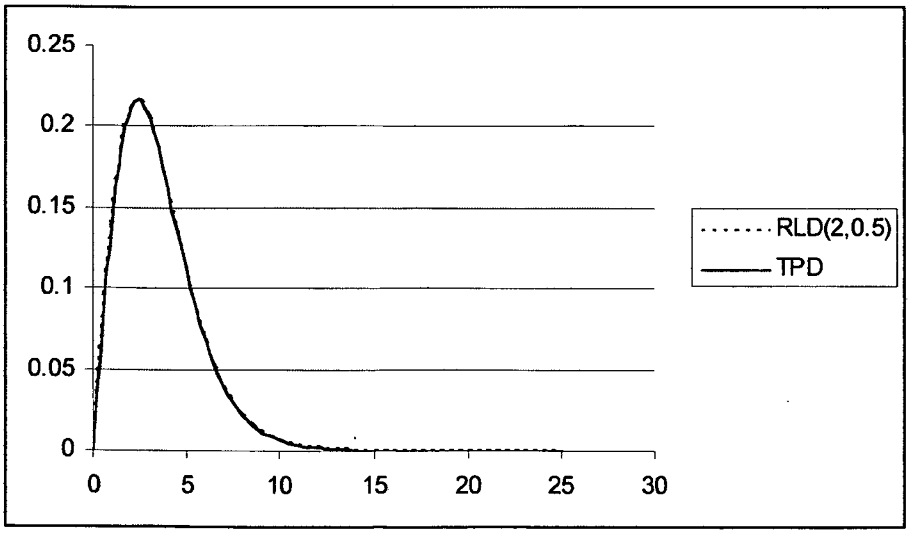

Figure: Comparison of RLD (Rayleigh Lognormal distribution) and Three Point distribution for μ =2.0 and η = 0.5, $\eta/\mu = 0.25$

It is noted that proposed three point distribution becomes the best as η/μ becomes smaller. We realized that this approximation should be employed for low values of η/μ.

Chapter 6

Future Suggestions and Conclusion:

We find the approximation for Rayleigh Lognormal distribution using three point distribution is quite versatile particularly for small values of η. However, for large value of η, the approximation may not yield proper results. It may also be mentioned that for large η, the pdf approximation of RLD may become bimodal distribution.

Based on the simulation results the QoS parameters like outage probability, average fading, can be obtained. The performance of the proposed approximation for different modulation scheme will be the future scope of this work.

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