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## CALL ADMISSION CONTROL IN MOBILE NETWORKS

Dissertation Submitted to JAWAHARLAL NEHRU UNIVERSITY in partial fulfilment of requirements for the award of the degree of Master of Technology in Computer Science

by

#### K. VENKATA RAMANA



SCHOOL OF COMPUTER & SYSTEMS SCIENCES JAWAHARLAL NEHRU UNIVERSITY NEW DELHI - 110 067 January 1997

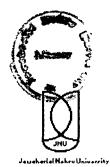
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#### DECLARATION

This is to certify that the dissertation entitled "CALL ADMISSION CONTROL IN MOBILE NETWORKS " which is being submitted by me to the School of Computer & Systems Sciences, Jawaharlal Nehru University for the award of Master of Technology in Computer Science, is a record of bonafide work carried out by me.

This work is original and has not been submitted in part or full to any university or institution for the award of any degree.

le levetata havene K.Venkata Ramana

(M.Tech, III Sem)



### जवाहरलाल नेहरू विश्वविद्यालय JAWAHARLAL NEHRU UNIVERSITY NEW DELHI - 110067

#### CERTIFICATE

This is to certify that the dissertation entitled " CALL ADMISSION CONTROL IN MOBILE NETWORKS " which is being submitted by Mr. K. VENKATA RAMANA to the School of Computer & Systems Sciences, Jawaharlal Nehru University, for the award of Master of Technology in Computer Science, is a record of bonafide work carried out by him.

This work is original and has not been submitted in part or full to any university or institution for the award of any degree.

Prof. G.V.Singh (Dean SC & SS)

Prof. Karmeshu (Supervisor)

School of Computer & Systems Sciences Jawaharlal Nehru University New Delhi

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I take this opportunity to thank all of my faculty members and friends for their help and suggestions during the course of my project work.

K. Venkata Ramana

..... dedicated to my beloved parents

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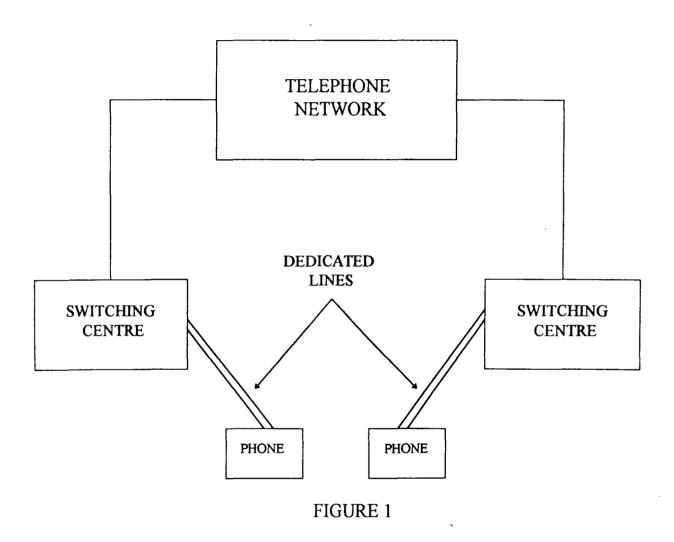
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The rapid growth in the demand for Mobile Communications has led the researchers and industry into development efforts towards a new generation of mobile systems (Tekinay and Jabbari,1991). The mobile communications itself has become a separate field of study called Personal communications which starts with a person carrying a device that sends, receives , stores and processes information. Personal communications includes cordless telephones, cellular telephones and mobile computers (Goodman).

The personal communication device sends and receives information according to the needs of the person like a personal computer. Each owner has a 'personal profile' which satisfies service features such as call waiting, caller identification, selective call forwarding and voice mail. Each person has a 'personal subscriber number' which is like a telephone number. This number belongs to only one person who can use it on any machine and at any place supporting the mobile communications (Satyanarayanan,1996).

#### 1. Telephone Network

A telephone network is a dedicated network of telephone lines as shown in figure1 By lifting the receiver, we dial the number and let the switching office know that we want to have a connection. Using this dialled number, the switching office connects the line attached to our phone with the line attached to the phone to which we want to get connected. After this connection is established, a dedicated line would be there till the receiver is kept down.



## 2. Mobile Network

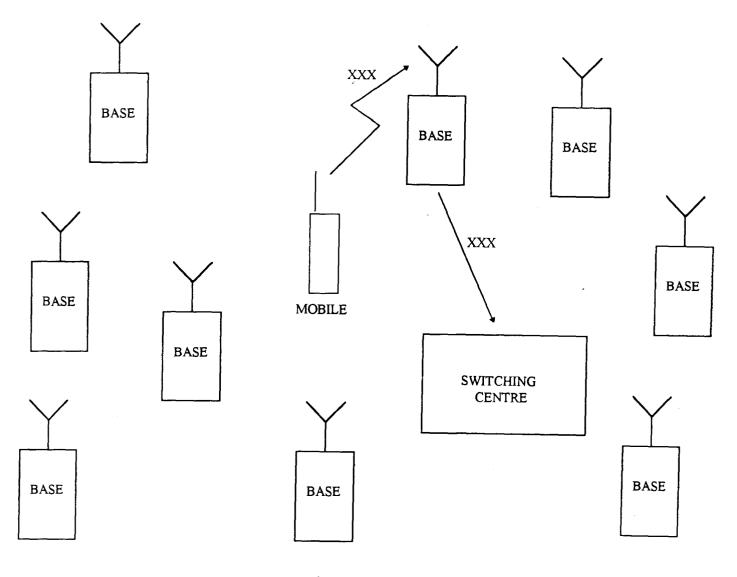
The process of making a call with a cellular phone is quite different. A cellular phone is not connected to the switching station. Instead it uses a radio channel to establish connection to the nearest office called 'base station'. A cellular service area contains a

two dimensional array of base stations separated by some distances ranging from about one kilometer to ten kilometer depending on the population density of mobile subscribers. The region served by each base station is called a cell, hence the term derives cellular telephone(Hac.1995).

When we turn on the cellular phone, its radio receiver scans a set of radio 'control channels' and measures the signal strength arriving at each base station. Generally the stronger signal comes from the base station which is nearer to the device. So, the radio in the cellular phone tunes to the control channel of the nearest base station. The control channel transmits information in both the directions.

#### 2.1 Call Initiation

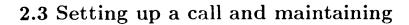
For initiating a call, a mobile uses the control channel to send the number it is calling to the nearby base station. As all the phones in a cell use the same channel to initiate a call, a cellular phone has to identify itself to the network. In addition to transmitting number and its identity, the phone has to establish its authorization to use the network. This is due to the fact that with signals going through air, the cellular phone company has to take care regarding the unauthorised use of the network. When a base station receives a call initiation message it transmits to the 'switching center' as in figure2 (adopted from the paper by Goodman). Cellular switching center can handle calls of about one hundred base stations. A switching center is connected to the worldwide telephone network. The switching center controls the radio channels at all of its base stations in assigning a channel to the phone calls. As control signals and cellphone call signals have to be transmitting through the same channel, around 5% of its channel would be reserved for transmitting control signals.

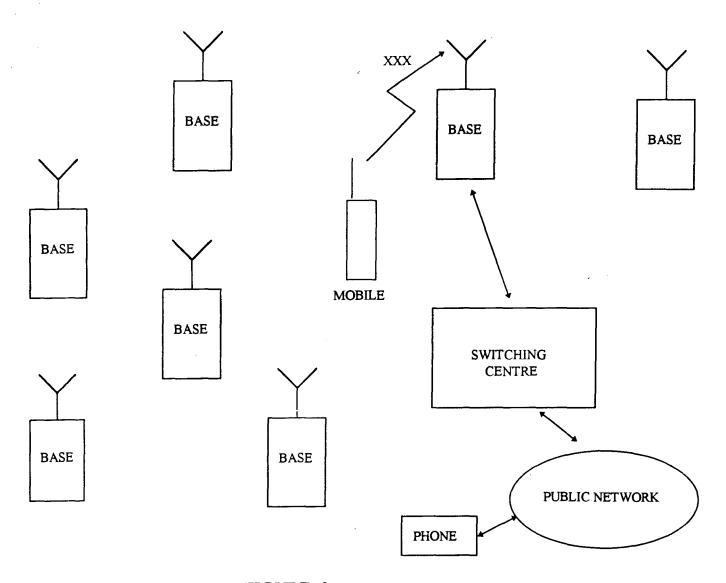




A radio channel would be carrying several calls simultaneously, but the two calls which are in the same channel must sufficiently be apart to prevent their signals interfering with each other. This minimum spacing is called channel reuse distance. It has a significant impact on the capacity of a cellular telephone network. If the channel reuse distance is small the channel can accomodate a large number of calls for the given geographical area (Baiocchi and Sestini,1996).

4





#### FIGURE 3

In setting up a call, the switching center takes the dialled number and decides which channel to be used. Then the phone tunes to that specified channel and connects it to the nearest base station where the other phone is present. After the voice travels from the other phone to the caller, the connection is said to be established. So, the caller's voice travels from the cellphone through the local base station to the switching center and thence to the other phone as in figure3 (adopted from the paper by Goodman). It is evident that a mobile user would be travelling while the connection is established. During this course, the phone may move away from the original base station resulting in transmitting a weak signal. The switching center knows that the signal received at the original base station is very weak for transmission and commands the radio to tune to another channel which is suitable for communicating with the adjacent base station. Thus the controller of the switching station hand-overs the call to another base station. This 'hand-off' or 'hand-over' is a critical operation which occurs in mobile networks. A call may have several hand-offs during the event of its communication.

#### 2.4 Search of a mobile phone

Another important aspect of a cellular phone is the search of a cellular phone. When someone dials a telephone, the network has to search the mobile to which communication path has to be established. For this the switching center sends 'paging' messages to several base stations. These base stations detects the presence of the particular mobile in its geographical area. If the mobile is there, it sends back a paging message about the presence. Then the switching office connects the phone dialled to the base station from where the paging message is received. This results in connecting that base station to the other phone. But this becomes impractical when the number of mobiles become very large and change takes place frequently as the paging messages may overflow the control channel capacity.

In present systems, the mobile network stores its location information at two places in order to avoid too much paging. The mobile's location is stored at the phone's present location and at the subscribers home switching center. Consider a situation where a mobile subscriber at New Delhi visits Paris and someone calls him from Paris, the call first goes to New Delhi where the database informs the switching center that the 'mobile' is in Paris. Then the call diverts back to Paris where a local connection is established.

## 3. Technical Challenges

In view of the above, we can say that personal communications differ widely from conventional wired communications adding complexity to the communication task. Goodman suggested a few differences between conventional and mobile networks which are listed below:

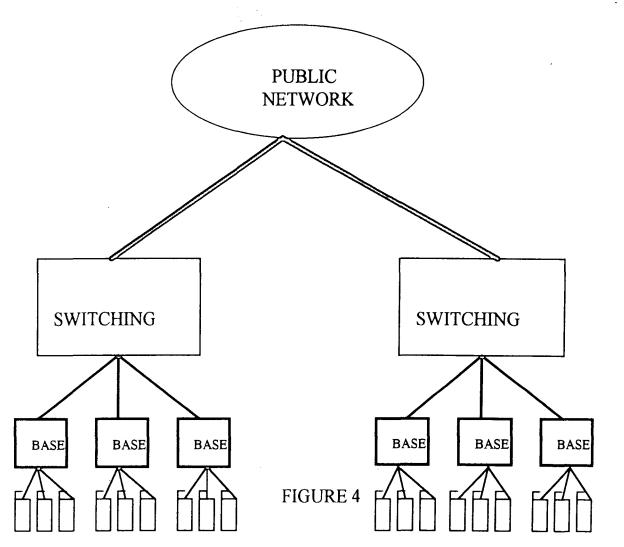
In a conventional telephony network, each subscriber has a dedicated pair of wires connecting the subscriber to the telephone network. But the cellular system has a shared transmission medium linking the subscribers and the base station.

	Conventional	Personal
Linktonetwork	High Quality, dedicated	Variable quality, shared
Capacity	Depends on economics	Depends on government
Powersupply	External	Self contained
Configuration	Infrequent	Always changing

A major constraint on personal communications is the limited capacity or bandwidth available for transmission of information between base stations and mobile terminals (Katzela and Naghshineh,1996). The bandwidth available will depend on the government agencies which regulates the use of radio spectrum. The conventional telephone network capacity can be increased by increasing the telephone wired lines. The power supply for a mobile radio must be provided in the system itself in the form of a battery cell. This imposes an obstacle in designing the size of the radio device.

Another major challenge is that the configuration of the personal communication network changes frequently as the mobile phone moves from one place to the other. The network has to take care about the mobility of these radio devices. In order to maintain the quality, the network may have to change the radiated power levels and/or change the base station. The network has to record the locations of moving subscribers, even though they don't have a call. This dynamic information helps a lot in determining the region where the mobile is present.

4. Network Architecture



An example of a Cellular network architecture is shown in figure4. The main elements of the cellular network are base stations, a switching network, public switched net-

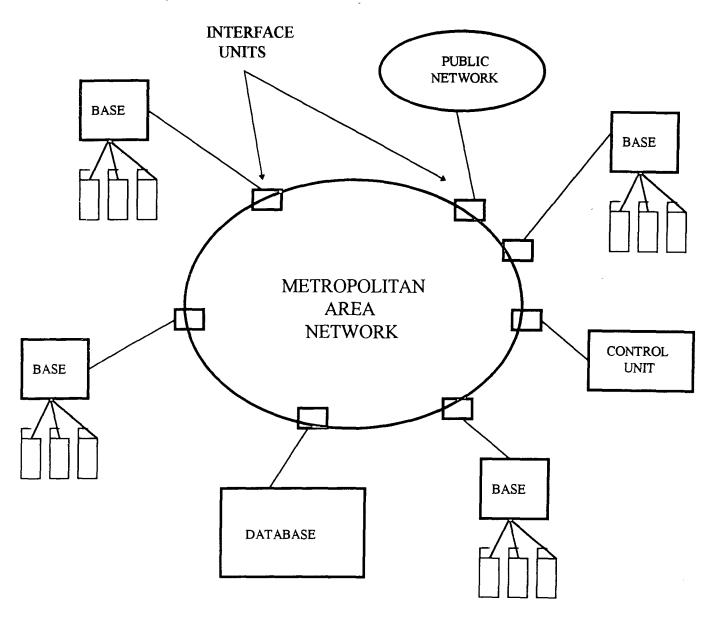
work(PSTN), lines connecting base stations, switching network and PSTN, and the mobile subscribers. Each base station directly communicates with the mobiles in its cell. The base stations are connected to the switching network via land lines. The switching network makes important decisions regarding channel allocation, call handoff and controls all of the communications in its service area. All the switching centers are connected to the PSTN. A mobile network is hierarchical in nature.

In the context of making a call, the subscriber enters the mobile phone number and presses the send button on the phone. Every base station has a reserved control channel capacity, through which the user sends a request for call initiation. Every base station frequently monitors the control channels and when a 'send' command is requested, the base station which receives the strongest signal takes the control. If the channel capacity in the particular cell area is available, the base station of the mobile from its database and depending upon the availability of the channel, the connection is established. When the conversation is completed, the user sends a disconnect signal to the base station, which in turn relays to the switching network. The switching center frees the channel and can allocate to an another user. During the course of conversation, if the user moves to another cell, then the problem of cell handoff should be considered. Suppose a user moves from  $ccll_1$  to  $ccll_2$  then both the base stations would be monitoring the signal and sends the control signals to the switching center. The switching center makes a decision of handoff considering the relative strengths of the signals at the two base stations.

#### 4.1 Circuit switching and packet switching

The connections among the base stations and switching center may be "packet switching" or "circuit switching. A packet switched system makes packets of information. Each

packet contains a payload containing the information for communication and an address which helps the network in delivering the information accurately and on time. It is like a postcard which has information and space for address. The network divides long communications into packets and sends each one at a time from one point to another (Cho and Marshall,1995).



#### FIGURE 5

In a circuit switched network, a dedicated path is established and the information is sent continuously over this path. Once the transmission is completed, the path or connection would be disconnected. A circuit switched network uses a control channel to

assign resources. A packet switched network is a logical connection. It need not have a centralized control, rather each point can act as a localized control for routing the packets (Goodman). An example of a packet switched network is shown in figure 5 (reproduced from the paper by Goodman).

There are a number of problems during the operation of a cellular network. Some of the problems is adjacent channel interference, cochannel interference, fading and channel reuse distance. Network management is an important aspect in the design of the network. An efficient network management results in offering a better quality of service to the subscribers.

## 5. Network Management

The principle task of a personal communication network is to move information to and from the people. In order to keep track of the bandwidth, energy and mobility, the network has to store and process its own internal information. As proposed by Goodman the most important problems of a personal communication network are

- 1. Call management
- 2. Channel allocation and resource management
- 3. Call admission control
- 4. Power control
- 5. Handover management
- 6. Mobility management

- 7. Database management
- 8. Interaction with signal processing

#### 5.1 Call management

Call management is necessary in any information networks. It involves in setting up and terminating communication sessions and providing special services such as tele conferencing, call waiting, call number identification, billing and others. In order to make personal communication systems similar to conventional ones, many call management techniques have been developed.

#### 5.2 Channel allocation and radio resource management

The radio resource management problem is to assign at every instant of time, a physical channel and assigning transmitted power levels, for the terminal and for the base station. Efficient utilization of the channel or spectrum is a major challenge in the design of cellular systems. Channel assignment strategies can be divided into three types namely fixed, flexible and dynamic (Tekinay and Jabbari,1991).

A fixed channel assignment strategy implies that a predetermined set of channels are reserved for each base station. When a call attempt is made the cell is admitted or rejected depending on whether channel capacity at that particular base station is available or not.

In a dynamic assignment policy, all the channels are held by the switching center and allocates the channel when a call request is made. Here the job of the base station is to just inform the switching center about the calls and handoffs. In the flexible channel assignment policy, part of the channel capacity is reserved for the base stations and the rest is held by the switching center. When a call request is made, the base station allocates the channel if available, otherwise it requests the switching center for allocation of the channel.

#### 5.3 Call admission Control

Call admission control is a dynamic policy which determines whether a call has to be admitted or not. It takes into account different strategies such as priority call handling, call handoffs, arrival and departure rates. When a handoff occurs the call admission policy has to give priority to the handoff call over new call as the mobile subscribers expect very low call dropping probability over the new call blocking probability. The call admission control policy has to monitor the dynamic nature of the number of handoffs, as the handoffs may be very high during the peak hours.

#### 5.4 Power control

The main aim of the power control is to deliver to each mobile a signal that is strong enough to overcome noise interference of any other signal in the mobile spectrum. Power control in a mobile terminal is also important in conserving energy. This power supply has an impact on the size of the mobile system. But due to the advancement of the technology, button sized cells have been developed and mobile systems become handy in size.

#### 5.5 Handover management

Handover is a phenomena that transfers an ongoing call from one cell to another as a user moves through a geographical area of a mobile network system. As smaller cells are employed to meet the demands for increased capacity(Lee,1991), the chances of cell boundary crossings increases. Each handover makes an extra burden for the network . Each time a handover happens, the cell has to be rerouted to the new base station. Smaller the number of handovers, smaller would be the switching load. Handovers have a direct impact on the perceived quality of service (QoS) as delayed handover deteriorates the QoS to below an accepted level.

Handovers are of two types, one is a hard handover which occurs when the old connection is broken before a new connection established. On the other and a soft handover is one where a connection is established even before the old connection is broken.

#### 5.6 Mobility management

The objective of the mobility management is to keep track of the each mobile so that they can be reached when necessary. During the early stages of mobile network design the switching center used to send paging messages to all base stations and upon response from one base station, connection will be established. This phenomena becomes quite complex. Researchers have explored advanced techniques for this and termed as " intelligent mobility management ". This strategy records the presence of a mobile in the mobile's home switching center. It maintains the mobility according to the statistical patterns of the mobiles during the past.

#### 5.7 Database management

This has an impact on the quality of service (QoS) at every stage of the call. The data update access time must be very very small so as to give fast switching to the mobile. The database must have less number of searchings and a distributed database scheme may be needed. The database must be designed in such a way that the control signals could be transmitted quickly than the billing signals.

#### 5.8 Interaction with signal processing

The network architecture and network management has a strong influence on the quality of mobile communication systems to meet subscribers expectations. Equally important are the efficient and effective working of the hardware devices and elements such as modulation, channel coding, source coding, equalization antennas, transmitters, receivers and terminal design.

An understanding of the system is very essential to carryout the performance related issues for efficient design of the system. Development of mathematical models and their analysis becomes very essential for a better understanding and control of the system. A mathematical model clearly gives an idea of how the actual system responds. In the following chapter we give a model of a distributed call admission policy in which the nearest neighbouring cells are considered. The analysis of the model provides policies for call admission control with uniform and symmetric arrival, departure and handoff rates. Chapter 3 deals with the generalization of the proposed model with non-uniform and asymmetric rates in which the distant cells are also considered. chapter 4 deals with the transient analysis of the call admission control policy.

# Distributed model for call admission policy with symmetric rates

Intensive research in the field of mobile communications has led to the development of advanced solutions for the problems of mobile communication networks. A large number of solutions for call admission control as well have been developed but none is a comprehensive solution. Every solution lacks something or the other.

#### 1. One dimensional model

Naghshineh and Schwartz(1996) proposed a distributed call admission policy based on constant arrival departure and handoff rates by taking into account the nearest neighbouring cells. They have considered a one-dimensional array of cells such as the streets and highways assuming a fixed channel allocation scheme for the system.

Notation

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 $C_n$ : Cell where the call admission request is made

 $C_r, C_l$ : Adjacent cells of  $C_n$ 

n,l,r: Number of calls in  $C_n, C_l, C_r$  respectively

 $\lambda$ : Arrival rate of calls at a cell

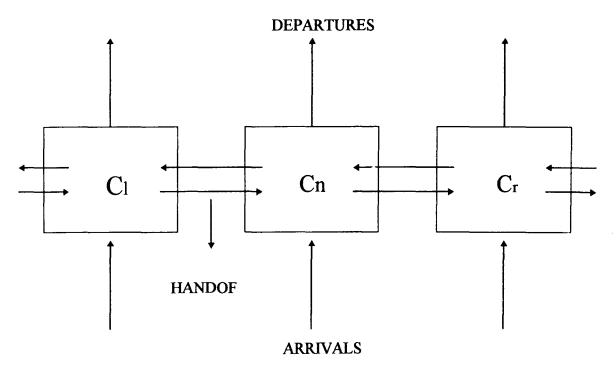
 $\mu$ : Departure rate from a cell

h: hand-off rate to a neighbouring cell

N: Maximum number of calls that the system can support

 $p_s$  : probability that a call stays in the same cell

 $p_{\frac{m}{2}}$ : the probability that it moves to an adjacent cell



#### FIGURE 6

The handoff dropping probability equals the overload probability  $P_0 = \sum_{N=1}^{\infty} P_i$  where  $P_i$  is the probability of having i calls in a cell. By admitting a new call and after handoffs to and from neighbouring cells the desired QoS has to be maintained. A new call is admitted to cell  $C_n$  at time  $t_0$  if and only if the following two conditions are satisfied, one is that

during a small interval T from  $t_0$  the overload probability of cell  $C_n$ , affected by handoffs into and out of the cell, must not exceed  $P_{QoS}$ . The second condition is that during the same time interval the overload probability of adjacent cells of  $C_l$  and  $C_r$  affected by handoffs out of and into the cell as also newly admitted calls must not exceed  $P_{QoS}$ . Naghshineh and Schwartz assume that the handoff to cell  $C_l$  and  $C_r$  have equal probability

Denoting by k the number of calls in a cell at time  $t_0$ , the probability that i out of these remain in the same cell is the binomial distribution  $B(i, k, p_s)$  and j out of k would be in adjacent cell  $C_l$  is given by  $B(j, k, \frac{p_m}{2})$  where

$$B(i,k,p) = \binom{k}{r} (p)^{i} (1-p)^{k-i} \qquad i = 0, 1, \dots, k$$
(1)

The distribution  $P_{n_0t_0+T}$ , which is the number of calls in cell  $C_n$  at time  $t_0 + T$  is the convolution of the three binomial distributions  $B(i_n.n,p_s)$ ,  $B(i_l,l,\frac{p_m}{2})$  and  $B(i_r,r,\frac{p_m}{2})$ . For large n,l,r it can be approximated by the Gaussian distribution  $G(m,\sigma)$  (see appendix1). Thus

$$P_{n_0 t_0 + T} \simeq G(m, \sigma^2) \tag{2}$$

where

$$m = np_s + (l+r)\frac{p_m}{2},$$

and

$$\sigma^{2} = \left( n p_{s} (1 - p_{s}) + (l + r) \frac{p_{m}}{2} (1 - \frac{p_{m}}{2}) \right)$$

ð

and the overload probability is given by the tail of the Gaussian distribution as

$$P_{0} = \frac{1}{\sqrt{2\pi}} \int_{N}^{\infty} exp\{-\frac{u-m^{2}}{2\sigma^{2}}\} du$$
(3)

which equals

$$= Q\left(\frac{N-np_{s}-(l+r)\frac{p_{m}}{2}}{\sqrt{\left(np_{s}(1-p_{s})+(l+r)\frac{p_{m}}{2}(1-\frac{p_{m}}{2})\right)}}\right)$$
(4)

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{1}{2}t^2 dt}$$
(5)

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is the integral over the tail of the Gaussian distribution. This integral can be expressed in terms of the error function. For a given  $P_{QoS}$  there would be a value such that  $P_{QoS} = Q(a)$ .

$$\Psi(y) = \frac{2}{\sqrt{\pi}} \int_{0}^{y} e^{-x^{2}} dx$$

$$= 1 - 2Q(y^{\sqrt{2}})$$
(6)

We can then solve for n which gives the admission threshold, the maximum number of calls that can be admitted satisfying the first admission condition. Thus we get a quadratic in n

$$\left\{N - np_s - (l+r)\frac{p_m}{2}\right\}^2 = a^2 \left\{np_s(1-p_s) + (l+r)\frac{p_m}{2}(1-\frac{p_m}{2})\right\}$$
(7)

Distributed model for call admission policy with symmetric rates where smallest root is the required solution i.e.

$$n = n_1 = \frac{1}{2p_s} \left\{ a^2 (1 - p_s) + 2N - p_m (r + l) - a\sqrt{a^2 (1 - p_s)^2 + 4N(1 - p_s) - p_m^2 (r + l) + 2p_m p_s (r + l)} \right\}$$
(8)

To calculate  $n_2$  the threshold value of n for  $C_n$  when there are arrivals at  $C_r$  and that too at a poisson rate  $\lambda$ , we make the following additional assumption.

There are E(n) in the cell to the right of  $C_r$  and  $i_l$  of these handoff to cell  $C_r$  during time interval  $(t_0, t_0 + T)$ . The assumption about E(n) has been made, for want of knowledge of the actual number, as a compromise between the amount of information regarding calls. for efficient utilization of network resources, given that there are r mobiles in cell  $C_r$  at time  $t_0$ , the probability of having k cells in cell  $C_r$  at the time  $t_0 + T$  is given by the convolution of the binomials  $B(i_n, n, \frac{p_m}{2})$ ,  $B(i_r, r, p_s)$ ,  $B(i_l, E(n), \frac{p_m}{2})$  and the poisson probability

$$\frac{(\lambda T)^{\nu} e^{-\lambda T}}{\nu!} \tag{9}$$

where  $\mu$  is the newly admitted calls at the cell  $C_r$ . This convolution can be approximated by the Gaussian distribution

$$G\left(rp_{s} + (n+E(n))\frac{p_{m}}{2} + \lambda T, \left(rp_{s}(1-p_{s}) + (n+E(n))\frac{p_{m}}{2}(1-\frac{p_{m}}{2}) + \lambda T\right)\right)$$

$$(10)$$

the right tail of which gives the overload probability for given  $P_{QoS} = Q(a)$ , we have as proceeding as before

$$n_{2} = \frac{1}{2p_{m}} \left\{ a^{2}(2 - p_{m}) + 4N - 4\lambda T - 2E(n)p_{m} - 4rp_{s} - a\sqrt{a^{2}(2 - p_{m})^{2} + 16N + 8p_{m}(\lambda T - rp_{s} - N) - 16rp_{s}^{2}} \right\}$$
(11)

This is the number of calls that can be admitted into cell  $C_n$  so that the second call admission condition is satisfied for  $C_r$ .

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Similarly we can determine  $n_3$ , the number of calls that can be admitted into cell  $C_n$  such that the second call admission for  $C_l$  is satisfied. The final threshold for  $C_n$  such that all the admission conditions are satisfied is given by

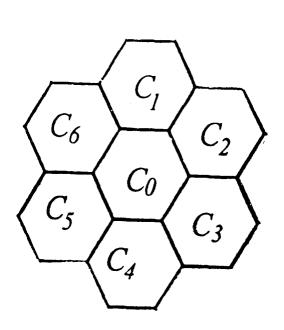
$$n = \min(n_1, n_2, n_3)$$



### 2. Extension to 2D model

The call admission control policy has been extended to 2-d hexagonal cells. The probability that a mobile handoffs to a neighbouring cell is  $\frac{p_m}{6}$  and the probability that it stays in the same cell is  $p_s$ . Similar to the 1-D case the call admission conditions are specified.

Here the neighbouring cell condition would be extended to all the six neighbouring cells.



The probability distribution of the number of calls in cell  $C_0$  at time  $t_0 + T$  is given by the convolution of seven binomials, which can be approximated by a Gaussian distribution

$$G(m_0, \sigma_0^2) \tag{13}$$

where

$$m_0 = n_0 p_s + \sum_{i=1}^6 \frac{p_m}{6} n_i \tag{14}$$

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Distributed model for call admission policy with symmetric rates

and

$$\sigma_0^2 = n_0 p_s (1 - p_s) + \left(\sum_{i=1}^6 n_i\right) (1 - \frac{p_m}{6}) \frac{p_m}{6}$$
(15)

for large  $n_i$  the overload probability  $P_0 = Q(a)$  can be approximated by the right tail of the Gaussian distribution. Thus we determine the threshold  $n_0^a$  for the first admission condition. Similarly for the second admission condition, the thresholds  $n_1^a, n_2^a, \ldots, n_6^a$  are calculated. The final threshold is given by

$$n = \min(n_0^a, n_1^a, \dots, n_6^a)$$
(16)

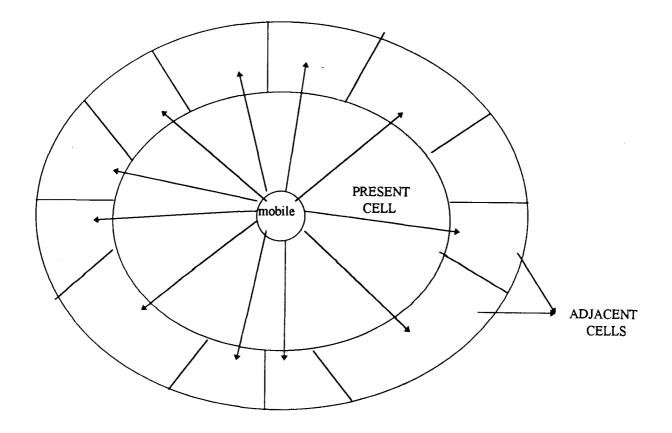
where 'n' is the maximum number of calls that can be admitted to cell  $C_0$  such that all the admission conditions are satisfied.

In this model the arrival, departure and handoff rates are taken as constant and symmetric and only the nearest neighbours are considered for handoffs. This model can be extended in a more general way by considering all cells, asymmetric handoff rates and variable arrival, departure rates for different cells.

## Generalized model for call admission policy with asymmetric rates

The model given by Naghshineh and Schwartz(1996) consider only the neighbouring cells for handoffs, but due to the connectivity of the cells, we have to consider the distant cells as well. Further the handoff rates have been taken as symmetric which in general not true as the calls do not move uniformly in 2-D space. For example, in congested areas mobiles may have slow down and spend a considerable amount of time in the same cell whereas in non-congested areas mobiles move very fast spending a small or negligible amount of time in the same cell. In case of negligible amount of time one may consider occurances of non-neighbouring transitions as well. However, the situation corresponding to non-neighbouring transitions exists in 2-D case as shown in figure.

The following is a generalized model for the distributed call admission control policy with asymmetric transition or handoff rates.





## 1.One dimensional model

We consider a one dimensional array of cells. Here we assume that the system employs fixed channel allocation (FCA) strategy.

Notation

- $n_i$ : number of calls in  $i^{th}$  cell  $C_i$  at time t
- $\lambda_i$ : arrival rate of calls

Generalized model for call admission policy with asymmetric rates

 $\mu_i$ : departure rate

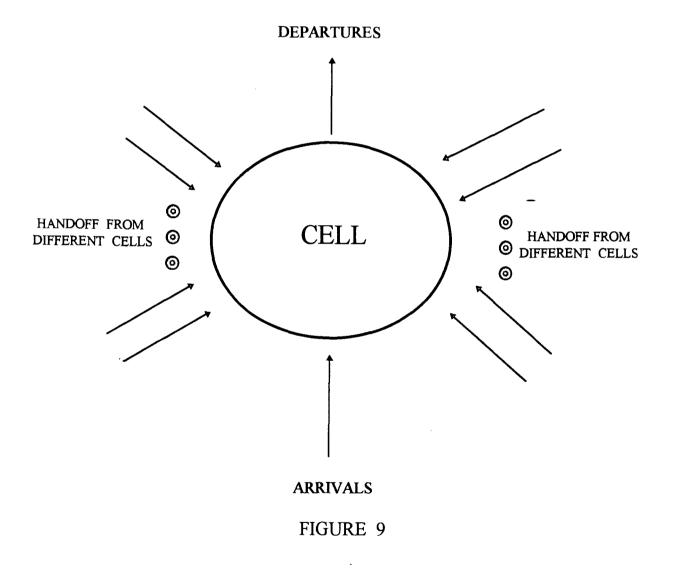
 $p_{ij}$ : probability of handoff transition from  $i^{th}$  cell to  $j^{th}$  cell,  $i \neq j$ 

.

 $p_{ii}$ : probability of staying in the same cell  $C_i$ 

 $q_{ij} : (1-p_{ij}), \ j \neq i$ 

N: maximum number of calls that the system can support



A call can make a transition from any  $i^{th}$  cell to  $j^{th}$  cell in a small interval of time.

Let  $C_n$  be the cell where a call request is made. The probability that out of the k calls, i would be in the same cell is given by the binomial distribution  $B(j, k, p_{ii})$  and j out of k would be in any other cell  $n_l$  is given by  $B(j, k, p_{kl})$  where

$$B(i,k,p) = \binom{k}{i} (p)^{i} (1-p)^{k-i}$$
(1)

The call admission conditions proposed by Naghshineh and Schwartz(1996) are generalized for one dimensional array of cells. In order to satisfy the first call admission condition, the probability distribution of the number of calls in  $C_i$  at time  $t_0 + T$  is denoted by  $P_{n_i t_0 + T}(n_i)$  using a convolution of binomial distributions  $B(n_{ii}, n_i, p_{ii}, B(n_{ji}, n_j, p_{ji})$ for all j, where  $n_j \ge n_{ji} \ge 0$ . We can approximate the Binomial distribution B(i, k, p) by a Gaussian distribution  $G(m, \sigma^2)$  with mean m = kp and variance  $\sigma^2 = kp(1-p)$ . Based on these assumptions, the number of calls in cell  $C_i$  at time  $t_0 + T$  can be approximated by Gaussian distribution given by

$$P_{n_i t_0 + T}(k) \simeq G\left(\sum_{j=1}^{\infty} n_j p_{ji}, \left(\sum_{j=1}^{\infty} n_j p_{ji} q_{ji}\right)\right)$$
(2)

The overload probability  $P_0$  is given by the tail of the Gaussian distribution denoted by

$$P_0 = \sum_{N+1}^{n} P_{n_{i_0} + T}(k)$$
(3)

Generalized model for call admission policy with asymmetric rates

where

$$\tilde{n} = \sum_{j=1}^{\infty} n_j \tag{4}$$

$$P_0 = Q\left(\frac{N - \sum_{j=1}^{\infty} n_j p_{ji}}{\sqrt{\left(\sum_{j=1}^{\infty} n_j p_{ji} q_{ji}\right)}}\right)$$
(5)

where Q(.) is the integral over the tail of the Gaussian distribution which can be expressed in terms of an error function. For a fixed  $P_{QoS}$ , there exists a value  $P_{QoS} = Q(a)$ . Then we have the relation

$$\left(N - \sum_{j=1}^{\infty} n_j p_{ji} - a_{\sqrt{\left(\sum_{j=1}^{\infty} n_j p_{ji} q_{ji}\right)}}\right) = 0$$
(6)

Solving for  $n_i$  in the above equation we get an admission threshold for the  $i^{th}$  call. This threshold  $m_i^i$  satisfies the first call admission condition for the cell  $C_i$ .

The number of mobiles  $m_1^i$  that can be admitted to cell  $C_i$  such that the second call admission condition is satisfied for cell  $C_1$  being approximated in a similar fashion. Given that the arrival process is poisson, the probability distribution of having k mobiles in cell  $C_1$ at time  $t_0 + T$ , denoted as  $P_{n_1t_0+T}$  is given by the convolution of the binomial distributions  $B(n_{11}, n_1, p_{11}), B(n_{j1}, n_j, p_{j1})$  for j=2 to  $\infty$ , and the poisson distribution

$$\frac{(\lambda_1 T)^{\nu} \epsilon^{-\lambda_1 T}}{\nu!} \tag{7}$$

The resultant of the convolution can be approximated by a Gaussian distribution has been discussed in chapter 2. We have

$$P_{n_1 t_0 + T}(k) \simeq G\left(\sum_{j=1}^{\infty} n_j p_{j1} + \lambda_1 T, \left(\sum_{j=1}^{\infty} n_j p_{j1} q_{j1} + \lambda_1 T\right)\right)$$
(8)

The overload probability is given by  $P_0$  which is the integral over the tail of the Gaussian distribution.

$$P_0 \simeq Q\left(\frac{N - \sum_{j=1}^{\infty} n_j p_{j1} + \lambda_1 T}{\sqrt{\left(\sum_{j=1}^{\infty} n_j p_{j1} q_{j1} + \lambda_1 T\right)}}\right)$$
(9)

Using  $P_{QoS} = Q(a)$  we can find a threshold value  $m_1^i$  which gives the number of calls that can be admitted to  $C_n$  such that the second admission condition for  $C_1$  is satisfied.

Similarly, we can find the number of calls that can be admitted to cell  $C_i$  such that the second admission condition is satisfied for cells  $C_2, C_3, \ldots, C_{\infty}$  is as  $m_2^i, m_3^i, \ldots, m_{\infty}^i$ .

The final threshold which satisfies all admission conditions (w.r.t each cell) for cell  $C_i$  is given by

$$m = \min(m_1^i, m_2^i, \dots, m_i^i, \dots, )$$
(10)

,

where m defines the maximum number of calls that can be admitted to cell  $C_i$  such that all admission conditions are satisfied. This generalizes the results due to Naghshineh and Schwartz(1996).

#### Nearest neighbour model

We can obtain the results for nearest neighbour model proposed by Naghshineh and Schwartz by applying the boundary conditions, uniform arrival departure and transition rates to the generalized model. In a one dimensional case of the nearest neighbourhood model only the adjacent cells would be considered. So by replacing  $p_{ii}$  by  $p_s$ ,  $p_{ij}$  by  $\frac{p_m}{2}$  and using the notations  $n, l, r, E(n), \lambda$ , the probability

$$P_{n_{it_0+T}}(k) \simeq G\left(\sum_{j=1}^{\infty} n_j p_{ji}, \left(\sum_{j=1}^{\infty} p_{ji} q_{ji} n_j\right)\right)$$
(11)

becomes

$$P_{nt_0+T}(k) \simeq G\left(np_s + (l+r)\frac{p_m}{2}, \\ \left(np_s(1-p_s) + (l+r)\frac{p_m}{2}(1-\frac{p_m}{2})\right)\right)$$
(12)

similarly the probability

$$P_{n_1t_0+T}(K) \simeq G\left(\sum_{j=1}^{\infty} n_j p_{j1} + \lambda_1 T, \left(\sum_{j=1}^{\infty} n_j p_{j1} q_{j1} + \lambda_1 T\right)\right)$$
(13)

becomes

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$$P_{rt_{0}+T}(k) \simeq G\left(rp_{s} + (E(n)+n)\frac{p_{m}}{2} + \lambda T, \\ \sqrt{\left(rp_{s}(1-p_{s}) + (E(n)+n)\frac{p_{m}}{2}(1-\frac{p_{m}}{2}) + \lambda T\right)}\right)$$
(14)

The above results deduced are similar to the ones for nearest neighbour model. So from these equations, we can find the the number of calls that can be admitted such that the call admission conditions are satisfied.

### 2. Extension to the 2D model

We can extend the steady state analysis of the 1D case to a 2D model. We consider a two dimensional array of hexagonal cells in a finite geographical area. Here we assume that the mobile call can handoff to any cell in a 2D array of cells.

Notation

 $n_{ij}$ : number of calls in  $(i, j)^{th}$  cell at some time t.

 $p_{ij,ij}$ : probability that the cell stays in the same cell

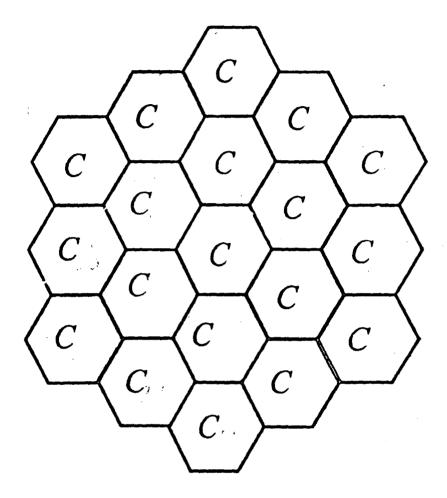
 $p_{ij,kl}$ : probability of transition from  $(i,j)^{th}$  cell to  $(k,l)^{th}$  cell,  $(i,j) \neq (k,l)$ 

 $q_{ij,kl}$  :  $(1 - p_{ij,kl}), (i, j) \neq (k, l)$ 

 $\lambda_{ij}$ : arrival rate to the  $(i, j)^{th}$  cell.

 $\mu_{ij}$ : departure rate from the  $(i, j)^{th}$  cell.

N: Maximum number of calls that the system can support



By applying the call admission conditions discussed in chapter 2, we can obtain the call admission threshold value for any cell. Let  $C_{ij}$  be the cell where a call admission request is made. The probability distribution of the number of calls in cell  $C_{ij}$  such that the first admission condition is satisfied becomes the convolution of the binomials, which can be approximated by a Gaussian distribution given by

$$P_{n_i j_{t_0} + T}(K) \simeq -G\left(\sum_{k=1}^{\infty} \sum_{k=1}^{\infty} n_{kl} p_{kl,ij}, \left(\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} n_{kl} p_{kl,ij} q_{kl,ij}\right)\right)$$
(15)

The overload probability  $P_0$  which is the integral over the tail of the Gaussian distribution. We find

$$P_0 = \sum_{N+1}^{N} P_{n_i j_{t_0+T}}$$
(16)

where

$$\tilde{N} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} n_{ij} \tag{17}$$

Equation (16) yields

$$P_0 = Q\left(\frac{N - \sum_{k=1l=1}^{\infty} n_{kl} p_{kl,ij}}{\sqrt{\left(\sum_{k=1l=1}^{\infty} n_{kl} p_{kl,ij} q_{kl,ij}\right)}}\right)$$
(18)

where Q(.) is the integral over the tail of the Gaussian distribution. For a fixed  $P_{QoS} = Q(a)$ , we can obtain a threshold value  $m_{ij}^{ij}$  which satisfies the first call admission condition.

The probability distribution of the number of calls in any other cell  $C_{mn}$  at time  $t_0 + T$ , such that the second call admission condition is satisfied is given by the sum of the binomials and a poisson, which is approximated by a Gaussian distribution. We have Generalized model for call admission policy with asymmetric rates

$$P_{n_{mn}t_0+T} \simeq G\left(\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}n_{kl}p_{kl,ij} + \lambda_{mn}T\right) \left(\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}n_{kl}p_{kl,ij}q_{kl,ij} + \lambda_{mn}T\right) \right)$$
(19)

By integrating over the tail and equating to Q(a) (a pre determined quality of service), we obtain  $m_{mn}^{ij}$  the threshold for which the second admission condition is satisfied. Similarly other threshold values for other cells in relation to the second call admission condition are calculated.

The final threshold value such that all the admission conditions are satisfied is given by

$$m = \min(m_{11}^{ij}, m_{12}^{ij}, \cdots, m_{21}^{ij}, m_{22}^{ij}, \cdots, \cdots)$$

$$(20)$$

where m gives the maximum number of calls that can be admitted to cell  $C_{ij}$ .

#### Nearest neighbour model for 2D

The nearest neighbourhood model can be easily obtained from this model by applying the boundary conditions, i.e., by replacing  $p_{ij,ij}$  by  $p_s$ ,  $p_{ij,kl}$  by  $\frac{p_m}{6}$  and using the notations  $n,l,r,\lambda$ , eq(15) reduces to eq(2.13) from which the threshold values are calculated.

The generalized model will give a better approximation in making the call admission control policy. This admission policy decisions would be made by the base stations for every interval of time by obtaining the information from various other base stations. The

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#### Generalized model for call admission policy with asymmetric rates

time dependent nature of the problem is not discussed in this section. The following section discusses about the transient analysis of the model, since the arrival, departure and handoff rates are not uniform during the day. The time dependence is reflected by making the parameters as explicit function of time.

This model can be regarded as generalization of the previous models which do not include any time dependent nature of the arrival, departure and handoff rates. The rates may assume large values during some rush hours and very low at lean period of the day. The next chapter gives a model in which the time dependent nature of the problem has been included.

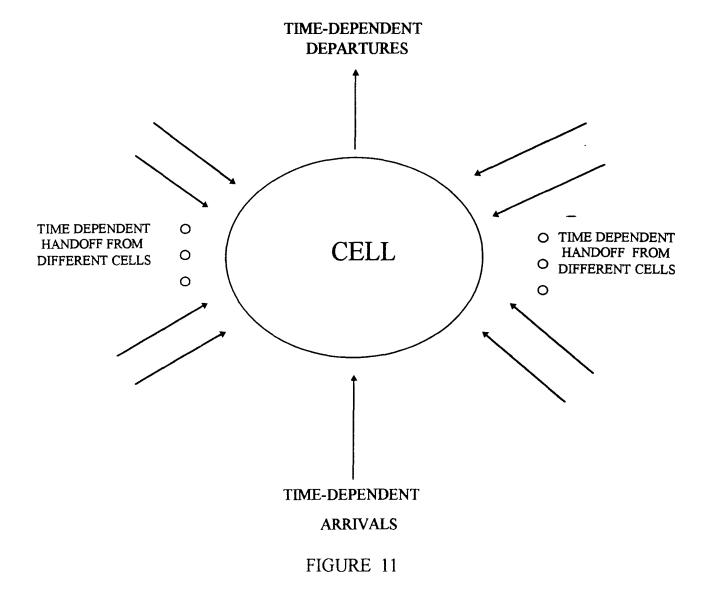
The solutions so far proposed are static in nature. The time-dependent nature of the system has not been embedded in the problem. Generally the arrival, departure and handoff rates are not uniform throughout the daytime. The rates may be very high during the peak hour situations and very low during the night times. Sometimes the arrival, departure rates would be high with a low handoff rates and vice versa. Thus the study of the system response to the dynamic nature of the traffic becomes very important. Also, the system may not be able to provide a better quality of service with static system approximations.

#### Notation

- $n_i$ : number of calls in  $i^{th}$  cell  $C_i$  at time t
- $\lambda_i(t)$ : arrival rate of calls at  $i^{th}$  cell

 $\mu_i(t)$ : departure rate of calls at  $i^{th}$  cell

- $\gamma_{ij}$ : probability of handoff transition from  $i^{th}$  cell to  $j^{th}$  cell,  $i \neq j$
- $\gamma_{ii}$ : probability of staying in the same cell  $C_i$
- N: maximum number of calls that the system can support



Let n be the number of calls in cell  $C_i$ . The change in the number of calls during a time internal  $(t, t + \Delta t)$  is given by

$$dn(t) = \left\{\lambda_i(t) - \mu_i(t) + \gamma_a(t) - \gamma_b(t)\right\} dt \tag{1}$$

where

$$\gamma_a(t) = \sum_{j=1}^{\infty} n_j \gamma_{ji}(t) \text{ and } \gamma_b(t) = \sum_{j=1}^{\infty} n_j \gamma_{ij}$$
 (2)

For capturing the fluctuations in the number of calls in a cell, we assume that the fluctuations in the arrival, departure and handoff rates occur on a very small time scale in comparison to the time scale of the system. So, the parameters of eq(1) can be written as

$$\lambda_{i}(t) = \bar{\lambda_{i}} + \tilde{\lambda_{i}}(t)$$

$$\mu_{i}(t) = \bar{\mu_{i}} + \tilde{\lambda_{i}}(t)$$

$$\gamma_{a}(t) = \bar{\gamma_{a}} + \tilde{\gamma_{a}}(t)$$

$$\gamma_{b}(t) = \bar{\gamma_{b}} + \tilde{\gamma_{b}}(t)$$
(3)

where  $\bar{\lambda}_i, \bar{\mu}_i, \bar{\gamma}_a, \bar{\gamma}_b$  are the deterministic parts and  $\tilde{\lambda}_i(t), \tilde{\mu}_i(t), \tilde{\gamma}_a(t), \tilde{\gamma}_b(t)$  are the fluctuations characterised by white noise, stationary Gaussian processes, which may be treated as a formal derivative of a Weiner process(Cox and Miller, 1968). The fluctuations are assumed be delta correlated (Karmeshu and Jaiswal, 1981) such that

$$E[\tilde{\lambda}_{i}(t)] = 0$$

$$E[\tilde{\mu}_{i}(t)] = 0$$

$$E[\tilde{\gamma}_{a}(t)] = 0$$

$$E[\tilde{\gamma}_{b}(t)] = 0$$

$$E[\tilde{\lambda}_{i}(t_{1})\tilde{\lambda}_{i}(t_{2})] = \sigma_{\lambda}^{2}\delta(t_{1} - t_{2})$$

$$E[\tilde{\mu}_{i}(t_{1})\tilde{\mu}_{i}(t_{2})] = \sigma_{\mu}^{2}\delta(t_{1} - t_{2})$$

$$E[\tilde{\gamma}_{a}(t_{1})\tilde{\gamma}_{a}(t_{2})] = \sigma_{a}^{2}\delta(t_{1} - t_{2})$$

$$E[\tilde{\gamma}_{b}(t_{1})\tilde{\gamma}_{b}(t_{2})] = \sigma_{b}^{2}\delta(t_{1} - t_{2})$$

Then the incremental change during time  $(t, t + \Delta t)$  is in the number of calls in the cell is given by

$$dn(t) = \left\{ \bar{\lambda}_i - \bar{\mu}_i + \bar{\gamma}_a - \bar{\gamma}_b \right\} dt$$

$$+ \left\{ \tilde{\lambda}_i(t) - \tilde{\mu}_i(t) + \tilde{\gamma}_a(t) - \tilde{\gamma}_b(t) \right\} dt$$
(5)

Using eq(4), eq(5) can be rewritten as

$$dn(t) = \{ \bar{\lambda}_i - \bar{\mu}_i + \bar{\gamma}_a - \bar{\gamma}_b \} dt + \{ \sigma_\lambda dw_\lambda(t) - \sigma_\mu dw_\mu(t) + \sigma_a dw_\lambda(t) - \sigma_b dw_\mu(t) \}$$
(6)

where  $w_{\lambda}(t), w_{\mu}(t), w_{a}(t), w_{b}(t)$  are independent Weiner processes.

For the sake of completeness, X(t) can be defined as Weiner process with drift  $\mu$  and variance  $\sigma^2$  with the following properties. A Weiner process X(t) has the property that for any non-overlapping intervals it has independent increments and for some interval  $(t_1, t_2)$ ,  $X(t_2) - X(t_1)$  is normally distributed with mean  $\mu(t_2 - t_1)$  and variance  $\sigma^2(t_2 - t_1)$  (See Cox and Miller). As parameters involved are assumed to be white noise Gaussian processes, n(t) is a markov process and the probability density function satisfies the Fokker-Planck equation (FPE)

$$\frac{\partial P(n,t)}{\partial t} = -\frac{\partial}{\partial n}\mu P + \frac{1}{2}\sigma^2 \frac{\partial^2 P}{\partial n^2}$$
(7)

where the drift  $\mu = \{\bar{\lambda}_i - \bar{\mu}_i + \bar{\gamma}_a - \bar{\gamma}_b\}$  and variance  $\{\sigma^2 = \sigma_{\lambda}^2 + \sigma_{\mu}^2 + \sigma_a^2 + \sigma_b^2\}$ 

FPE(7) has to be solved with initial condition

$$\lim_{t \to 0} p(n, t) = \delta(n - n_0) \tag{8}$$

and appropriate boundary conditions.

#### 1. Unrestricted Process

We start by solving FPE(7) when the boundary conditions are natural i.e. the process n(t) is unrestricted. The natural condition imply that

$$\lim_{n \to \pm \infty} p(n,t) = 0$$
 (9)

For the solution of the FPE we closely follow the analysis contained in Cox and Miller(1969). Introducing the stochastic variable Z(t),

$$Z(t) = \left(\frac{(n(t) - n_0 - \mu t)}{\sigma}\right)$$
(10)

FPE reduces to

$$\frac{1}{2}\frac{\partial^2 p(\boldsymbol{y},t)}{\partial z^2} = \frac{\partial p(\boldsymbol{y},t)}{\partial t}$$
(11)

where p(y,t) is the probability density function of the process y(t) with the initial condition

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$$\lim_{t \to 0} p(y, t) = \delta(y = y_0); \quad y_0 = 0$$
(12)

It may be noted that (11) is the well known heat conduction equation

Multiplying eq(11) by  $e^{-\theta y}$  and integrating from  $-\infty$  to  $\infty$ , we get

$$\frac{1}{2}\theta^2 M = \frac{\partial M}{\partial t}.$$
(13)

where

$$M(\theta, t) = E\left[e^{-\theta Y(t)}\right] = \int_{-\infty}^{\infty} p(y, t)e^{-\theta y}dy$$
(14)

The initial condition in eq(12) reduces to

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$$M(\theta,0) = \int_{-\infty}^{\infty} \delta(y=y_0) e^{-\theta y} dy = 1$$
(15)

From equations (14) and (15) we find

$$M(\theta, t) = e^{\frac{1}{2}\theta^2 t} \tag{16}$$

which is mgf for the Gaussian probability density function. Thus the process Z(t) follows a Guassian process with mean 0 and variance t. Thus from eq(10) we find that n(t) is a Gaussian process with mean  $n_0 + \mu t$  and variance t.

$$p(n,t;n_0) = \frac{1}{\sigma\sqrt{2\pi t}} exp\left[-\frac{\left(n-n_0-\mu t\right)^2}{2\sigma^2 t}\right]$$
(17)

where  $(-\infty < n < \infty)$ 

### 2.Reflecting barrier at the origin

Now we consider the case corresponding to a reflecting barrier at n = 0. This is necessitated by the fact that the number of calls cannot become negative. This is a more realistic situation which is not considered in the paper by Naghshineh and Schwartz(1996).

We now proceed to evaluate the condition for the reflecting barrier at n = a. FPE(7) can be rewritten as

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$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial n} J(n, p) \tag{18}$$

$$J(n,p) = \mu p - \frac{1}{2}\sigma^2 \frac{\partial p}{\partial n}$$
(19)

where J(n,p) is the probability current. For a reflecting barrier the probability current at n = a vanishes i.e.

$$\left[J(n,p)\right]_{n=a} = 0 \tag{20}$$

substituting J(n,p) from eq(19), we obtain

$$\left[\mu P - \frac{1}{2}\sigma^2 \frac{\partial P}{\partial n}\right]_{n=a} = 0 \tag{21}$$

The situation considered in this subsection corresponds to Weiner process n(t) with drift  $\mu$  and variance  $\sigma^2$ . Process n(t) is confined to the positive half line with reflecting barrier at origin. This case has been analyzed in Cox & Miller. FPE(7) has to be solved in conjunction with the reflecting barrier at n = 0 and initial condition

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$$\lim_{t \to 0} p(n, t) = \delta(n - n_o) \tag{22}$$

Employing the method of images (Cox & Miller ), we obtain

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$$p(n,t) = \frac{1}{\sigma\sqrt{2\pi t}} \left[ exp\left\{ -\frac{n-n_0 - \mu t)^2}{2\sigma^2 t} \right\} + Aexp\left\{ -\frac{(n-n_0 - \mu t)^2}{2\sigma^2 t} \right\} + \int_{-\infty}^{-x_0} exp\left\{ -\frac{(n-\xi - \mu t)^2}{2\sigma^2 t} \right\} k(\xi) d\xi \right]$$
(23)

which satisfies the eq(18) and the initial condition eq(12). To determine the constant A and the function  $k(\xi)$  we demand that the boundary condition (21) is satisfied.

The condition (21) is rewritten using (23) as

$$\frac{1}{2}\sigma^2 \frac{dk}{d\xi} - \mu k(\xi) = 0 \tag{24}$$

.

$$(n_0 - \mu t)exp\left(-\frac{n_0\mu}{\sigma^2}\right) - \left\{n_0A + \mu t + t\sigma^2 k(-n_0)\right\}exp\left(\frac{n_0\mu}{\sigma^2}\right) = 0, \quad (t > 0)$$
(25)

It follows that

$$A = \epsilon x p \left( -\frac{2n_0 \mu}{\sigma^2} \right), \qquad k(\xi) = \frac{2\mu}{\sigma^2} \epsilon x p \left( \frac{2\mu\xi}{\sigma^2} \right)$$
(26)

The solution for the pdf becomes

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$$p(n,t) = \frac{1}{\sigma\sqrt{2\pi t}} \left[ exp\left\{ -\frac{(n-n_0-\mu t)^2}{2\sigma^2 t} \right\} + exp\left\{ -\frac{4a\mu t - (n-n_0-\mu t)^2}{2\sigma^2 t} \right\} \frac{2\mu}{\sigma^2} + exp\left(\frac{2\mu x}{\sigma^2}\right) \left\{ 1 - \Phi\left(\frac{(n+n_0+\mu t)}{\sigma\sqrt{t}}\right) \right\} \right],$$

$$(27)$$

where  $\Phi(x)$  is the standard normal integral.

letting  $t \to \infty$ , we find that for  $\mu < 0$ , the equilibrium distribution reduces to

$$p(n;\infty) = \frac{2|\mu|}{\sigma^2} exp\left(\frac{-2|\mu|n}{\sigma^2}\right) \qquad (n > 0, \mu < 0)$$
(28)

which is an exponential distribution independent of the initial value  $n_0$ .

Threshold conditions

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We can represent eq(17) by a Gaussian process with mean  $m(t) = n_0 + \mu t$  and variance  $\sigma^2(t)$ 

$$p(n_0, n; t) \simeq G\big(\boldsymbol{m}(t), \sigma^2(t)\big) \tag{29}$$

Thus for any cell *i* and for convenience writing  $p(n_0, n; t) = p(n; t) eq(29)$  may be written as

$$p(n_i;t) \simeq G\left(m_i(t), \sigma_i^2(t)\right) \tag{30}$$

One can proceed on similar lines as discussed in previous chapters for deriving the threshold conditions. It may be noted that in view of the fact that the mean and variance are explicit functions of time, the threshold conditions would also depend on explicit time.

## Conclusion

The call admission policy developed here gives a more realistic description of the problem in terms of time-dependent situation. The time dependent nature of the system with asymmetric rates have also been investigated. The transient analysis provides description at any point of time and this in turn gives call admission control policies within the time-dependent framework. Such an analysis becomes important when one is dealing with rush hour situations. Based on transient analysis, one is able to obtain insight into time-dependent call admission control policies. The policy can be implemented in the form of software at each base station which makes time-dependent decisions concerning the geographical region served by the base station.

The time dependent solution has been found for an unrestricted process in terms of Gaussian process. It is easy to obtain the convolution of independent Gaussian processes. This has made analytical treatment tractable. However when the stochastic process is reflected by imposing a reflected barrier at n=0, the solution process is no longer a Gaussian process. However the convolution of such processes can be computed for obtaining call admission control policies in an area of future interest.

There is an enormous potential for extensions in the call admission control policy by including the priorities of calls in relation to handoffs and in relation to more revenue.

Another extension would deal with dynamic channel allocation type as discussed in chapter 1. In dynamic channel allocation, part of the channel capacity is reserved by the switching center. If the channel capacity is not available at the base station, then the call requests would be diverted to the switching center which allocates part of the channel to the requesting base stations in a periodic manner. The overload probability of such a system would be the calls rejected by the base station as well as the switching center. For the design of such a system, it may need two admission control policies one at the base station and the other at the switching center.

The model developed here can be applied directly at the base stations, but applicable to switching center with additional provisions. However, models need to be developed for dealing of the time-dependent situations in the dynamic allocation policies.

# Appendix

The mgf of the binomial distribution B(n,p) is

$$q = (1-p)$$

$$M(t) = (pe^{t} + qe^{-pt})^{n}$$

$$= e^{npt}(pe^{qt} + qe^{-pt})^{n}$$

$$= e^{npt}exp\{n\log(pe^{qt} + qe^{-pt})\}$$

$$= e^{npt}exp[n\log\{p(1 + qt + \frac{1}{2}q^{2}t^{2} + ...) + q(1 - pt + \frac{1}{2}p^{2}t^{2} + ...)\}]$$

$$= e^{npt}exp\{n(1 + \frac{1}{2}pqt^{2} + ...)\}$$

$$= e^{npt}exp\{n(\frac{1}{2}pqt^{2} + O(t^{3}))$$

For large n, take  $t = O(\frac{1}{\sqrt{n}})$ , then

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$$M(t) = exp\{npt + \frac{1}{2}npqt^2\}$$

Corresponding to the mgf of the normal distribution, convolution of three independent binomial distribution  $B(n_i, p_i)$  i = 1, 2, 3, ... has mgf

$$\prod_{i=1}^{3} (p_i \epsilon^t + q_i)^n$$

For large  $n_i$  this leads to

$$exp\{\sum_{i}(n_{i}p_{i}t+\frac{1}{2}n_{i}p_{i}q_{i}t^{2})\}$$

which is the mgf of the normal distribution

$$G\left(\sum n_i p_i, \frac{1}{2} \sum n_i p_i q_i\right)$$

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