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Study of Chaotic Time Signals to Determine Whether They Arise From the Same Underlying Dynamics

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The research work embodied in this dissertation entitled "**Study of Chaotic Time Signals to Determine Whether They Arise From the Same Underlying Dynamics** " has been carried out in the School of Environmental Sciences, JNU, New Delhi. This work is original in the sense that it has not been submitted in part or full for any other degree or diploma of any other university.

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Rajeev Kumar Azad

CONTENTS

Abstract

An interesting question which arises in many disciplines - physics, chemistry, ecology, climatology - is whether two or more experimentally measured time series are coupled by same underlying dynamics. Many methods have been developed to answer this question. Some of these are based on reconstructing the attractor from the time series and hence to determine the dimension of the attractor. If the two series are the result of the same dynamics, both should yield the same result. The problem here is that these methods are cumbersome and tend to be not very robust in the presence of noise. The symbolic dynamics (coarse graining) method discussed in this thesis appears to be much more effective and robust in determining whether the different signals are the result of the same underlying dynamics. The method has been seen to work quite well even in the presence of noise. The method applies the partitioning of phase space into 'states' (coarse graining) of a chaotic attractor. Application of statistical properties, like information entropy to this partitioning leads one to conclude whether the two different signals have their origin in common dynamics. The method's suitability in extracting delay information from experimental chaotic time series of delay equation has also been seen. The method appears to be a useful tool in gathering delay information even if noise of appreciable magnitude is present.

CHAPTER ONE

INTRODUCTION

Theoretical studies have made significant impacts in subjects like physics and chemistry. In recent times, even more complex sciences like molecular biology have started using theoretical techniques in trying to understand some fundamental aspects. In environmental sciences, especially in ecology [1-4], theoretical modelling efforts have contributed to developing an understanding of the temporal evolution of some simple ecosystems using techniques used in the study of non-linear dynamics.

The basic theme is fairly straight forward. The system under consideration is modelled by differential equations which are non-linear and coupled. The aim is to understand how such a system would evolve in time.

Another way-probably more relevant in ecology is to measure the time evolution of a particular variable or set of variables and then ask the question - could the time series be the result of some underlying dynamical system and if so could the same dynamical system be responsible for the behaviour of the set of variables ?

In this thesis we review the techniques developed by Lehrman et al. [5] which help in answering some of the above questions. In particular, Lehrman et al. have developed a technique to determine whether or not two time series (experimentally measured) could have resulted from a common underlying dynamics.

A precondition for such a study is that the time series under study be chaotic. The implications of this method in time-delay equation are also examined. In some situations it is seen that response to a stimulus occurs but with a delay. This method is adopted to see whether one can get an idea of how much delay occurs.

The above techniques would have many applications in environmental sciences. The dynamical systems in ecology have also been found to show chaotic behavior [6,7]. The experimental time series as observed for a variety of environmental aspects have been actually shown to behave chaotically. For example, El Nino events which involve a widespread warming of the equatorial Pacific Ocean surface water, have been successfully modelled using partial differential equations [8]. The El Nino events occurs irregularly and affects the world-wide climate remarkably. Recent studies have shown that the unpredictability of this phenomenon is due to spatiotemporal chaos [8-12]. Clearly, El Nino dynamics is due to the coupling of some environmental factors (variables), the time series of which are chaotic. It would be relevant to investigate whether these time series arise from the same underlying dynamics. Such studies could then help in establishing the factors which are coupled.

As stated earlier further relevance can be sought for by looking into systems with time delays. Time delay means that to a certain stimulation, the response or action comes but with a delay. The concept of the retarded action applies to disciplines as widely separate as laser, physiology, ecology etc. In physiology, delays applies to the feedback situations where there is a time gap between the sensing of some disturbances and the arrival of an appropriate response [13]. Such physiological traits have been modelled by non-linear differential equations with a delay called delay differential equations. For example the models of respiration and also cell maturation show the significant delay [14].

The importance of delay can also be seen in ecological models [6,15]. One such is a simple logistic delay equation of single species :-

$$dx/dt = r\,x(t)\,[1 - x(t - T)] \qquad (1.1)$$

where x represents the population density, r is constant and T is the delay. Here the current rate of change of population density depends upon the population density at some time in the past.

Such delay equations can be highly chaotic. It would be a matter of interest if one can gather the delay information from experimental time series of chaotic systems with delay equation. We test the Lehrman et al. method for time delay equation also. It is observed that the method works quite well in obtaining the delays in such time-series.

It is thus seen that this model finds good utility in understanding some of the useful aspects of chaotic motions. In particular, it can be used in studying the dynamical systems governed by our environment.

We have organized our study in the next four chapters. Chapter two deals with chaos - its meaning and how it is an inseparable part of non linear dynamics, its deterministic nature being used to model systems with examples of Figenbaum attractor, Henon attractor and Lorenz model. Then some of the qualitative as well as quantitative tools used commonly to identify and study the chaotic motions have been briefly illustrated. Lastly we discuss some of the routes a system undergoes before entering into chaos.

In chapter three we review the Lehrman et al. method useful in determining the time evolution of chaotic signals from the same underlying dynamics. The method applies the symbolic dynamics (coarse graining) where each member of the discrete time series is substituted by a symbol or number according to some rules.. The motive is to extract the information from the simplified series without compromising any way with the result one requires. To illustrate the method, two models, viz. Lorenz model and high dimensional model have been considered. We discuss how the coarse graining method is useful in the study of such models. In this context, the information content as contained in the symbolic series and its use further in getting the conditional information of two correlated variables is discussed. We present the results of numerical simulations performed to review the method as suggested by Lehrman et al.

Chapter four is the application of symbolic dynamics method for two attractors-Part I deals with the Rossler attractor and Part II with the time delay differential equation. It is seen that method works quite well with the Rossler attractor. We vary some of the parameters of symbolic method as used in the case of Lorenz model to see the conditions in which this study is best suited. Then we numerically simulate the delay differential equation to check the validity of this method in this case also. It is inferred that the method is good enough to gather the delay information. The method is robust even in the presence of noise.

Chapter five is the discussion of the results. We analyse the results of the numerical experiments as done on different models (chapter three & four). The robustness of this method in studying systems of diverse fields is also discussed. We indicate its main advantages over some other methods. We show how fourier transform method helps in getting an approximate idea of sampling time interval required to get better results of coarse graining method. In the light of this method's suitability in gathering delay information, we ponder over its possible usefulness in getting delay from experimental time series of El Nino event. We also discuss how the method may be useful in getting the time shift of two chaotic time series evolving from same dynamical system.

CHAPTER TWO

A BRIEF REVIEW OF CHAOS

I. Introduction

Newton's laws of motion have become the foundation stone of the understanding of dynamics since its inception over three centuries back (Newton's Principia, 1687). Till some years back, it was the general idea that if the forces between the particles are known and also the initial positions and velocities, one can predict the motion of a system far into the future. However, with the coming of computers, it is seen that such predictions are not always possible.

In particular, the study of non linear systems shows this aspect. The advent of computers has helped in the rapid development of studies of such dynamics. It has been found that even very simple deterministic dynamical systems show complicated behaviour. It is found, that for certain ranges of values of parameters in the system, the system shows a behaviour where its motion is restricted to a region of phase space and it keeps off fillng the phase space. The system is not cyclic. The trajectories are extremely sensitive to initial conditions. The region of phase space where the system operates is called a strange attractor and the motion of the system is referred to as chaotic.

Chaotic dynamics are observed in a wide spectrum of field viz. physical, biological, ecological etc. The problem of turbulent flow of fluids has always remained a matter of interest. Beyond a velocity called critical velocity, the regular pattern of flows goes turbulent. What is most interesting is that a set of simple mathematical equations can provide an analogous behaviour as seen in such chaotic and turbulent motions. It is now a established fact that chaotic dynamics are inherent in many of the non-linear physical phenomena and the subject has itself evolved into a separate entity called Non-linear Science. The rising column of smoke, wind past the aircraft wing, a storm in the atmosphere of Jupiter or Earth provide good example of chaotic motions. Despite enormous complexity, we can nonetheless see the underlying regularity and order. For example, the storm contains large, rather uniform regions. Chaotic patterns are characteristically varied in their details, but they may have quite regular general features. A lake which has dried up and thus formed complex pattern due to the cracks appearing on the surface shows that the pattern is almost the same at different places, but it repeats itself with unpredictable variations and thus is chaotic. Such chaotic patterns are sensitive to the conditions under which they are formed. Chaotic motions differ from random motions in the fact that they have no random or unpredictable inputs or parameters. They are referred to the motions in deterministic physical and mathematical systems whose time history has a sensitive dependence on initial conditions. The underlying structure to such dynamics can be searched in the phase space where the structure may appear to have positive non integral dimensions, termed as fractal structure [16]. For such motions, the forecasting uncertainty grows exponentially. The study of chaotic motions can help in understanding the source of such random like behaviour. To quantify such deterministic noise, tools or measures such as Lyapunov exponents and fractal dimensions are generally used [6,16].

II. How to Establish Chaos

With the study of chaotic signals, both the qualitative and quantitative ideas to identify chaotic vibrations have come up. Some of the methods [6,16,17,18] generally used are the following :

(a) Study of the time history of the signals

The observation of series of a signal can tell whether the signal is periodic or chaotic. The problem is one needs to observe the pattern for quite a long time, as there is likelihood of chaotic patterns appearing during much later time which is not being observed. The other difficulty is that if the signal is not periodic, it might be quasiperiodic which is not chaotic either, since it can be expressed as sum of two or more incommensurate periodic signals.

(b) Observation of the phase plane

The phase plane traced out by the periodic motion is closed one. But for chaotic motions, the path traced on the plane never closes or repeats. The trajectories of the orbits are confined to a limited portion of the phase space and keeps on filling the portion.

(c) Fast fourier transform (FFT)

Any periodic or non periodic signal can be expressed as a synthesis of sine or cosine signals.

$$f(t) = (1/2\pi) \int F(\omega) \exp(i\omega t)\, d\omega. \qquad (2.1)$$

$F(\omega)$ is often complex, hence its absolute value $|F(\omega)|$ is taken. The plot of $|F(\omega)|$ against the frequency is taken. For periodic or quasiperiodic motions, narrow spikes or lines are seen indicating that the signal has discrete set of harmonic functions $\{\exp(\pm\omega_k t)\}$, k=1,2,... . The beginning of chaotic regime is indicated by rather broad range of continuous frequency spectrum. The full chaos can be seen with the appearance of continuous frequency spectrum dominating the narrow spikes. This is applicable for low dimension non linear system. For systems with high degrees of freedom, we can still get broad range of frequencies even though the motion is nonchaotic.

The numerical calculation of $F(\omega)$ is often tedious, so the spectrum analyzers use the discrete version of $f(t) = \{f(t_k) = f_0, f_1, \ldots\ldots, f_n\}$ applying an effective algorithm called the Fast Fourier Transform.

(d) Poincare map

To understand a dynamical system, differential equations modelling it are solved. The trajectories thus obtained in the phase space depict the evolution of such systems. Poincare section [18] may be visualised as a surface (one dimension less than the dimension of phase space) which cuts across the trajectories in a region of phase space. For example, it is a plane if the phase space is three dimensional. Subsequent crossings of trajectories on this section is found from a function which is a finite difference equation and called as Poincare map. If the motion is periodic with period one, for example a circle, the Poincare section is a point. If it is a period two motion, the map consists of two points . For a motion of the type

$$\mathbf{x(t) = c_1 sin(\omega_1 t + d_1) + c_2 sin(\omega_2 t + d_2)} \qquad \mathbf{(2.2)}$$

where ω_1/ω_2 is an irrational number; and if the sampling is at a period corresponding to either frequency, the map is found to be closed curve [Fig.2.1] [16]. Such motions are quasiperiodic. If ω_1/ω_2 is a rational number , the map will consist of a finite set of points.

If the motion is chaotic, one can take two cases - first if the motion is undamped, the map appears to be a cloud of unorganised points. Secondly if it is damped, the map appears to be having infinite set of highly organised points. Such points appear to be moving in parallel arrays as can be seen in Fig. 2.2[16].

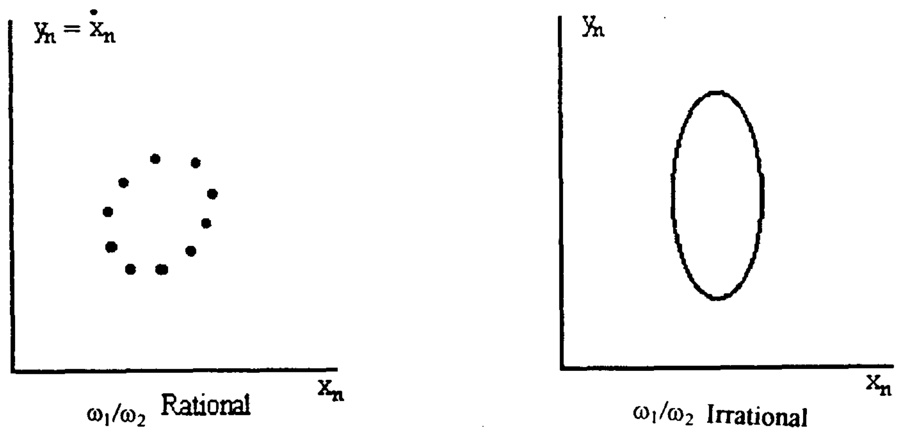


Fig. 2.1 Poincare map of a motion with two harmonic signals with different freqencies.

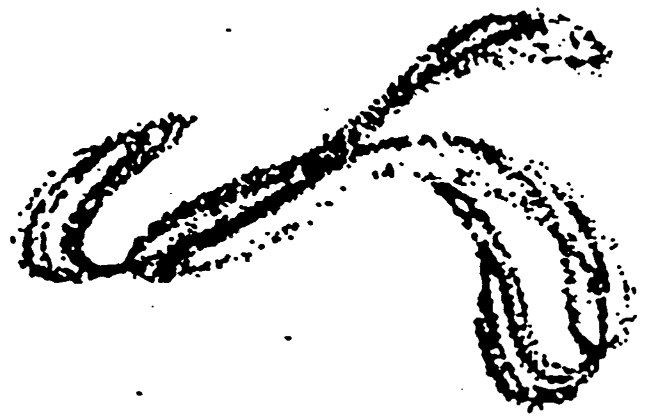


Fig. 2.2 Poincare map of a chaotic attractor.

Quantitative methods

(e) Lyapunov exponents

Chaotic motions are highly sensitive to initial conditions. It means if two trajectories start very close to each other, they will be much farther apart after considerably small times. The moving apart follows exponential nature.

If d_0 is the distance between two such nearby points, after some small time t, the distance d by which the points would be separated is given by :

$$d(t) = d_0 2^{\lambda t} \qquad (2.3)$$

The constant λ is called Lyapunov exponent.

The exponential divergence of nearby trajectories happens locally, for d(t) cannot go to infinity as the system itself is bounded. Hence to determine λ, the average of the exponential growth at many points along a trajectory has to be taken [16]. A reference trajectory is taken (see Fig. 2.3) and then a point on a nearby trajectory leads one to calculate $d(t)/d_0$. When d(t) becomes larger enough to depart from exponential behaviour, one moves to a new nearby trajectory and defines a new $d_0(t)$. One may thus define the first Lyapunov exponent as

$$\lambda = [1/(t_N - t_0)] \sum_{K=1}^{N} \log_2 (d(t_k)/ d_0(t_{k-1})) \qquad (2.4)$$

The positive value of the exponent i.e. $\lambda>0$ implies the motion is chaotic.

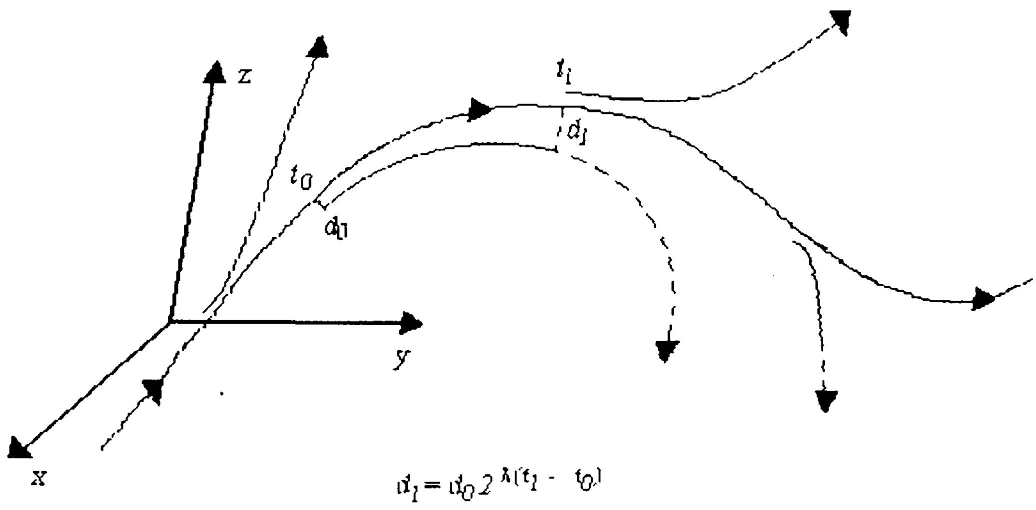


Fig. 2.3 Sketch of the change in distance between two nearby orbits used to define-

- the largest Lyapunov exponent.

(f) Fractal dimension

This is further a quantitative measure of the strangeness of the attractor. To illustrate this let us take the example of Koch curve. One takes a line segment of length one. It is then divided into three equal parts and the middle segment is replaced by two segments of length 1/3 as shown in figure 2.4 [16]. Thus the total length of the new boundary is increased to 4/3. This process is repeated now for each of the four segments and so on. Every time this process is repeated, the length increases by 4/3 and finally the total length approaches infinity. After many such steps, the curve appears fuzzy. In the limit one has a continuous curve but nowhere differentiable. Still the curve has properties of area. The idea is one gets a sense that dimension of the curve is less than 2 but greater then 1, i.e. it is fractal.

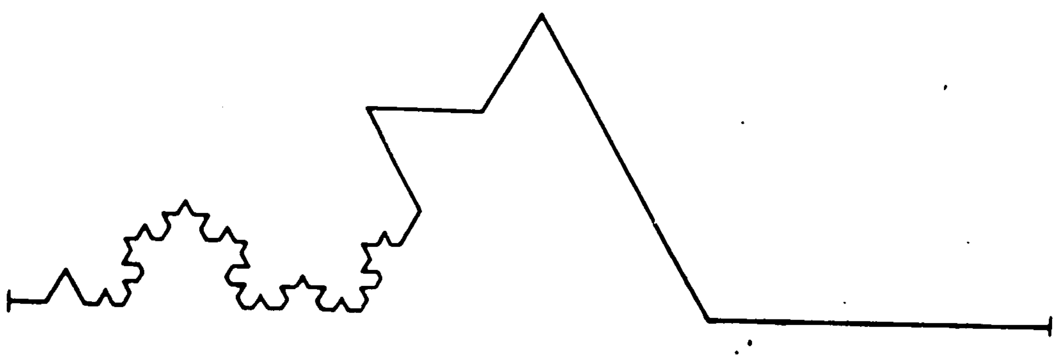


Fig. 2.4 Partial construction of Koch curve.

Suppose one has a uniform distribution of N_0 points along a line and one need to cover this set of points with cubes of length ε (Fig.2.5) [16]. We are concerned with the minimum number of cubes $N(\varepsilon)$ needed to cover the set. Considering N_0 to be large, $N(\varepsilon)$ can be seen to scale as

$$N(\varepsilon) \approx 1/\varepsilon$$

If we take the uniform distribution of N_0 points in two dimensional surface, $N(\varepsilon)$ will scale as

$$N(\varepsilon) \approx 1/\varepsilon^2$$

The dimension can thus be seen to follow the scaling law:

$$N(\varepsilon) \approx 1/\varepsilon^d \qquad (2.5)$$

d is now substituted by d_c to denote capacity dimension. Thus,

$$d_c = \lim_{\varepsilon \to 0} \log N(\varepsilon) / \log (1/\varepsilon) \qquad (2.6)$$

If the dimension d_c of a set of points turns out to be noninteger, it may be called as fractal.

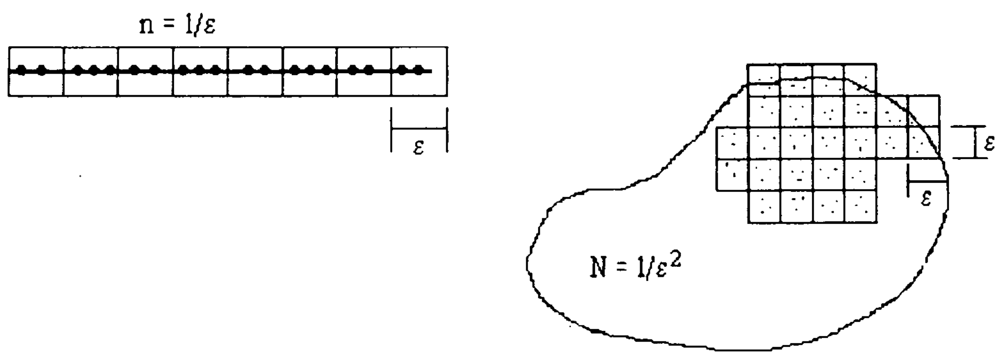


Fig. 2.5 Sketch illustrating the covering procedure for linear and planar distributions of points.

III. Routes to Chaos

Period doubling bifurcations

The simplest one-dimensional example to illustrate chaotic dynamics is the logistic equation or the population growth model [16,19] :

$$x_{n+1} = ax_n - bx_n^2 \qquad (2.7)$$

x_n is the population of nth generation, a and b are constants.

The first term on the right hand side exhibits the growth effect while the non-linear term comes due to competition when the population shows overcrowding. Clearly, the negative impact of this term balances the unlimited growth of the population.

We may write the equation in nondimensional form,

$$x_{n+1} = \lambda x_n(1 - x_n), \qquad (2.8)$$

λ is growth rate parameter.

Consider the situation for different values of λ.

Case 1. $\lambda < 1$

Each successive year the population decreases until it settles down to zero value (Fig. 2.6 A).

Case 2. **λ just greater than 1.**

The population grows and settles to a steady value after some time. In this case $x_{n+1} = x_n = x^*$,

$$x^* = \lambda x^* (1 - x^*) \quad \text{implies}$$

$$x^* = 0 \quad \text{or} \quad x^* = 1 - 1/\lambda$$

Solutions to the logistic equation for different values of the growth rate parameter

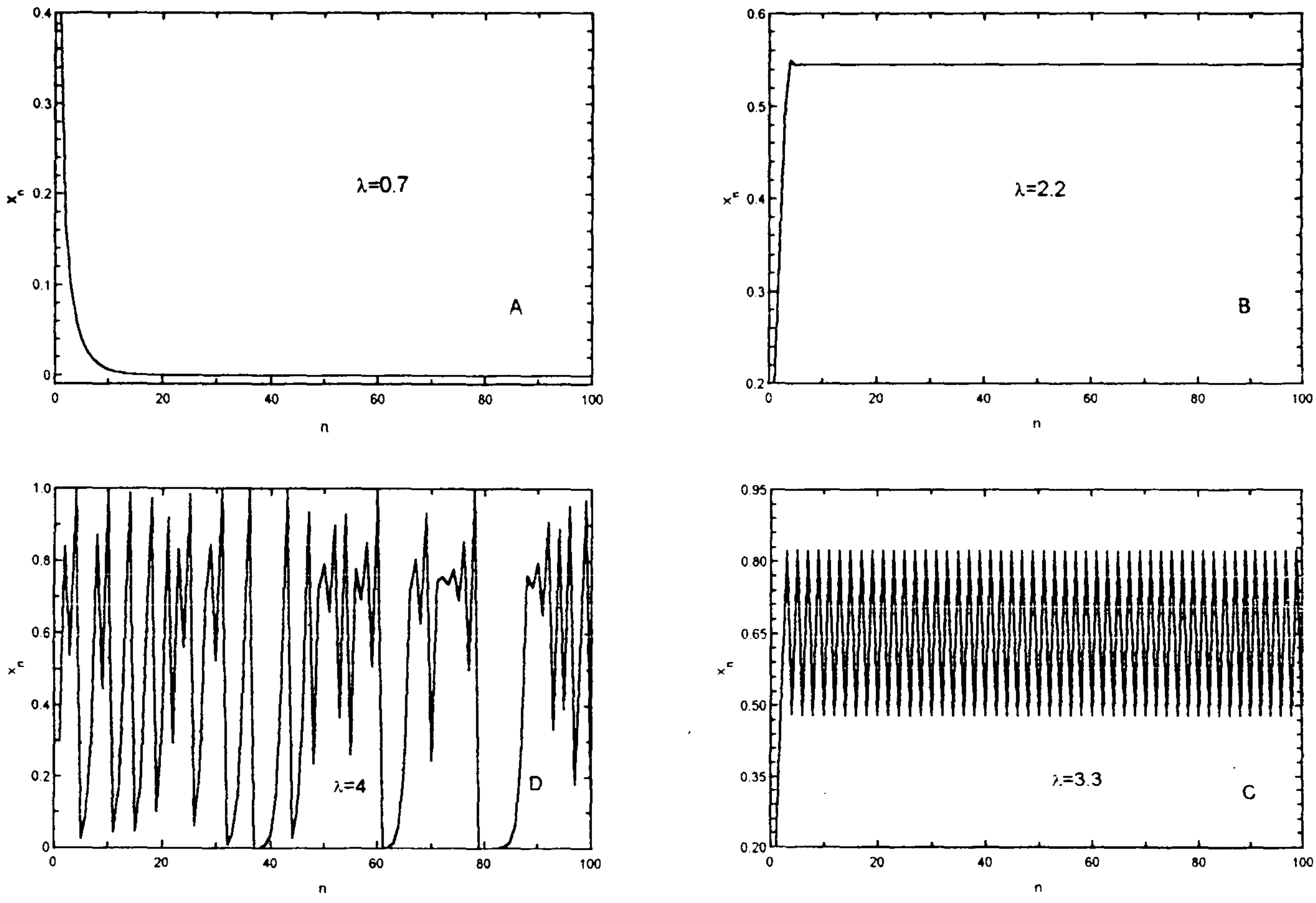


FIG. 2.6

For positive growth rate, discarding x*=0, we see for λ=2.2, $x \to 0.5454$ after long period of time (Fig. 2.6 B).

Case 3. λ=3.3

As λ is further increased, beyond the value 3, the steady state behaviour now disappears. Choose λ=3.3. After few years, the population settles down into a time dependent regular pattern. The population fluctuates between two extreme values - one high, one low. Such situation is called period two doubling or two cycle (Fig. 2.6 C). As λ increases above 3.4, a cycle of four appears, a bit more increase would lead to eight cycle, a tiny bit more and the period double yet again until at λ=3.59946....... and infinite number of period doubling has occurred, a situation which might be called chaos. We see that the chaos appears as a result of many successive doublings of the period and hence we may call this the period doubling route to chaos.

Case 4. λ=4

As seen above, we obtain full chaos, the long term behaviour does not settle down to any simple periodic motion. In fact all the x-values between zero and one show up. It becomes time independent (Fig. 2.6 D).

Fig. 2.7 shows that the successive values of λ at which cycles of length 1,2,4,8,..... first appears are denoted by λ_1, λ_2, λ_4, λ_8,...... and the one at which the cycle of infinite length appears is denoted by λ_∞. This is the onset of chaos.

The work done by theorists and some experimental verifications thereafter show that the route to chaos as shown by this simple model might be more generally applicable. For such systems, as the control parameter analogous to λ is changed, the system after undergoing successive doubling phenomena becomes unstable and goes chaotic.

Bifurcation diagram of the logistic map

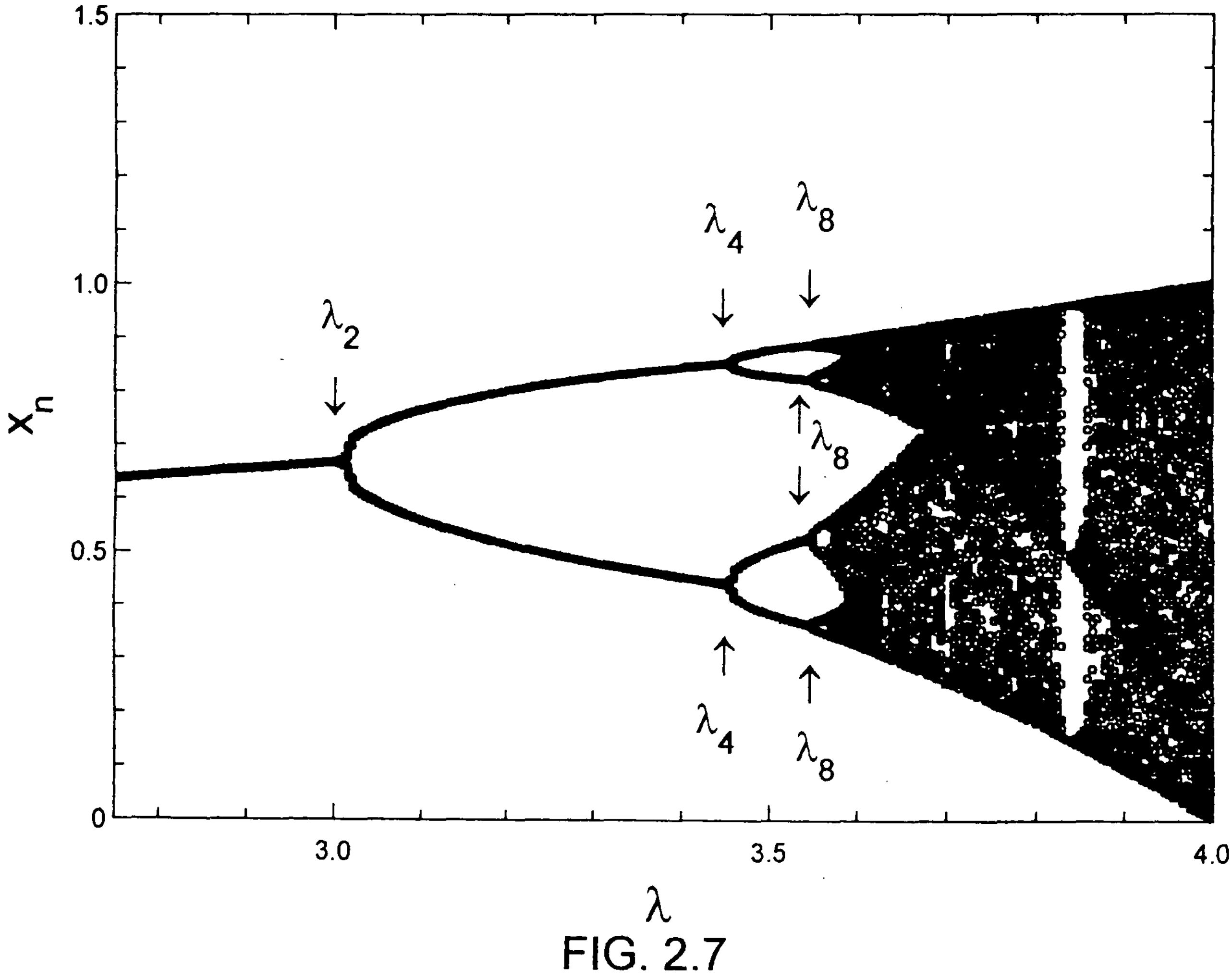


FIG. 2.7

Figenbaum showed that λ values at which very long cycles would appear turned out to be much more predictable. For long cycles, the spacing forms a geometrical series in which the successive terms are divided by a constant factor called δ. This δ has been found to be universal. Its value is δ=4.8296.

As we have already seem that for a certain value of growth rate parameter λ, the ratio of existing population to the maximum one approaches a fixed value. We may call this value as the attractor since the ratio after some time is attracted towards this fixed value. As the value of λ is further increased, we get period two cycle where the ratio is attracted towards two fixed values and so on. On increasing λ again we reach a point after which one obtains infinite period cycle. One encounters all the values between 0 and 1 of this population ratio. Thus the attractor is the entire interval between zero and one. For chaos the attractor can be an interval or a collection of different intervals. Such an attractor where one sees structure inside of structure inside of sructure and so on, is called a strange attractor. Such nested behaviour is described as scale-invariant or fractal. Scale-invariant means that when we blow up a certain portion of such structure and then blow up again, we obtain structure which is similar to pervious one . Hence it is invariant. They are called fractals because the dimension of such strange objects is not just an integer, but instead a positive non integral number.

The limit at which one moves from finite to infinite cycle of length, there one obtains an attractor called Feigenbaum attractor.

The Henon map as obtained from set of equations

$$\mathbf{x_{next} = \lambda x\ (1\text{-}x) + y}$$

$$y_{next} = xb \tag{2.9}$$

shows that the strange attractors occur for many values of b and λ [19]. Also here the motion is chaotic. Blowing up the phase space diagram again and again shows similar structure every time, i.e. the attractor is scale invariant and fractal.

Quasiperiodic route to chaos

We have seen the period doubling route to chaos. There are some other routes also. One such is quasiperiodic route [16,17]. Now when the parameter is changed, the stable periodic orbit may not enter into higher periods as observed in period doubling, but it may become a limit cycle[1]. Such transitions are called Hopf bifurcations. Further change of parameter may see two more Hopf bifurcations and we get three simultaneous coupled limit cycles and then the chaotic motions become possible.

When two simultaneous periodic motions with frequencies ω_1 and ω_2 are present and they are incommensurate, i.e. the ratio ω_1/ω_2 is an irrational number, the motion is termed quasiperiodic. The Poincare map of such motion is a closed curve. The motion is on the surface of a torus where the trajectories tend to fill it and the Poincare map is a plane cutting the torus as shown in Fig. [2.9] [16]. As the parameter is further varied, the torus structure of quasiperiodicity may break and the motion goes chaotic.

[1] A limit cycle [20] is a closed path of a non linear system which is approached spirally from either the inside or the outside by a non closed path either as $t \to +\infty$ or $t \to -\infty$. Fig. 2.8 shows a limit cycle where a non closed path C_1 approaches closed path C from the outside.

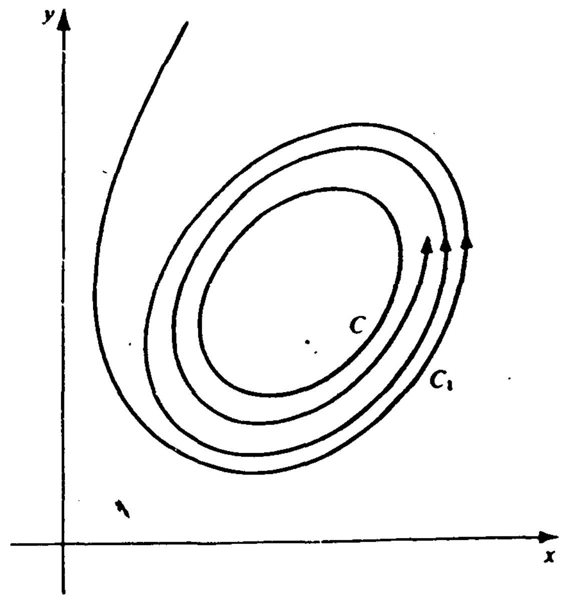


Fig. 2.8 A limit cycle

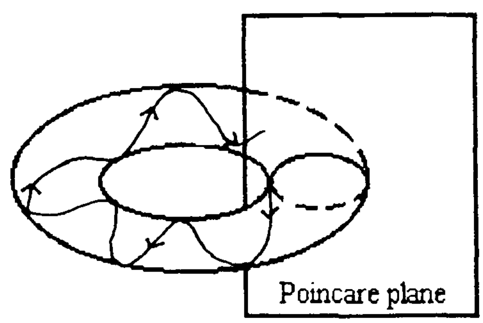


Fig. 2.9 Sketch showing the coupled motion of two oscillators and the Poincare plane used to detect a quasiperiodic route to chaos

Intermittency

Intermittency is yet another route to chaos [16]. It refers to such situations where bursts of chaos occur after long periods of periodic motion. As the parameter is varied, the bursts become more frequent and longer [Fig. 2.10]. Such chaotic motions have been observed in experiments on convection in a cell with a temperature gradient (Rayleigh Benard convection).

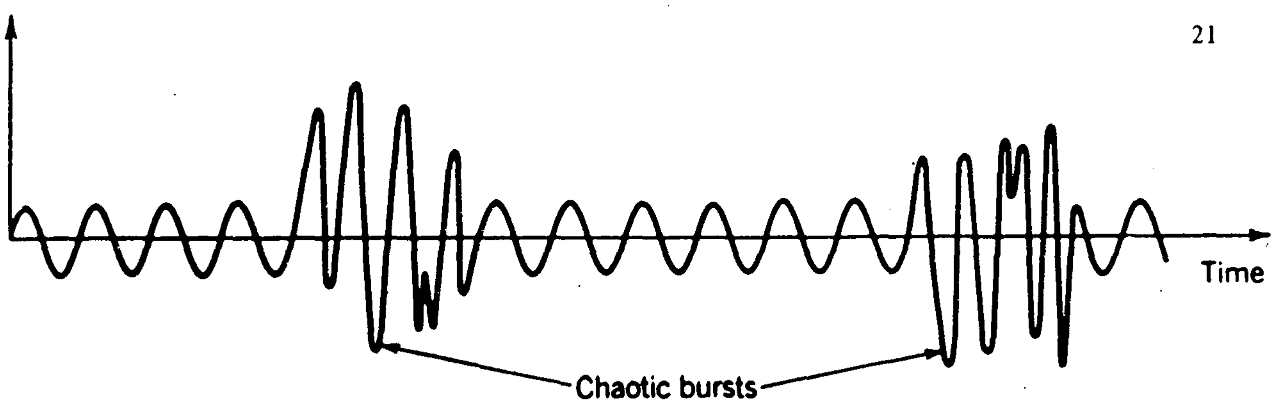


Fig. 2.10 Sketch of the intermittent chaotic motion.

IV. Lorenz Attractor

As we would take the particular attractor as one example for study in subsequent chapters, it is worthwhile to make introductory remarks. The unpredictability as observed in the flow of fluids past an obstacle, air flow near the aircraft wing and a score of other such turbulent features led scientists to think over it again and again. In 1963, an atmospheric scientist E.N. Lorenz of MIT proposed a simple model for thermally induced fluid convection in the atmosphere [21]. When the fluid is heated from below, it becomes lighter and rises up while denser fluid from above tends to come down. As a result, convective rolls are produced and what one observes is a complicated swirling motion. The simple model as presented by Lorenz uses only three variables. It is described by the set of equations :-

$$dx/dt = a(y - x) \quad (2.10)$$

$$dy/dt = rx - y - xz \quad (2.11)$$

$$dz/dt = xy - bz \quad (2.12)$$

where a=10, r=28 and b=8/3

In the phase space, it represents orbits which are confined to small region as shown in Fig.(2.11). Not a single point is ever crossed twice. It always form new orbits as the time evolves. The attractor has inherent simplicity despite being complex. The attractor has two loops - right and left. As the time progresses, the system moves through each of these two kinds of loops in turn. The structure thus formed appears like two wings of a butterfly. But the structure is very sensitive to initial conditions, for even a slight change in initial values will cause a complete reshuffling of the loops at later times.

The above is a brief review of some relevant aspects of chaos. In what follows, we study the application of the technique of Lehrman et al. to time series which are chaotic.

Phase portrait of Lorenz attractor

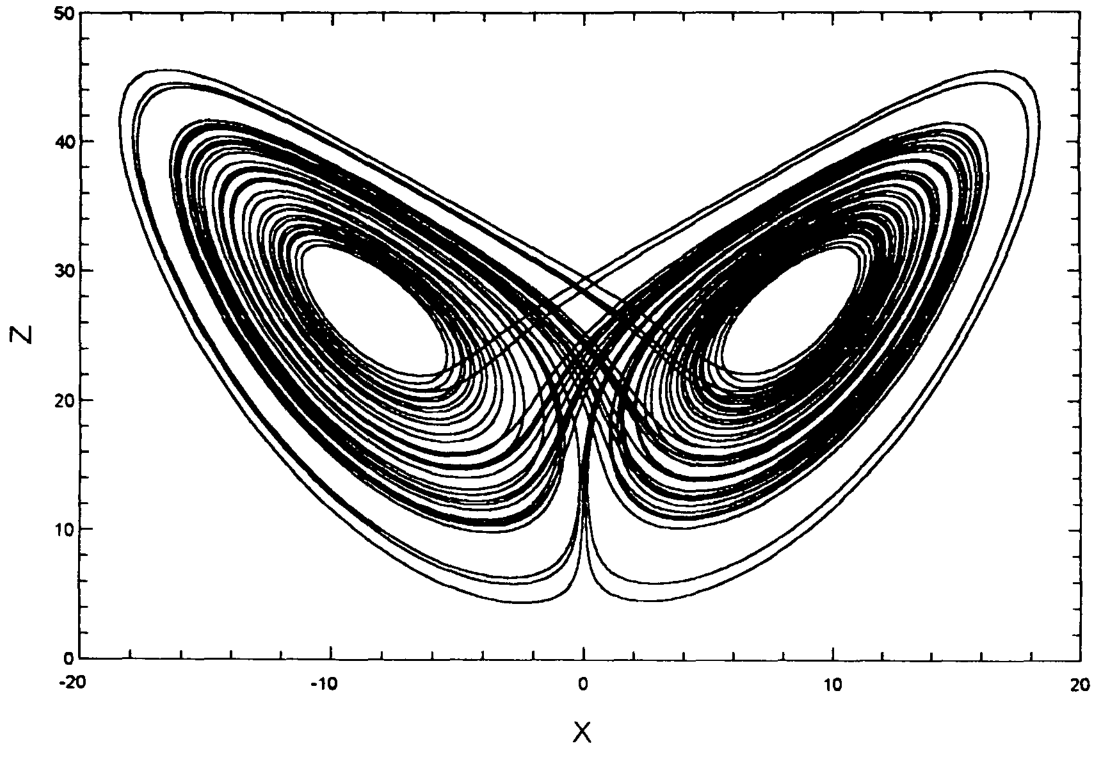


FIG. 2.11

CHAPTER THREE

REVIEW OF LEHRMAN et al. METHOD ON SYMBOLIC ANALYSIS OF LORENZ MODEL AND HIGH DIMENSIONAL MODEL

We present in this chapter the review of the method adopted by Lehrman et al. [5] to study the dynamical coupling of chaotic signals. The method applies symbolic analysis [22-24] of such signals to determine whether they follow the same underlying dynamics. The method works well even if external noise is added to the signal.

This technique is applied to Lorenz model and high dimensional model. We have discussed the Lorenz model briefly at the end of second chapter. The Lorenz model in the presence of additive noise can be written as:

$$dX/dt = -aX + aY + \delta X \qquad (3.1)$$

$$dY/dt = -XZ + rX - Y + \delta Y \qquad (3.2)$$

$$dZ/dt = XY - bZ + \delta Z \qquad (3.3)$$

Here X(t) is proportional to the amplitude of the fluid velocity while Y(t) and Z(t) are related to temperature fluctuations in a simplified Benard thermal convection model.

Further a=10, b= 8/3, r=28 . The terms δX, δY and δZ represent the additive noise.

We solve equations (1) - (3) by fourth order Runge-Kutta numerical method with time step Δt=.005 and the initial conditions X_0=6, Y_0=6, Z_0=13.5. Then every time step Δt, a random variable with Gaussian distribution is added to the signals X(t), Y(t), Z(t). The variances of all three random variables are equal and are parametrized as :

$$\sigma_x = \sigma_y = \sigma_z = R\sqrt{<(\Delta Y)^2>}$$

$$\sqrt{<(\Delta Y)^2>} = 0.325 \qquad (3.4)$$

The average values are calculated in the absence of external noise.

We check the method for three different noise ratio R=0.5, 1 and 1.5.

For the present study of symbolic analysis method, we first take the two signals X(t) and Z(t) as generated by Lorenz model without external noise. The time series of the two signals for t=25 and time step Δt=.005 is shown in Fig (3.1). The behaviour of the two signals appear quite different, though they have evolved following the same dynamics, i.e. on the same attractor- the Lorenz attractor.

The purpose of the method is to show, by analysing the time series data only, that the two time series could have arisen from the same underlying dynamics. Other methods exist which show a similar result - one could calculate the dimension of the attractor from each of the time series. But these other methods are fairly cumbersome and are less robust in the presence of noise indicating that with experimentally measured time series, these methods would be less accurate.

We discretize the time series signals as

$$X_n = X(t_0 + n\tau)$$

$$Z_n = Z(t_0 + n\tau) \qquad (3.5)$$

Here n=0,1,2,.......

Time records of X & Z signals of Lorenz model

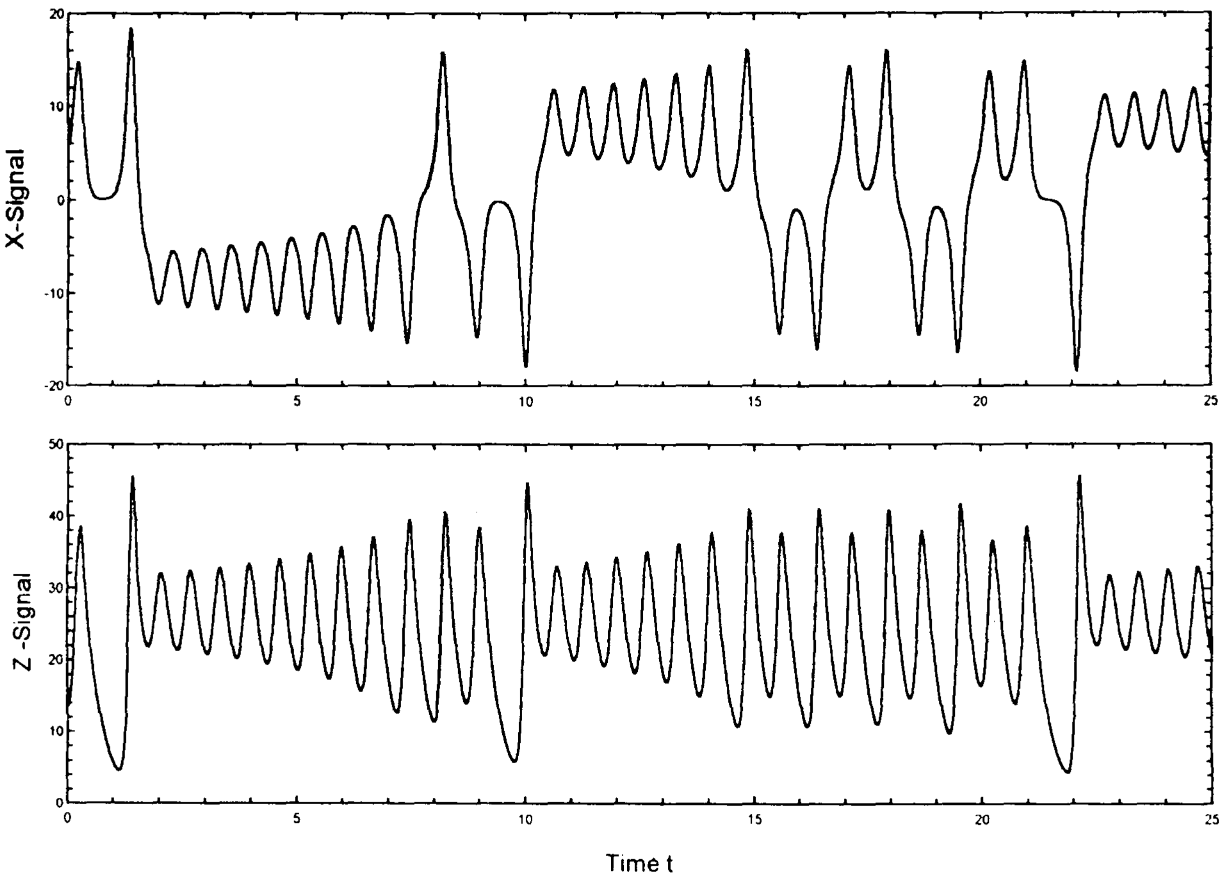


FIG. 3.1

For this case we choose $\tau=1$, i.e. we choose every 200th signal points in the time series of signals with $\Delta t=.005$. The most suitable value of τ is taken after verifying where the main result looks to be most pronounced. To get an approximate idea of τ, we obtained the Fast Fourier Transform (FFT) of our signals. This method that we applied for it is illustrated below :

Suppose we sample a signal f(t) according as

$$\mathbf{f_n = f\,(n\Delta)} \qquad \textbf{(3.6)}$$

where n=....., -2,-1,0,1,2,......

Δ is the time interval used for sampling. The reciprocal of Δ is called the sampling rate, i.e. the number of samples taken per unit time.

The Nyquist critical frequency [25] is defined as follows :

$$\mathbf{f_c = 1/(2\Delta)} \qquad \textbf{(3.7)}$$

If we sample a sine wave with Nyquist critical frequency f_c, the sampling would imply that the first sample taken would lie on one of its crest, the second on the trough of that cycle,the third on the crest of the next cycle and so on. It means there are two samples per cycle. The importance of this frequency is that if the continuous function f(t) , sampled at an interval Δ, happens to be bandwidth limited to frequencies less in magnitude than f_c, i.e. $|f|\leq f_c$, then the function is completely determined by its samples f_n. This is known as sampling theorem [25]. To illustrate the usefulness of this for our method, we take the first 4096 points of X-signal of Lorenz model sampled at $\Delta t=.005$. We obtain the fourier transform of this data as :

$$\mathcal{X} = \mathbf{FFT(X)} \qquad \textbf{(3.8)}$$

The frequency axis may be defined by noting that for any point D ($D \leq 4096/2$), the corresponding frequency is

$$\mathbf{f = \{(1/\Delta t)/4096\}\ D} \qquad \mathbf{(3.9)}$$

which we do for the first 2047 points (the remainder of the 4096 points are symmetric).

Thus

$$\mathbf{f = (200/4096)\ D} \qquad \mathbf{(3.10)}$$

where D = 0,1,....., 2047

Now we plot the transform x as y-axis and D as x-axis as shown in Fig (3.2).

The graph shows that the transform x falls off rather rapidly with time. The value of D after which x seems to be much lower is noted as D_c and the corresponding frequency as the critical frequency f_c. Thus from eqn (3.10) we get

$$\mathbf{f_c = (200/4096)\ D_c} \qquad \mathbf{(3.11)}$$

The frequencies greater than f_c corresponds to signal values rendered insignificant due to excessive noise. This frequency (when converted to time) gives an approximate idea of when the autocorrelation goes to zero. This time calculated as indicated above, is used as the lag in the calculations. Any time greater than this would be appropriate.

This frequency f_c may be treated as Nyquist critical frequency. Then eqn (3.11) can be written as

$1/(2\Delta) = (200/4096)\ D_c$, which gives

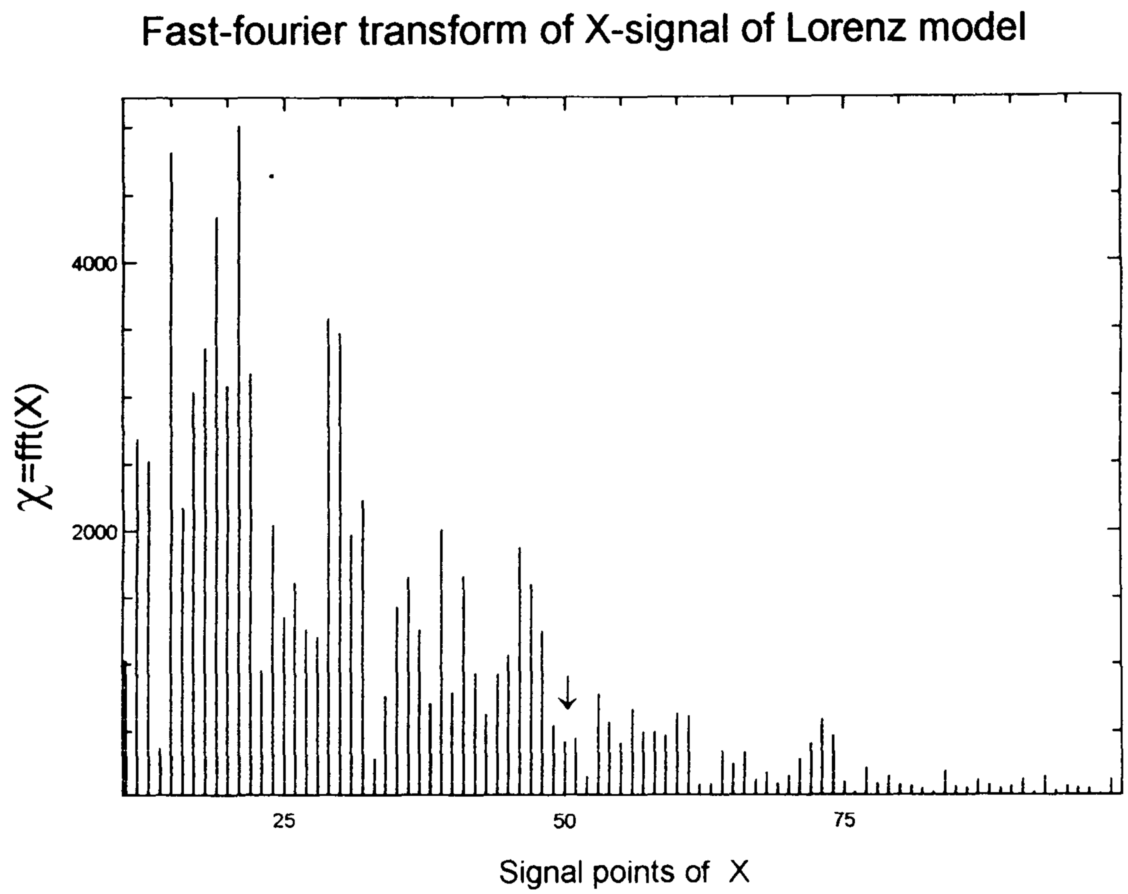

Fast-fourier transform of X-signal of Lorenz model
4000
2000
χ=fft(X)
25
50
75
Signal points of X

FIG. 3.2

$$\Delta = 4096/(400\ D_c) \qquad (3.12)$$

Δ here is the analogue of our τ. Clearly the value of τ would be greater than the Δ thus calculated. As shown in Fig. (3.2), the value of D_c is taken as 50. From eqn (3.12), Δ comes out to be ~0.2.

The good approximation of τ can be obtained for by looking for some values of τ greater than 0.2. The best value of τ is one which makes our result most prominent.

Now to analyse two different signal patterns, we need to have an appropriate language in order to extract maximum required information without losing the essence of pattern behaviour. We use here the symbolic language for the signals. To our signal points, X_n, Z_n, we associate symbolic representation. The symbolic dynamics, in fact, present the partitioning of the phase space (coarse graining) such that the information concerning the particle orbits is embedded in the partitioning [23,24]. In the method, for the discrete signal points X_o, X_1, X_2,....., one associates a sequence of integers S_o, S_1, S_2,....., according to some prescribed rule. Then to a shorter sequence of length L taken out of this sequence, one associate an integer l, again defined by some rule. The core meaning of this phase partitioning is that if D_l be the set of all X in some initial domain D such that any orbit with X_o=X will evolve in time to produce the same sequence (S_o, S_1, S_2,.....) of length L and thus exactly the same value of l, then D_l is said to represent a coarse grain element of phase space. It is to be noted that the sequence define a symbolic dynamics if for a given l each set D_l is simply connected. This means that there are not two or more different sets of all X corresponding to a particular D_l. Thus a particular truncated sequence is characterized uniquely by an integer l,

$$I = \sum_{k=0}^{L} M^{L-k}\, S_k \qquad (3.13)$$

where M is the number of symbols or integers used to divide the entire sequence into different domains as will be illustrated now. S_k corresponds to the kth symbol or integer of the truncated sequence of length L.

One may thus divide the X_n time series corresponding to different symbols/integers according as

$$S_n = \begin{cases} A \quad (0) \quad X_{min} < X < X_{c1} \\ B \quad (1) \quad X_{c1} < X < X_{c2} \\ C \quad (2) \quad X_{c2} < X < X_{max} \\ \cdot \\ \cdot \end{cases} \qquad (3.14)$$

X_{c1}, X_{c2},....., are critical points which divide the whole series into different domains.

Rochester and White [24] have shown that the proper selection of critical points is necessary for the symbolic dynamics method to work. They studied the partitioning of the Henon map. It is thus established that the critical points must be boundaries of symbol domains; otherwise the nonmonotone character of the map leads to domains D_l which are not connected, meaning thereby that many such domains would have the same symbol sequence. Thus very different orbit are associated with the same sequence and the partitioning would not reproduce the correct statistical properties of the map. Thus the rule for partitioning is that any critical surface must separate symbol domains.

Lehrman et al. emphasizes that, for coarse graining method to work in establishing the dynamical coupling, one need just rough approximation. It means that the better optimum language is one that keeps the degeneracies of domains D_l to a minimum.

The determination of values of such critical points is discussed later.

For our discussion we take integers instead of symbols. For the Lorenz model, we convert the time series by using just two integers 0 and 1 i.e. in eqn (3.13), M=2. We take out all sequences of length L, as for example if L=5,

1 1 1 0 0 1 0 1 0 0 11. . .

we can have as many as M^L such different sequences. Here they are 2^5=32 different sequences.

Clearly the sequences thus obtained have unique value of l, where

$$l = \sum_{i=1}^{5} 2^{L-i} S_i$$

$$= 2^4 S_1 + 2^3 S_2 + 2^2 S_3 + 2 S_4 + S_5 \qquad (3.15)$$

Here l may take any value from 0 to 31. The same procedure has to be followed for Z_n time series also.

Our next consideration would be to determine the critical points.

We break here briefly to discuss the information theory [26,27] that is used to get the results.

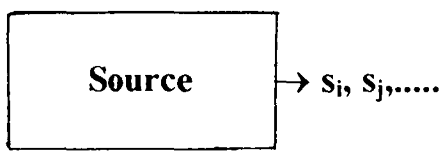
If E be some event which occurs with probability P(E), then the information received when the event E has occurred is

$$\mathbf{I(E) = \log 1/P(E)} \quad \textbf{units of information} \qquad \textbf{(3.16)}$$

If one takes a logarithm to base 2, then the resulting unit of information is called a bit

$$\mathbf{I(E) = \log_2 1/P(E)} \ \textbf{bits} \qquad \textbf{(3.17)}$$

Let us think of a discrete information source emitting a sequence of symbols from a fixed finite source alphabet $S = \{s_1, s_2, \ldots, s_q\}$. Successive symbols are selected according to some fixed probability law.



An information source

We assume that successive symbols emitted from the source are statistically independent. Such as information source is called zero memory source and is completely described by source alphabet S and the probabilities with which the symbols occur :

$$P(s_1), P(s_2), \ldots, P(s_q)$$

If symbol s_i occurs, the amount of information is equal to

$$I(s_i) = \log 1/P(s_i) \quad \text{bits} \qquad (3.18)$$

Since the probability of this to happen is $P(s_i)$, the average amount of information obtained per symbol from the source is

$$\sum_{S} P(s_i)\, I(s_i) \quad \text{bits}$$

$\sum_{S}$ is the summation over the q symbols of the source S.

This quantity, the average amount of information per source symbol is called entropy H(S).

$$H(S) = \sum_{S} P(s_i) \log 1/P(s_i) \qquad \text{bits} \qquad (3.19)$$

Considering our discussion on the symbolic representation of time signals, the information content may be defined as entropy E where,

$$E = -\,1/L \sum_{I} P_I \ln P_I \qquad (3.20)$$

P_l is the probability of finding a particular sequence l, i.e. the number of sequences corresponding to this l divided by total number of all sequences.

To get the critical points X_{ci} for a particular signal, one need to maximize this entropy with respect to some ranges of sample points chosen. For example one may choose one range of sample points so that every time a sample point X_c so selected converts the time series into integer symbols 0,1 according as if $X < X_c$, then $X = 0$, else $X=1$. and then see at which value of X_c the entropy is getting maximized. The value of X_c that maximizes the entropy denotes the critical point of signal series. Similarly two critical points for the series can be found if we maximize the entropy using two different ranges of sample points. If we increase the number of critical points from one to two, we see that information content getting increased. We may see further increase with the increase of critical points until we reach a stage where addition of critical points will not increase the information content anymore and thus for this set of critical points X_{ci}, an optimum language for the symbolic analysis is found.

We obtain the critical points for both X_n and Z_n data. We then convert it into states $l_x(n)$ and $l_z(n)$. If the X_n and Z_n data come from different dynamics, the evolution of $l_x(n)$ and $l_z(n)$ are not correlated. It means that if the variable X occupies the state l_0, the variable Z can occupy any of the states available to it. But if X_n and Z_n data follow the same dynamics, then if X occupies the state l_0, Z can occupy only neighboring states. This happens because these states are different symbolic coarse grainings of the same orbit. But if we time shift these two sequences $l_x(n)$ and $l_z(n)$, i.e. for sequences like $l_x(n)$ and $l_z(n+n_0)$, the correlation no longer exists. In order to verify this assertion, we compute here the conditional entropy defined as

$$E(Z/X) = -1/N_l \sum_{l_x} 1/L_l \sum_{l_z/l_x} P(l_z/l_x) \ln P(l_z/l_x) \quad (3.21)$$

where $P(l_z/l_x)$ is the probability for the variable Z to occupy l_z state if the variable X occupies l_x state. N_l is the total number of different l_x sequences. The first summation is done over all accessible l_z states for a given l_x state and the second summation is done for all available l_x states.

A sharp minimum is observed in E(Z/X) vs shift parameter n_0 curve in Lorenz model when $n_0=0$ i.e. when X and Z signals are evolving simultaneously and hence they are correlated. This correlation is destroyed by taking nonzero integral values of n_0.

For the Lorenz model, we have studied four cases- one when there is no noise and rest three when noise of different noise ratio R is added.

We take $\tau=1$. We get an approximate verification of this from FFT method as described earlier. Time T=4000, with $\Delta t=.005$, we simulate 800000 signal points. We use one critical point X_c and Z_c and the result appears quite outstanding. The full optimization may need some more critical points. The number of integer symbols being used are 0 and 1, i.e. M=2. The graph of E(Z/X) vs n_0 for four cases is shown in Fig. (3.3).

Case A

This is ideal situation for model when noise δX, δY, and δZ are zero (R=0). Here $X_C=0.07$ and $Z_c=21.24$.

A sharp minimum appears at $n_0=0$ but if time shifting is done, E(Z/X) increases and nearly stabilizes at some far away n_0 values.

Conditional entropy as a function of shift parameter n_0 for the X & Z signals of Lorenz model

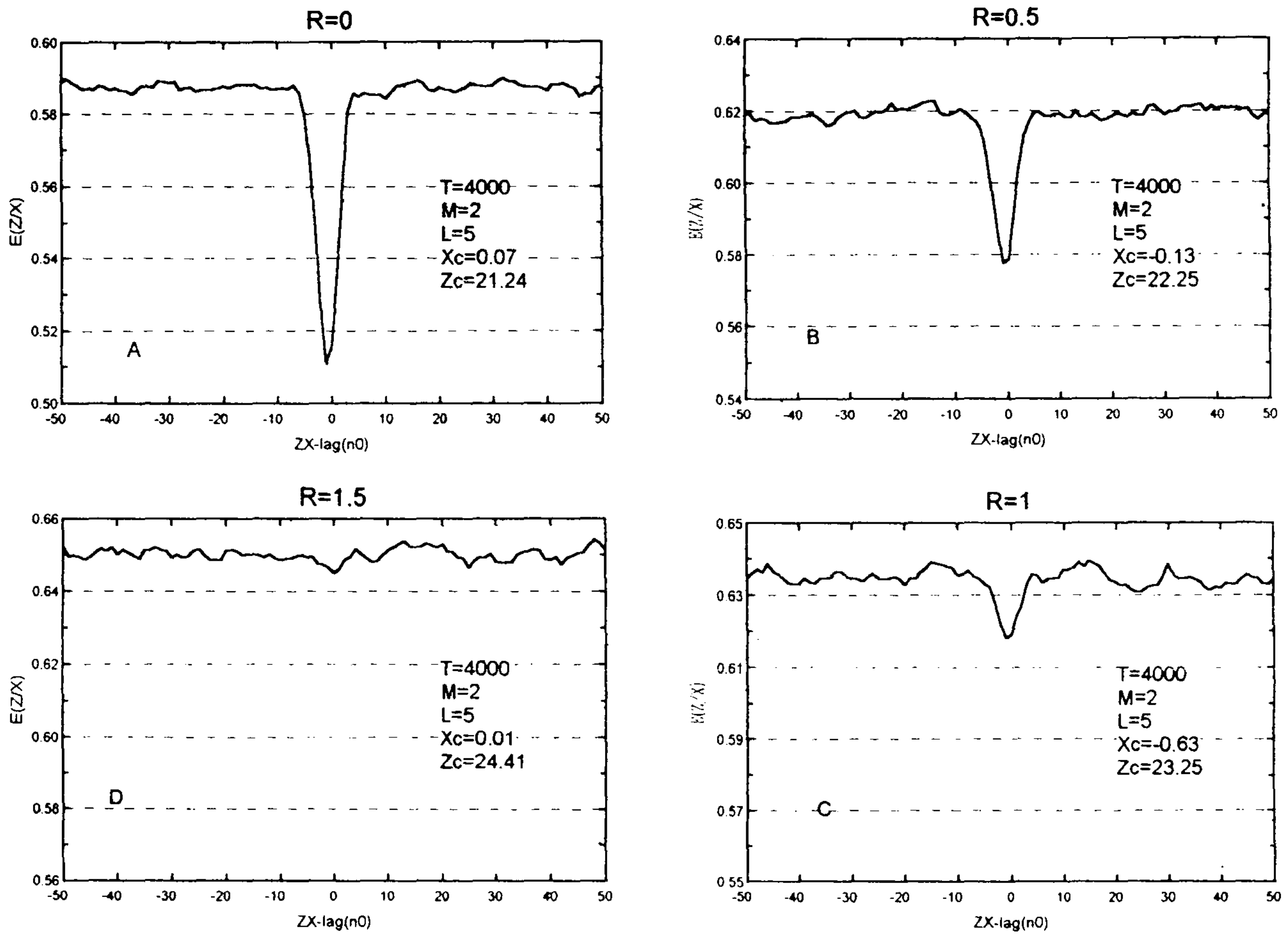


FIG . 3.3

Case B

Noise ratio R=0.5, X_c=-0.13, Z_c=22.25. This minimum gets shifted up substantially and comparatively stabilises rather a bit earlier.

Case C

R=1, X_C= -0.63, Z_C=23.25. The peak appears but the sharpness is considerably reduced.

Case D

R=1.5, X_C=0.01, Z_C=24.41. Due to heavy noise the peak is greatly reduced and stabilizes much quicker.

From the four cases, we observe that the peak gets less pronounced as R is increased. The presence of peak in all cases demonstrates that correlation is not easily disturbed by noise but the effect of noise leads to the increase in information content E(Z/X) rather. Hence the result appears quite robust even in the presence of noise.

We proceed further to apply this method for more complex signals shown in Fig. (3.4) and generated by a high dimensional model.

The equations of motion are

$$dX_k/dt = - X_{k-2} X_{k-1} + X_{k-1} X_{k+1} - X_k + F \qquad (3.22)$$

Time Records of signals 1 & 3 for the high dimensional model

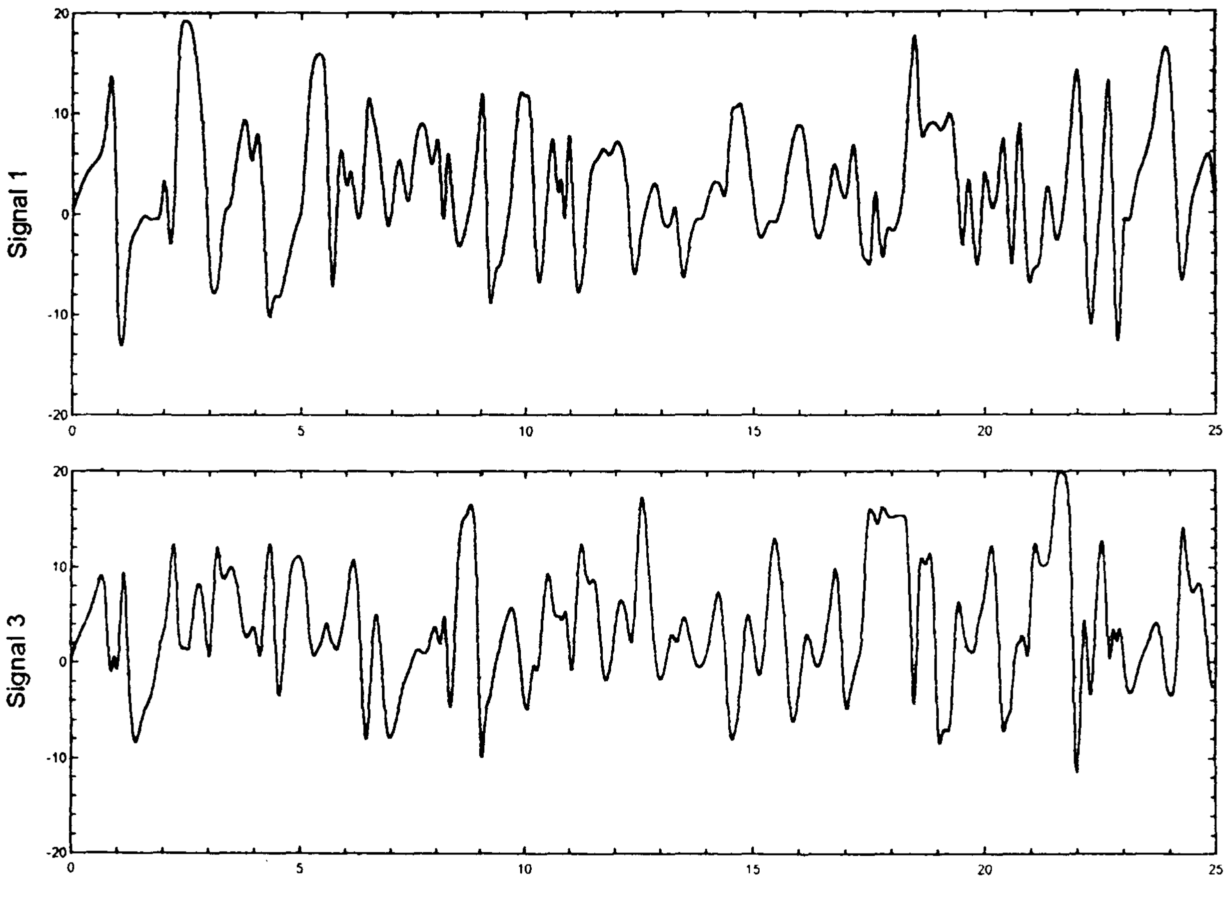


FIG. 3.4

It models the basic physics of a turbulent system. Here k is an integer. The variables X_k may be considered as values of some atmospheric quantity in K sectors of a latitude circle with periodic condition $X_{k+K} = X_k$. The external forcing and internal dissipation are described by the constant F and linear term. The quadratic terms simulate advection which conserves the total energy of the system $X_1^2 + X_2^2 + \ldots.. X_K^2$.

We take K=10 and F=15. We thus have ten coupled non-linear equations of motion corresponding to signals from X_1 to X_{10}. To solve it, we use fourth-order Runge-Kutta numerical method with $\Delta t = 0.01$. Time T=20000. The language is here optimized by taking two critical points for each signal, i.e. for the ith signal X_i,

$$\begin{aligned} &\textbf{If } X_i < X_{c1}, \textbf{ then } X_i = 0 \\ &\textbf{If } X_{c1} < X_i < X_{c2}, \textbf{ then } X_i = 1 \qquad \textbf{(3.23)} \\ &\textbf{and If } X_i > X_{c2}, \textbf{ then } X_i = 2 \end{aligned}$$

Thus the number of integer symbols taken is 3, i.e. M=3. X_{c1} and X_{c2} are the two critical points used for signal X_i. The critical points for signal 1 are found to be 0.39 and 5.34 and that for signal 3 are 0.50 and 5.38. Using FFT analysis, the value of τ has been found approximately around 0.15. We check the result for three values of τ as shown in A, B, C of Fig. 3.5 for signal 1 and signal 3 (s1 & s3). We see the results satisfy the symbolic study for the high dimensional model.

Conditional entropy as a function of shift parameter n_0 for signals 1 & 3 of the high dimensional model

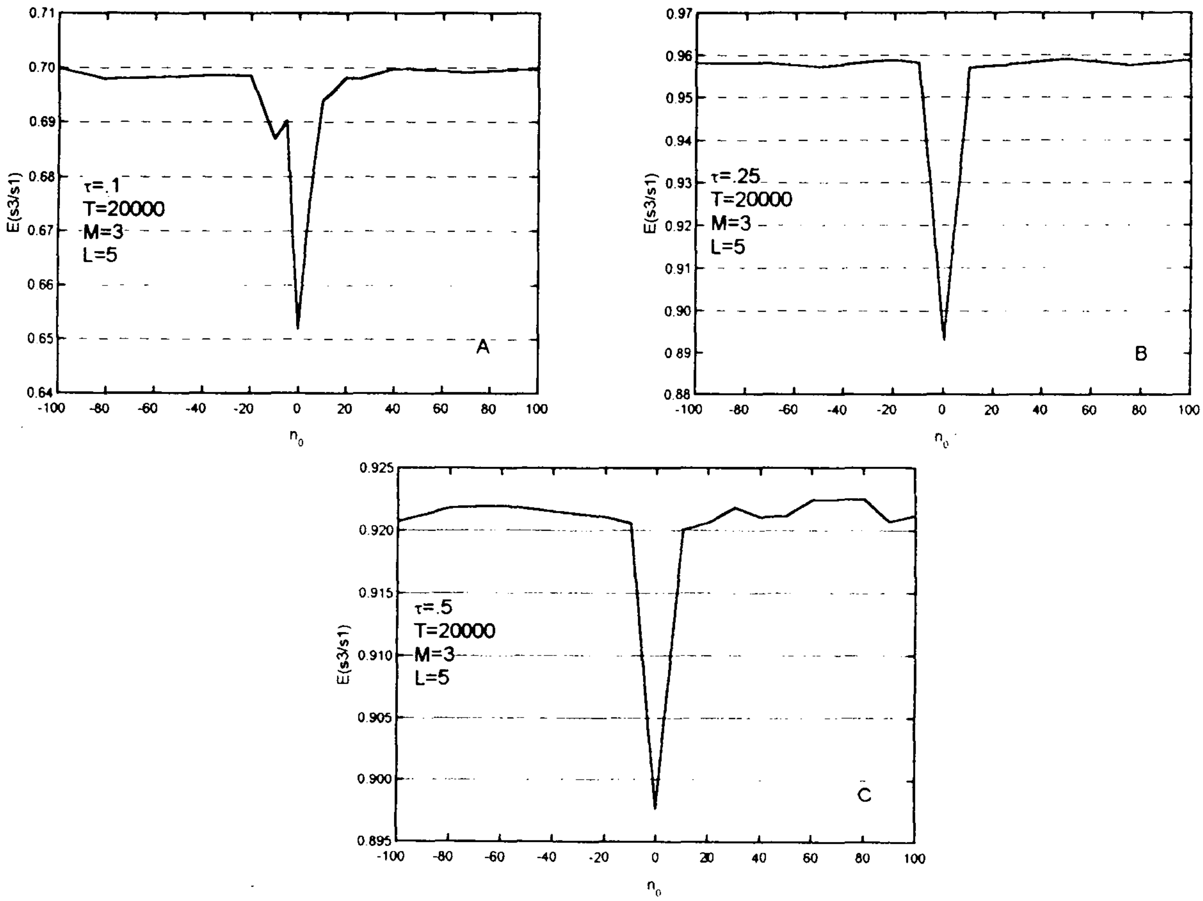


FIG. 3.5

CHAPTER FOUR

APPLICATION OF SYMBOLIC STUDY TO OTHER DYNAMICAL SYSTEMS

I. Rossler Attractor

Rossler proposed another simple model describing the dynamics of chemical reactions in a stirred tank [6,16].

$$dX/dt = -(Y+Z) \quad (4.1)$$

$$dY/dt = X + aY \quad (4.2)$$

$$dZ/dt = bX - cZ + XZ \quad (4.3)$$

where a, b, c, are constants

This is also a three dimensional system with a single non linear cross term XZ.

This system may be thought to model the flow around one of the loops of the Lorenz attractor and is thus a model of a model. Here a=0.38, b=0.3. With c=4.5, we see the chaotic motion. c is taken as the bifurcation parameter. The phase portraits of the attractor is shown in Fig. 4.1. The flow forms a single spiral embedded in a disc, with trajectories from the outer part of the spiral twisted and folded back into the inner part of the spiral.

We apply symbolic analysis for the Rossler system to verify the method as applied to Lorenz attractor in the previous chapter.

Phase portraits of Rossler attractor

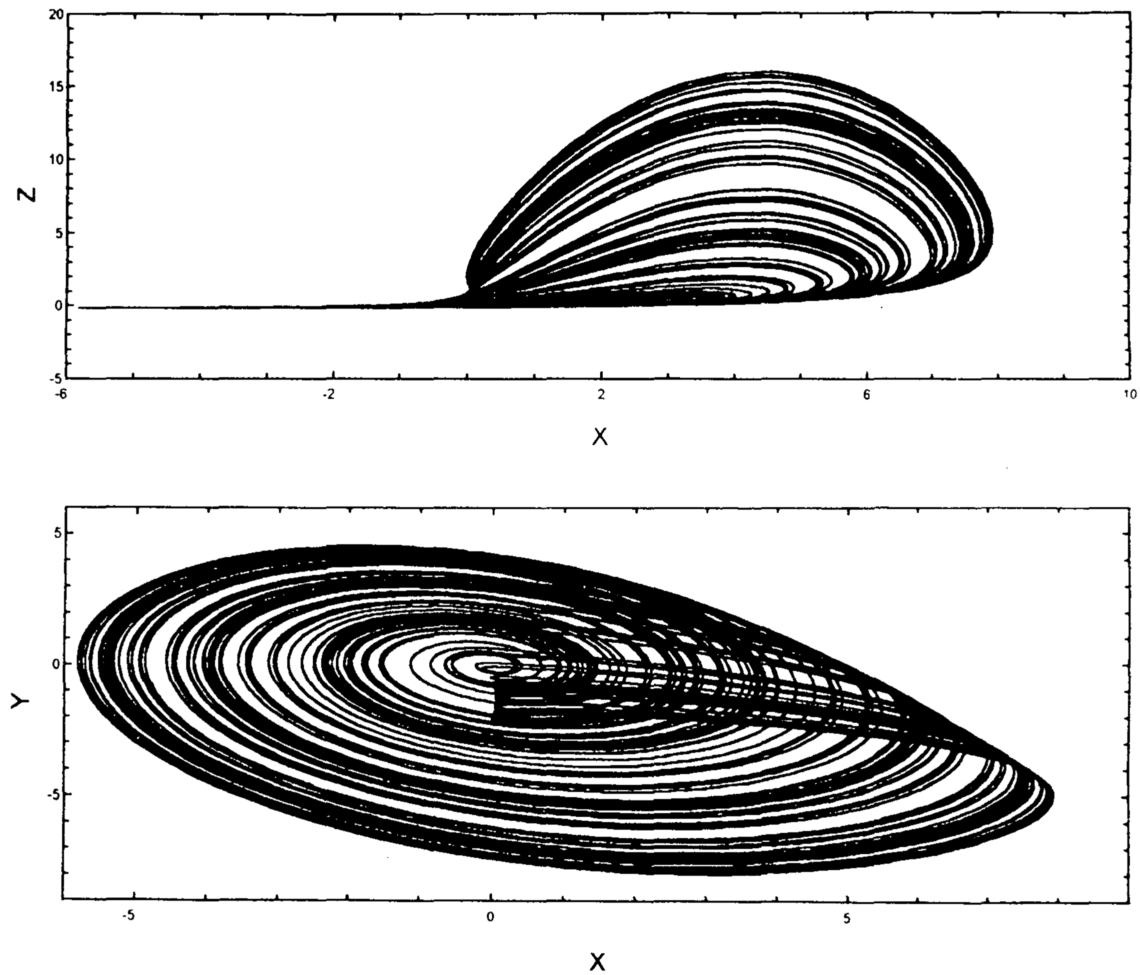


FIG. 4.1

The equation (1) – (3) is solved numerically by Runge-Kutta fourth order method with time step $\Delta t=0.005$. We take the initial condition $X_0=Y_0=Z_0=1.0$. The time T=3900. We discretize the X and Z signals. Using FFT analysis, the most appropriate value of τ has been found as 1.5. We then determine the critical points X_c & Z_c. Fig. 4.2 shows the information entropy E as a function of X_c points for M=2 and L=5. The information content is maximum at $X_c \approx 0.95$. Hence this is chosen as the critical point appropriate for partitioning.

We analyse the coarse-graining method for four cases here as shown in Fig. 4.3.

The graphs from (A) to (C) are ones in which three different lengths of short sequences, i.e. L=3, 5 and 7 are taken. We take one critical point for each of the cases. We want to verify the value of L most appropriate for our study. Before that we show here how the maximum value of information entropy changes for X and Z signals for three values of L. We tabulate the result shown below.

L	EX	EZ
3	0.633	0.615
5	0.562	0.557
7	0.504	0.507

Table 4.1

EX and EZ denote the maximum value of information entropy for X and Z signals respectively. We observe that the entropy decreases as the length L is increased.

Information content as a function of X_c points of X-signal for the Rossler model

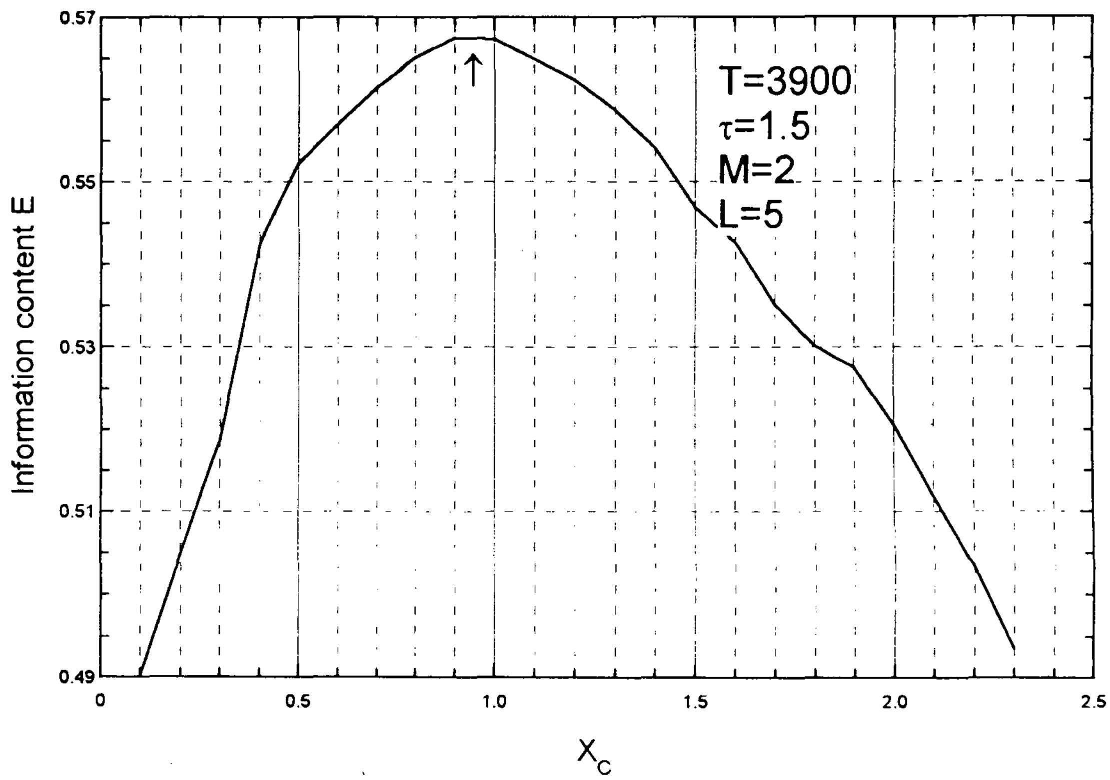


FIG. 4.2

Conditional entropy as a function of shift parameter n_0 for the X & Z signals of Rossler model

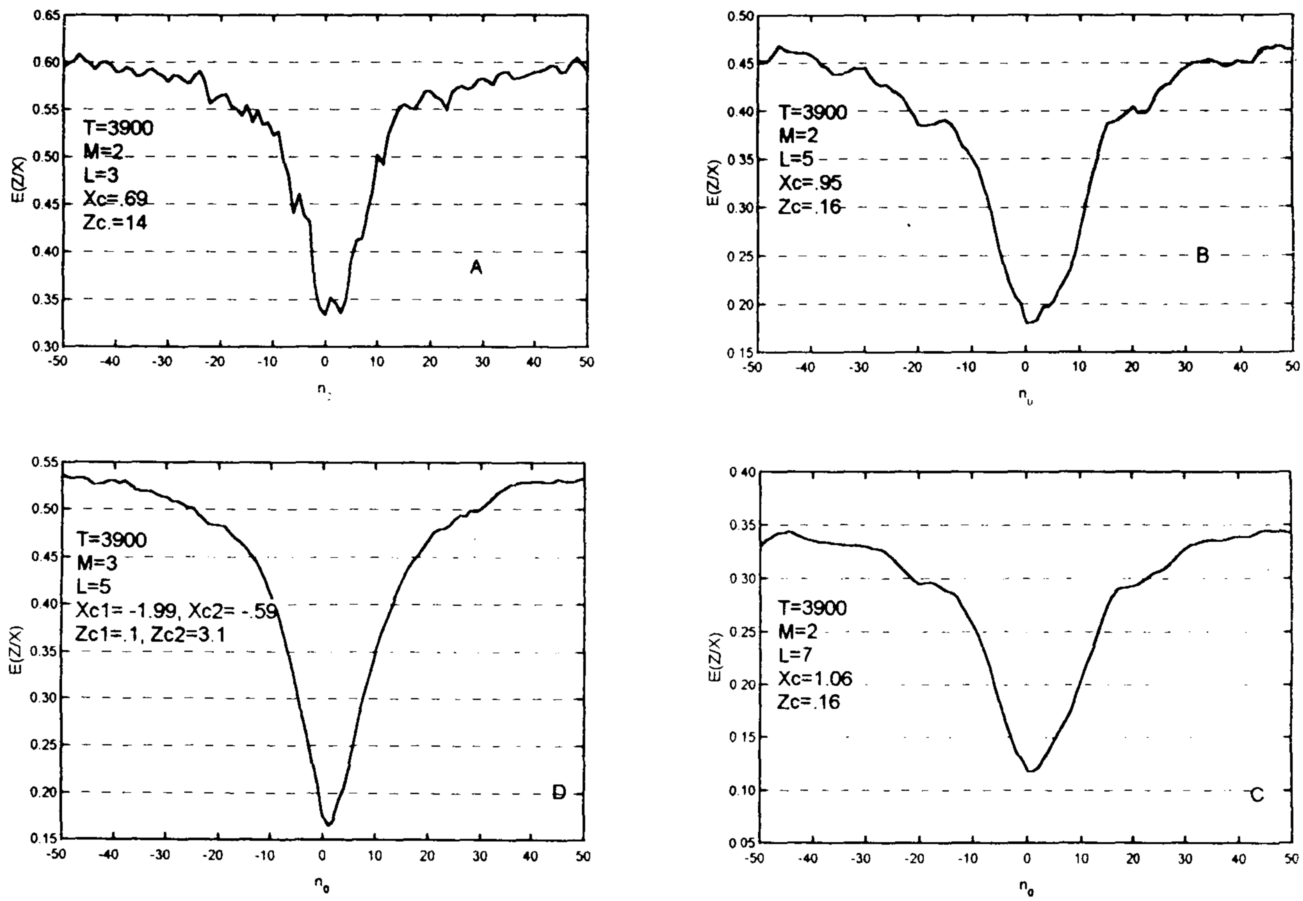


FIG. 4.3

The information content,

$$-1/L \sum_{l} P_l \ln P_l$$

is maximum when all the P_l's are equal. When one chooses a small L, say 3 with one critical point, the number of possible different sequences is 2^3. It appears that most or all of these sequences appear with non zero probabilities. As L increases, the number of possible sequences increases and some of them appear with almost vanishing probabilities thereby reducing the information content.

A glance at graphs (A) – (C) shows that the most pronounced peak occurs when L=5. Hence short sequences of length L=5 yield better approximation for present symbolic dynamical study.

Graph (D) shows a special case when two critical points are taken. Here L=5. We compare the information entropy of X and Z signals corresponding to one and two critical points.

Critical Point(s)	EX	EZ
1	0.562	0.557
2	0.725	0.701

Table 4.2

From Table 4.2, we see that information entropy corresponding to two critical point is higher. The increase in critical point implies increase in number of symbols used to redefine the series and hence number of unique values of l used in partitioning the phase space increases. (l=0 to 2^5-1 for one critical point, l=0 to 3^5-1for two critical point). Hence the information content is higher in the case with two critical points. The comparison of conditional entropies for the two cases has been made in graph (B) & (D). Graph (D) shows a much pronounced peak in comparison to graph (B). It implies that two critical points make the symbolic language more suitable for this study.

This study of the Rossler equation indicates that the Lehrman et al. method is fairly general. Further we have shown that choosing 'τ' using the FFT is reasonable.

II. Time Delay Equation

Time delay applies to situations where there is occurrence of delayed action in response to certain casualty. This time-lag is very crucial in determini- ng the nature of action [13,28]. Time delay finds wide relevance in manifold domains as diverse as optics[29], physiology [14] and population biology [15,30]. The time delay as inherent in so many natural or man made dynamical systems result is in rich dynamics. Mackey and Glass (14 ,18) have studied the delayed mechanisms in physiological control systems. They pointed out that a number of chronic and acute diseases show non-periodic behaviour and the oscillatory instabilities shown by them has been established through mathematically complex models. They emphasize that simple mathematical models of physiological systems show periodic as well as non periodic dynamics similar to those encountered in human diseases. They take the simple ordinary differential equation,

$$dX/dt = \lambda - \nu X \qquad (4.4)$$

where X is a variable to be controlled, t is time, and λ and ν are positive constants giving the production and decay rates respectively, of X. In the limit

$t \to \infty$, $X=\lambda/v$ starting from any initial condition. This system reaches a point of stable equilibrium. In fact to describe physiological systems, λ and v are not taken constants, but depend on the value of X at some earlier time. In considering a homogeneous population of mature circulating cells of density X, it was assumed that the production was a non linear function of the density at a time τ'in the past, $X_{\tau'}$–

$$dX/dt = f(X_{\tau'}) - vX \qquad (4.5)$$

$f(X_{\tau'})$ is considered in two different forms:

$$f(X_{\tau'}) = \lambda\theta^n/(\theta^n + X_{\tau'}^n) \qquad (4.6)$$

$$f(X_{\tau'}) = \lambda\theta^n X_{\tau'}/(\theta^n + X_{\tau'}^n) \qquad (4.7)$$

where θ, n are constants.

Fowler and Kember [31] uses the Mackey-Glass equation in the form,

$$\varepsilon\, dX/dt = -X + f(X_{t-\tau'}) \qquad (4.8)$$

$$f(X) = \lambda X/(1+X^c) \qquad (4.9)$$

We thus have a first order delay differential equation which may show chaotic behaviour. It has an exponential relaxation term $-X(t)$ and an external forcing term $f(X_{t-\tau'})$ which is non linear with a delay τ'. ε is the ratio of the relaxation time to the delay and thus is a dimensionless parameter. ε is taken as very small and hence solutions can be taken close to the limit,

$$X = f(X_{t-\tau'}) \qquad (4.10)$$

We look for the solution of equation (4.8) – (4.9). The chaotic attractor is seen for $\lambda = 2$, $c = 10$, $\tau' = 1$, $\varepsilon = 0.05$ [31,32]. We iterate the equation with time step $\Delta t = 0.005$.

It must be emphasized here that in a delay differential equation, one need to specify initial conditions over a period of time and hence the dimension of such equation is infinite [13]. It is now an accepted fact that the dynamics of non linear equations are characterized by dimension which is not always an integer but it may be less than the number of independent variable used to describe the equation. Farmer [33] studied the delay equation (4.8) – (4.9) and showed that it was possible to determine the dimension and for certain parameters, the dimension appeared to increase linearly with time delay. Thus substantially high delay would lead to high dimensional delay equation.

The time series of the system being studied is shown in Fig. (4.4). The phase portrait is shown in Fig (4.5).

The objective of our study is to determine the time delay in an experimental time series similar to Fig (4.4), assuming that the non linear function $f(X_{t-\tau'})$ is not known.

For this we apply the symbolic analysis method (coarse graining) as developed by Lehrman et al. and described in chapter 3. We see that the method works extremely well for delay time series also and it is quite robust even in the presence of noise.

The time series of our signal X(t) is descretized as

$$\mathbf{X_n = X(t_0 + n\tau)} \qquad \mathbf{(4.11)}$$

$$n = 0, 1, 2, 3, \ldots\ldots$$

Time series plot of the chaotic time delay system

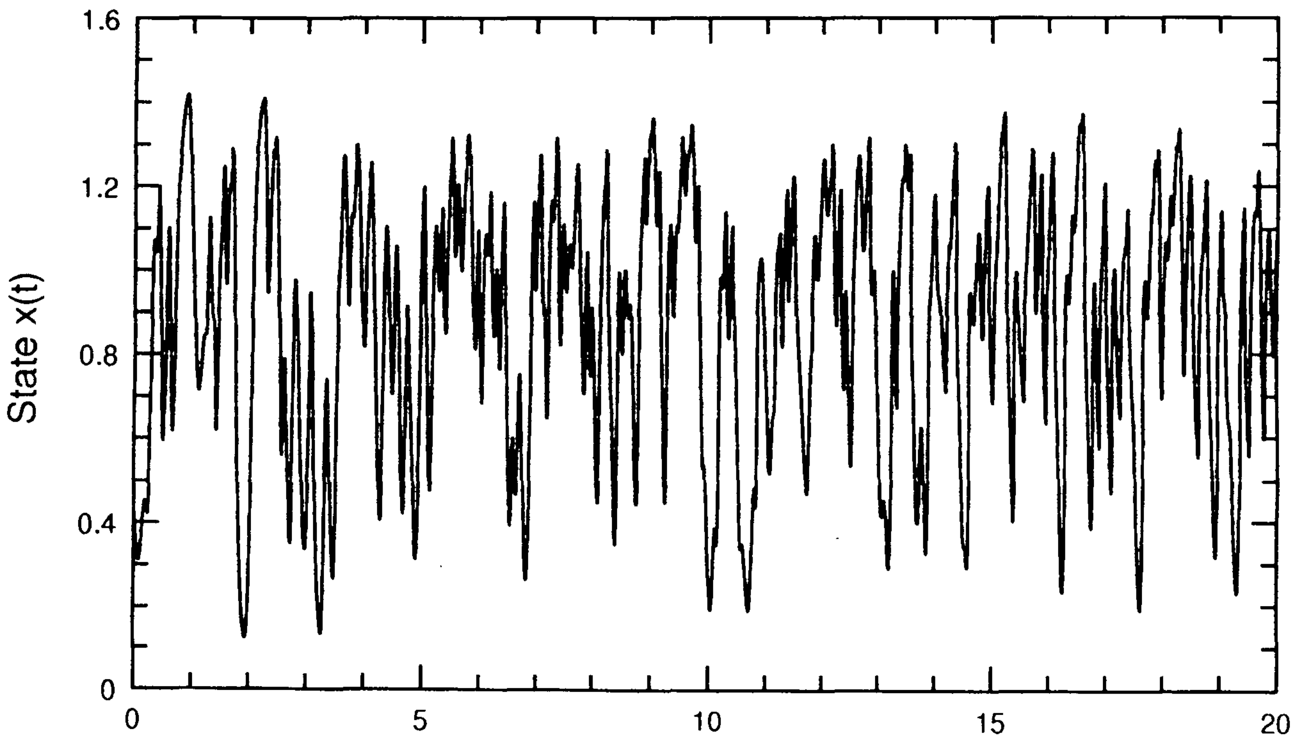


FIG. 4.4

Phase portrait of the time delay chaotic system

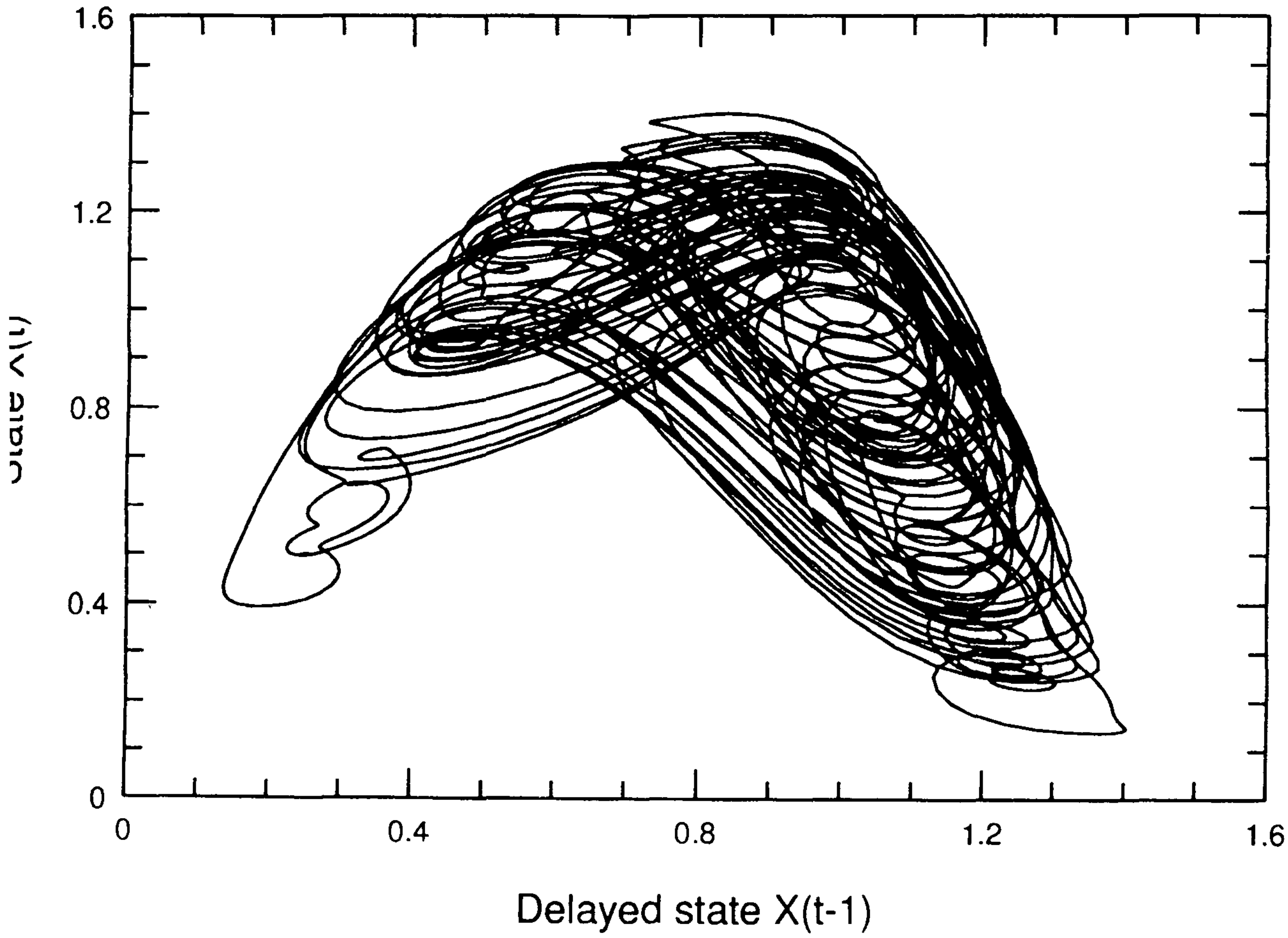


FIG. 4.5

The delay equation is converted into difference equation [31,32] and taking $dX/dt \approx (X_t - X_{t-\delta})/\delta$, the equation is iterated as

$$X_t \approx [\varepsilon/(\varepsilon+\delta)]\ X_{t-\delta} + [\delta/(\varepsilon+\delta)]\ f(X_{t-\tau}) \qquad (4.12)$$

Where δ is sufficiently small, $\delta<\varepsilon$. We take $\delta=(1/4)\varepsilon$. We take all the initial values as 0.1. The delay equation is iterated with time step 0.005. Using FFT analysis, the value of τ has been approximated as 0.1. The first 1000 data samples are discarded to allow the trajectory to fall on the system attractor and then the next 10000 samples are taken. We take one critical point for the symbolic language of the delay signal . The critical point has been found as $X_c=0.92$.

The conditional entropy $E(X/X_d)$ where X_d implies delayed state $X(t-\tau')$ is calculated for different values of delay τ' by shifting the time series with respect to itself. The graph is shown in Fig. (4.6). We observe a sharp minimum for time delay $\tau'=1$. It thus establishes that the coarse graining method works well to determine the time delay in experimental time series.

We check now how the method works when noise is added. Let the measurement process be described by

$$Y_t = X_t + n_t \qquad (4.13)$$

Conditional entropy as a function of time lag τ' of the time delay equation

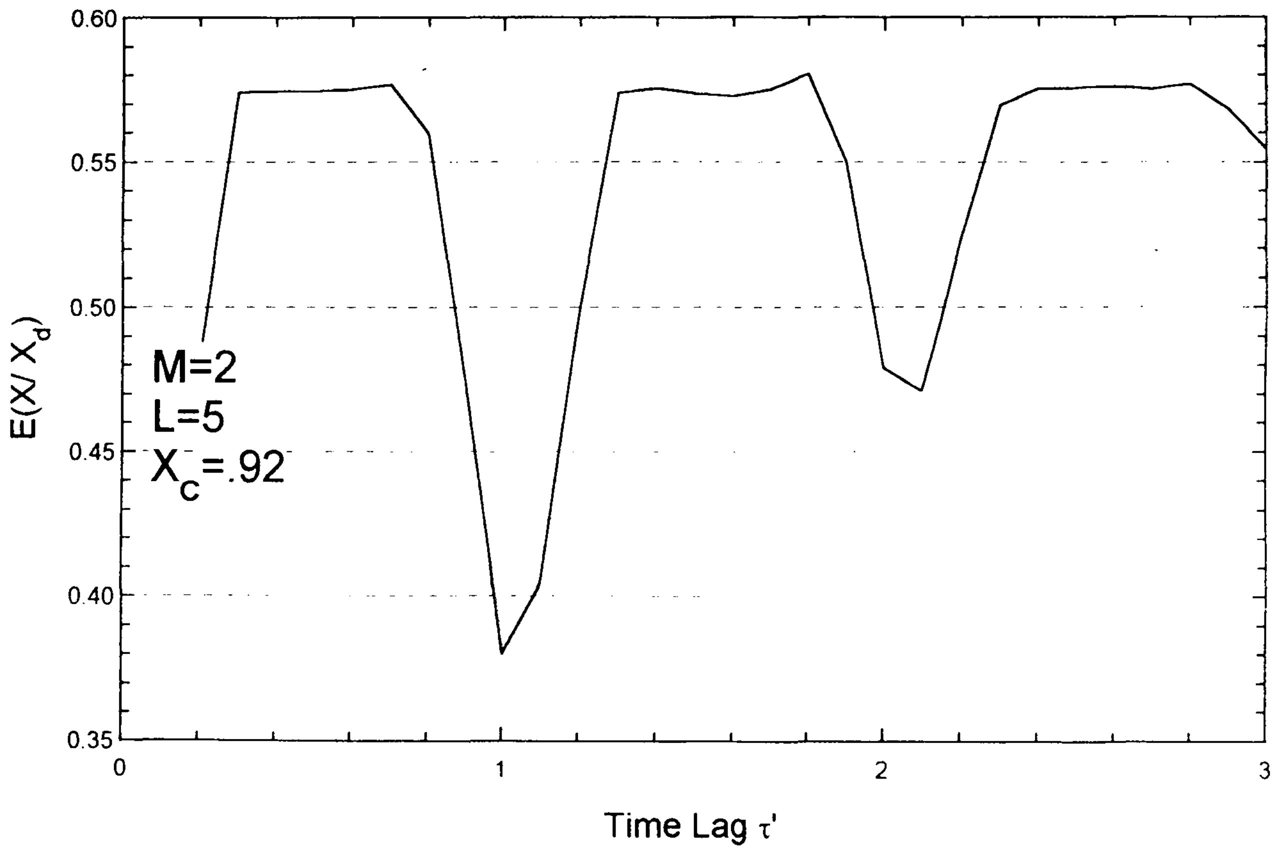


FIG. 4.6

Where Y_t is the measurement value of X_t and n_t is a white measurement noise with zero mean and variance σ^2_n [32]. The measurement noise intensity is characterised by the signal to noise ratio (SNR). SNR = $\sqrt{\sigma^2_s}/\sqrt{\sigma^2_n}$. σ_s is the standard deviation of the noise free chaotic series and σ_n that of noise. We generate noisy time series described by equations (4.8), (4.9) and (4.13) with SNR = 2, ($\sigma^2_s \approx 0.1$, $\sigma^2_n \approx 0.025$) as shown in Fig. (4.7). The graph of $E(Y/Y_d)$ versus time delay τ' is shown in Fig. (4.8). The critical point for the time series is found as Y_c=0.95. We again observe sharp fall when the delay τ' equals 1 which means that the method works well in the presence of noise.

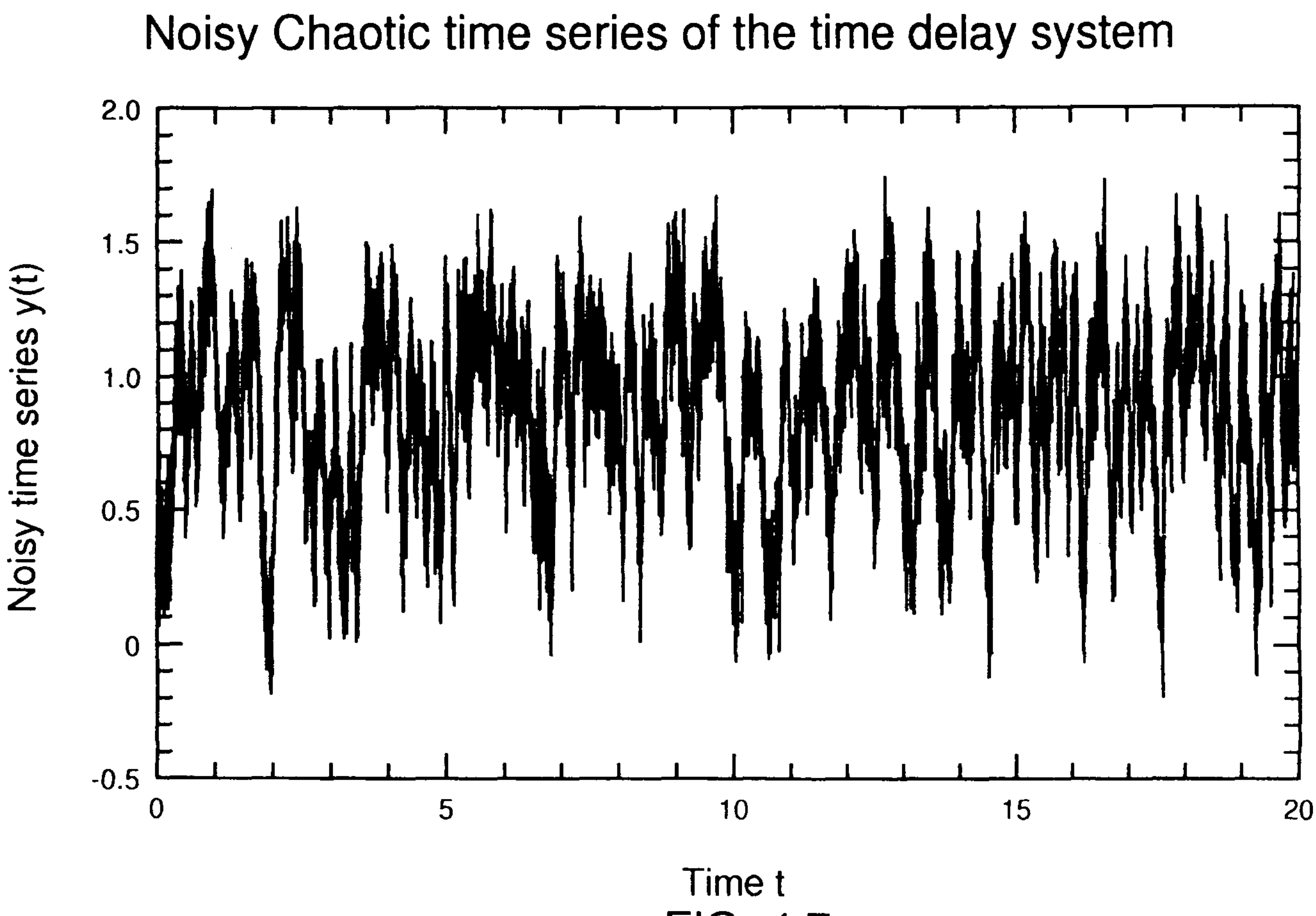

Noisy Chaotic time series of the time delay system
Noisy time series y(t)
2.0
1.5
1.0
0.5
0
-0.5
0
5
10
15
20
Time t

FIG. 4.7

Conditional entropy as a function of time lag τ' for a noisy time series

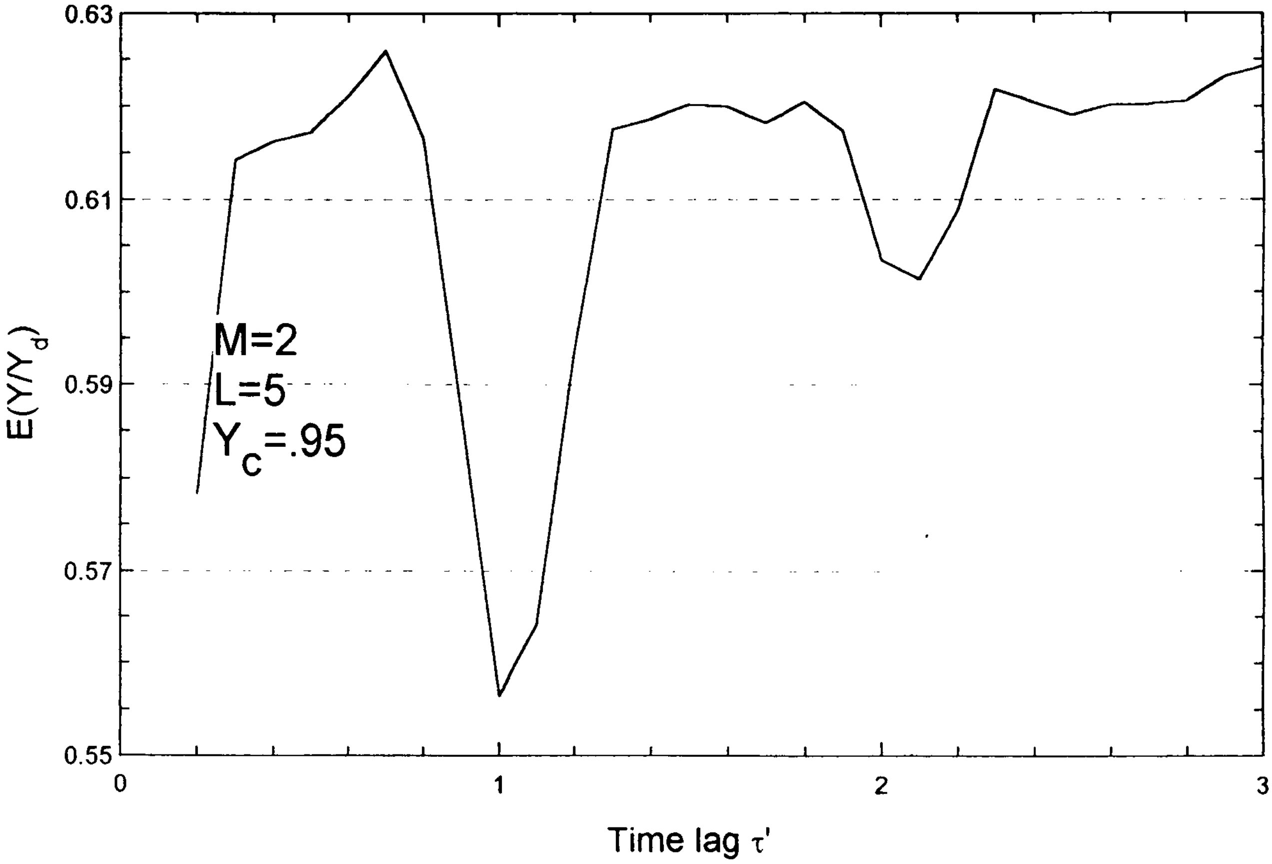


FIG. 4.8

CHAPTER FIVE

DISCUSSION

The numerical studies on non linear model system – Lorenz, Rossler and a high dimensional one show that the method of Lehrman et al. does, in fact help in establishing whether two chaotic time series could originate from the same underlying dynamics. Other methods which rely on calculation of the dimensions of the attractor from each of the time series using the method of Grassberger and Procaccia [34] also do the same. However what is seen is the robustness of the present method in the presence of noise.

This approach investigates the correlation of the informations of two (or more) series. It has been elaborated by defining the conditional entropy for two signals. A sharp minimum is seen when time shift is zero for Lorenz, high dimensional and Rossler model. It implies that the conditional entropy, i.e. the information of one signal with respect to the other is minimum. The meaning is, to a certain value of one signal, the other cannot take any of all possible values, but is constrained to take only certain values. Hence the conditional information is confined and therefore is minimum, showing the two series are correlated. Otherwise, if the series are not at all correlated, to a certain value of one signal, the other has the freedom to take any of all the possible values; thus increasing the conditional probability of two such signals and the conditional information is maximum. We have seen the breaking of such correlation by time shifting of two series with respect to each other. Obviously this will tend to decrease the constraints and the conditional information will increase until we reach a shift where the information tends to a maximum value and any further shift will no longer increase the information.

We have, checked the validity of the method in the presence of noise. Noise of increasing magnitude is added to the signals. The persistence of sharp minima shows the robustness of this method. The other advantage is one does not need to determine the dimension of the dynamical system [5]. Some other methods, viz. a geometrical method [35] applied for reconstructing phase space from a single variable and another, a different method of symbolic analysis of noisy chaotic signals [36] also exist.

The work on Rossler model shows the need of optimization of symbolic language by taking more than one critical point for each time series. With two critical points the peak is much more pronounced than that for one critical point. Clearly with the addition of noise, the peak would tend to disappear faster in the latter in comparison to the former case. The information does not keep on rising as the number of critical points is increased. Beyond a certain number of critical points (usually 2 or 3) the information stabilises. The advantage of this method is that, even if one works with fewer critical points than the number which give the maximum information, it works. In fact one need to make only rough approximation of the optimum langauge.

It is worthwhile to mention some other aspects which we investigated in connection with this study.

We checked how the method works for different sampling time intervals τ. It is seen that the selection of appropriate value of τ is a must for the coarse graining method to work well. One gets a well pronounced peak for a right value of τ. We searched for a method to get an approximation of τ. In chapter three, we have shown how the analysis of fourier transform of discrete series helps in achieving this. The value of τ should not be less than the sampling interval found from this fourier transform method. The most appropriate value is one which gives the best peak. The method is verified for all the models we studied and it appears to work quite satisfactory.

We also tested the validity of coarse graining method to analysing time delay equations. The method appeared successful in getting the delay information from such equations. The graph shows the peak to be most pronounced at the delay interval. It is quite effective even after the addition of noise. Thus a technique based on coarse graining method and information entropy has been developed to gather the delay information from an experimental chaotic time series.

The experimental time series of El Nino have been found to show delay [12]. Thus one possible utility of this technique can be in getting delay information as contained in El Nino phenomena.

Another possible application could be the following. Consider two experimentally measured time series. These time series could have been measured at different starting times. The above method will then, probably, help us in determining the time difference between the starting times. Of course, the two series must be chaotic and governed by the same dynamics.

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