

DESCRIPTIVE ANALYSIS OF SIMPLE ELECTRICAL CIRCUITS

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C E R T I F I C A T E

This dissertation entitled "Descriptive Analysis of Simple Electrical Circuits" embodies work carried out at the School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi-110067.

This work is original and has not been submitted in part or full for any other degree or diploma of any other University.

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CHAPTER I

I N T R O D U C T I O N

If one looks at an electrical circuit as a representation of relationships between a set of concepts, one moves closer to the traditional concerns of psychology, linguistics, and philosophy. Partially at least, Artificial Intelligence succeeds these traditional concerns, in so far as it is attempting to answer some of the questions customarily raised in psychology, linguistics and philosophy.

To answer the question what is Artificial Intelligence, there are at least two approaches. One way would be to study various definitions that have been offered and discussions so generated. The other approach could be to look at the outstanding work done in the tradition marked by Mc Carthy, Turing (52), Minsky (33, 34), and others. Also it may be interesting to look at the criticisms directed against Artificial Intelligence.

One way of avoiding the definition but still capturing its meaning was offered by A.M. Turing (52). He suggested a test for machine intelligence. A man and a machine were kept in separate rooms and were asked questions which needed intelligence to be answered. The interrogator would not have knowledge about which one was the man and which was the machine. They were allowed to correspond with each other through telecommunication. If at the end of the test, by analysing the answers the interrogator failed to distinguish between the machine and the

man, then the machines could be said to be intelligent.

There does not seem to be much point in defining "intelligence" or in trying to distinguish between its natural and 'artificial' manifestations. Rather a good work on Artificial Intelligence might provide insights into such questions as a side-effect. Of course, this can provide an additional reason for being interested in Artificial Intelligence.

The earliest attempts in Artificial Intelligence centred around games (11, 29, 47, 48, 50). A game such as chess is well-defined and non-trivial. It is well accepted that intelligence is necessary to play such games. The literature abounds with special purpose programs for playing chess, tic-tac-toe, go-moku, checkers, etc.

As a natural consequence, the theorem proving became an eligible subject matter of Artificial Intelligence. The break-through began with the geometry proving programs (39, 40) and now there are advanced programs to prove theorems in various branches of mathematics (18, 19, 20).

Frequently expressed criticism against Artificial Intelligence was that it bothers too much about "puzzles" and not "problems". It turns out that while handling problems the effort needed to solve them is some exponential function of the size of the problem. This typically results in 'explosions' of cases to be considered beyond the

bounds of any computer. Perhaps the most formidable challenge before Artificial Intelligence is the development of this capability to handle the 'explosion'. It would be, of course, interesting to compare similar capability of human being.

Perhaps it is here, while comparing the human capabilities, the importance is to be given to emphasize the representation of the problem. Again it is in this context the question of syntax (form) and semantics (content) of a representation comes in. These are relative concepts. Considering the syntax of some higher level programming language, its semantics may be given (by compiler for instance) in some lower level language (i.e. an assembler level language) which can, in turn, be seen as a syntactic object, whose semantics is (perhaps) machine level language, and this can also be viewed as syntax relative to a still lower level, the states of a machine, and so on. It seems, therefore, that machine is more abstract and the semantics involves more built-in or stored information and less search. These relations seem to be based sometimes on analogy and sometimes on the necessity.

It turns out that such an analogy exists between a pictorial representation of an electrical circuit and an abstract description of the same circuit in terms of a set of equations. The electrical circuit itself is an abstract description of an electrical phenomenon and so on, at different levels.

In this dissertation an attempt is made to study the electrical circuits in the background of the laws of electrical engineering. The laws of electrical engineering together with past experiences (if any) can form a knowledge base to operate upon and make logical deductions which may be useful or interesting for analysis of circuit.

CHAPTER II

P A T T E R N R E C O G N I T I O N

The domain of recognition of visual scenes is well-suited for the study of intelligence. Though it is difficult to define 'intelligence,' pattern recognition is generally accepted as an intelligent activity. As Kanai and Chandrasekharan (27) say'... since the search for regularities is the principal concern of all scientific enquiry and recognising pattern is intrinsic to all intelligent activity, it would appear that the field of pattern recognition encompasses all scientific enquiry and intelligent behaviour.'

Human brain is a complex pattern recognition system. Efforts have been put in trying to build models which try to copy human reasoning and decision-making which is needed in recognising patterns.

2.1. Patterns

A pattern may be defined as a model, guide or plan used in making or designing things. Almost everything within reach of our senses can be taken as a pattern - a character, a photograph, a biological wave form, speech pattern, tastes etc. According Meisel (31) ...' In their widest sense, patterns are the means by which we interpret the world. ... some patterns having a physical manifestation 'eg. a character, a photograph, a biological wave form etc., while '...others have only abstract existence' like patterns of social and economic behaviour.

Though automatic recognition of pattern is done at present in many areas such as mail-sorting, blood-cell counting, etc., there are many fields yet to be considered. The major approaches to pattern recognition are described as follows:

2.2. Statistical approach

Two vital concepts in pattern recognition are classification and description. The problem of classifying an unknown pattern into any one of the known categories, is dealt with by applying statistical approach.

A main feature of this approach is the application of statistical probability theory as a powerful mathematical tool (1, 5, 8, 26, 27, 36, 49) in decision making in pattern recognition.

The real world data from a picture space of infinite dimension is sensed and represented as a point in the feature space. The feature space is divided into mutually disjoint regions, each corresponding to a pattern class. Some characteristic measurements called 'features' are then made on the set of N-dimensional vectors to 'learn' the common attribute of the input pattern class. Learning is done by estimating the probability densities on the distribution of the set of samples in the feature space. Once a class is learnt, a new

vector can be recognised by noting its properties common to that class. This is done by evaluation of the already learnt conditional probability density of each class at the point in the vector space that represents the new input to be classified. Applications include character recognition, crop classification, medical diagnosis, classification of electro-cardiograms etc.

In this approach, the classification is based on a set of selected measurements, extracted from the input pattern. Usually, the decision of what to measure is rather subjective and also dependent on practical situations (for eg. the availability of measurements, the cost of measurements, etc.). Unfortunately at present, there is very little general theory for selection of feature measurements.

One of the inherent inadequacies in this methodology is lack of generation and description. As analysis and description, plays an important role in pattern recognition, mere classification is inadequate for the purpose.

In case of very complex pictures, eg. in scene analysis, the number of features becomes very large. The structural relationship

between different component patterns becomes important. Syntactic approach takes care of this problem.

2.3 Syntactic approach

The structural information of the pattern is the resource of this approach (15, 16, 17, 36, 37). Complex patterns are broken up into simpler subpatterns which in turn are broken up into still simpler subpatterns. The simplest unit is called pattern 'primitive'. The structural relations specify how the primitives are juxtaposed to qualify as a certain object. A set of rules specifying these relations are given by the so called 'grammar' of the pattern description language. As it is far easier to recognise the primitives than the whole pattern the problem becomes much simpler. After the primitives are identified the recognition process is accomplished by performing a syntax analysis or parsing of the 'sentence' describing the given pattern. Analysis of complex scenes like circuit diagram, bubble chamber tracks, architectural designs are examples of successful application of this approach. In this approach the pattern can be represented in the form of a tree and described by a grammar. Recursive nature of grammar and application of Formal language theory are useful aspects of this approach.

2.4 Semantics:

Inefficiency of purely syntactic or statistical approach lies in the fact that the importance of the role of context and semantics has been ignored. In purely syntactic approach, it is not possible to handle contextual requirements and restraints. Semantics extracts meanings from the symbol structures and uses these meanings to aid problem solving. It is difficult to describe extensional class of pictures involving non pictorial ^{para}phrases.

Garman (22, 23) tackles the problem of scene analysis by decomposing a complex scene into parts by considering the scene to be made up of three-dimensional paralleloiped. He deals with different kinds of vertices and the adjacent regions and draws inference. Concept of context is dominant in his approach.

Among other pioneering work in this field have been done by Narasimhan (36, 37), Evans (13, 14), Clowes (7), Rosenfeld (43), Winston (57, 58 59), Sadehanda (45, 46).

Description is another important concept in pattern recognition. When it is necessary to use the result of one step as an input to the other, it is desired that the output should be given in a descriptive manner.

2.5 Problem Solving

Almost everything we solve with the help of computer is a problem. Apart from problems of numerical analysis, problem solving includes a number of different types of problems such as theorem-proving, game playing, analysing a visual scene, etc. which need intelligence to be solved.

If a machine is to behave in an intelligent manner, it must be able to observe and comprehend its environment. Pattern recognition plays an important role in this context. Two major parts in problem solving are -- representation, and search for solution.

A problem should be represented in such a way that it becomes easy to implement on a computer and the search procedures can be applied. Heuristic search techniques which improve upon the solution of a particular problem often give good results but lack generality. Systematic procedures based on human problem-solving techniques can, therefore, be used to give a generalised method. A human problem solver often solves a problem in a straightforward way without looking for all possible solutions. This kind of approach of a human is sometimes intuitive and sometimes analytical. As the problem becomes complicated, intuition fails to give satisfactory result.

Analytical methods become more suitable in that case. Mode of representation of a problem often makes the task of human easier. Depending upon the handling of the knowledge-base so created, every step in solving the problem can be made systematically. Systems have already been designed based on this type of analytical reasoning of human to solve complicated problem (52,56). This dissertation is also an attempt to work in similar light on solving some simple problems of electrical network.

CHAPTER III

C I R C U I T A N A L Y S I S

As already stated in an earlier chapter, here we are attempting to design a system to solve some circuit theory problems from the point of view of handling a data base.

Electrical engineering and particularly, electrical network analysis is now a very well established area and there exists abundant literature (2, 3, 54). But looking at this area from the data base point is of recent origin (52), and likely to be both interesting and useful.

Circuit description is given in the form of a matrix Δ (described in the next section). One-step deductions based on fundamental circuit laws are coded in the similar manner. A multistep problem is simplified and broken up into several one-step problems. The network can be simplified without affecting the quantity to be found. The simplification is done by combining series and parallel resistances as well as Y-V transformations. These transformations together with the one-step deductions constitute the data base in the system.

Strategies based upon circuit conditions pick up relevant one-step deduction from this data base. Result of one-step deduction is then fed into the next step. Equations encountered in this process are solved automatically and the solutions are used in further deductions. The process continues till the desired results are obtained.

3.1 Representation

All information available about an electrical circuit is represented in terms of nodes and elements. An electrical circuit or network is an interconnected array of resistances, inductances, capacitances, etc., together with electrical sources. A branch is connected between two nodes. In special situations two or more branches can be combined into a single equivalent branch. In the formation of a network, branches are connected together at their nodal points.

The description of the circuit is given in the form of a matrix A .

The information available regarding the interconnections of branches as well as the device parameters are given in A . Description of circuit of fig 3.1 is given below

$$A = \begin{array}{|c|c|c|c|c|} \hline & 1 & R & 1 & 2 \\ \hline & 2 & C & 1 & 5 \\ \hline & 2 & R & 2 & 3 \\ \hline & 3 & C & 2 & 5 \\ \hline & 3 & L & 1 & 4 \\ \hline & 4 & R & 4 & 5 \\ \hline & 4 & I & 2 & 5 \\ \hline & 1 & V & 10 & 5 \\ \hline \end{array}$$

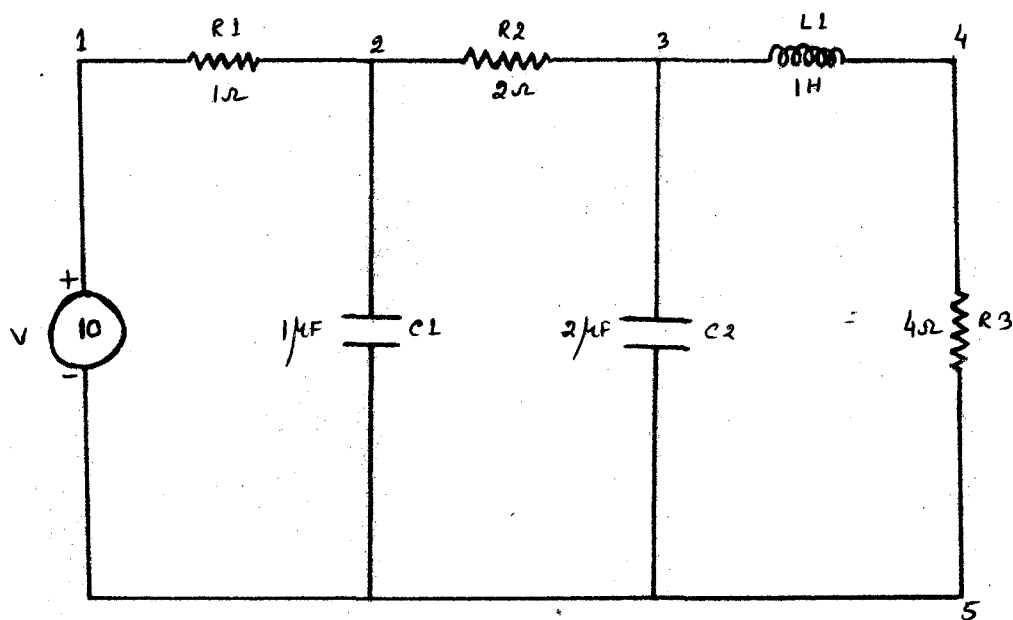


Figure 3.1

Each row of A gives information about a branch. The first and fourth column elements of a row represent the nodes to which the branch is connected. The second column element gives the type of element or the quantity whose value is given by the third column elements i.e. R for resistance, L for inductance, etc. For example, between the nodes 1 and 2 the branch is having a resistance of 1 Ohm. Similarly, a current of 2 amperes is flowing through the branch between 4 and 5. The units for a particular type of element should be consistent i.e. ohm for resistance, henry for inductance, microfarad for capacitance, etc. When given in any other unit, it must be converted.

If inductances and capacitances are present in the circuit, the third column elements must be replaced by corresponding impedances. For example, if there is an inductance L , it must be replaced by $X = L\omega$. This can be given by a modified matrix A' .

3.2 Laws and One-step Deductions

The fundamental laws that are to be applied are Ohm's law, Kirchoff's laws, etc.

3.2.1 Ohm's Law

Ohm's law for steady direct current states (42) that in any electric circuit, the current is directly proportional to the applied

emf. and inversely proportional to the total circuit resistance'.

$$I \propto V$$

$$V = IR$$

where V is the emf., I the current and R , resistance.

3.2.2 Kirchhoff's Current Law

Kirchhoff's current law is usually stated (42) as 'The algebraic sum of the currents entering a node equals the algebraic sum of the currents leaving it'.

3.2.3 Kirchhoff's Voltage Law

Kirchhoff's voltage law is usually stated (42) as 'In any closed loop the algebraic sum of the emfs equals the algebraic sum of the potential drops'.

Applying the above laws, some straightway deductions can be made on a circuit. Some examples of one-step deduction in resistive network are given below.

- (i) If the voltage on both terminals of a resistance and the value of the resistance are known, the current through it can be found out. As shown in figure 3.2 the current through the branch (1, 2) can be found if the resistance R_1 and the voltages at its two ends (1 and 2) are known.

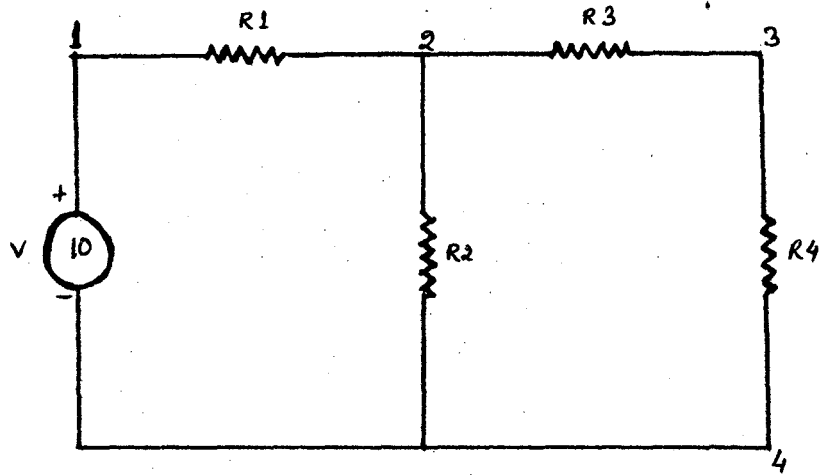


Figure 3.2

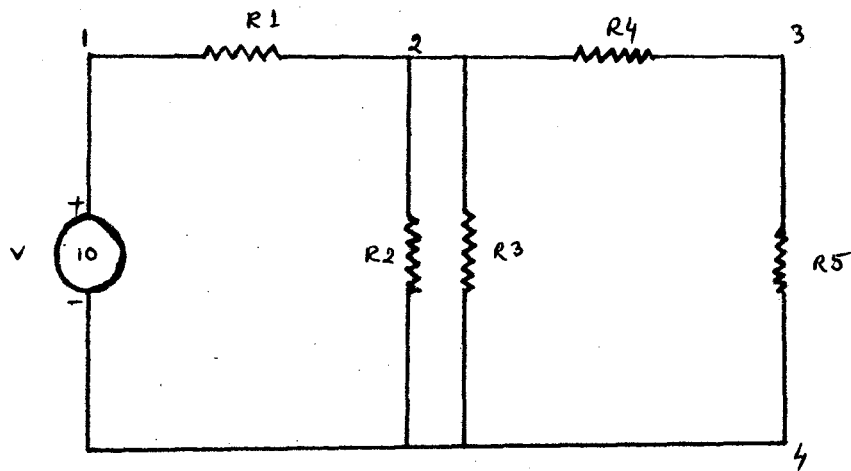


Figure 3.3

(ii) If the current through a resistance, and the voltage at one of its terminals are known along with the value of the resistance, the voltage at the other terminal can be found out. This can also be illustrated in figure 3.2. If the current through R_1 , its value and voltage at 1 are known, voltage at 2 can be assigned.

(iii) If the voltage on both terminals of a resistance are given, and the current through it is known, then the value of the resistance can be assigned. For example, if the voltages at 2 and 4 are known and the current through R_2 , then the value of the resistance R_2 can be found (figure 3.2).

(iv) If all but one of the currents flowing into a node are given, the remaining current can be assigned.

If all but one, say through R_4 , currents from a node, say 2 are known (in figure 3.3) i. e. if the currents through R_1 , R_2 and R_3 are known, current through R_4 can be found out.

3.3. Circuit Simplification

Sometimes it is not possible to make a one-step deduction straightway due to complexity of the circuit. In that case, certain simplification of circuits makes the problem easier. That is, we can use an one step deduction after simplification. These simplifications may be as follows.

Since the series combination of any number of resistances does not alter the current through a resistance, the equivalent resistance should be found if the voltages are given at the end of a chain of resistances. One-step deduction of type (i) can be applied thereafter to find the current. As the current through a particular branch depends upon the voltages at the two nodes and its resistance, any simplification can be made in any branch parallel to it without altering that particular resistance. For example, if there are two branches in parallel to a branch through which current is to be found, then those two branches can be combined and replaced by the equivalent branch.

Other simplifications such as series combinations of resistances constituting a parallel branch also help applying one-step deductions.

Similarly, we can convert a Y - combination of resistances to Δ , or Δ to Y according to need. That is, if we have a Y combination and want to reduce the number of nodes then we convert the Y-to- Δ and proceed.

Some examples of simplification can be illustrated in figure 3.3. If the problem is to find the current through the resistance R_3 then combination of R_4 and R_5 is permitted as the voltages at 2 and 4 are not altered.

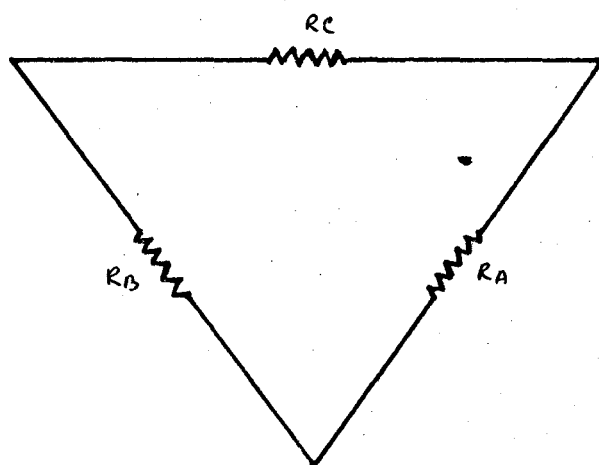
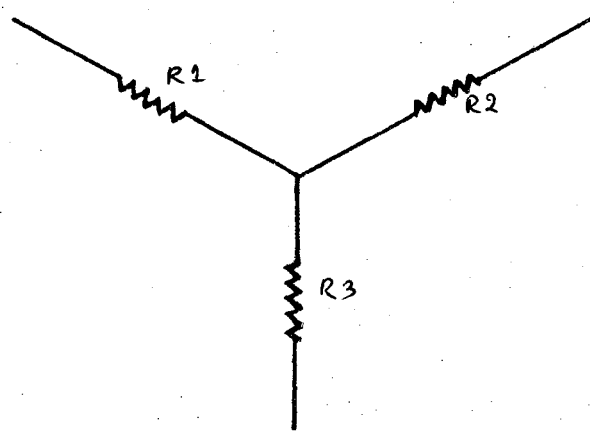


Figure 3.4 Y - Δ Conversion

In case of finding current through R_5 , R_2 and R_3 can be combined first and then their combination along with R_1 and R_4 can be converted to V .

$Y - V$ conversion is given by (as shown in figure 3.4)

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{\sum R_Y}{R_1}$$

$$R_B = \frac{\sum R_Y}{R_2}$$

$$R_C = \frac{\sum R_Y}{R_3}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

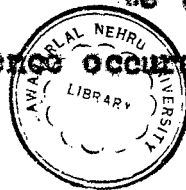
$$R_2 = \frac{R_C R_A}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

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If we are to find the current through R_4 and the voltages at 2 and 4 are known, the simplest way is to combine R_4 and R_5 and make a one-step deduction.

It is sometimes advantageous to assume a quantity at a node or across an element if the circuit is complex. We then proceed similar to 'Gallemin's Trick' till coincidence occurs and the required quantity found.



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3.4. The Data-base

One-step deductions based on electrical laws, viz. Ohm's law and Kirchhoff's law, and the laws for simplification or modification of a circuit-constitute the major part of the data base.

One-step deductions are coded in a similar way as the circuit.

One-step deduction for finding current based on Ohm's law (OHMI)

is coded as below

$$\begin{bmatrix} 1 & R & 10 & 2 \\ 1 & V & 20 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & R & 10 & 2 \\ 1 & V & 20 & 2 \\ 1 & I & 2 & 2 \end{bmatrix}$$

If we are to find the current through the branch between 1 and 2, we scan the matrix. If the first and fourth column elements (which indicate the nodes) of some row are 1 and 2, and the second column element is 'V' then we search for a similar row with second column element 'R'. The value of the current is found by dividing the third column element of the row indicating the voltage by the third column element of the row indicating the resistance. One more row has to be added to the matrix giving the current. The sign \Rightarrow shows that the matrix on the right is the modified form of the matrix on the left.

Another one-step deduction which finds current is based on Kirchhoff's current law (KIRI). If the one-step deduction (i) fails to find current, we search the matrix for rows having second column element 'I'. If all but that particular branch currents which are connected to any one of the two nodes 1 and 2, say, are given, we can find that current.

Other one-step deductions for finding voltage and resistance are coded as described in the following way. The one-step deduction (ii) for finding the voltage at a particular node is coded as follows (let the reference node be 5) .

$$\begin{bmatrix} 1 & I & 10 & 2 \\ 1 & R & 2 & 2 \\ 2 & V & 5 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & I & 10 & 2 \\ 1 & R & 2 & 2 \\ 2 & V & 5 & 5 \\ 1 & V & 25 & 2 \end{bmatrix}$$

The description on the left shows that the current through and the resistance of the branch connected between 1 and 2 are given. Voltage drop across the branch (2, 5) is 5 volts. As 5 is the reference node, voltage at 2 is 5 volts. The modified matrix giving the voltage at 1 is shown on the right hand side of (\Rightarrow) sign.

Similarly other one-step deductions are coded and deductions are made on them to find out unknown facts about the circuit.

The laws of series, parallel as well as Y - V transformation are also coded in the same manner (as one-step deductions) and are included in the data-base.

The series combination is coded as

$$\begin{bmatrix} 1 & R & 5 & 2 \\ 2 & R & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & R & 10 & 3 \end{bmatrix}$$

Here we only remove the node 2 from these two rows of the matrix.

The resistances are in series if the intermediate node is not connected to any other branch. By scanning the matrix, if we can make sure that node 2 occurs only twice, then we can replace the two branches by one as given in the right hand side.

Similarly, a parallel combination can be replaced by an equivalent branch.

$$\begin{bmatrix} 1 & R & 5 & 2 \\ 1 & R & 5 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & R & \frac{25}{10} & 2 \end{bmatrix}$$

Both the nodes must repeat in this case. Y - V conversion can also be done in this manner which will be described later.

3.5 The Method

Let us consider a ladder network of figure 3.5 to illustrate the procedure.

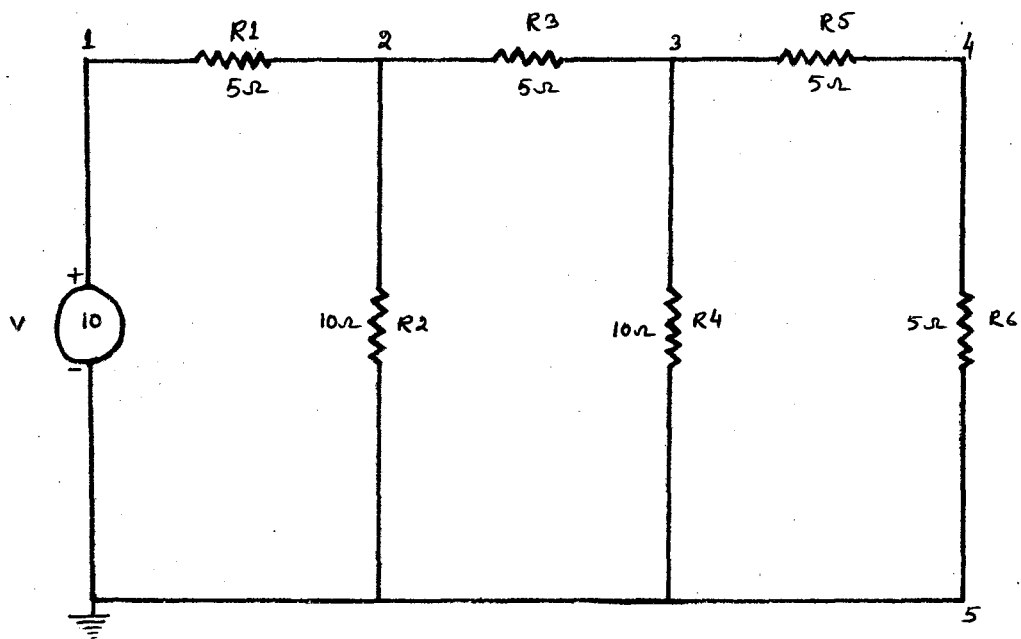


Figure 3.5

The circuit is described as

$$A = \begin{bmatrix} 1 & R & 5 & 2 \\ 2 & R & 10 & 5 \\ 2 & R & 5 & 3 \\ 3 & R & 10 & 5 \\ 3 & R & 5 & 4 \\ 4 & R & 5 & 5 \\ 1 & V & 10 & 5 \end{bmatrix}$$

Let the problem be to find current through the branch connected to nodes 3 and 5.

- (1) The node connected to the maximum number of nodes is taken as reference node. By scanning the first and fourth column, it is found to be 5 i.e. 5 is the reference node.
- (2) As the problem is to find current, routines of one-step deductions of type (i) and (iv) are called.

By scanning the matrix A for voltage (i.e. second column element 'V'), no voltage is found at 3 or 5. As already assumed, voltage at 5 is 0 volt but that of 3 is not known. No current (I) is also given in A . So both (i) and (iv) are ruled out.

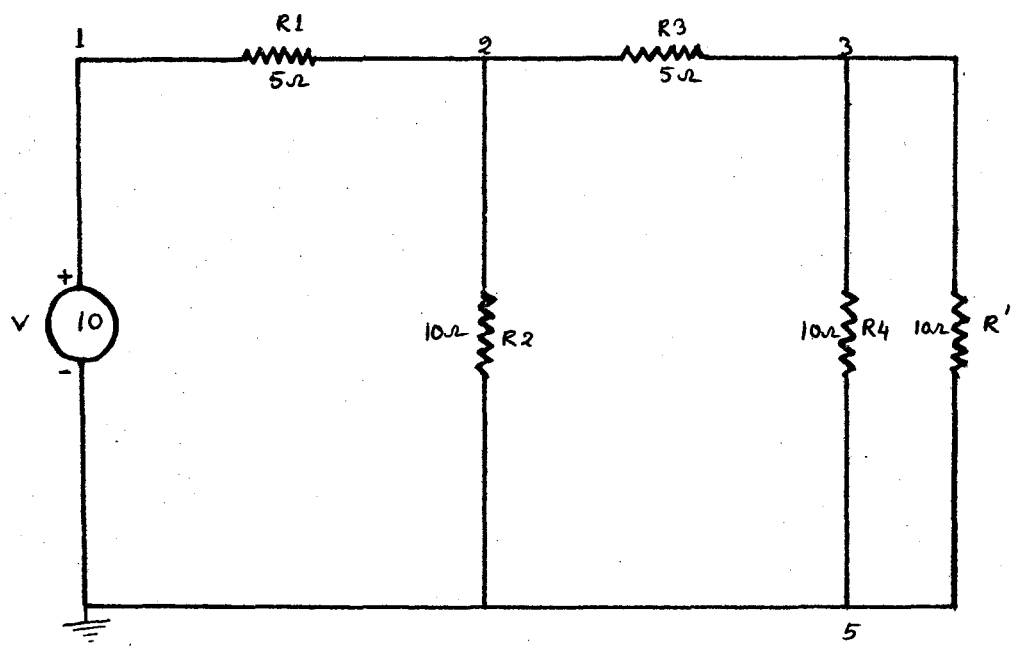


Figure 3.6

- (3) To simplify the circuit, we combine R5 and R6 as they constitute a branch parallel to R4. This becomes clear from the fact that the nodes connected to R4 are also the nodes at the two ends of R5 and R6. They are combined by the series routine and the modified representation of figure 3.6 is as follows.

$$A = \begin{bmatrix} 1 & R & 5 & 2 \\ 2 & R & 10 & 5 \\ 2 & R & 5 & 3 \\ 3 & R & 10 & 5 \\ 3 & R & 10 & 5 \\ 1 & V & 10 & 5 \end{bmatrix}$$

- (4) To simplify the circuit further we scan the matrix and find that the branches (1, 2), (2, 3) and (2, 5) constitute a Y which can be altered to a V. As this conversion does not alter the resistance R4, the Y - V conversion routine is called and the simplified description is given as

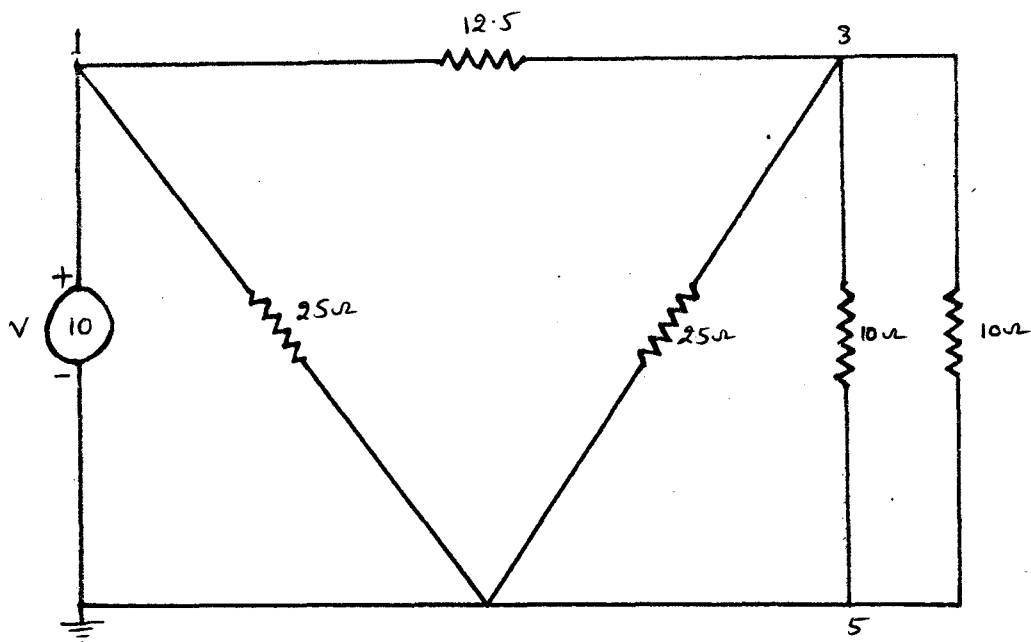


Figure 3.7

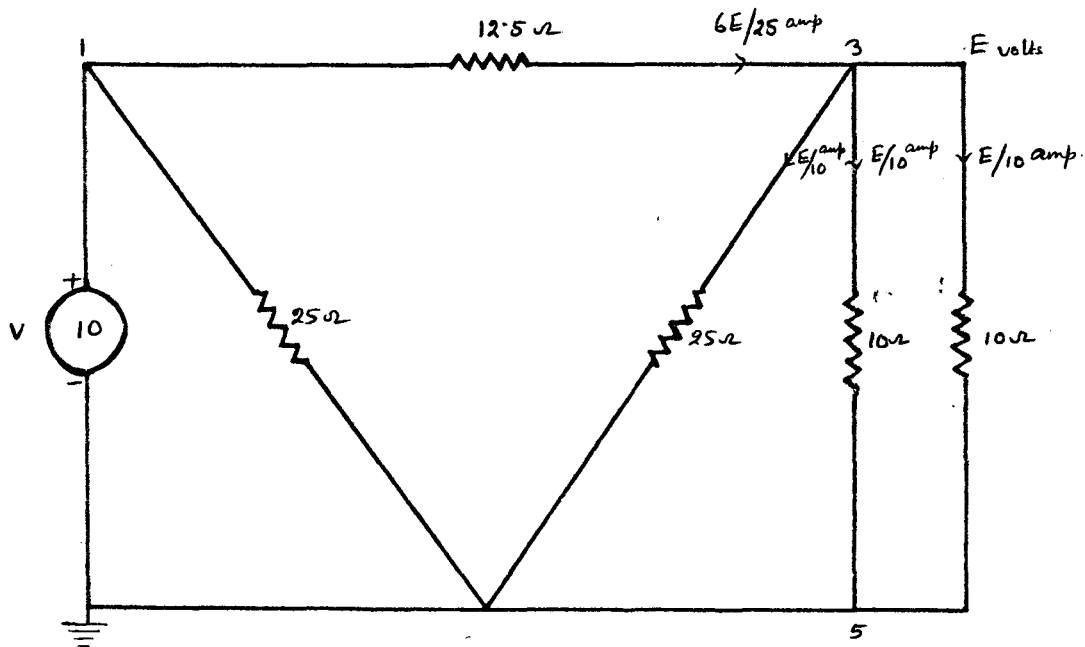


Figure 3.8

$$A = \begin{bmatrix} 1 & R & 25 & 5 \\ 1 & R & 12.5 & 3 \\ 3 & R & 2.5 & 5 \\ 3 & R & 10 & 5 \\ 3 & R & 10 & 5 \\ 1 & V & 10 & 5 \end{bmatrix}$$

(5) As no other simplification is possible, we again call the one step deduction (i) and (iv). As voltage at 3 is not known, (i) can not be applied. So a voltage at the node 3 is assumed. Let it be E volts.

(6) One-step deduction (i) is made and current through R4 is calculated in terms of E.

$$A = \begin{bmatrix} 1 & R & 25 & 5 \\ 1 & R & 12.5 & 3 \\ 3 & R & 25 & 5 \\ 3 & R & 10 & 5 \\ 3 & R & 10 & 5 \\ 1 & V & 10 & 5 \\ 3 & I & E/25 & 5 \\ 3 & I & E/10 & 5 \\ 3 & I & E/10 & 5 \end{bmatrix}$$

- (7) To find the value of E , currents through the two other branches connected to 3 and 5 are calculated by applying one-step deduction (i).
- (8) One-step deduction (iv) is applied to find the current through the branch (1, 3). While applying this one-step deduction, an equation of I in terms E is encountered which is solved mechanically, which will be described later.

$A =$

1	R	25	5
1	R	12.5	3
3	R	25	5
3	R	10	5
3	R	10	5
1	V	10	5
3	I	$E/25$	5
3	I	$E/10$	5
3	I	$E/10$	5
1	I	$6E/25$	3

- (9) This current, the value of the resistance, and the voltage at one node being known, the voltage at the other end is found by calling the one-step deduction (ii). The equation arrived at in this step is also solved automatically in a similar manner.

After finding each quantity in terms of E , the matrix A is checked to see if a coincidence has been taken place.

At this stage, we find that the voltage at 1 being given (10 volts), a coincidence occurs and the value of E is found.

The current through (3, 5) follows.

3.6 The Algorithm

The algorithm can be written as follows.

Given a network and its description, try one step deduction depending upon the problem.

- (1) If the problem is to find current, try one-step deduction of type (i) and (iv). If not possible go to simplification.
- (2) If the problem is to find voltage, try one step deduction of the type (ii). If not possible go to simplification.
- (3) If the problem is to find resistance, go to one-step deduction of the type (iii).

- (4) **Simplification:**
Try to simplify the circuit without affecting the required quantity. Go to (i) or (ii) or (iii).
- (5) **If the solution is not possible, assume a voltage at the end of the network. Find the required quantity in terms of the unknown by applying one-step-deduction.**
- (6) **Proceed finding possible quantities by making one-step deductions until coincidence (a situation when a quantity calculated in terms of an unknown is actually given) occurs.**
- (7) **Find the unknown from the coincidence.**
- (8) **Find the required quantity.**

CHAPTER IV

SOLUTION OF EQUATIONS

Simple algebraic equations involved in circuit analysis can be solved mechanically by making each step systematically. One of the major problems encountered in solving equations involving trigonometric, exponential and logarithmic functions is that of combinatorial explosion in search space. This problem is common to all problems of Artificial Intelligence. One way of looking into it is to break up the problem into smaller parts. The result of local feature detectors are combined into larger units or even complete objects. As the problem becomes complicated, the number of objects becomes very large. The number of possibilities of trial-and-error search becomes explosive. Heuristic techniques are applied to control the breadth of a trial-and-error search. By heuristics, we mean the systems having selective features that improve the problem-solving efficiency. Heuristic methods are, in general, very efficient but problem-specific. Systematic searching using human reasoning may be used to reduce search where each step is predetermined by some strategy depending upon the goal.

Coming back to the solution of equation, an equation can be represented in the form of a tree. Starting from the terminal, all branches are searched, and constraints are put. If all the conditions are satisfied, then we arrive at a possible result and search

for other solutions. As the number of possible steps increases, the combinatorial search may become very large. This can be avoided by making each step based on some strategies depending upon the goal. All the strategies must have access to all the legal moves or possible steps. If the legal moves are coded correctly, the equation becomes equivalent to the transformed one after applying the legal move.

4.1 Description

The description of equations becomes necessary to solve equations non-numerically. Equations are described as a string of characters. These characters may be variables, operators or functions. They must be recognised individually. The properties of the operators and behaviour of the functions must be noted so as to make correct legal moves. Legal moves are coded similarly. The equation has to be examined and depending upon its goal, a strategy applies a legal move.

4.2 Strategies

Strategies are the aims of particular steps that are allowed in solving an equation. A step or a legal move is made

if that is needed. While solving an equation, this need is determined by the strategy and appropriate legal moves are picked up. For example, if an unknown is occurring a number of times in an equation, it is necessary to combine them or to reduce the number of occurrence. The strategy of collection does the job by collecting two of them together. The legal move to do it will be the one depending upon how the unknown occurs or which operator or function it is associated with. There are strategies like that of isolation, attraction, substitution, etc.

4.3 Legal moves

Legal moves are the possible steps of action taken in order to solve an equation. The legal moves must be carefully and correctly coded to ensure perfect equivalence between an equation and its resultant after making the move.

Strategy and legal move can be explained by the following example.

$$A \sin^2 X + A \cos^2 X + B \sin X = C \dots\dots\dots (1)$$

where C is a constant.

By applying the strategy of collection, we obtain

$$A(\sin^2 X + \cos^2 X) + B \sin X = C \dots\dots\dots (2)$$

This is done by making the legal move

$$ua + va = (u+v)a$$

Equation (2) is further simplified by attracting the two occurrences of X within bracket. The strategy of attraction makes the legal move

$$\sin^2 \theta + \cos^2 \theta = 1$$

and we get

$$A.1 + B \sin X = C$$

The strategy of isolation can now be applied to solve the equation as the unknown X occurs only once

$$B \sin X = C - A$$

$$X = \sin^{-1} \frac{C - A}{B}$$

Coming back to the problem of analysis of circuits, let us discuss the procedure adopted for solving the equations involved.

4.4 The Procedure

In solving an electrical network, equations arise from one-step deductions. These equations are solved and the solutions fed into the next step for further processing.

In certain circumstances, if a parameter is not given, it is assumed and the necessary computation done in terms of it. The unknown can be found if after a number of steps, a point is

reached where a quantity evaluated in terms of the unknown is actually given. This is called coincidence. When a coincidence is reached, the unknown can be found.

Taking a specific case, let us consider an equation that occurs while finding current by applying the one step deduction (iv) based ^{on} Kirchhoff's current law.

$$XE + YE = I$$

where E is an unknown quantity. This equation is simplified by applying the strategy for collection

$$(X+Y)E = I$$

This current I can now be fed into another step to find a voltage, say. And we arrive at another equation

$$\frac{R_1}{R_2} (X+Y)E + E = E'$$

Let E' be given.

Strategy of collection simplifies the equation by the same legal move

$$\left\{ \frac{R_1}{R_2} (X+Y) + 1 \right\} E = E'$$

Strategy of isolation can now be applied to find the value of E

$$E = \frac{E'}{\frac{R_1}{R_2} (X+Y) + 1}$$

The value of ϵ so obtained can then be put in some other part of the procedure to solve the problem.

CHAPTER V

C O N C L U S I O N

The domain of electrical engineering is a fertile area where the techniques developed in Artificial Intelligence can profitably be applied. Typically, a circuit problem is well structured, defined and yields to clear path of reasoning as in the case of games, a well known subject matter for the workers in Artificial Intelligence. In designing a system one can adopt any one of the two following philosophies. One can go in for learning systems with little inbuilt capability but endowed with rich capabilities to learn from experience and the other systems which are designed with all capabilities desired to begin with. It is clear that both of these have their applications depending on the problem situations and often a judicious combination of these two lines of approach have enabled the design of interesting systems.

There are two other divisions in the approaches - heuristics and mimicry. Heuristic techniques are developed to improve upon the solutions of particular problems. While solving a problem by this technique, human thinking or ability of a human in solving the same problem is not taken into account. In the latter procedure, the human reasoning and decision-making is simulated.

The system developed in this dissertation is an attempt to handle the data base containing the facts about simple electrical circuits. The approach is based on the simulation of human problem-solving method. Simple circuits with resistances, inductances, capacitances, and voltage sources have been tried. Emphasis has been given on reducing the search space rather than on the analysis of circuits. It has been tried to reason how an engineer or a technician plans to solve problems of electrical circuit from the schematic diagram. The operation of devices must be known for casual reasoning of human. For example, an engineer may reason, "If I knew this voltage and this resistance, I could deduce this current. From that I could get that current....". This process is propagated without even knowing the actual parameters. The system in the same manner makes a step predetermined by some strategy depending upon the problem situation and the goal. The combinatorial search thus can not be random and the search space is reduced. The parameters determined by one-step deductions are propagated until a coincidence is reached and the actual value found out.

can

The system can modify or simplify a circuit without disturbing all other parts. Added knowledge thus modify only related parameters.

The other part of the system in a similar way tries to minimise the search space in solving equations that are encountered in circuit analysis. A human mathematician solves an equation in a very straightforward way by applying his reasoning, sometimes even unconsciously. The system tries to simulate similar reasoning of a mathematician to make an appropriate step in solving an equation. These steps or legal moves are chosen by high-level strategies depending upon the goal. Random search is thus avoided. The procedure has been applied successfully in solving simple equations that arise in analysing electrical circuits.

The system does not take care of dependent sources. It can be extended to include complicated circuits with diodes, and transistors. The states of these devices must be considered in analysing the circuit. If the system is able to find out the conceptual dependencies of its parts then addition of knowledge can be easily handled.

In solving equations, on the other hand, semantics or meaning of a physical quantity can be of importance. For example, if algebraic identifier is known to represent a physical length then we can assume that it is a positive real number. This type of knowledge can be helpful in solving problems of physical science and

engineering. If while calculating, searching, or solving some equation, the value of a resistance is found to be negative or imaginary, then that solution or search should be discarded.

The procedure adopted for equation solving in the system can be further extended to solve trigonometric and simultaneous equations. Elimination and substitution strategies can be applied. The order and condition of elimination of variables are to be accounted for in the strategy for elimination. The equations are to be ranked according to how easily they can be solved. Recognition of common subterms in two equations or strategy to square and add terms in trigonometric equations can also be thought of. The system does not touch the circuits giving transient currents which are found frequently and of importance. Extension of ^{the} system to include this phenomenon of transient current may be useful.

The system is knowledge based with no learning. The problems are solved with the programmed knowledge supplied at that time. The system seems to be a helpful step in the field of automatic problem solving .

A P P E N D I X

I M P L E M E N T A T I O N

The system designed in this dissertation has been attempted to be implemented on the EC-1020B installed at the Jawaharlal Nehru University. The routines are written in Fortran IV. The outline sketch of the design of the system with some important subroutines has been described in the next section.

- (1) Read the matrix A, the quantity to be found Z (I or R or V), and the nodes (X,Y) which this branch is connected to.
- (2) Search matrix A row-wise to find which node is connected to maximum number of other nodes. Make that node reference node (RB).
- (3) If the problem is to find current i.e., Z=I, call the subroutine OHMI or KIRI.
- (4) If the problem is to find resistance i.e. Z=R, call the subroutine OHMR.
- (5) If voltage at a node is to be found, i.e. Z=V, call the subroutine OHME.
- (6) If the required quantity, W is not found, and Z=I, replace series resistances (if any) by their equivalent by calling the subroutine SERIES.
- (7) If the problem is to find current or resistance, replace the Y combination of resistances (if exists) by Y by calling the subroutine YDEL.
- (8) Go to (3), (4) or (5) if Z = I, R or V respectively.

- (9) If $W = 0$ i.e. the quantity is not found, assume a quantity, say a voltage B , at a node connected to the branch in question.
- (10) Find the required quantity, W in terms of the unknown, B .
- (11) Apply possible one step deduction by calling the subroutines OHMI, OHMR, OHME or KIRI and find another related quantity. Check if it is given in the matrix A .
- (12) If yes, call the subroutine EQUL by which the unknown can be found from the value given.
- (13) Find W from this value of B .

The following are some of the important subroutines.

OHMI

- (i) Search the matrix A row-wise. If the second column element of some row is V (given by VV in flow chart (2)).
- (ii) Check if the first column element is X and the fourth column element is Y , or if the first column element is Y and the fourth column element is X .
- (iii) If yes, search for a similar row with the second column element equal to R .
- (iv) If yes, W can be found by dividing the third column element of the row indicating voltage (V) by the third column element of the row indicating resistance (R).

OHMR (given in flowchart (3)) and OHMB (given in flow chart (4)) are similar to OHMI as all of them are based on Ohm's Law.

KIRI

- (i) Scan the matrix A row-wise. If the second column element of some row is I (given by II in subroutine (2')),
- (ii) Check if the first or the fourth column element is X (or Y).
- (iii) If yes, check the fourth column element of all the rows which are equal to X(or Y) and the second column element is I.
- (iv) If the third column elements of such rows are found to be some multiple of an unknown, say E, then call EQUN.
- (v) Otherwise sum up all the third column elements of such rows (SUM1).
- (vi) Then check the first column elements of all the rows which are equal to X(or Y).
- (vii) If the third column element of these rows are multiple of E, call EQUN.
- (viii) Otherwise, sum up all these elements (SUM 2).
- (ix) If SUM 1 and SUM 2 are some multiple of E, call EQUN.
- (x) Otherwise W is found by subtracting SUM 2 from SUM1.

SERIES

- (i) Scan the matrix A row-wise. Initialize NI=0. Check if the second column element of some row (ith, say) is R (given by RR in flow chart(8)).
- (ii) If yes, check if first column element of some row (say, ith) is equal to X and fourth column element is equal to Y. If yes go to (iv).
- (iii) If no, check if the first column element is equal to Y and fourth column element is equal to X. If no go to (ii) till all rows are scanned.
- (iv) Check if there is any other row (ik th, say) whose first column element is equal to first column element of ith row and the fourth column element is not equal to the first column element of the ith row. If yes go to (vi).
- (v) If no, check if the fourth column element/^{of} ik th row is equal to the first column element of the ith row and the first column element of the ik th row is not equal to the fourth column element of the ith row. If yes go to (vi). If no go to (iv) and scan till all the rows are finished.
- (vi) $NI = NI + 1$
If all the rows are scanned, and NI is equal to I then $RX = A(1,3) + A(ik,3)$.
- (vii) If $A(1,I)$ is equal to $A(ik,I)$ then $X = A(ik,3)$

IF (1,I) is equal to A(ik,4) then X = A(ik,I)

A (ik,I) = A(ik+I,I)

A (ik,4) = A (ik +I, 4)

A (ik,2) = A (ik + I, 2)

YDEL

- (i) Scan the matrix A row-wise. Initialize NI=0.
If second column element of some row is R (given by RR in the flowchart (7)) then
- (ii) check if the first column element of that row (say ith) is equal to X and the fourth column element is not equal to Y. If yes go to (iv)
- (iii) If no, check if the fourth column element is X and the first column element is not equal to Y. If yes, go to (iv)
- (iv) Scan the matrix to see if there is any other row (ikth) with second column element equal to R, in which the first column element is equal to the first column element of ith row and fourth column element is not equal to the fourth column element of ith row.
That is, $A(ik,I) = A(1,I)$
 $A(ik,4) \neq A(1,4)$
If found, go to (vi).
- (v) If no, check if the fourth column element of ikth row is equal to the first column element of ith row
That is, $A(ik,4) = A(1,I)$
and $A(ik,I) \neq A(1,4)$
If found go to (vi).

- (vi) $NI = NI + 1$
 Go to (iv). Continue scanning till all rows are searched. Store the value of ik when $NI = 1$ and $NI = 2$. Let these be ik_1 and ik_2 .
- (vii) If $NI = 2$, then
- $$Z_1 = \frac{A(1,3) \times A(ik_1,3) + A(1,3) \times A(ik_2,3) + A(ik_1,3) \times A(ik_2,3)}{A(1,3)}$$
- $$Z_2 = \frac{A(1,3) \times A(ik_1,3) + A(1,3) \times A(ik_2,3) + A(ik_1,3) \times A(ik_2,3)}{A(ik_1,3)}$$
- $$Z_3 = \frac{A(1,3) \times A(ik_1,3) + A(1,3) \times A(ik_2,3) + A(ik_1,3) \times A(ik_2,3)}{A(ik_2,3)}$$
- (viii) If $A(1,I)$ is equal to $A(ik_1,I)$ then $A(ik_1,I) = A(1,4)$
 If $A(1,I)$ is equal to $A(ik_2,I)$ then $A(ik_2,I) = A(1,4)$
 If $A(1,I)$ is equal to $A(ik_1,4)$ then $A(ik_1,4) = A(1,4)$
 If $A(1,I)$ is equal to $A(ik_2,4)$ then $A(ik_2,4) = A(1,4)$
 $A(1,3) = Z_1$
 $A(ik_1,3) = Z_2$
 $A(ik_2,3) = Z_3$

BQUN

An array C of five elements is defined (if SUM 1 and SUM 2 are equal to zero) by

C(1) = numerical part of the third column element of a row whose first or fourth column is X (or Y) and second column element is I.

C(2) = 'B'

C(3) = '+'

C(4) = Numerical part of the third column of a similar row

C(5) = 'B'

If SUM 1 and SUM 2 are not zero, then

C(1) = numerical part of SUM 1

C(2) = 'E'

C(3) = -

C(4) = numerical part of SUM 2

C(5) = 'E'

If C(3) is equal to '+', then SUM 1 (or SUM 2) is equal to C(1) + C(4) times 'E'.

If C(3) is equal to '-', then W is equal to (SUM 1 - SUM 2) times 'E'.

Increase the order of row of the matrix by 1 i.e.

$N = N+1$

A (N, 1) = X

A (N, 4) = Y

A (N, 2) = I

A (N, 3) = W

HOWL

Define an array CI if a coincidence occurs

CI (1) = numerical part of W

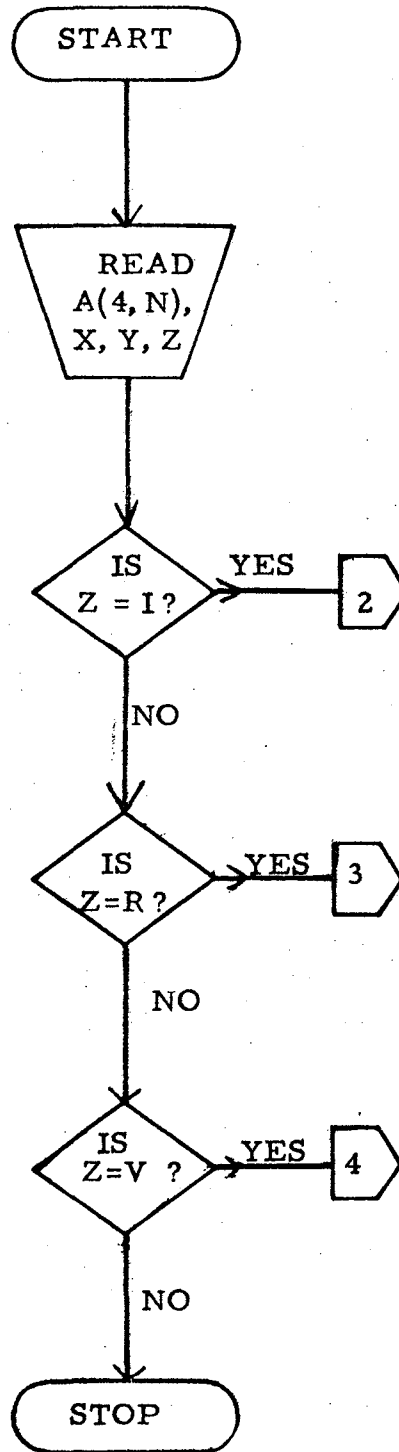
CI (2) = 'E'

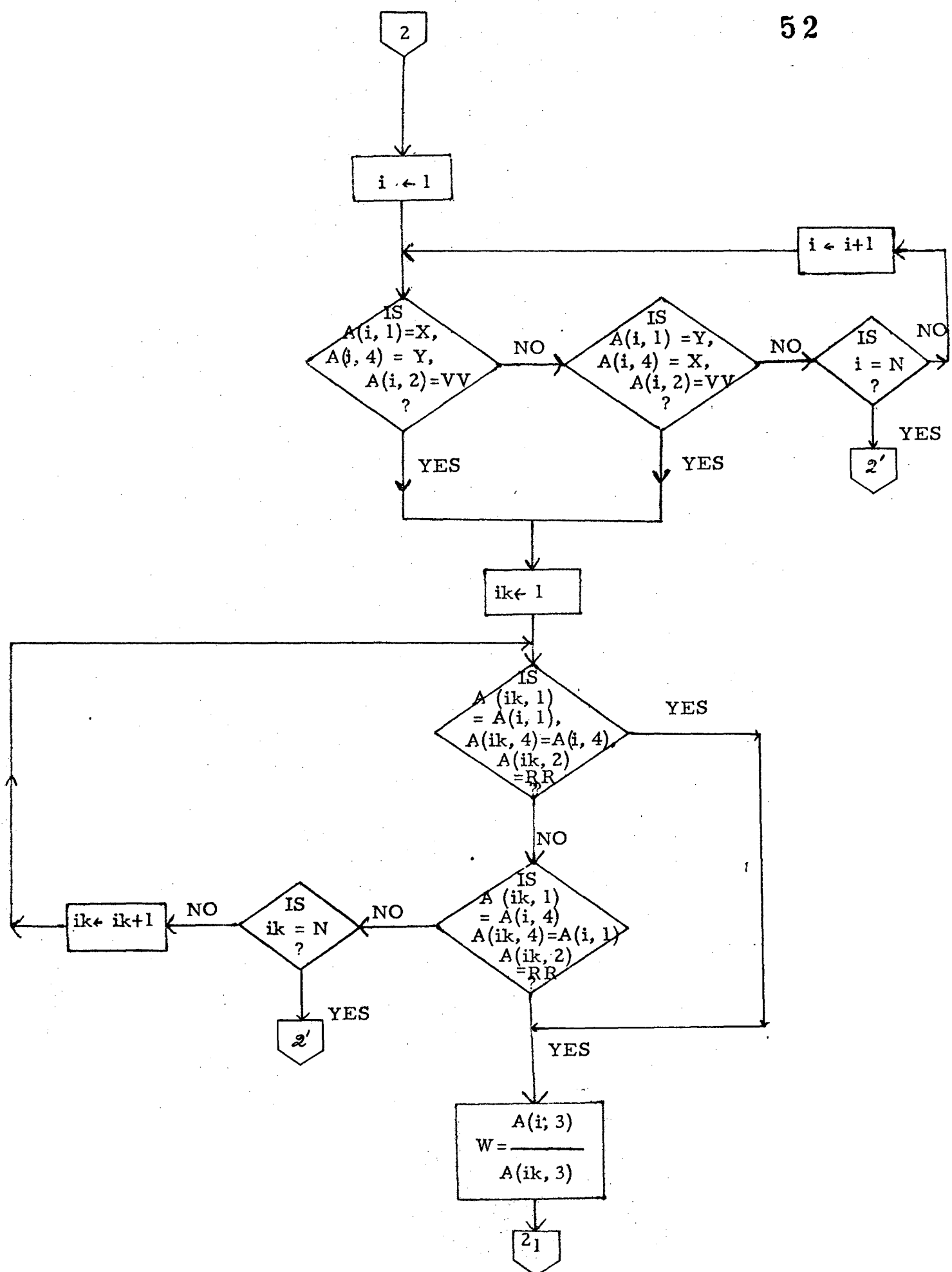
CI (3) = '-'

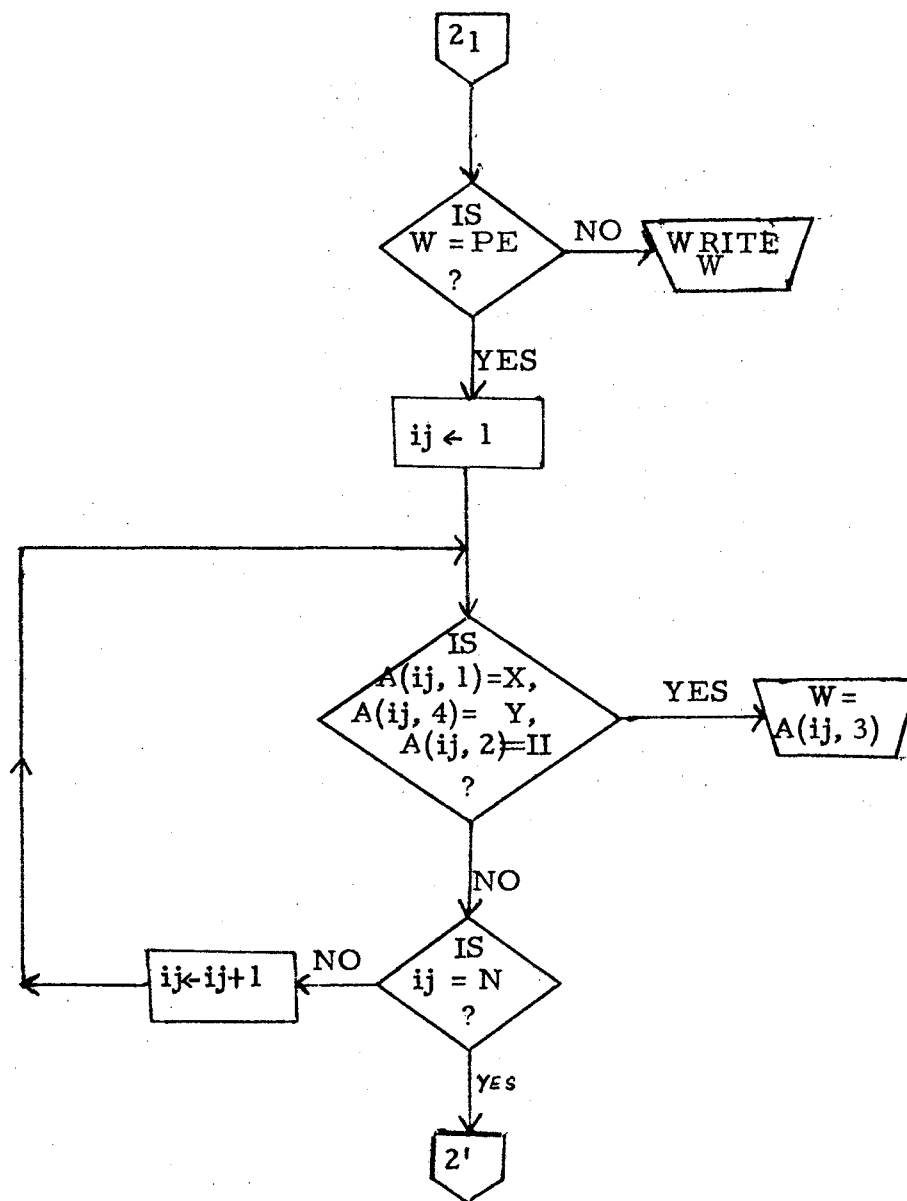
CI (4) = The value of N actually given in A

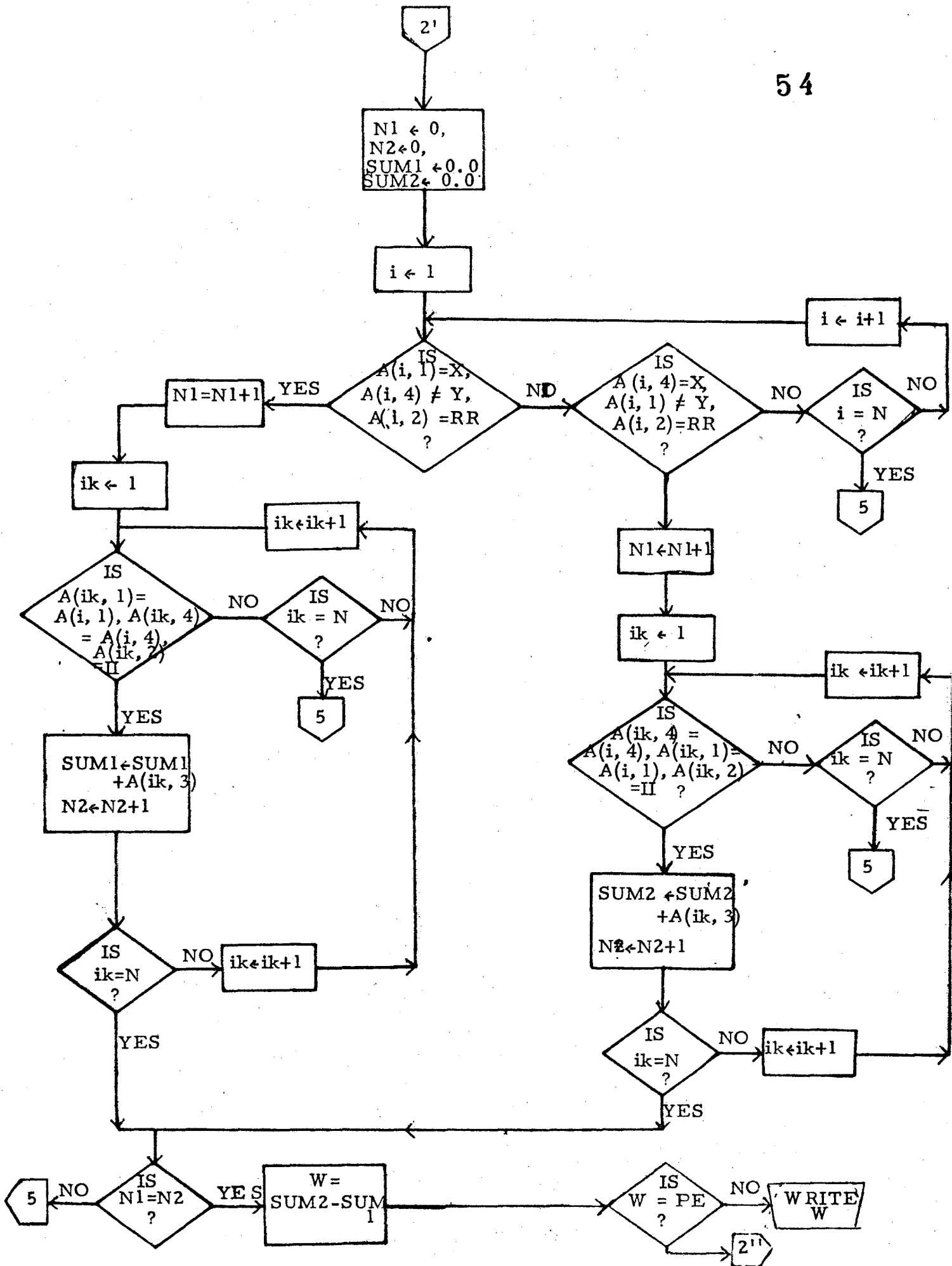
The value of the unknown, E is given by dividing CI(4) by CI(1).

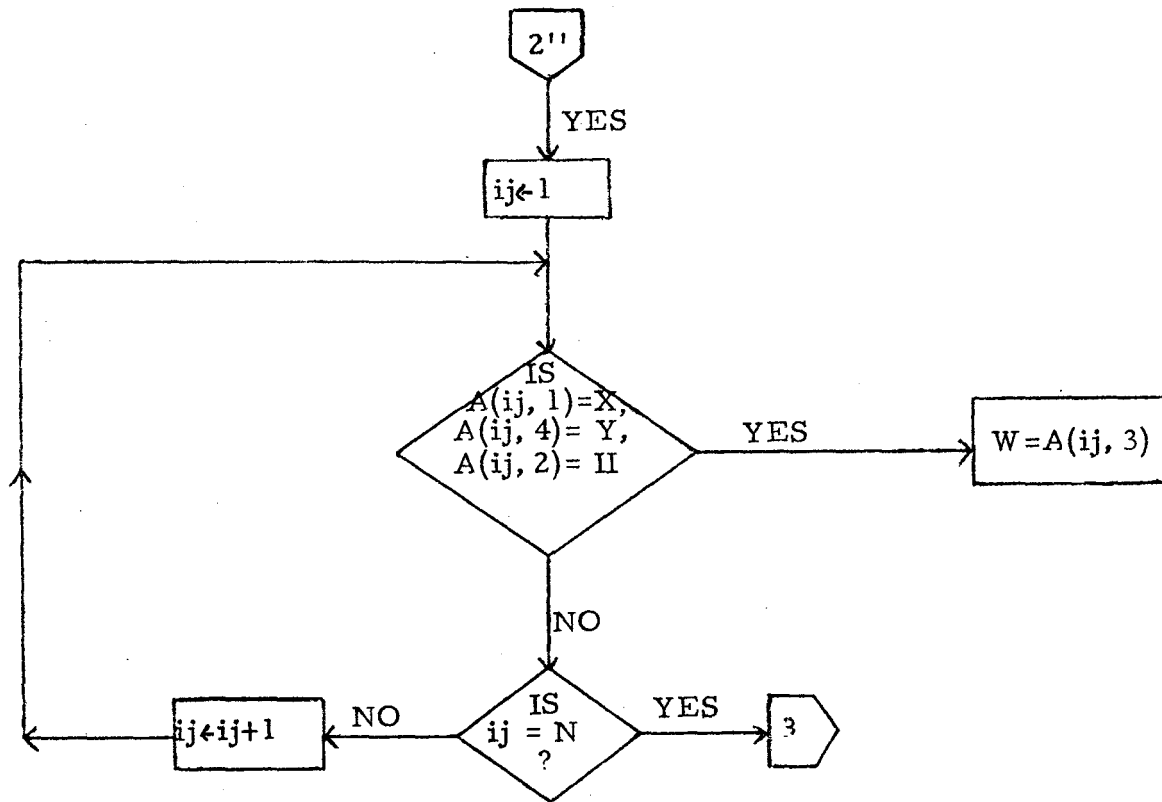
The flow diagram of the system is as follows.

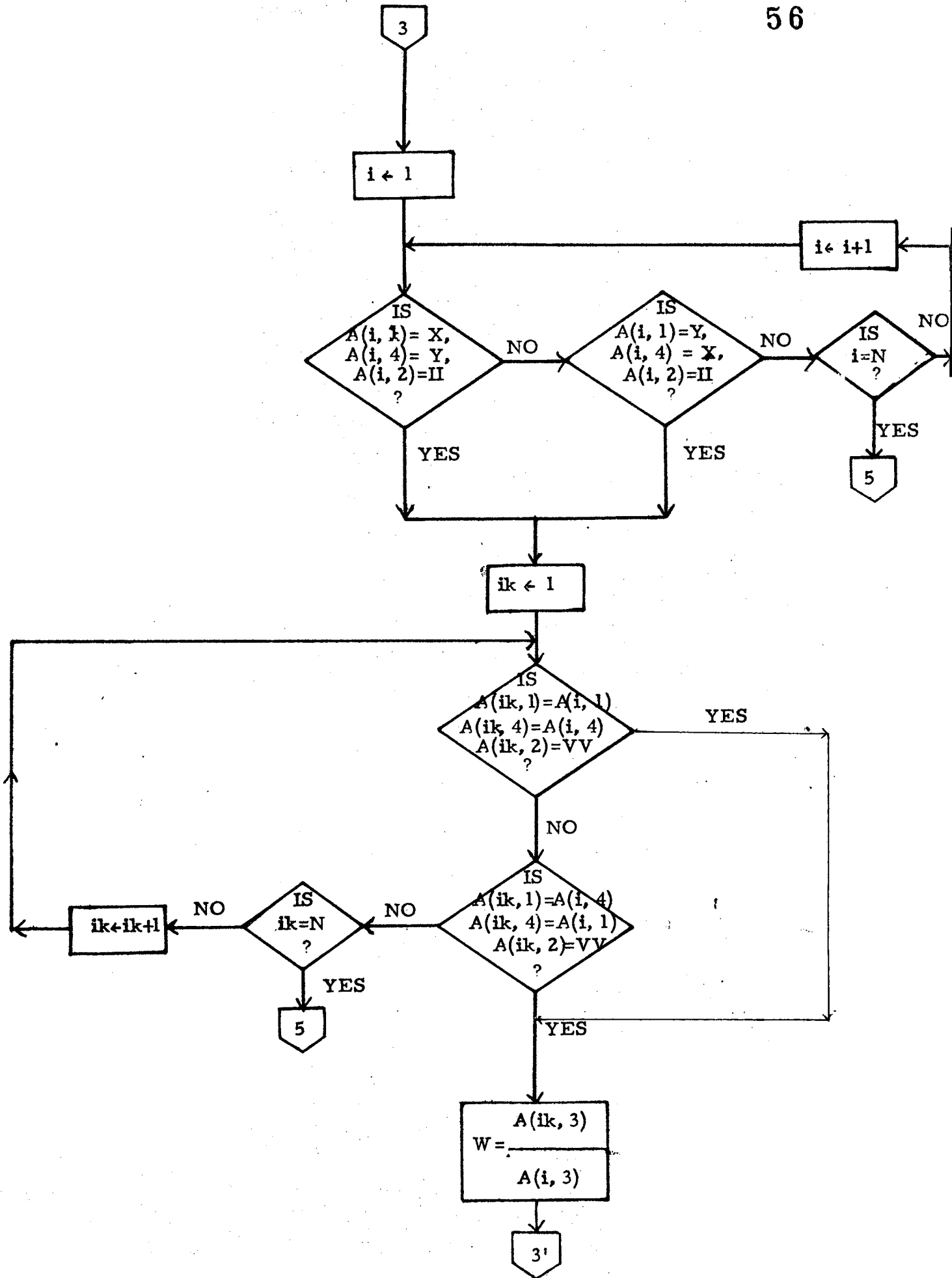


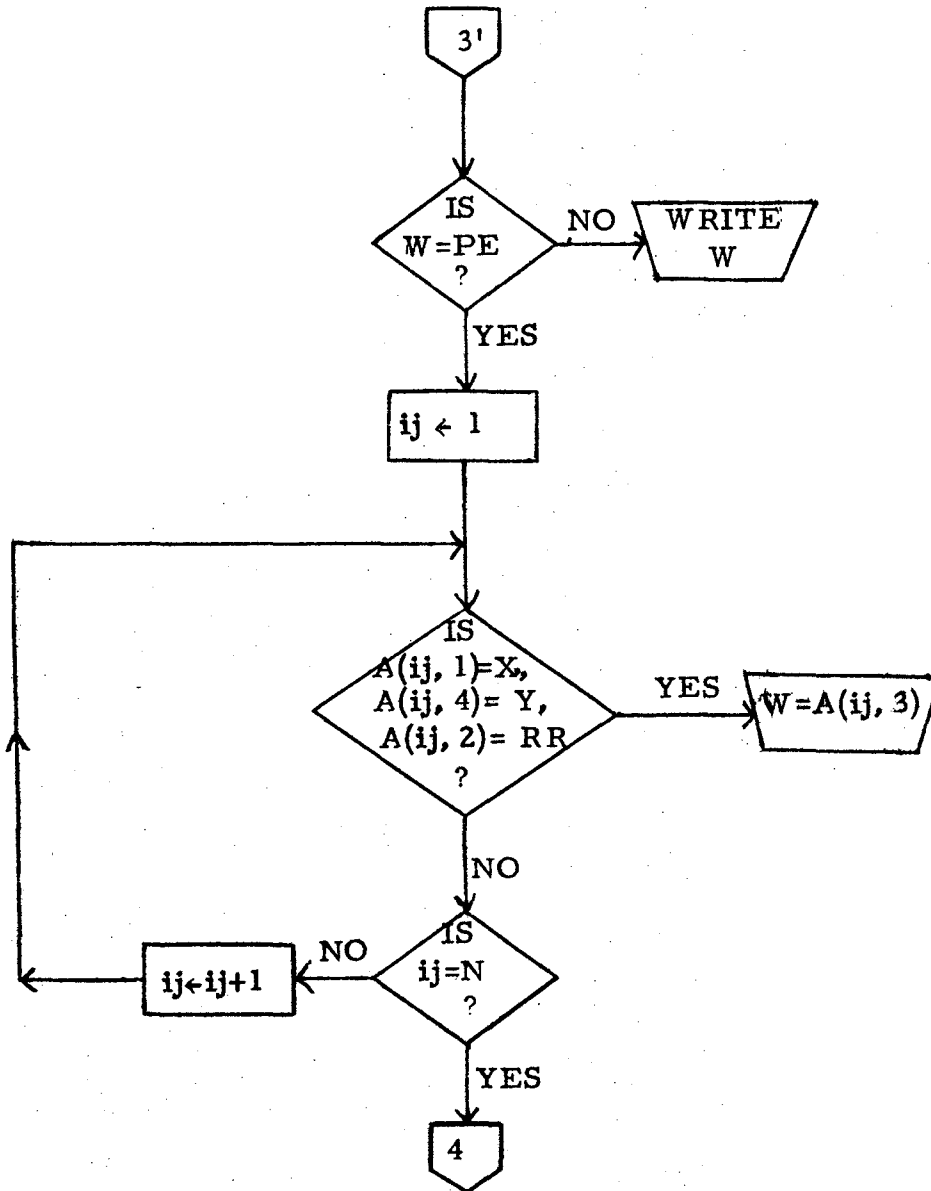


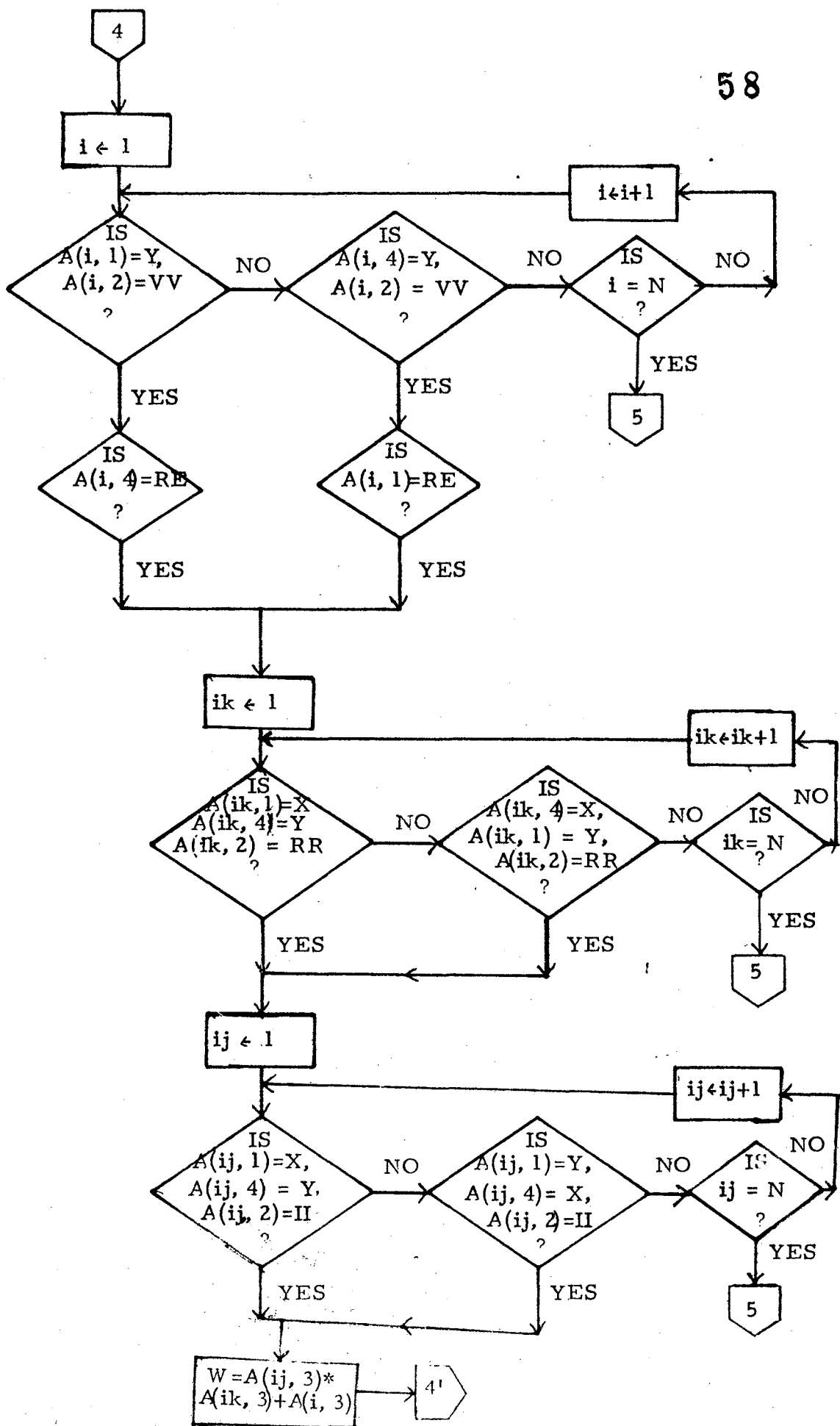


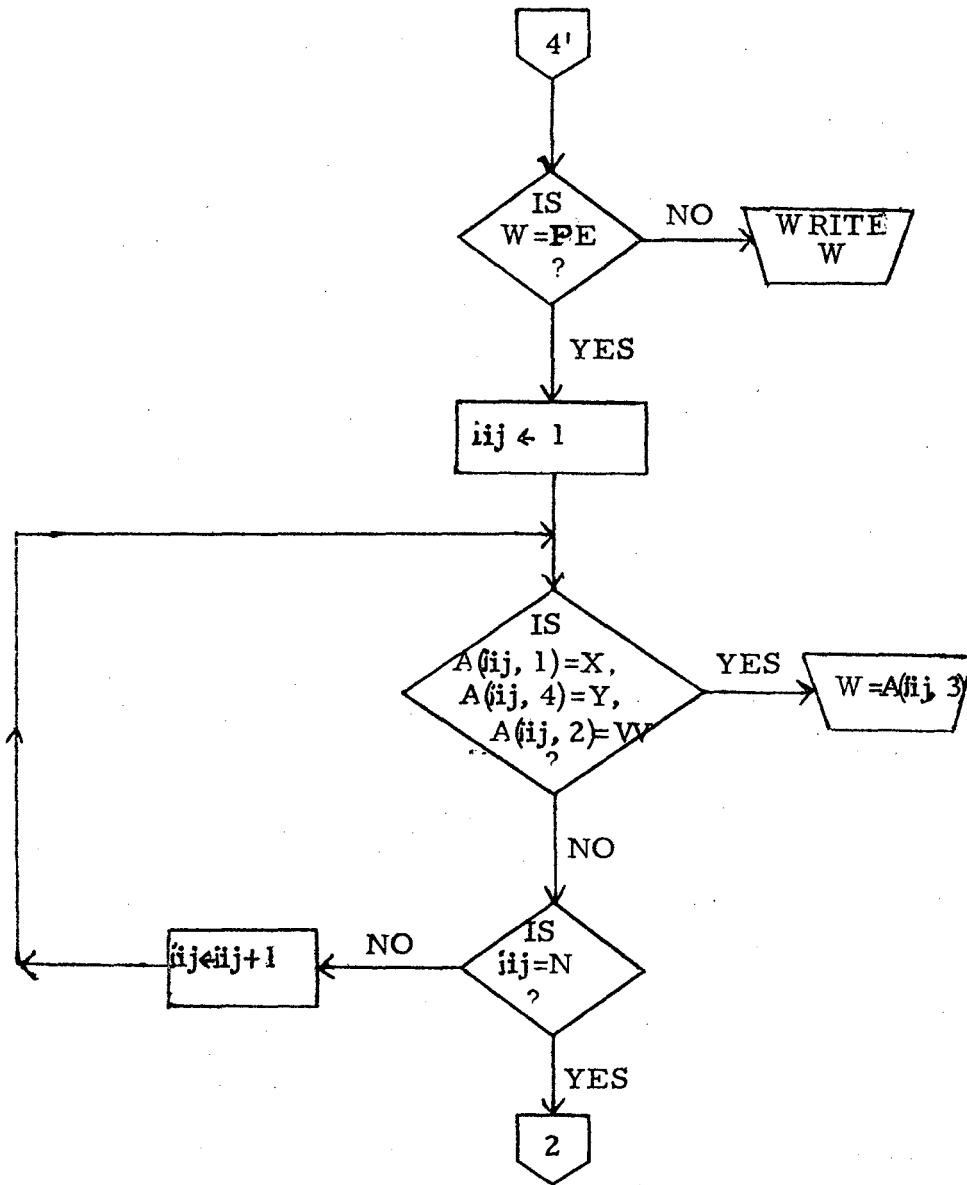


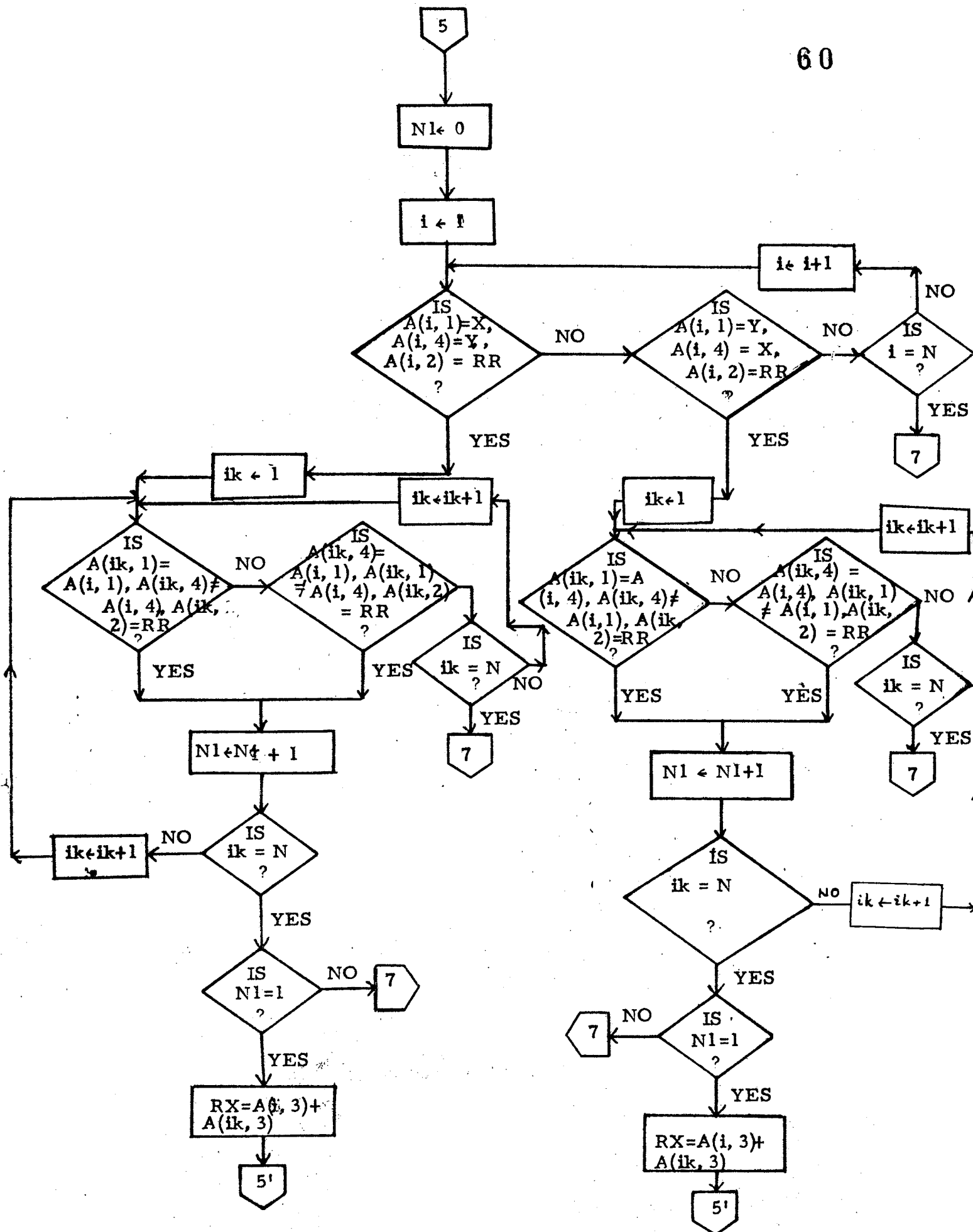


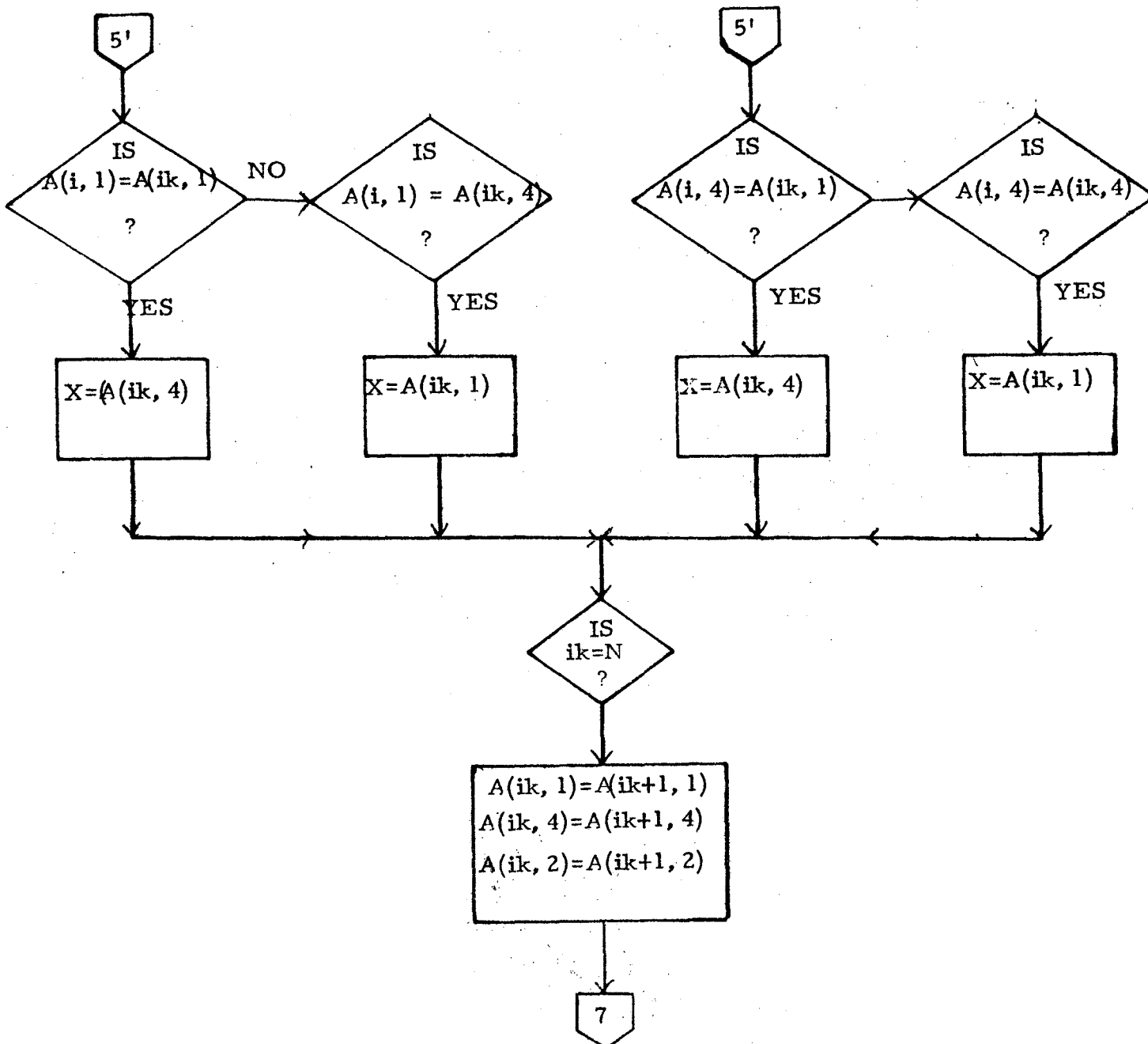


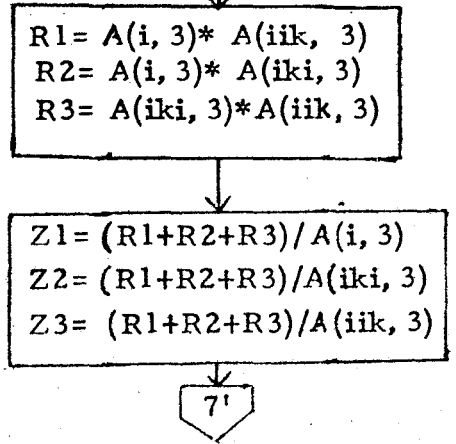
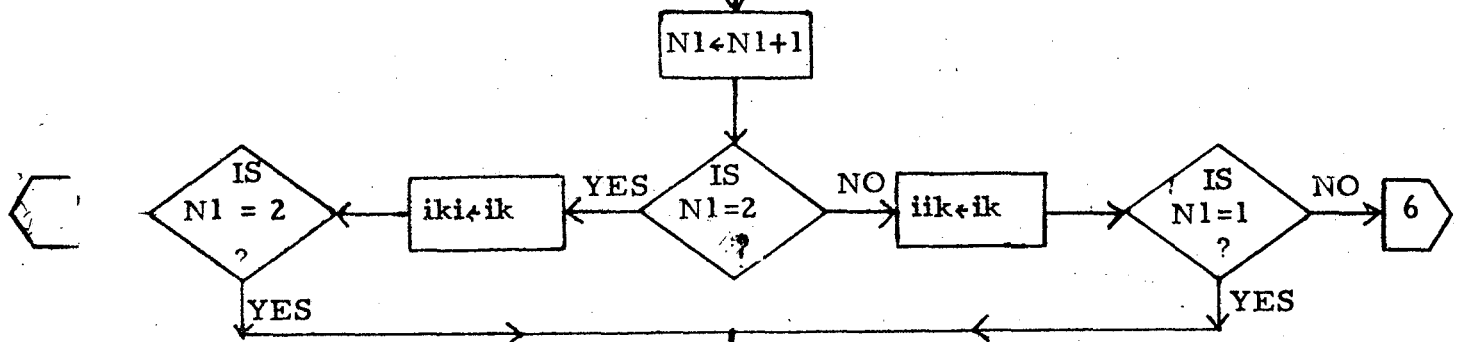
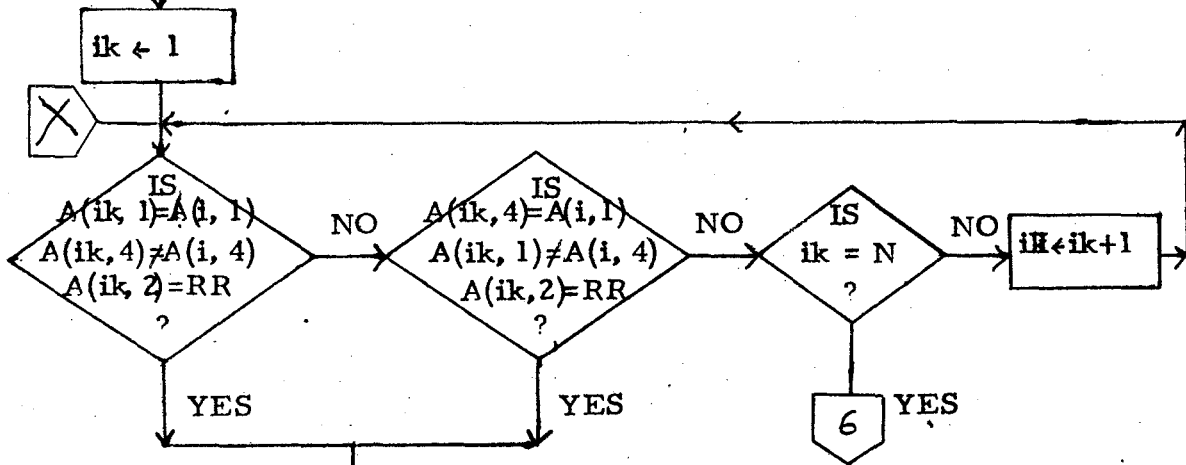
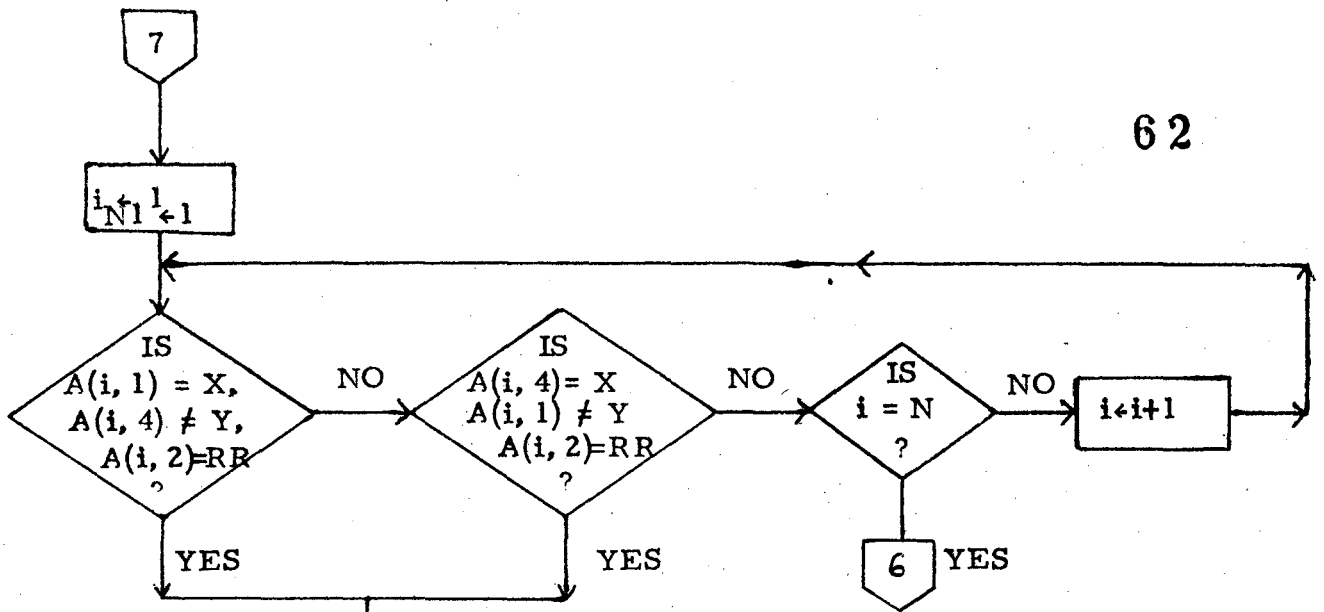


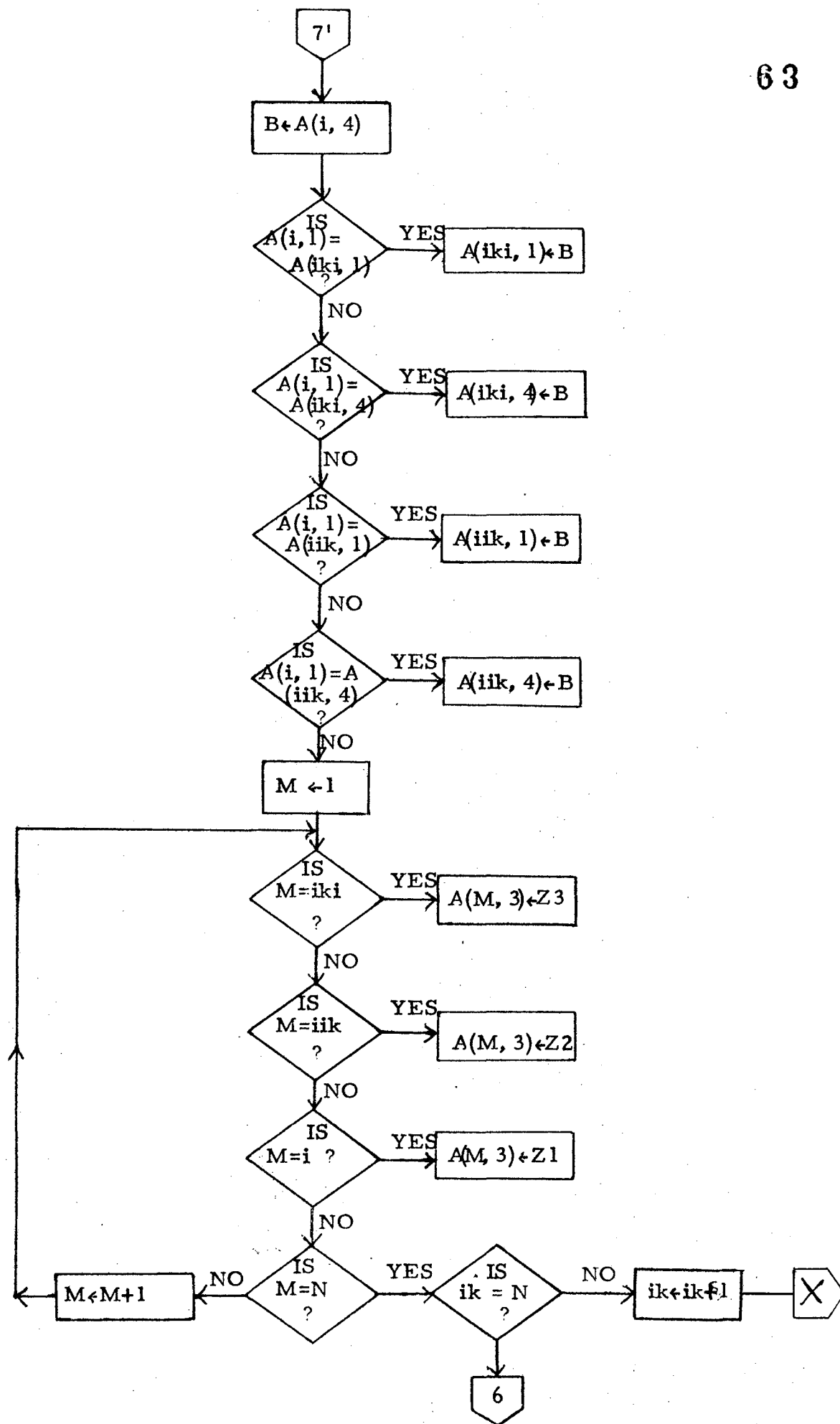


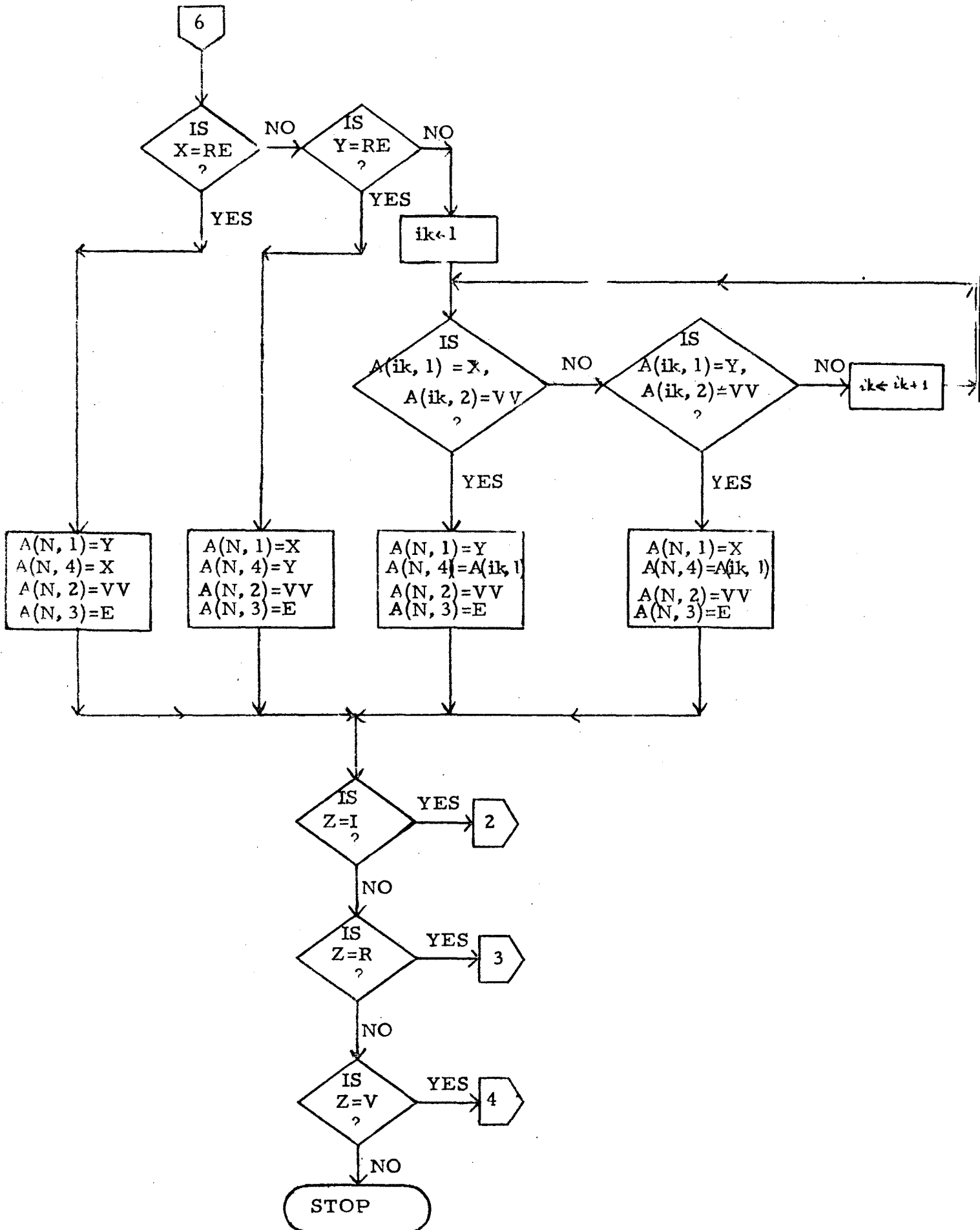












R E F E R E N C E S

AND

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