SEGMENTATION IN PICTURE PROCESSING

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C E R T I F I C A T E

This dissertation entitled "Segmentation in Picture Processing" embodies the work carried out at the School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi-110067.

This work is original and has not been submitted in part or full for any other degree or diploma of any other University.

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TABLE OF CONTENTS

S Y N O P S I S

Recognition of visual scenes and patterns is generally accepted as the best example for study of intelligence. Earlier works on pattern recognition were aimed at classifying unknown patterns. Later it was realised that mere classification could not produce satisfactory output due to several reasons. For instance, when pattern recognition activity is cascaded in several stages so that the output of one stage is input to the next, a descriptive scheme becomes more powerful. It may also be preferrable to have a descriptive output instead of mere 'YES', 'NO' and 'DON'T KNOW'. Before a pattern can be recognised, it is always necessary to preprocess the picture. Picture processing includes digitisation i.e., conversion of an analogue picture to digitized one, compression of data, enhancement to improve degraded and noisy picture and segmentation for isolating objects. Segmentation of arcs and curves is an essential part of picture analysis.

This dissertation attempts to give a method of segmentation of juxtaposed objects. This method has been tried in many conjugated objects which includes biomedical images (muscle fibre as seen in transverse section) and TiO_2 overlapping blobs. The picture is taken as preprocessed. This edge-based approach of segmentation is based on matrix manipulation.

The points of concavities at the border of the picture are determined first. Duplicacy is then removed and insignificant points of concavity are erased. Then the direction of path of segmentation from the significant concavity points is determined and paths are allowed to grow towards the interior of the pattern until they meet each other. Segmentation is thus achieved.

A general discussion on picture processing is given in the Chapter II. In Chapter III, a review is given on segmentation of closed curves and arcs. The method of segmentation of suxtaposed objects is described in Chapter IV. This Chapter also includes the conclusion where the limitations and possibilities of extension of the method are discussed. Implementation of the algorithm is given in appendix. References and selected bibliography are listed in the last section in alphabetical order.

CHAPTER I

INTRODUCTION

INTRODUCTION

Visual images are probably the most important means by which men experience their environment. Human being can process a limited amount of data under suitable circumstances. As Eden (11) puts it regarding visual image processing in animals and men, "...... in many circumstances, an organism's performance is orders of magnitude faster and more accurate than the best machines that have been designed to date." But the data obtained from an image is generally associated with a considerable amount of noise. To help men extracting useful information, some machine-aided 'a priori image improvement' is required. Such techniques of enhancement are applied before presentation of images to human beings. Another motivation behind automatic picture processing is to relieve man of repetitive monotonous jobs like reading printed text or checking inventory or detecting manufacturing defects. To avoid risk in working is dangerous, dirty, or remote environment like underwater drilling rigs or in steel plants where an on line decision is sometimes necessary to be taken, machine processing becomes essential. Visual information is very important in various fields like space exploration, aerial survey, biomedical images, finger prints, engineering drawings, etc. So the transmission and storage of these informations is equally important. As the amount of data is quite large, redundency must be removed. For transmitting economically, coding with reduced bits becomes most effective.

One of the problems encountered in picture processing is the problem of recognition. Patter recognition can be said, in a very general sense, to simulate human reasoning and decision making. The human brain is a complex pattern recognition system which can classify unknown patterns into known groups. As recognition involves intelligence, pattern recognition can be taken as an intelligent activity. Kanal and Chandrasekharan (31) say "..... since the search for regularities is the principal concern of all scientific enquiry and regognition of patterns is intrinsic to all intelligent activity, it would appear that the field of pattern recognition encompasses all scientific enquiry and intelligent behaviour."

In picture processing, a problem of the kind --- 'given an unknown pattern, which of the number of categories it will belong to,' is quite common. The approach here is (definitely) decision.- theoretic or statistical (4),(9),(66). Each pattern is associated with a group of attributes and each attribute with values. Thus a picture is transformed into a point in the n-dimensional attribute-value space. This n-space is divided into mutually disjoint regions, each region corresponding to a particular class. Given the unknown pattern, its attribute values are computed and using this value, the class to which it belongs can be determined. In this approach each pattern is considered as an independent entity. The structural relationship between different patterns are totally ignored. In case of very complex patterns the number of attributes becomes very large. Moreover, the structural relationship among different parts becomes an useful aspect. In problems of scene-analysis, therefore, the patterns are broken up into simpler sub patterns which are broken up into still simpler sub patterns. The simplest subpatterns are called pattern 'primitives' which are far easier to recognise than the original pattern. A set of rules governing the composition of primitives into patterns are specified by the 'grammer' of the pattern description language. Once the primitives are recognised, the pattern recognition is done by parsing of the 'sentence' describing the given pattern to see whether it is syntactically correct with respect to the given grammer. Minsky (42) and Fu (16), (15), (41) are among many workers in this area. The advantage of this method lies in the recursive nature of grammers. The techniques of formal language theory can be directly used to the problem of compactly representing and analysing patterns by using tree-structure.

Picture recognition is only one aspect of the large problem of picture analysis and description. Instead of getting an output of 'YES', 'NO' and 'DON'T KNOW', it is often desirable to get a description of the input. Image analysis or scene analysis includes text analysis, analysis of photomicrograph of mitotic cell, handling of blocks by robots, etc. Another important factor in analysis of picture of semantics or meaning. Guzman (20) analyses a complex scene by decomposing into parts by considering the scene to be made up of three-dimensional parallelopiped. He deals with different kinds of varities and gives weightage to adjacent regions to draw inference. His approach has a dominent role of context.

In biomedical field, automatic image analysis has gained popularity because very often it does a more accurate job of measuring and counting cell analysis, nuclear medicine, diagnostic radiology are some of the fields where computer has become essential. In computerised tomography, a computer built into the computer tomography apparatus can measure size, location and radiodensity of the pathology on the computerised tomography scanning device. On occasions like these when measurable parameters determine the behaviour and characteristic of biomedical materials, the automatic image processing is more sensitive, quantitative and rapid means of relating performance to structure.

In all cases of scene analysis, description begins from basic primitives and thus segmentation takes the central role in pattern recognition. There have been attempts to develop techniques to operate a specific domain of picture. Some contributors are Rosenfeld and

Johnston (61), Ledley, Rintala and Hsu (56), Arcelli and Levialdi (5), Eccles, Mc Queen and Rosen (12). Many of these techniques need some preprocessing on input pictures. It may sometimes happen that two or more objects are juxtaposed and overlapping and appear together within a single boundary (56), (5), (12). In these cases the boundaries of such objects consist of sharp inward curvatures or concavity. These inward curvatures signify the presence of two or more objects. In order to separate them, segmentation at the concavity point is necessary.

This dissertation is an attempt to develop a method of segmentation of closed curves arising in the recognition of certain pictures. This has immediate application in the area of biology (transverse section of muscle fibre) and industry (TiO_2 blob separation). The approach is based on matrix manipulation. There are two parts of the system. The first part finds the concavities in the picture. Duplicacy is then removed. Once the points of concavity are extracted, the non-maxima or the points with less concavity are supressed. Thus the significant concavities are found. The second part does the segmentation. The direction of paths of segmentation from the significant points of concavity are determined. Segmentation paths are then allowed to grow from the concavity points along these directions. The paths grow stepwise from each significant point of concavity. These paths meet each other and segmentation is achieved.

A brief outline of the system is sketched below (Fig.1.1)

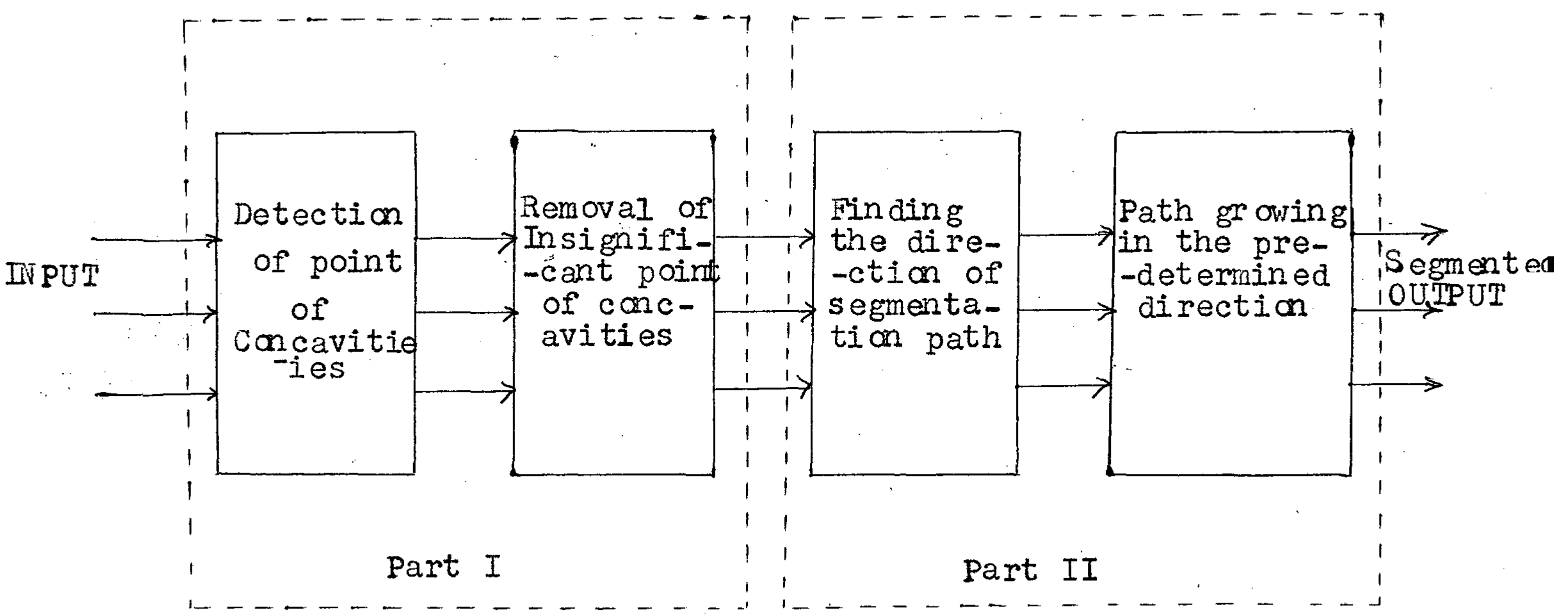


Fig. 1.1

CHAPTER II

PICTURE PROCESSING

2.1 Digitization

A picture can be considered as a continuous function which need to be digitized before processing by a digital computer. Digitization may be said to be the sampling of the gray levels in the picture at an M x N array of points where M and N are arbitrary numbers. Each element of this array is called pixel (picture element). Total number of points represent the spatial resolution of the conversion. Since the value of the gray levels at these points may vary in a continuous range, the gray level must be quantized. By quantization we mean that the range of gray level is divided into K intervals so that the gray level at any point can take only one of these values. This analog-to-digital conversion is done by optical scanners like drum scanner, flying spot scanner etc.

2.1.1 Sampling

A continuous picture can be represented by a finite string or array of numbers, called samples. It should be always possible to reconstruct the picture from these numbers. Small errors involved in the reconstruction should not distort the scene. For a good reproduction of the original picture M, N and K should be large but within the limit of spatial and gray scale resolution capabilities of the receiver. One way of sampling a picture is to find the values of the gray levels at discrete array of points. The picture function can be represented in terms of sample function and an interpolation function. Both these functions are taken as Fourier transformable. Rosenfeld and Kak (64) have shown that by Fourier expansion and using properties of Fourier series, it is possible to find necessary and sufficient condition for exact reconstructability of the function from its samples.

Another way of sampling is by expanding the picture function in terms of a set of orthonormal function and taking the co-efficients of the expansion as picture samples.

If a picture is transmitted in a communication link where an orthonormal set of functions is available at both the transmitting and receiving ends, then the transmission of the co-efficients is sufficient. These co-efficients can be used for reconstruction of the picture. For an error-free reconstruction, an infinite number of these co-efficients are required but for practical purposes only a finite number is transmitted. The orthonormal functions are usually ordered so that high-ordered terms contribute to the fine detail in a picture and neglecting them may lead to a loss of resolution.

Fourier sampling, Standard sampling etc. are examples which use orthonormal sampling. Results for the sampling errors using Fourier, Standard, and optimum sampling are discussed by Habibi and Wintz (22).

2.1.2 Quantization of Picture Samples:

In quantization, the range of values of the samples are dicided into intervals and all the values within an interval are represented by a single level. When the samples are obtained by using an array of points or by standard sampling, a fine degree of quantization is necessary in regions where gray level changes slowly. Otherwise false contours arise due to abrupt change in gray level between different levels. Roberts and Lippel (58), (35), discusses this problem by adding irregular noise prior to quantization.

In general, the quantization levels are chosen evenly. But in case of picture, where sample values in a certain range occur frequently, finely-spaced quantization levels are used in that range. Coarsely spaced quantization levels are placed elsewhere.

Average accuracy of the quantization can be increased this way without increasing the number of levels. This method is known as tapered quantization. Theory of optimum quantization which justifies the reason for tapering quantization scale is given by Max (38). Non-uniform quantization where samples are made to pass through compressor before quantizing uniformly and again passing through expander before reconstruction is discussed by Smith (70).

2.2 Compression

Compression is the technique of reducing an input picture to represent it by as few bits as possible for the purpose of transmission or storage. In general, the picture is represented by uncorrelated data i.e. any element cannot be predicted from the rest. If the volume of the uncorrelated data is quite large, then they must be ranked according to degree of significance of their contribution to both the information content and the subjective quality of the picture. When this is done, those elements of data which are unimportant can be neglected. Such deletion is a major contribution towards picture compression. A number of investigators have studied the application of linear orthogonal transformation in picture compression. Some of them are Ahmed, Natarajan and Rao (2), Habibi and Wintz (22), Habibi (21), Pratt

(52), (53), Tasto and Wintz (72) and Wintz (75). Most of them have applied Karhunen-Loeve, Fourier, Hadamard and Haar transforms.

Contour coding is one of the most efficient method of compression for pictures with small number of regions having constant gray level. This method consists of sending only the information regarding boundaries between the constant gray levels to the receiver which in its turn reproduces the contour.

Another method of picture compression seperates a picture into 'highs' and 'lows'. The 'highs' picture can be obtained by taking either gradient or Laplacian of the picture and the 'lows' picture is obtained by low-pass spatial filtering of the picture. The 'highs' picture consists of the edges in the picture while the 'lows' consists of a picture with no sharp edges. The high-frequency information obtained from the 'highs' picture when combined with the 'lows' picture, gives back essentially the original picture. Application of this technique is found in Graham's (17) work.

Roberts (58) has suggested a technique of picture compression in which some noise of uniform amplitude distribution, and peak-to-peak value equal to one quantum step is added to the picture samples before quantization, and identical noise is subtracted at the receiver. The result looks like an unquantized output.

2.3 Enhancement

Enhancement is done in order to compensate the degradation of the quality of a picture when converted from one form to another. These techniques are generally used to suppress selected features of a picture or to emphasize such features at the expense of others to increase the picture's usefulness. There are many useful functions which include increasing contrast or changing the gray-scale range, removing certain distortions and modifying the spatial frequency of the picture. Work in image quality and enhancement is done by Huang (27), Levi (36), Mc Camy (39), Prewitt (54), Biberman (7).

2.3.1 Gray Scale Modification

In case of unevenly exposed pictures, the gray level of each point of the picture is different from that of the other. One of the most useful techniques to achieve uniformity is the gray-scale modification whereby the gray level of each point is changed. In case of pictures where gray level of each point is not different but is uniform over a certain region of the picture, it is only necessary to increase the contrast. Gray scale transformation in this case, is applied so as to change the gray level in a uniform way throughout the picture or some regions of the picture.

If we have two pictures of same scene taken under different lighting conditions and want to compare them, we can make use of histogram modification technique. Histogram as defined by Rosenfeld and Kak (64) is "given a picture f, let $P_f(Z)$ denote the relative frequency with which gray level Z occurs in f, for all Z in the gray level range (Z_1, Z_k) of f. The graph of $P_f(Z)$ as a function of Z, normalized so that $\int_{Z_1}^{Z_k} P_f(Z)\, dZ$ is equal to the area of f, is called the histogram of f". One way of compensating for the difference in gray level at identical points of the two pictures is to transform the gray scale of one picture so that its histogram matches with that of the other. When a picture's histogram is transformed so that all gray levels occur equally often, the result tends to higher contrast. Because the points in densely populated regions of gray scale are forced to occupy a larger number of gray levels whereas the points in sparse regions of scale are forced to occupy fewer levels. The overall effect is of contrast enhancement. Histogram modification techniques are discussed in detail by Hall (24), Haralick (25), and Troy (73).

In case of pictures with optical or mechanical distortions, geometric correction is most helpful. Piecewise linear approximation or even higher-order approximations can be used to the unknown distortion and hence correction is made. Much work in enhancement including geometric correction technique is done in Jet Propulsion Laboratory (46).

2.3.2 Sharpening

Blurring is a averaging, or integration, operation which weakens high spatial frequencies more than low ones. Hence sharpening or deblurring is done mainly by applying differentiation operator, particularly, Laplacian operator as done by Prewitt (54), and high-emphasis spatial frequency filtering as done by Arguello et. al. (6). In order to sharpen blurred features, such as edges and lines, that run in any direction, isotropic operator should be applied. Laplacian operator, being rotation invarient is very useful in sharpening pictures. A subtractive combination of a picture and its Laplacian gives fruitful results. In order to reduce noise sensitivity, instead of Laplacian, a linear combination of the second partial derivative in two perpendicular directions are used. This helps to smooth the picture in the direction along an edge. These methods should be used in absence of knowledge about blur.

To compensate for the degrading effects of light, motion of the image on the film, and the spatial frequency-response limitations of the recording medium, high-spatial frequency content of a picture must be pushed up. High-emphasis filtering techniques has a transfer function with increasing spatial frequency, upto the point where noise would begin to dominate; it then drops off, more or less abruptly to zero. Good result has been found by using 'transfer function compensation' by Arguello (6).

2.3.3 Smoothing

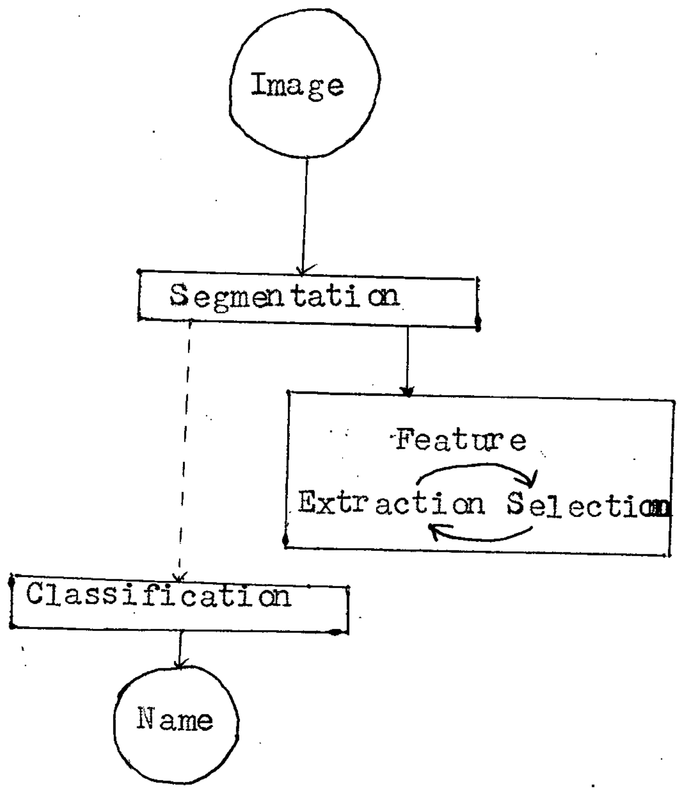
Differentiation and high-emphasis filtering can not be used to sharpen pictures with noise since noise generally involves high rate of change of gray level which usually becomes stronger than the picture signal at high frequencies. Smoothing techniques should be applied to remove noise. Care should, however, be taken while applying these operation as smoothing gives rise to blurring.

In case of pictures where it is possible to distinguish noise from non-noise, noise can be easily removed by changing the gray level of the noise to average of the levels of neighbouring picture points. In case of two-valued pictures where it is possible to count the number of neighbours of a noise point, a black point can be changed to white if it has too many white neighbours and vice-versa. Instead of comparing gray level of a noise point with that of every neighbouring point, it is done by averaging. Since the whole process depends upon the choice of neighbourhood, the border points should be carefully considered.

If it is not possible to distinguish the noise from the rest of the picture, the process of averaging is done which weakens the noise. When we have several noisy copies of the same picture or a symmetrical picture it is useful to apply pointwise averaging. But when only a single copy is available, local averaging is done by giving each point a new gray level which is the average of the original gray levels in some neighbourhood of the point. In case of two-valued picture, noise that is smaller than the picture detail can be removed by the process of shrinking and reexpanding (61). This can be done by changing all black points to white if they have any white neighbourhood and then changing all white points to black if they have any black neighbourhood. This process deletes not only isolated points, but also thin lines. A selective local averaging works better in such a situation. Whenever an edge or a line is present, the directional average should be taken as done by Graham (18). If the noise is signal dependent, variable-sized neighbourhood should be considered to obtain a uniform reduced noise level throughout the picture as discussed by Pizer (49), Vetter (50). One way of obtaining variable-sized neighbourhood is to calculate the sums of the gray levels in a series of expanding neighbourhood of a point, and stop expanding when the sum reaches a present threshold. Quantum-limited pictures which consist of clusters of dots can be smoothed by this technique. In order to ensure that the neighbourhood does not cross the edge of the picture, neighbourhood can be grown around a point by accepting neighbouring points if these gray levels differ only slighly from that of the point. This method is applicable to homogeneous regions of non-noisy pictures.

2.4 Segmentation of Picture

Segmentation of picture or image is that part of image analysis which concerns itself with the spatial differentiation of the various objects constituting a visual scene. It can be regarded as being a form of preprocessing prior to object feature extraction, selection and then classification as shown in figure below.



To analyse the picture in different ways, different segmentation techniques are applied. Some of them can be used directly on a picture while others can be applied on partially segmented pictures. Segmentation procedures can be broadly classified into four major categories as stated in 2.4.1, 2.4.2, 2.4.3 and 2.4.4.

2.4.1 Histogram techniques act on a local property which occur frequently on the picture. A histogram may be defined as a plot of the number of picture points at each possible intensity level. This technique is useful in case of pictures having different regions of distinct homogeneous gray-levels. Due to difference in intensity of gray level, there will be distinct peaks in the histogram whereby the picture can be easily seperated. Various attempts have been made in this area by Narasimhan and Fornango (45) and Rosenfeld et. al. (62). Histograms are generally used in finding the threshold for segmenting a picture.

If a picture has got objects whose gray levels are different from one another, we can use the simple method of thresholding to seperate them. By thresholding at a point we get a two-valued picture e.g. 1's at object points and 0's elsewhere. But it is always very difficult to choose the threshold because if we threshold too high, too many object points will be classified as background whereas if we threshold too low, the reverse will happen. One good way of selecting the threshold is to use the histogram of the picture. The histogram will have at least two peaks (binodal), one corresponding to the object and the other to the background. If the gray levels intermediate between the object and the background do not occur frequently, we can have a flat valley. The best way is to select the bottom of the valley to give a good threshold. In case of pictures where there are too many valleys, bottom of each valley can be taken to segment the picture. To find out the appropriate local feature to draw the histogram, histograms of a number of local properties are drawn and the one whose valley touches zero is taken. Thresholding can be applied when a picture has a kind of uniformity in gray level. In case of drawing histogram, apriori knowledge of a local characteristics is to be known. Moreover, the performance can be known when the results are mapped back onto the image space in case of a histogram based technique.

2.4.2 Region-based Technique

Region-based techniques function directly on the picture space rather than on histogram space. They try to find out similarities between neighbouring regions and thus grow the regions up. The joining is done depending upon a given property and the process continued until all points are joined to some region.

In order to find out points of identical gray level in a picture of arbitrary gray level, matching technique is very useful. There may be various simple or complex patterns to be matched against a picture. Matching is not a difficult task if there is no noise present in the picture. Due to presence of noise, exact match is never possible. Only maximization match criterion like signal-to-noise ratio can be found.

One way of matching is by cross-correlation between two functions of gray levels over certain region discussed by Harris (26).

Any two picture can be matched if an unlimited amount of distortion is allowed. But the pictures should match with minimum distortion. A template can be thought of as composed of many subtemplates, some of which can match with the picture. In order to find a combination of partial match with least tension, mathematical programming can be used. One of the best ways of doing this is to find the matches of the subtemplates and then to build up a combination of them with least tension by using heuristic search techniques. If distortion is not present, it becomes time-consuming to search all possible positions. The best short cut is to reject a mismatch completely after trying once. This will reduce the number of possibilities and hence time.

Aggrawal (1) gives a technique of image segmentation using prototype similarity. His procedure starts with picking up of a few candidate regions in the image in terms of which the whole image can be represented. Symbolic representation is used to segment the image into meaningful components.

2.4.3 Edge-based Technique

Another technique which searches the picture space is edge-based technique (25, 46). Here the dissimilarity between regions is searched out. There are many ways of finding an edge which is represented by a discontinuity or change in gray level of a region. The change may be abrupt or more or less gradual. The ideal case is of a step edge where the gray level is stepped up or down sharply but due to presence of various noise, this does not happen. The edge detection operator responds not only to the edge but also to noise. Various mathematical optimization techniques are therefore used in designing an edge detector.

Derivative operator gives high values at points where gray level is changing. Thus operator such as Laplacian can be used as edge detector. If the edge is blurred instead of being abrupt, the gradient gives maximum value at the steepest point where we can segment it. But in case of a picture with a long ramp, the gradient will have a constant value on the ramp while the Laplacian will have zero value. Finding the shoulders of the ramp, we can select the mid-point to segment the picture. Modification of derivative operator is used by Prewitt (54) for edge detection.

There are many methods of edge detection based on probabilistic theories regarding where an edge should be found. Shirai (67) worked with the knowledge about the structure of polyhedral blocks. Edges were known to be straight and points of curvature were known to be vertices. The system could begin with a prominent feature like an outside edge, and proceed until a vertex is found. The form of this vertex can be used to predict the location of another edge. Thus even the weakest edge can be detected since apriori knowledge about its location is available.

Jarvis (29) has made an attempt to segment pictures by the combination of line, region, and semantic structure in the image.

Kelley (32) gives an edge detection method using planning. He prepares a smaller digital picture with less details from the original. Edges of objects are then located in the reduced picture. The edges found in the reduced picture are used as a plan for finding edges in the original.

2.4.4 Edge-and-Region-Based Technique

Some fruitful results are found from a combination of all the above mentioned methods. Both edge-based and region-based techniques being complementary, they can be used to interact sequentially to give better performance. Region-based techniques can be used to find enclosed area of a closed pattern and smooth border of which may be obtained by using an edge-based technique. Rintala and Hsu (56) has used this technique in tracing the cell border of totally overlapped cells. His system finds the enclosed regions by region-growing technique. Having found the enclosed regions, the system tries to find the smoothest border of a cell by applying search techniques on the border.

Histogram techniques can be used to find a region where some property may be found to dominate and region-based algorithm used to find the finer results. Different processes may interact in various ways depending upon the problem involved. Less computation becomes often necessary in sequential techniques as the result of one phase acts as the selected source of computation in the next. These techniques have wide application in problems of cell analysis.

Instead of computing a complex technique at every point, some inexpensive computation is done to detect some objects. More complex computation is done on the detected points to extend or track the objects. This sequential method of applying result of one computation for further investigation has the cost advantage over the parallel procedure. Moreover, tracking methods are quite flexible.

If we have objects which are thin, dark and continuous curves whose slopes never differ much from 90°, we can extract them by tracking them row-wise. Detection and tracking involve local property like local contrast. Since gradient can be used as a measure of contrast and it has a direction too, tracking can be done along the perpendicular direction. Tracking criterion may depend upon the position of point or its neighbourhood.

In Raster tracking (20), detection criterion is applied to each point of first row and tracking method is applied to each accepted point. Detection criterion is applied to points not belonging to accepted region. When bottom is reached tracking is stopped.

Raster tracking becomes unvalid for tracking curves which are almost horizontal which can be overcome by using another perpendicular raster. This doubles the cost of the previous method. One way of avoiding this problem is to use omnidirectional curve-following. From the current point on a tracked curve, a curve-following algorithm examines a neighbourhood of that point selects the best candidate for the next point. If the curve branches, one of the next point is kept for further investigation whereas the other is used for tracking down.

It is possible to grow the objects by joining accepted points. All neighbours of an accepted point are examined and any of them that meet the tracking criterion can be included. New points are accepted and joined to the point until no acceptable neighbours exist.

Another way of region growing, is done by partition building as done by Pavlidis (48), which starts with an initial partitioning of the picture and gradual improvement of it by splitting and merging the regions until a good partitioning is obtained. Starting with an initial partition, the variance of each region is checked. If it exceeds a limit E, say, the region is further partioned into two or more parts so as to make varionce of each partition below E. If two regions are close by and their variance is less than E and whose mean gray levels are close enough, then they are joined to make a single region with variance less than E. An important extension of the techniques described is to go back and undo a decision which gives bad results after some steps. One advantage of tracking method is the utility of newly acquired information during further processing.

CHAPTER III

SEGMENTATION TECHNIQUE : A REVIEW

Segmentation of a complex picture gives rise to a subset of it which can be further segmented into individual arcs and closed curves. The problem of segmentation of closed curves arises when the boundaries of objects overlap. Problems of this kind are encountered frequently in bio-medical field e.g. distinguishing monocytes by means of shapes of their nuclei, skeletal muscle fibres in transverse section, automatic detection of extra large malignant cells from slides when proper strain techniques are available to outline the border distinctly etc. as well as in industry e.g. controlling the size of TiO_2 powder particle during the production of paint components.

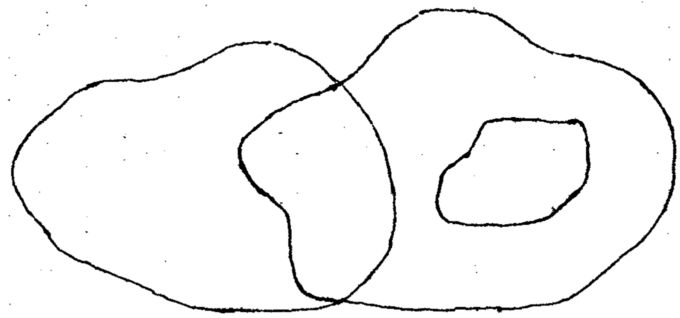
Depending upon the nature of the problem, the techniques of segmentation varies. The basic principles of the methods are almost same as that of picture segmentation. Some of the methods are discussed below.

3.1 Matching

Matching is a very old technique used in optical character recognition, map-matching navigation, etc. The basic principle of this method lies in measuring the degree of match between the sequence of slope (i.e. chain code) of the 'template' arc and a part of the sequence of the given curve. Search techniques become very useful here as in the case of picture matching. Work in this area is reported by Harris (26).

Rintala and Hsu (56) have done extensive work on overlapping cells. His program uses direct search heuristic technique to extract feature information. Application of his program is found in 'hematology'. The first part of the program finds out enclosed regions of the pattern. He starts with a zero and tests if it is completely surrounded by 1's. An enclosed region is thus grown and the number of 0's in the suspected enclosed region are counted out. If during the process, it was found that the suspected enclosed region was connected to some external region i.e. a 2 was encountered during search, the 0's counted were all changed to 2's which signified the boundary. Otherwise the 0's are changed to 3's (fig. 3.1). Part I of the program is sufficient for finding out the number of cells and their approximate areas.

Part II of the program locates the border of a cell. This part starts with the result of Part I of the program. Starting with a 1, the neighbouring 3's are searched to find the next element of the cell border. Difficulty arises when overlapping cells are allowed. While tracing a cell border when a point is reached from where there is more than one path to be followed, it becomes difficult to decide which one of the paths



0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	1	0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0
0	1	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	1	0
0	1	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0	1	0
0	0	1	0	0	0	0	0	1	0	1	0	1	1	1	0	0	1	0
0	0	1	1	1	0	0	0	1	0	1	0	0	0	0	0	1	0	0
0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	2
2	2	2	2	2	2	1	1	1	2	1	3	3	3	1	1	2	2	2
2	2	1	1	1	1	3	3	1	1	3	3	3	3	3	3	1	1	2
2	1	3	3	3	3	3	1	3	3	1	3	3	1	1	1	3	1	2
2	1	3	3	3	3	3	1	3	3	1	3	1	3	3	1	3	1	2
2	2	1	3	3	3	3	3	1	3	1	3	1	1	1	3	3	1	2
2	2	1	1	1	3	3	3	1	3	1	3	3	3	3	3	1	2	2
2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Fig. 3.1 Enclosed Regions in the Overlapped Pattern

should be considered as the continuation of cell border. The 'straight line search' of the program assumes smoothness assumption on a cell border. The straight line search determines the validity of a cell border by a 'right-oriented 3 check' which checks if there is a 3 somewhere to the right of each cell border point if a clockwise trip is made around the border of a cell. The Right Search and the Left Search are similar to straight line search for using in special situations. The straight line search uses the previous chain code direction as the initial search direction whereas in the Right Search the initial search direction is one less than that of the straight line search. This procedure finds the most curved cell instead of smoothest cell. The left search uses as the initial search direction, a direction one greater than that of the straight line search. The Left Search finds the originality of a cell found by the straight line search and the right search (fig. 3.2).

Rintala's program works well in most patterns but there are pathological cases where it may fail. This is due to ambiguity present (fig. 3.3) in certain cases.

3.2 Thresholding

The technique of thresholding can be applied to problems like verification of handwritten signatures,

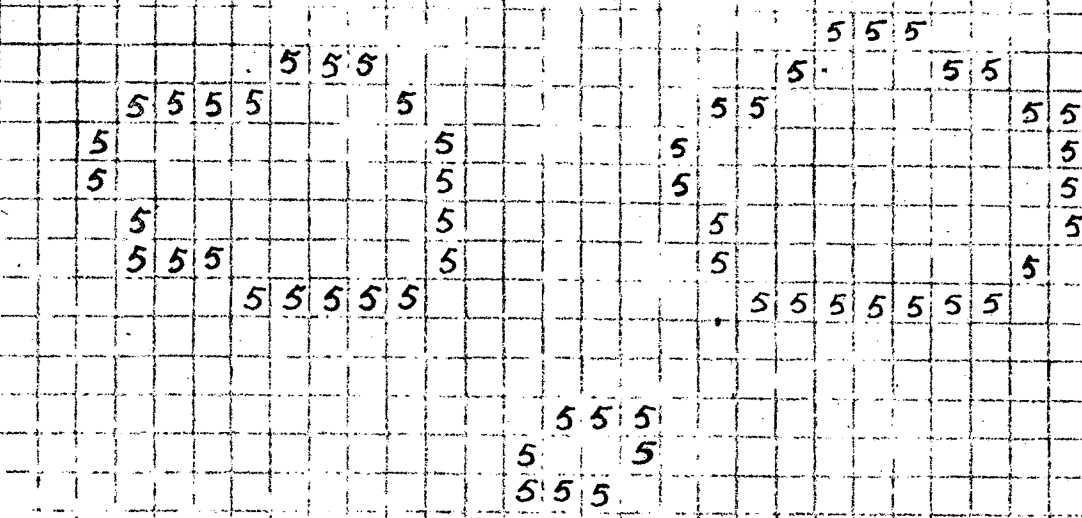


Fig. 3.2 Cells of the Pattern

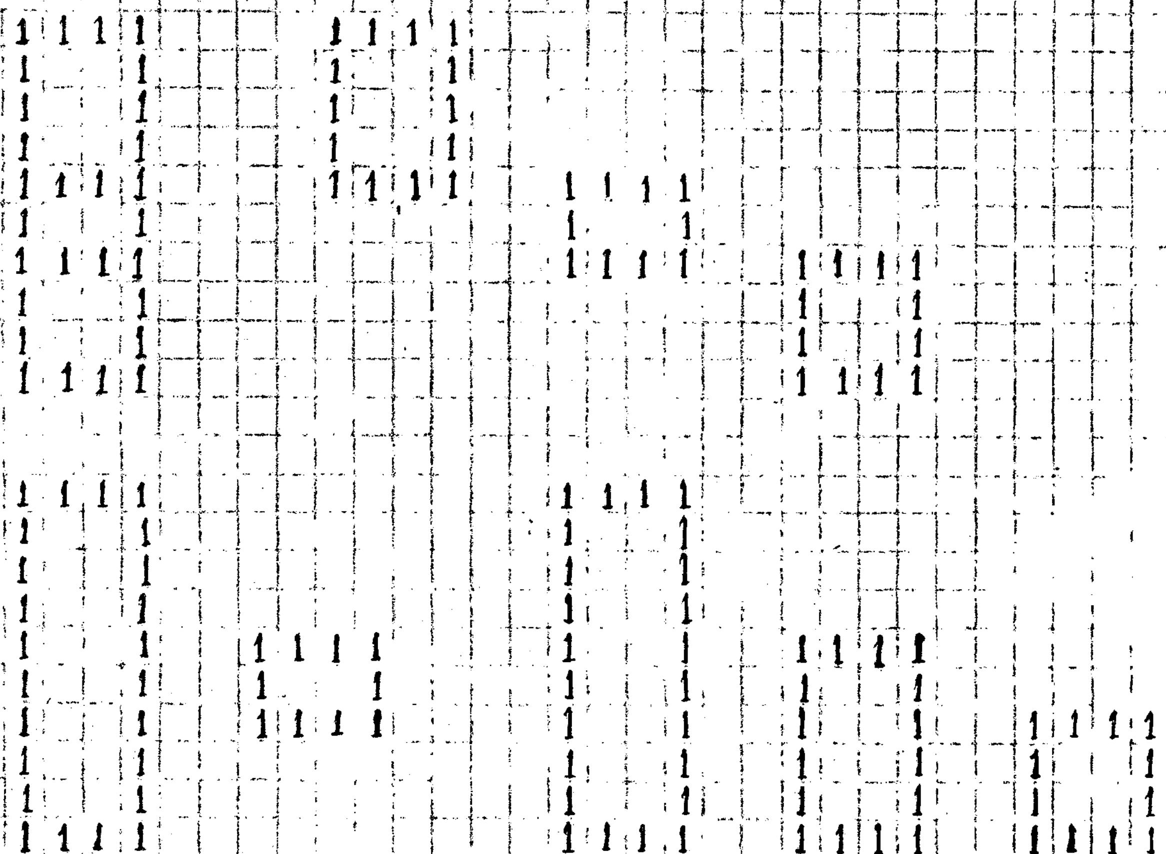


Fig. 3.3 An Ambiguous Case

seperating a drawing into directional strokes, etc. In this method, a part of the curvature which lies within a given range can be extracted by suppressing the points at which the slope lies outside this range. Nagel's (43) work on handwritten signature can be mentioned here.

3.3. Partition Building (piecewise approximation)

In this technique, curves are broken arbitrarily into pieces. Real curves of standard form are then tried to be fitten to these pieces. Polygon approximation is one of the frequently used methods of this kind. A set of points are picked up arbitrarily from the curve and are joined to form an inscribed polygon. If the curve does not lie close to any side of this polygon then that piece is further subdivided. Pavlidis (47), (55) and Feng (13) have worked with this approach. By measuring corner quality for each vertex, the shape of the original curve can be found. Using this information Lee and Fu (34) have given a generalised approach.

A simple technique can be applied in case of segmentation of a boundary curve of an object. Given a point on the curve, the number of points in a given neighbourhood of the point is counted. If the number of points is about half the area of that neighbourhood, then the curve must be relatively straight at that point.

If the number of points is very small or very large compared to half the area, then the object must have sharp convex or concave angle respectively at that point. Here again non-maxima or non-minima can be suppressed to locate the angle sharply.

All the above techniques are used in finding out the maximum curvature on the boundary of objects. After finding the maximum concavity or convexity, segmentation of the curves or arcs are done by joining the significant maxima or minima points.

3.4 Edge detection

Technique of edge detection is based on detecting curvature maxima. Significant maxima is detected by taking average slopes over adjacent, non overlapping segments of a curve and suppressing non-maxima. This method is used by Rosenfeld and Thurston (65).

Rosenfeld and Johnston (61) gives another procedure for finding significant curvature maxima. They measure the cosine of the angle between trailing and leading vectors. These vectors span a variable number of points on the boundary. The number is decreased from a fixed value until the maximum value of the cosine is found. This value is taken as a measure of the curvature.

Ladley (33) gives a different method for measuring the angle. He considers boundary segments as consisting of nine adjacent boundary points. Each segment is associated with two vectors : the trailing vector from the back end to the segment centre, and the leading vector, from the segment centre to the front end (fig. 3.4). The angle between the leading vector and the trailing vector is taken approximately proportional to the curvature of the segment. The vectors are laid off on a rectangular grid with their tails at the origin. The number of counts in going from the head of the trailing vector to the head of the leading vector along the outside of the grid is taken approximately proportional to the angle between the vectors i.e. proportional to the curvature of the segment. The clockwise or anticlockwise count gives the convexity or concavity of the segment.

Another edge detection technique is based on finding points of inflection. Points of inflection are zero crossings of the curvature which separate the curve into convex and concave parts. The digitized boundary is always subject to noise. Noise can be removed by convolving it with a rectangular filter. The boundary of a curve can be expressed as a chain of points or curvatures. After removing noise it becomes possible to choose out points with heavy negative curvature.

The nodes are classified and matched and line segmentation drawn between matched pairs.

Eccles, Mc Queen and Rosen (12) used such a technique in segmenting juxtaposed objects consisting of nominally discrete and separate objects, such as 'skeletal muscle fibres as seen in transverse section'.

The boundaries of the objects are expressed in terms of a sequence $\{Ui\}$ where Ui is a code for the direction from (x_i, y_i) to (x_{i+1}, y_{i+1}). The sequence Ui, called the direction chain, and the eight possible directions on a rectangular grid is dealt with extensively by freeman (14).

Considering two consecutive elements of direction chain, the curvature element Pi is defined as P_i = octmod $(U_i - U_{i-1} + 11) - 3$ where the function octmod(s) gives the absolute value of S, modulo 8.

Having found the negative-curvature points, they are categorised as significant(s), non significant (N) or noise according as two threshold levels (fig. 3.5). Segmentation between S-S node pairs and N-S node pairs is allowed but between N-N pairs disallowed. This process can be used in overcoming the difficulty of false segmentation path encountered by Arcelli and Levialdi (5). Paths of segmentation are drawn depending upon internodal distance and the angles between the directions in which the nodes point and the line joining them. This is done in order to overcome difficult situations found in certain conjugated objects. After segmenting primary (between segmentation path and boundary) and secondary (between two segmentation paths) lobs (fig. 3.6) the residual part can be treated as original object and the procedure applied. Another edge-based approach is applied by Arcelli and Levialdi (5) to separate overalpping convex blobs present in a binary input picture. This program finds application in controlling the size of TiO_2 powder particles during the production of paint components. Concavity arises due to overlapping blobs. The points of concavity are detected and segmentation paths are drawn from them.

Work in this dissertation is in the same line. It is an attempt of segmenting a picture where objects occur within the same boundary. Concavities are obtained due to juxtraposition of convex objects which are extracted. The approach basically is edge-based. Segmentation paths are drawn from the significant points of concavity towards the interior of the picture until these paths meet each other and segmentation is achieved.

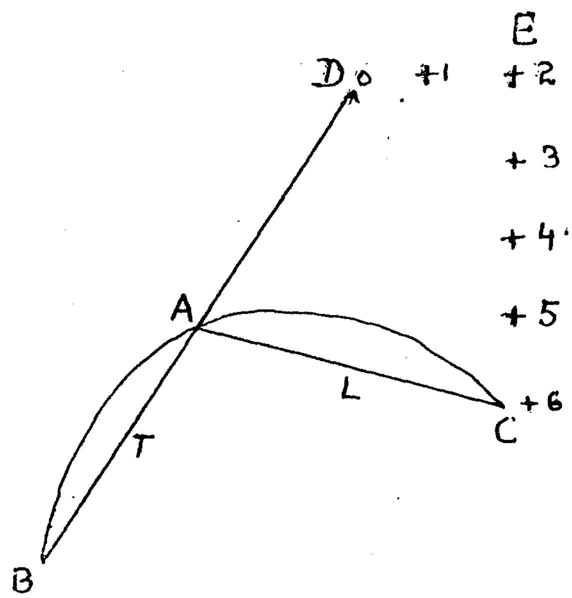


Fig. 3.4 Ladley's Representation : For An Arbitrary Point The Trailing Vector, B A, And The Leading Vector, A C Are Shown.

Fig. 3.5 Some Possible Shapes Of Conjugated Objects

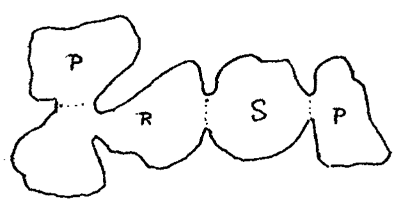


Fig. 3.6 Segmentation of Conjugated Objects Showing Primary (P), Secondary (S) and Residual Lobe (R).

CHAPTER IV

SEGMENTATION

4.1 Segmentation of Juxtaposed objects

The system, as already described in earlier chapters, has been designed for segmenting juxtaposed or conjugated objects. It consists of two parts, each carrying out specific tasks. The output of the first part is taken as input to the second. The system starts with the following assumptions:

(i) A sufficiently noise-free picture is available .

(ii) The pattern consists of objects of same or different size which are conjugated together so as to appear to have the same boundary.

(iii) The boundary has significant inward curvatures.

The pattern is then digitised (74) in order to convert it to discrete form. This is done by superimposing a grid or matrix of squares over the figure. A 1 is placed in each square where the area covered by the pattern exceeds a certain threshold. The threshold is taken as .5 (i.e. half the box) in this case. Zeros are filled in elsewhere. In the process of digitization sometimes there is a fair possibility of losing information concerning the original pattern. Care has been taken to reduce this loss and it is achieved by increasing the number of squares per unit area.

The picture is now obtained in a discrete form (in arrays of 0 and 1) which may consist of convex objects agglomerated together. This matrix of 0's and 1's is fed as input to Part I. To avoid computations for the 0's at the boundary of the matrix, the dimension of the exact picture matrix is increased by six by adding the outermost boundary of 1's and keeping two intermediate covers of 0's. This is illustrated in fig. 4.2 by taking the simple conjugated object of fig. 4.1.

Part I: Concavity Detection

Concavity is a local property. To test whether a picture is concave or not, as Arcelli (5) says ".... existance of three points is sufficient, such that q is in the line segment joining p and r and p is in A (the picture), q is not in A and r is in A".

The digitized matrix is taken as input. This matrix is scanned rowwise and concavity detection criterion is applied on each zero element. With each zero as centre, the 3 x 3 matrix is checked. The sum of all elements of this matrix is calculated. If the sum is found to be greater than or equal to 4, then the zero at the centre is replaced by the sum. No action is taken otherwise. This process is repeated

1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1

Fig. 4.1

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	1	1	1	1	1	1	0	0	0	0	0	1
1	0	0	1	1	1	1	1	1	0	0	0	0	0	1
1	0	0	1	1	1	1	1	1	0	0	0	0	0	1
1	0	0	1	1	1	1	1	1	1	1	1	0	0	1
1	0	0	1	1	1	1	1	1	1	1	1	0	0	1
1	0	0	0	0	0	0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 4.2

for all the 0's. The elements of the matrix whose values are greater than 1 are the points of concavity. Different possible concavities of fig. 4.3 are shown after processing in fig. 4.4.

As a result of the above procedure, double concavities may appear in adjacent positions. In order to remove doubles, all 4-elements are processed. Taking a 3 x 3 window with the 4-element as centre, if any number greater than 4 is found in any of the eight neighbouring boxes, then the element 4 is replaced by zero. This check is repeated with each 5-element as the centre. If there are two equal numbers in the 3 x 3 matrix, any one of them may be taken as the point concavity. After completing this process, doublets are removed and significant points of concavity are obtained. The possible configurations of doublets and their processing are shown in fig. 4.5 and 4.6 respectively.

Part II: Path Growing

In order to have correct segmentation, the appropriate direction of the path must be determined. By correct segmentation, we mean a segmentation which is not different from the manual one. The process of finding the direction of segmentation paths is based on the approximation of curvature in terms of

1	0	0
1	0	0
1	1	1

(a)

1	0	1
1	0	1
1	1	1

(b)

0	0	0
1	0	1
1	1	1

(c)

0	0	1
1	0	1
1	1	1

(d)

0	0	0
1	0	0
1	1	1

(e)

Fig. 4.3 Possible configuration of concavities

1	0	0
1	5	0
1	1	1

(a′)

1	0	1
1	7	1
1	1	1

(b′)

0	0	0
1	5	1
1	1	1

(c′)

0	0	1
1	6	1
1	1	1

(d′)

0	0	0
1	4	0
1	1	1

(e′)

Fig. 4.4 Configuration of processed concavities

0	0	0	1
0	0	4	1
0	4	1	1
1	1	1	1

0	0	0	1
0	0	4	1
1	5	1	1
1	1	1	1

0	1	1
0	4	1
0	5	1
1	1	1

0	1	1
0	4	1
0	4	1
0	1	1

Fig. 4.5 Possible doublet configuration

0	0	0	1
0	0	0	1
0	4	1	1
1	1	1	1

0	0	0	1
0	0	0	1
1	5	1	1
1	1	1	1

0	1	1
0	0	1
0	5	1
1	1	1

0	1	1
0	0	1
0	4	1
0	1	1

Fig. 4.6 Configuration after removing doublets

neighbouring points. To find the direction of path the output of Part I is further processed. All the significant points of concavity are changed to 2 for convenience in further processing. Again a 3 x 3 matrix with (the significant point of concavity) 2 as centre is checked. Sum of the elements of each of two vertical and two horizontal rows which do not contain the point of concavity is calculated seperately. Path will grow from the point of concavity along the perpendicular from point of concavity towards the row having sum equal to 3. If there are two intersecting rows having sum equal to 3, path is grown from point of concavity across the box common to both the rows. When there are three rows each having sum equal to 3, then the path is allowed to grow across the box of the row whose rest two boxes are shared by the other two rows. All these cases are illustrated in figure 4.7.

In the process of finding segmentation path along the above found direction, the box next to the point of concavity along the path direction is replaced by a number 3. The process is repeated for all points of concavity. Now with this 3 as centre, its seven neighbourhood points are checked (the box in the path direction is now considered). If there is any number greater than 2, the path is terminated and segmentation

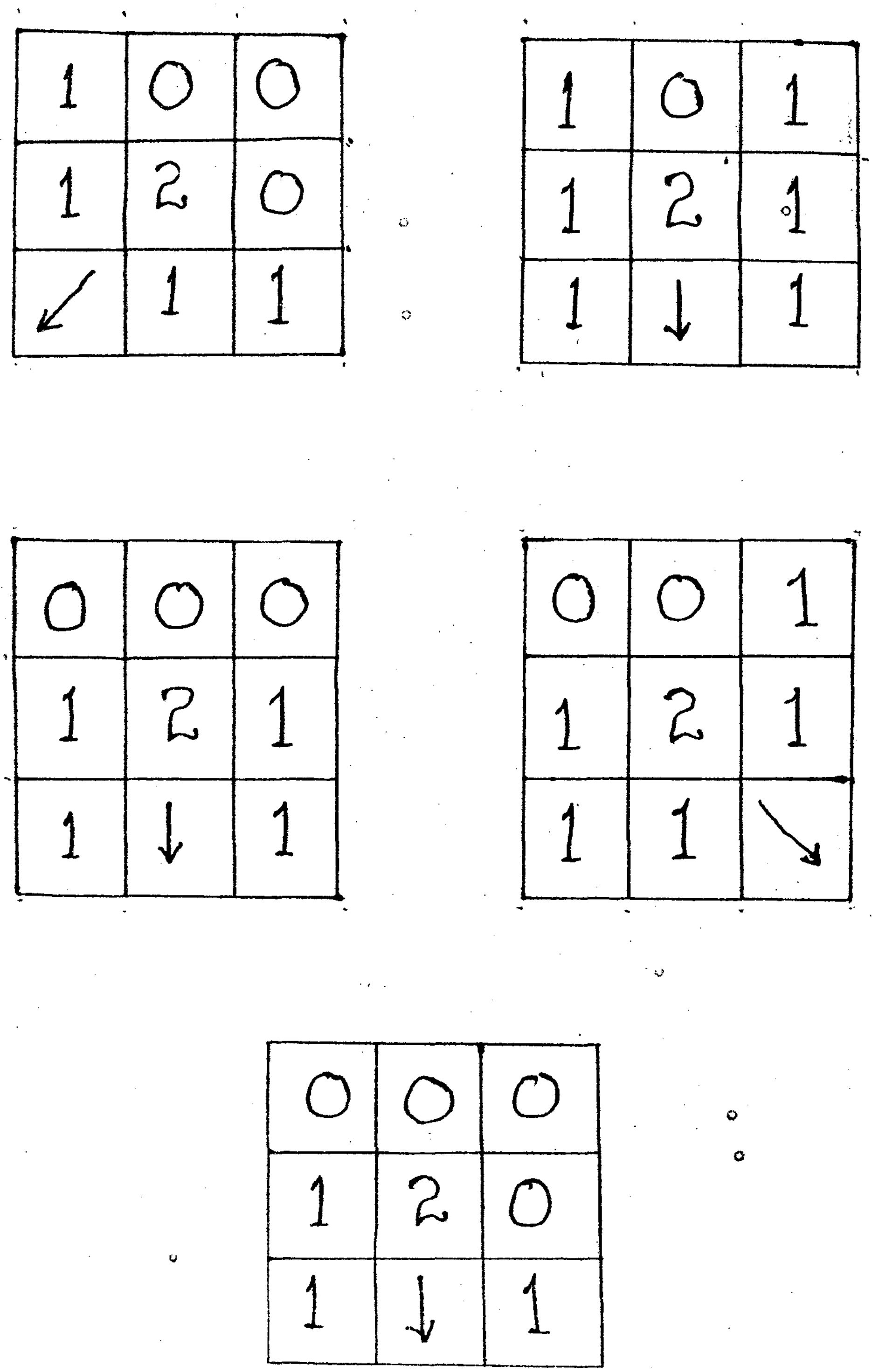


Fig.4.7 Direction of Paths

for this point of concavity is achieved.

If the path is not terminated, we continue growing the path in the same direction by placing number 4,5,......, n in successive steps. To achieve segmentation, the neighbourhood check is made after each step.

To illustrate the process, the graph plotter output of an object and its primary curvature chain, when traced from bottom left in anticlockwise direction is taken. This picture of muscle fibre, as given in Eccles's (12) paper, is magnified and drawn on a graph paper. The hand coded image is shown in fig. 4.8. The successive steps of concavity detection, extraction of significant concavity, path growing in the appropriate direction and segmentation, are examplified in fig. 4.9, 4.10, 4.11 and 4.12 respectively.

The success of segmentation depends upon the fineness of quantization i.e. capacity of the binary matrix to retain the characteristics of the picture. One of the main characteristics of a conjugate object being the sharp concavities at the boundary, the points of concavity should be preserved for obtaining successful results.

Although majority of the paths merge, successfully, some wrong segmentation paths may grow. This is due to

0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0
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0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
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0	0	0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0
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0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0
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0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0

Fig.4.8 Digitized Pattern

```
0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0
0 0 0 0 0 0 0 0 0 5 1 1 1 1 1 1 1 1 1 0
0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 6 1 1 1 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 1 1 0
0 0 0 0 0 0 0 0 0 5 1 1 1 7 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0
0 0 0 0 1 1 1 1 4 5 1 1 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
0 0 1 1 1 1 1 1 1 1 1 1 6 5 1 1 1 1 0 0
0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 0 5 6 7 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
0 0 1 1 1 1 1 1 1 6 4 0 0 0 1 0 0 0 0 0
0 0 1 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 0 0
0 0 1 1 1 1 1 1 1 1 5 5 1 1 1 1 0 0 0 0
0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0
1 1 1 1 1 1 1 1 1 7 1 1 1 1 1 0 0 0 0 0
1 1 1 1 1 1 1 1 1 5 1 1 1 1 0 0 0 0 0 0
1 1 1 1 1 1 1 1 0 0 1 1 1 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0
```

Fig.4.9 Detected point of Concavities

0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	6	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0
0	0	0	0	0	0	0	0	0	5	1	1	1	7	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	0	5	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	6	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	7	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	6	0	0	0	0	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	1	5	0	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	7	1	1	1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0

Fig.4.10 Significant Concavities after removing doublets

0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	1	1	3	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	3	1	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	1	1	1	0
0	0	0	0	0	0	1	1	1	1	3	1	1	1	1	0	0	1	1	0
0	0	0	0	0	0	0	0	0	2	1	1	3	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	0	2	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	3	1	1	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	2	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	2	3	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	3	1	1	1	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	2	0	0	0	0	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	1	2	0	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0

Fig.4.11 Direction of Segmentation paths

0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	1	1	3	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	1	1	1	1	4	5	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	4	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	3	1	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	1	1	1	0
0	0	0	0	0	0	1	1	1	1	3	1	1	1	1	0	0	1	1	0
0	0	0	0	0	0	0	0	0	2	1	4	3	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	0	2	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	3	1	1	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	4	4	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	2	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	2	3	4	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	4	1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	3	1	1	1	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	2	0	0	0	0	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	1	2	0	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0

Fig.4.12 Segmented Pattern

the presence of certain amount of noise which gives rise to false concavities. In such a case, the path instead of meeting any other path, will grow forever and meet the boundary of the picture. While checking the seven neighbours, if a zero element is encountered along the direction of the path, the path is declared false and is erased after segmentation is complete. This is illustrated in figure 4.13.

This method has also been applied to the data of overlapping TiO_2 blobs given by Arcelli (5). Successful segmentation is achieved as shown in fig. 4.14.

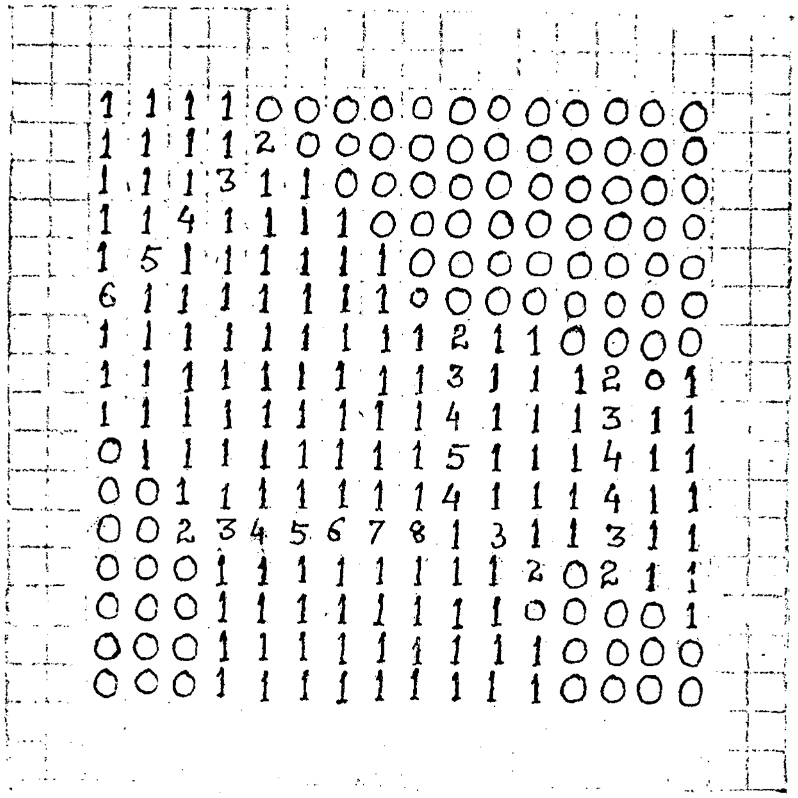

Fig. 4.13 False Segmentation Path

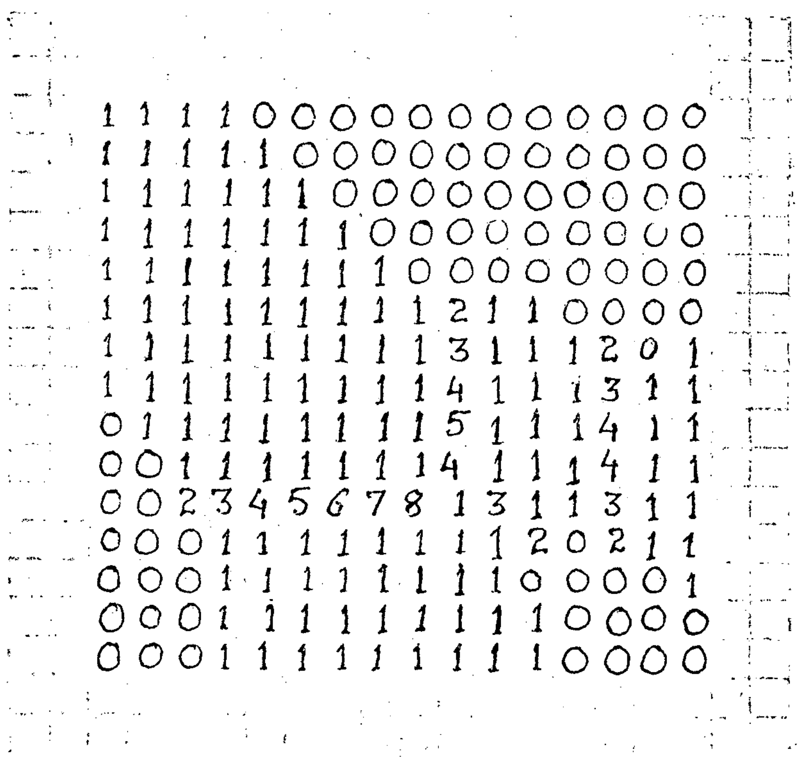

Fig. 4.14 Test figure of TiO_2

4.2. CONCLUSION

A method has been developed to single out objects which appear partially overlapped. It is based on the assumption that the concavities due to partial overlapping of the boundary of objects exist in the picture. Emphasis has been given on the detection of points of concavity, removal of insignificant point of concavity, finding the direction of segmentation path, and growing of path of segmentation along that direction towards the interior of the picture until segmentation is complete.

The program has been tested with many examples and segmentation has been found to agree with manual ones. One of the examples is conjugated objects as seen in transverse section of muscle fibres as illustrated by Eccles (67). Even when the specimen consists of discrete and seperate objects, it appears to be a single object due to their suxtaposition. The specimen taken consists of objects formed by two or more suxtaposed elements. The boundaries of these objects contain sharp concavity points. There is no overlapping of consistituent parts of objects.

Another example tested is that of overlapping convex blobs illustrated by Arcelli (20). These blobs are the TiO_2 powder particles which are to be singled out to determine their size which becomes necessary to control the size of the powder particles during the production of paint components.

Even though most of the results are correct, false segmentation paths are sometimes developed due to the presence of false concavity. This happens only because of presence of noise and can be removed by applying appropriate enhancement procedures for noise removal. With the coming up of newer and improved techniques of image sharpening this limitation can be hoped to be removed.

Automation in the acquisition and interpretation of data in microscopy has been a focus of biomedical research for almost a decade. Computer assisted processing of blood slides, computer system for systematically storing, analyzing and extracting information for scintograms in nuclear medicine, automatic radiological image enhancement systems etc. are already in use. In computerised tomography, which is nothing but the means of conversion of physical data to a digitized form, a computer built into the computerised tomography apparatus can measure the size, location and radiodensity of the pathology appearing on the computerised tomography scanning image. Automatic texture analysis of biomedical images like lung x-ray becomes often necessary for diagnosing the degree of severity of coal worker's pneumoconosis. However, in spite of many serious attempts, mechanical perception of microscopic fields with a reliability that would inspire routine application still eludes us.

We have simulated a parallel-computation system in serial access. The system can operate on each point independently. It can be improved extensively with the availability of parallel computation. Though our system can be applied to partially overlapped boundary of objects, the preliminary success encourages us to try to modify our method not only to improve the reconstruction of the shape and size of the singled objects but also to apply it to totally overlapping objects. This method then will give us means to differentiate the boundary of the nucleus and the cytoplasm of a cell. These techniques may also help in isolating malignant cells and thus likely to be of use in radiotherapy.

APPENDIX

IMPLEMENTATION OF SEGMENTATION

APPENDIX

The preprocessed pictorial data is entered on to the EC-1020 B system by a single hand-coding procedure. The preprocessing is done by converting the picture from continuous form to discrete form (digitisation). The data is compressed so as to conserve the storage space. Enhancement and restoration techniques are applied to improve the degraded, blurred, or noisy picture. However, the implementation described here is restricted to handling only the preprocessed pictures.

The routines of the segmentation program were developed in FORTRAN IV. The program consists of Part I and Part II. The outline sketches of the important subroutines of the two parts have been described in the subsequent sections.

Part I

Part I of the program finds out the points of concavity first. Then it selects the significant points of concavity by removing doubles. It has the following steps:

(a) Read A (I,J).

(b) If A (I,J) is equal to zero, analyse a 3x3 window with A (I,J) as centre.

(c) If the sum of all elements of the 3x3 window is greater than or equal to 4, then replace

A (I,J) by the sum. (These elements which are now greater than or equal to 4, are the points of concavity).

(d) If all A(I,J) = 0 are not scanned, go to (a).

(e) Put K = 4

(f) If A(I,J) = K, analyse the 3x3 window with A(I,J) as centre. If any other element is greater than or equal to K, put A (I,J) = 0.

(g) Increase K by one i.e. K=K+1 and go to (f).

(h) Continue until K=6.

(i) To remove doublets, repeat steps (f), (g) and (h) for all A(I,J) = K.

(j) Put 2 where the entry is greater than 1. (Now all elements having value 2 are significant points of concavity).

Then we process Part II.

<u>Part II</u>

Part II of the program consists of Scan II A, finding the direction of the path, and Scan II B, growing the path until segmentation is achieved.

<u>SCAN II A</u>

(a) Scan A (I,J) row-wise.

(b) If A(I,J) = 2 then consider the 3x3 matrix with this A (I,J) as centre.

(c) If A(I,J) is not equal to 2 then increment one element and go to (b).

(d) Take the sum of all elements in the rows and columns adjacent to A(I,J).

(e) If sum = 7 then check position of 0 and replace the element opposite to 0 by 3. Go to (h).

(f) If the sum of all elements of one column and one row is 3, check which column and which row is having the sum 3. Make the corresponding common element 3. Go to (h).

(g) If one column or one row his having the sum equal to 3 then find the middle element of that row or column and make it 3.

(h) Repeat the steps (a), (b), (c), (d) for all the elements of A(I,J). Go to Scan II B.

SCAN II B

(a) Scan A(I,J) row wise.
N = 3

(b) M = N+1
NN= N-1

(c) If A(I,J)= N then scan all the adjacent elements of A (I,J).

(d) If there is only one adjacent element which is equal to NN and no element is equal to N, then make the opposite of that element M. Continue till the row is finished.

(e) Increment the row by 1 and go to (b) till all the rows of matrix have been scanned.

(f) Increment N by 1 i.e. N=N+1 and go to (b).

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AND

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