AUCTIONS AND CORRUPTION: SOME RELATED ISSUES

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CERTIFICATE

This is to certify that dissertation entitled **"Auctions and Corruption : Some Related Issues"** submitted by me in partial fulfillment of the requirement of the degree of **Master of Philosophy** of **Jawaharlal Nehru University** is my original work and has not been previously submitted, in part or full, for the award of any degree of this University or any other University.

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We recommend that this dissertation be place before the examiners for evaluation.

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I stand responsible for any errors and omissions in this thesis.

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Chapter 1: Introduction

Auction

One of the principal questions in Economic Theory is the determination of the price of a good. The determination of a feasible price requires knowledge of market structure. Theory suggests different kinds of market structures like perfect competition, monopoly etc [1]. We will study a market structure, a sort of monopoly, but with competition among the buyers. This competition is due to the fact that seller has only one indivisible unit of good to sell. Buyers want to pay a minimum possible price to acquire the unique sellable object whereas the seller wants to put the good into the hands of a buyer who is willing to pay the most. Such a selling mechanism is called an auction [2]. Selling a rare antique or a wine which is more than hundred years old are examples of auction. Almost all the government purchases are done through auctions (or procurement).

One of the main aims of the theory of auction is to develop techniques so that the good lands up in the hands of the bidder who value the object most[3]. Any technique which fails to find the "buyer in interest" is called inefficient and efficiency is one of the main issues in the theory of auctions. Some of these techniques are well analyzed, for example a First Price auction or a Second Price auction. We will define some fundamental techniques covering these two in the later stage of this work. In auction theory these technique are

[1]As a basic reference for Market theory : Varian. H , "Macroeconomic Analysis" Third Edition

[2]The textbook reference for the basic theory; See Krishna, V.(2002) : "Auction Theory" , Academic Press .

[3]Such an auction is called an efficient auction

well known as Selling Mechanism. An auction mechanism is like a computer software. Once developed, one can find the output by fitting the inputs. In case of mechanism we need to fit the rules of auction and other information to find the outcomes of the auction (which basically includes a bidder with highest valuation and expected payment). In the next chapter we show how different auction mechanisms work.

Corruption

To initiate this entire study we need to understand the term "Corruption". A corruption is an act of acquiring private shares from other's property or from public property. A corruption makes some one better off at the cost of others. In many cases people practice corruption only because they are best placed to do so. We can modify it by saying corruption gives positive payoff to one individual free of cost. Corruption become blatant when attaining higher individual utility entails taxing of scarce resources. Corruption in many cases is the misuse of a position equipped with high social responsibility and trust for a dishonest gain. One common form of corruption is paying bribe for illegal provision of public services. Paying bribe is a form of corruption and can be viewed as a contract where one party pays other for an illegal privilege. The bribe receiver can make the contract entirely on his own terms. Hence bribery payments becomes forceful and one has to pay it. Consider an example of reckless drivers commonly found paying bribe to the traffic managers because the actual fine (fixed by law) is very high. It is a bribe to reduce the cost or price. There are some other forms of corruption (fraud or favoritism) but in many cases for the purpose of analysis we use bribe payments for the price or cost reduction.

Some Evidences

Corruption takes place frequently in government procurements and in many cases it is well documented. According to a report wall Street Journal explores that the winner firm *Hochfiet AG* of the tender for construction of airport in the Berlin had allegedly acquired application files of one of the rival firm *IVG* with the support from some government officials [4] .In 1996 Singapore government found a firm named *Siemen* paying bribe to get information about rivals' bid for a power station construction. The firm was banned for the five years and couldn't participate in any public procurement.

In many countries, defence department purchases arms and other equipment which accounts for a significant part of government expenditure. Because of the giant transaction, defense generally uses single-source (or non -competitive) method for procurement. Since the method is opaque, bribery contract is likely to be involved [5] .According to a report by Courtney (2002) [6] , US department of congressman estimated that a 50 percent of all bribe are paid for defence procurement. Another report by IMF(2000) on corruption and military purchase shows that procurements of defence equipments are 14 percent more costly due to corruption.

[4] Wall Street Journal, August 1999.(through some online source)

[5] "The extent of single sourcing and attendant corruption risk in defence procurement: a first look" : Regina Wilson, Dominic Scott and Mark Pyman . Presented at the conference "Public procurement", University of Nottingham, 19-20 June, 2006

[6] Courtney, C, Cockcroft, L and Murray, D, (2002): "Corruption in the Official Arms Trade." Transparency International (UK) Policy Research Paper 001,

Bofors scandal can be recalled as a major corruption in government procurement auctions in India. Many high ranked official and ministers were accused of receiving bribe from *Bofors AB* for a winning bid to supply $155mm$ howitzer.

Under the urban development plan *Delhi Development Authority* construct and sells apartments to the public. Department sells it using some lottery procedure after receiving application from those who are interested. An operation by Economic offence Department in 2008 discovered scams in the selling of these apartments. Many officials and private broker got caught. It is allegedly said that officials accepted false applications from private brokers.

The World Bank submitted a report in 2005 in which it is being mentioned that government procurement accounts for a substantial share of the world economy (12 to 15 percent of world GDP). The government procurements are highly corruption prone and manipulation is easy due to the lack of regulation. As per the estimates given in the report, the global volume of bribe in case of government procurement is around two hundred million US dollars per year.

Corruption and Auction

The above examples are taken to be the prima-facie evidence of corruption in auctions. We are here interested to give a theoretical understanding of corruption and its effects on the outcomes produced as suggested by the standard theory of auction [7]. Having an agent to conduct the process of

[7] In the next chapter we will discuss the standard model of auction.

auction is very practical. A seller may be a complex organization with many other objectives or it can be an individual with limited abilities. In both the cases the seller requires an expert who can act on behalf of him. The agent accepts the responsibility in lieu of some nominal wage but acts independently. With greater control on auction and the perfect scope of making illegal gains the agent betrays from the contract. Here corruption refers to the lack of honesty and cooperation of the agent. The agent can manipulate the prices and give privilege to any bidder who may not with the highest value. Or an agent can simply reduce the price i.e. he offers a lower price to the bidder who value the object most[8].

So an auction with an agent is a fragile structure from a seller's point of view. It results in lower than expected revenue for the seller and creates deadweight loss. In practical cases corruption may result in serious dangers. It may be the case that a winner is not the bidder with the highest value. He may win the procurement of a school building construction with the help of a corrupt agent. A poor construction may result and this may lead to deaths due to collapse of the school building.

Objectives

Given the importance of auction and prevalence of corruption the objective of this work is to develop a model of an auction in the presence of corruption. Our goal here is to understand the basic theory of auctions as well as the theory of auction under the presence of corruption. Using these we can compare the outcomes of the auction in two different scenarios.

[8] Lower price means lower than the willingness of highest bidder.

Corruption is an illegal act and there are laws to prevent it. But a law is effective only if some one is found guilty for the illegal activities. A law is expost solution, but the theory of economics is about exante contracts. Our objective here is to find some ways which allows the seller to get rid of corruption. We will develop a set of arguments in this regard. We are here keen to develop some technique to stop corruption.

Law enforcers can not observe corruption activities directly and demand evidences. A corruption is basically a measurement issue. Our motive is also to develop an idea of how to check the presence of corruption and the extent of spoils using the least possible information.

Above are the basic questions raised in this work. We will try to logically answer these questions. We will first give a brief review of literature in the next chapter i.e. in Chapter 2. In Chapter 3 we develop a basic auction model under the presence of corruption to arrive at the equilibrium bids, seller's revenue and spoils of corruption.We use the equilibrium values under these to formulate strategies that restricts corruption. Chapter 4 is a brief discussion of the econometrics literature on auction theory. In this chapter we discuss the econometric tools that could possibly be applied in coherence with our results in chapter 3 to identify the presence of corruption given limited data availability. Lastly we will provide some concluding remarks.

Chapter 2: Literature Survey

In the first chapter we have defined an auction. Auction by its meaning is very simple to be understood even by a layman. Now a days auction has become very useful and common practice in market mechanisms. U.S sells Treasury bills worth of billion dollars using a sealed bid auction. In India tea auctions are very popular. The most modern form of auction is "internet auctions", where bidder submit their bid online. There is huge international market where people buy latest products by bidding.

Though the history of auction is thousands of year old, it was only few decades ago when auction got its first theory in economics. It was all due to the seminal paper by Vickery (1961) which is supposed to be the first formal work in this direction. Starting from Vickery's paper and till today, auction theory has developed enormously. Along with the theoretical relevance its practical applications make it a separate branch of study.

The preliminary objective of this work is to analyze an auction affected by corruption. To give focus on the fundamental objective it is necessary to have a comprehensive idea about this subject. As an attempt towards this, we will first discuss the benchmark model of auction. Then we will discuss the idea of different kinds of auctions and the behavior of concerned participants. We next move to discuss the limitations of benchmark model. Here we will observe how the outcomes are changing with a little departure from standard theory. In the final section we discus the various aspects of price manipulations in an auction model. This section will provide us a basis to move forward in our further discussion.

Standard Auctions

We will begin our discussion by defining four common forms of auctions. In a situation where seller starts with a very high price and then lowers it until any bidder agree to pay, is called Dutch auction. An English auction is opposite to the Dutch auction. Here seller starts at a very low price and then gradually raises the price. Seller stop when all except one bidder get eliminated from the competition. Other than these two auctions we also have sealed bid auctions where bidder submit their bids to the auctioneer in a close envelope. A bidder who submits the highest bid and wins the object by paying his bid, it is called a First price Auction. But if the rule of auction is such that the winner pays the second highest bid, then it is called as Second price auction. Whatever be the payment rule, it must be announced by the seller before the auction starts. There are several other auction formats. We are interested to observe the behavior of the agents under the four auction formats mentioned above.[9]

We now discuss the benchmark model of auction theory which is also known as Symmetric Independent Private Value (SIPV) model [10]. This model is based on some crucial assumptions. A seller offers an object to n number of buyers. The maximum price that a bidder is ready to pay is the valuation of the bidder. Valuation of a bidder is his own private information and depends upon tastes and preferences. So a bidder does not know others' valuations.

[9]For a precise definition see Milgrom, P. and R.Weber(1982): "A theory of Auction and Competitive Bidding," *Econometrica*, Vol 50, 1089 – 1122.

[10]For detail discussion see Krishna, V.(2002): "Auction Theory," *Academic Press* or Menezes, F., Monteiro, P.K. (2005): "An Introduction to Auction Theory," *Oxford University Press.*

Even if a bidder comes to know about the valuation of other bidders it will not make any difference. The object has no resale value. Since all other bidders and seller does not know the valuation of any particular bidder, it is a random variable. The second assumption of the SIPV model is that valuations are independently distributed. Thirdly, the form of probability distribution is same for each bidder and it is known to all the agents. This is the assumption of symmetry. The fourth is the assumption of risk neutrality. All the bidders and the seller is risk neutral. An auction format is said to be efficient if it allocates the object to a bidder with the highest valuation. We primarily deal with the standard auctions in which a winner is the highest bidder. So any standard auction which satisfies above four assumptions is said to belong to the class of SIPV auctions. The four auction formats defined at the beginning satisfies the requirement of a SIPV model.

Realization of own valuation is same as saying realization of the type of a bidder. Knowing his own type a bidder chooses his strategy of bidding. A SIVP model is a static game of incomplete information [11] .We will explain how to achieve the Bayesian Nash equilibrium for each of the auction formats. There are two important conclusions about these four auction formats, which may be mentioned here.

Bidding strategy is equivalent in a Dutch auction and in a First Price Auction. A winner in a Dutch auction is one who chooses and pays the highest level of price. This is the way a winner and price are determined in a sealed bid First price auction. So a Dutch auction and a First Price auction are

[11] To see the definition of Game of incomplete information see Gibbons, R. (1992): "A premier of Game theory," *Pearson Education Limited.*

strategically equivalent [12].

In a Second Price auction a bidder's weakly dominant strategy is to bid his own valuation (later we will see the reason behind this). In an English auction it is rational for a bidder not to leave the game until price reaches to his own valuation. These two auctions are weakly satisfying the strategic equivalence, because here it is essential to know the own valuation [13].

One of the fundamental results of Vickery (1961)[14] is the bidding strategy of a bidder in a Second Price auction. It is weakly dominant strategy for a bidder to bid his own valuation. Bidding more than own valuation can never be profitable. Suppose a bidder bids his own valuation then he will make a profit equal to the difference between his valuation and the second highest bid if he wins. A lowering of bid may decrease the probability of win. Even if bidder wins by lowering his bid, his profit will not increase.

In case of First Price auction a winner has to pay his bid, so given other's bid a bidder faces a simple trade off. An increase in bid increases the probability of win and decreases the profit of a bidder. Equilibrium then must offset this imbalance. Any arbitrary bidder follows a bidding strategy which is increasing in his valuation. The strategy must be that of expected payoff maximizing. Overbidding can never be optimal because of negative payoffs

[12]Two games are strategically equivalent if given a strategy in one game there exist a strategy in the other game.

[13]The proof is given in Milgrom, P. and R.Weber(1982): "A theory of Auction and Competitive Bidding," *Econometrica*, Vol 50, 1089 – 1122

[14]Vickrey, W.(1961): "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, Vol 16, 8 – 37.

and a bidder with valuation equal to zero never submit a positive bid. Since players are symmetric it is obvious that the optimal bidding function (and not the bid) must be identical for all the competitors. Given that others are following the same bidding function, they will bid according to their own valuation. It is optimal for a bidder to put a bid which maximizes the expected payoff. In general a bidder always bids below his own valuation in a First Price auction.

In a Second Price auction each bidder bids his own valuation but actual payment is lower. In a First Price auction each bidder bids lower than own valuation, but actual payment is equal to the reported bid. Such kind of inverse relations is a trade off between choosing a First Price auction and a Second Price auction from a buyer's perspective. Given a seller is risk neutral this trades off makes him indifferent between these two auctions. This is because expected revenue is same across the two auctions. In theory it is well known as Revenue Equivalence Property [15].

Vickery generalizes this result for all four auction rules [16]. Riley and Samuelson (1981) analyze this result and add some more valuable results in the theory. They show that risk neutrality and independent and identical distribution of valuations is necessary for revenue equivalence.

In a Second Price auction the seller's expected revenue is the expected value of second highest valuation but in a First price auction all bidders bid below

[15] For a detailed proof of this result see Riley, J. and Samuelson, W. (1981): "Optimal Auctions," *American Economic Review*, Vol 71, 381 – 392.

[16] Vickery, W.(1962): "Auctions and Bidding Games," *Recent Advances in Game Theory*, Princeton University Press, Conference series 29, 15 – 27

their own valuation but the highest bidder has to pay his bid. We may argue that a average of these bids are the second highest valuation. But theoretically it depends upon the stochastic order and on the distribution of private valuation[17]. Riley and Samuelson (1981) made the result more robust by adding the concept of reservation prices. A reservation price is the lower limit of bids announced by the seller. Any bidder having valuation below reservation price would be eliminated from the game. Riley and Samuelson also demonstrated that a revenue maximizing seller always sets a reservation price equal to his own valuation. A reservation price results a scale up in revenue. The revenue equivalence property holds well with reservation price but at a higher level of revenue.

An alternative to reservation price is entry fee. In that case every bidder has to pay a nominal fee in order to participate. Suppose a bidder's valuation is r , which was earlier equal to the reservation price. An entry fee equivalent to the expected pay off with valuation r gives similar kind of outcomes.

Though the revenue equivalence can be established in many different ways but it's due to Myerson (1981)[18] who had put auction theory in a more general framework, gave a very robust argument in favor of this result. Myerson in his famous article "Optimal Auction Design" explain why and how this equivalence holds when we are considering a broad category of selling mechanisms.

[17]See the text book by Krishna, V (2002).

[18]Myerson, R .(1981): "Optimal Auction Design," Mathematics of Operation Research , Vol 6 , 58 – 73

According to Myerson a selling mechanism consists of three components. First component is the set of messages which in case of auction are the bids. Second component is an allocation rule, for example in standard auction a sellable object is allocated to the highest bidder. Finally a payment rule, for example in a Second Price auction winner has to pay the second highest bid. Every mechanism is also a game of incomplete information and an optimal strategy is the message that maximizes expected pay off (Optimal bid). Suppose a sender directly reports his estimated valuation to the seller, rather sending any message, it is called a direct mechanism. If in a direct mechanism a bidder reports the actual valuation as an optimum response then the mechanism is said to have a truthful equilibrium.

A truthful revelation is only feasible if it generates better pay off. A better pay off here implies that the utility from revealing true value must be as good as the utility derive from any other bid (No matter below or above the true valuation). If a direct mechanism satisfies this condition then it will become incentive compatible. The first fundamental result of Myerson is that if a direct mechanism is incentive compatible then the expected payments depends upon the allocation rule. A bidder's only concern is to maximize the expected utility, which depends upon the probability that the bidder will get the object. For example the payment rule in a Second Price auction is different from the payment rule in a First Price auction but the allocation rule is the same, as a result the expected revenue is equal.

The next result is in the context of Second price auction. An optimal mechanism is one that maximizes expected revenue subject to Incentive Compatibility and Individual rationality. Individual rationality means that a bidder will not loose his utility if he does not participate. Myerson proved that a Vickery or a Second Price Auction is regular [19] which satisfies symmetry, is an optimal mechanism.

Critics of SIVP model

In the previous section we observed a structure where a given number of risk neutral buyers with privately known value of an object compete with each other to get the object. Their values are identically and independently distributed. A seller always sells the object to the buyer who submits the highest bid and expects a same level of revenue irrespective of payment rule. If that were all, auction will be of no interest. The whole analysis is based on several assumptions. Relaxation of any of these assumptions lead to a change in bidding strategy and consequently the expected payment.

There may be a situation where bidders get some signal about other's valuations. Milgrom and Weber (1982)[20] developed a common value model where a bidder has some influence over other's valuation or in other words values are affiliated. This model is more general and SIPV model can be looked as a special case of it [21]. Here in this case too the Dutch price and the First price auction are strategically equivalent. But the equivalence does not hold well between an English auction and a Second Price auction. It is because due to the affiliation of values, bidders are not certain about their own valuation.

[19]A mechanism is said to be regular if given the probability distribution of bidder's valuation, the associated hazard function is increasing.

[20]Milgrom, P. and R.Weber(1982): "A theory of Auction and Competitive Bidding", *Econometrica*, Vol 50, 1089 – 1122.

[21]where (technically) the index of affiliation is equal to zero.

A confirmation of own valuation is necessary for the strategic equivalence between English auction and Second Price Auction.

The optimal strategies under affiliated model depends upon the expected value of joint distribution of self valuation and others' valuation. The interesting fact about the statistically dependent value model is that one can rank all the four formats of auctions in terms of expected revenue. English auction stood first followed by the Second Price auction and then the Dutch and First price auction shares the third position. Another important result is about efficiency. Efficiency in this case can be restored by adding one more assumption viz. the valuation must satisfy the single crossing property. A winner in affiliated value model is one who has the maximum information about valuations i.e. the highest signal. But efficiency implies allocation of object with the highest value and not with the highest signal. The interpretation is that a bidder should be less affected by other's valuation.

SIPV model can be criticized on many grounds. The benchmark model deals with a sale of single unit of a good. But there can be situations where more than one item is getting auctioned. These auctions are called multiple object auctions. Whenever a buyer bids on more than one item, efficiency may break down because the mechanisms will no longer be truth revealing. Restoration of the desirable properties require additional assumptions.

One of the crucial assumptions for most of the fundamental results of SIPV model is the bidder's attitude towards risk. Bidders are assumed to be risk neutral and they are interested in maximizing the net gain. We will now discuss some important issues regarding the outcomes of an auction when bidders are risk averse. Due to the risk aversion bidders have a Neumann Morgenstern [22] utility function and maximizes their expected utility of profits. Maskin and Riley (1984)[23] took a first serious attempt in this regard and showed that how bidding strategy and expected revenue will change in this case. Another interesting effort was taken by Steven Matthews (1987) [24]. Matthew claims that making assumptions about the preferences of buyers is the first step to view the auction theory from a buyer's perspective.

When the potential buyers are risk averse the revenue equivalence will no longer be valid. Buyers bidding behavior remain unchange if the bidding rule is Second Price auction. The reason is similar to risk neutrality case. But in case of First Price auction a risk averse bidder, if lowers his bid will get a smaller increment in utility level. But the lowering of bid may increase the probability of loss and given risk aversion a bidder never prefers uncertainty. If a bidder decreases the bid then loss in utility will be higher than risk neutral utility level. As a result bidder always submits a higher[25] bid. Risk averse condition in fact scale up the bidding under the First price auction. An overbidding due to risk aversion results in higher expected revenue for the seller, and results in failure of revenue equivalence. If buyers become extremely risk averse then even in a First Price auction buyers bide their own valuation to restore no profit- no loss condition. So an increase in degree of

[22]Risk aversion and expected utility theory was explored by these authors. A thorough treatment of this issue is given in Neumann J. V. and Morgenstern O (1944): "Theory of Games and Economic Behavior," *Princeton University Press.*

[23]Maskin, E. and Riley, J (1984): "Optimal Auctions with Risk Averse Buyers," *Econometrica*, Vol 52, 1453 – 1518

[24]Matthews, S. (1987): "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View", *Econometrica*, Vol 55 , 633 – 646.

[25]higher than standard bid when buyers are risk nuetral

risk aversion allows a buyer to maximize his revenue with a lower reservation price [26].

Above arguments ensure that a revenue non-equivalence implies a seller always prefer a First Price auction. It is now relevant to mention here that the price paid by a buyer in a Second Price auction is riskier than the price of the First Price auction and a buyer may not prefer Second Price auction [27]. But under the risk aversion payments are higher in a First Price auction and it may be that buyers prefer the Second Price auction. These two arguments are counteracting each other. Seller's preference of First price auction over Second Price requires information about bidder's utility function. However Matthew (1987) shows that for buyer's it depends on the Arrow - Pratt measure of risk aversion [28]. It is being proved that a risk averse bidder prefer First Price auction if his utility function follows Increasing Absolute Risk Aversion(IARA) and Second price if utility follows Decreasing Absolute Risk Aversion(DARA). However a buyer remains indifferent if his utility function reveals a Constant Absolute Risk Aversion (CARA).

As we know that in a First Price auction bidder bids more aggressively if they are risk averse. This extra amount is intuitively considered as a risk premium or and then the total bid will be the certainty equivalent. Suppose CARA holds then the certainty equivalent and so the total bid does

[26]The proof is given in Riley, J., AND W. Samuelson (1981): "Optimal Auctions," *American Economic Review*, Vol 71, 381 – 392.

[27]Milgrom, P. and R.Weber(1982): "A theory of Auction and Competitive Bidding", Ecometrica , Vol 50 . 1089-1122.

[28]For the basic definitions of Arrow -Pratt risk aversion see Varian, H. : "Microeconomic Analysis" Third Edition.

not depend upon wealth. So the aggressiveness in bidding behavior may completely normalized by CARA. Consequently the buyers become exante indifferent between First and Second price auction.

In case of risk averse buyers, though revenue equivalence does not hold, it still allocates the object efficiently. We now assume the asymmetry among the bidders. Asymmetry implies that at least one of the bidders chooses his valuation from a different distribution function. If so then the bidding strategy will depend upon the stochastic dominance of the player's probability distribution. A weaker bidder always bids aggressively in a First Price auction [29]. Revenue equivalence does not hold in an asymmetric bidder's case. It is completely ambiguous and difficult to speculate which form of auction yield a higher return. It completely depends upon the distribution function of each of the bidder. It is important to note that even with asymmetry among the bidders, it is optimal to bid their own valuation in a Second Price auction. Since the optimal strategy of a Second price auction does not depends on bidder's probability distribution [30], so a Second Price auction always allocates efficiently. Asymmetry causes non identical bidding strategies in a First Price auction. They choose different strategies satisfying other regularity conditions [31]. It is always possible that in a First price auction object may mislocate and go with lower valuation. Hence a First Price auction may not always allocate efficiently.

[29] Plum, M. (1992): "Characterization and Computation of Nash Equilibria for Auctions with Incomplete Information," International Journal of Game Theory,Vol 20, 393-418.

[30] Vickery, W. (1961): "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, Vol 16, 8 – 37.

[31] i.e. bids are increasing in valuation and a bidder with zero valuation never participates.

Auction and Price Manipulation

In the last two sections we critically observe the standard model of auctions. The SIPV and any of its extensions give us space to analyze the different kind of selling mechanisms. We get several generalized outcomes which are useful for further research.

This section gives us a basis to discuss the fundamental aim of this whole work, which we may put in form of two questions. First, what kind of cheating a standard auction may face and secondly, what would be the outcome of an auction under the fear of cheating. By cheating we simply mean an attempt to manipulate prices. We shall discuss the different kinds of price manipulations that may exist in an auction model. We firstly observe the effect of cartel among the bidders.

The analysis of collusion among the bidders in a Second Price and English auction was first initiated by Graham and Marshall (1987)[32]. Later McAfee and McMillan (1992)[33] established it for general auction formats. Collusion implies a ring formation by the subset of potential buyers to reduce the prices. The purpose of collusion is to reduce prices by reducing competition. The modus operandi of ring is to collect a subset of bidders and induce them to play a cooperative game by revealing their private information. The first step of collusion is to select a particular member who will submit a meaningful bid, the rest submit a bid equal to zero or below reservation price. In a way

[32]Graham, D., and R. Marshall (1987): "Collusive Behavior at Single-Object Second-Price and English Auctions," Journal of Political Economy, Vol : 95, 1217-1239.

[33]McAfee, P., and J. McMillan (1992): "Bidding Rings," *American Economic Review*, Vol 82, 579 – 599.

a ring pretend to be a single bidder. If a ring manage to win the object, it implies that the bid maker from the ring must be one with highest valuation among all the bidders (inside or outside the ring). So if a ring master [34] is one who has the highest valuation of the object then the ring allocates the object efficiently. Issue is how to share the spoils of cartel.

Consider a Second Price Auction. Even in the presence of a ring it is weakly dominant strategy for a bidder to submit own valuation. Because once a ring is formed and chooses its ring master the new game is same but with smaller number of bidders. Truth revelation is independent of the number of bidders. If the ring wins the object, ring master has to pay an amount equal to the highest bid among those who are outside of the ring. It may be possible that the actual bidder with second highest value has entered in the ring. So with a positive probability the expected payment of the winner will be lower. The gap between non-collusive expected payment and ring's expected payment is equal to the total expected gain of the ring.

However for the outside bidders the expected payment and probability of win does not get affected due to cartel. The seller is the bearer of entire loss. Suppose in a situation when all bidders are members of a ring then seller's loss will further increase and he will only get the reservation price. So a ring with higher number of bidders is more profitable. A seller may strategically oppose the cartel by setting a very high level of reservation price. A choice of proper reservation price may eliminate the deficits of the seller (if not completely, up to a certain extent). Graham and Marshall (1987) argued

[34]The term "ring-master" indicates the member of a ring who has right to make a meaningful bid. Later we will also see that how to choose this ring master.

that sellers can respond against collusion by choosing an optimal reservation price. In this regard they raise three points. Reservation price that maximizes seller's expected revenue increases as the size of cartel increases. But the increase in ring's size decreases the expected payoff of ring members. The third argument is that corresponding to any cartel size there exist a reservation price that maximizes seller's expected revenue. These three results together generates a situation of bilateral monopoly where an all inclusive cartel and with its corresponding reservation price is the Nash equilibrium.

The efficiency of a cartel requires a truthful revelation of valuation of ring members. In case of Second Price auction this requirement holds naturally i.e. a Second price auction is self enforcing. The cartel agreement in a First Price auction is not self enforcing. A formation of ring creates asymmetry between ring members and those who are outside of ring. Asymmetry creates inefficiency in first price auction. In a profit making cartel, a ring master submits a bid lower than the lowest value of the ring. Ring wins if the bid is higher than the highest bid outside the ring. Due to the asymmetry, probability of loss is higher and a cartel may not be gainful. A cartel may not be stable because it provides incentive to deviate. Consider a collusion of all potential bidders. In that case the ring always submits a bid equal to reservation price. So any arbitrary ring member whose valuation is greater than reservation price, have an incentive to deviate. He may submit a meaningful bid (higher than reservation price) and get the object, resulting higher payoff for him. This shows how collusion contains the basis of its own destruction.

To ensure efficiency a cartel requires a mechanism within the cartel. The mechanism should optimally choose the ring master and a rule of dividing spoils. Graham and Marshall (1987) define a devise for optimal selection of the ring master. It is known as pre-auction knockout (PAKT). In order to choose who will be the ring master, cartel can conduct a pseudo auction within the ring. Bids in this case are the transfer payments. Each bidder is required to transfer a lump sum to the ring which should ideally be the difference between cartel and non cartel prices. In a Second Price auction players reveal truly and it is easier to determine the exact transfer payment. But in case of First Price auction each member has to submit a bid for transfer payment. A winner will be the one who promise the highest level of transfer[35]. Suppose a cartel is incentive compatible then it allocates the object to the member with highest valuation in the ring i.e. to the ring master. Probability that the ring master will win the object is independent of cartel formation. So a mechanism with efficient cartel has a same allocation rule as it would be in the presence of any cartel. Following Myerson (1981) [36] if allocation rules are same in the two different mechanisms then expected payment must be the same (Revenue Equivalence) or it should differ up to an additive constant. The expected payment of the ring master would be split in two parts. One part goes to the seller and second is the transfer payment. The mechanism must guarantee that every bidder inside the ring will have a positive transfer payment from highest bidder over the cost of seller. In a very recent work by Pavlov (2008) [37], the author explains that a collusion agreement is a sub mechanism of original selling mechanism. This

[35]This rule is technically sophisticated but fragile because it does not ensures the stability of cartel

[36]Myerson, R .(1981) : "Optimal Auction Design," Mathematics of Operation Research , Vol 6 , 58-73

[37]Pavlov, G. (2008): "Auction Design in the presence of collusion" *Theoretical Economics*, Vol 3, 383 – 429.

study formally argue that Myerson's mechanism fails to be collusion proof and a cartel turn out to be interim efficient. However because of the possible asymmetry among cartel members it is also possible to design an auction which generates same level of revenue to the seller.

One important point about the cartel is it's instability. A cartel attracts other incumbents through additional profits. An increase in size of cartel reduces the benefits and provides incentive to cheat by existing members (The example of all inclusive collusion in case of First Price auction.). Robinson (1985) [38] argued that a cartel mechanism is incentive compatible if all the members share the same information about the member. A Second Price auction with a bidding ring is always stable but not in case of the First Price auction. Robinson's results are robust and hold even for affiliated value model.

In the previous chapter we have discussed that a seller may appoint a middle man who acts independently on behalf of the seller and conducts the process of auction. For example, in India Railways offer tenders for many of its construction works. Procuring railway tender is a large scale industry. Railway itself is a body and the auctions of railway contracts are done by the railway officials. Here the officers are the auctioneer or the middle man who conduct all the auctions. Railway is the seller and has no practical role in the process of auctioning its tender.

So we are assuming a separate entity between seller of the object and the auctioneer. Since auctioneer has superior command on the auction [39], he

[38]Robinson, Marc S. (1985): "Collusion and the Choice of Auction", *The RAND Journal of Economics*, Vol. 16, 141 – 145

[39]Auctioneer has a partial control because the auction format and the reservation price

may manipulate the prices in exchange of a bribe from the buyers. Such an auctioneer is called a corrupt auctioneer. Having an auctioneer must add extra cost on seller in terms of fee paid to the auctioneer (which for many theoretical purposes can be normalized to zero) but the auctioneer always have scope and incentive to affect prices.Clearly there is a mismatch between the objective of the seller and the auctioneer. The history of auction theory, developed in the last two decades, has number of theoretical and empirical work in this direction [40].

A corrupt auctioneer approaches the bidders and offers information about the other bidders in lieu of bribe. A bribery auction results in loss of expected revenue for the seller and gives incentive to the bidders. The object of corruption is in many cases for making supernormal profits but in other cases is due to favoritism [41] . We fill first follow the model of Burguet and Perry (2002) [42] where they use a simple example of two players in a procurement model and analyze the bidding behavior when one of the players gets extra privilege from the auctioneer.

is decided by seller it self.

[40]Laffont, J.J. , Tirole, J (1991): "Auction Design and Favoritism," *International Journal of Industrial Organization*, Vol 9, 9 – 42.

[41]when the auctioneer illegally support a particular bidder may be on a personal ground or to make a long term briery contract.

[42]Burguet, R. and Perry, M.K (2002): "Bribery and Favoritism by Auctioneer in Sealed Bid Auction." Mimeo

The model described by Burguet and Perry (2002) is a procurement [43] model where there are two suppliers. One of them is dishonest in a sense that he agreed to go for the bribery contract offered by the auctioneer to acquire information about the other bidder. The other one is an honest supplier. Bidders are asymmetric in terms of their cost of production but costs are independently distributed. The bidders' different behavior towards corruption further increases the asymmetry. A winner is one who submits the lowest bid. Suppose the dishonest bidder submits the lowest bid. A bribe payment by the dishonest bidder allows revising his bid up to honest bid level. So the dishonest seller receives a higher price. However favoritism may occur even if the dishonest bid is not minimum. The only requirement is that the cost of dishonest supplier should be smaller than the honest bid. So the auctioneer and the dishonest supplier can have the surplus to distribute.

Asymmetry makes the bidding strategies nonidentical in a First Price Auction (Procurement). Bribery results in more asymmetry. A dishonest bidder can either win by submitting a lower bid or by agreeing bribery contract whereas an honest bidder can only win by bidding against cost. So the optimal strategy of the honest bidder is independent of the proportion of bribe and other player's bid. However the optimal bid of a dishonest bidder is decreasing with respect to the proportion of bribe. Any aggressiveness of an honest bidder while bidding is only due to cost asymmetries. The dishonest

[43]Procurement is vertically opposite to an auction. Here bidders are the suppliers and they compete each other for a contract. The buyers receives bid and accept the supply from the lowest bidder. For a theoretical treatment of this issue see Dastidar, K.G. (2006): "Auction with Endogenous Quantity Revisited," *Contemporary Issues and Ideas in Social Sciences.* The scope of this paper is much wider and here we are only referring the introductory part.

bidder bids less aggressively.

Asymmetry in a First Price Auction generates allocation indeterminacy. A bidder who bids more aggressively may get the object if his valuation (or in this case cost) is lower (higher). With bribery, probability of inefficient allocation is much higher. So in a First Price procurement with a middle man, it is costly for a a low cost incumbent to secure win.

However Bureget and Che (2004)[44] introduces a scoring rule to mitigate the harmful effects of corruption. Their model is more general since they introduce quality factor. In this case a buyer allocates weight on the combination of price and quality quoted by a particular supplier. In this model with auctioneer a no corruption equilibrium is possible because buyers are more conscious in selection of supplier. We are limiting our discussion only to the price analysis. Adding quality of the object require more comprehensive study of literature [45].

Menzes and Monteiro (2006)[46] developed a general first price Auction model with all the assumption of SIPV model. The only addition is the auctioneer and his who may offer bribe to the highest bidder. This model assumes two different cases of bribe. First is the proportional bribe where auctioneer and highest bidder divide the spoils according to a predetermined ratio. Second

[44]Burguet, R., Che,Y.-K., (2004): "Competitive procurement with corruption," *Rand Journal of Economics*, Vol 35, 50 – 68.

[45]Cripps, M. , Norman, I. (1994): "The design of auctions and tenders with quality thresholds: the symmetric case," *The Economic Journal*, Vol 104, 316 – 326.

[46]Flavio, M. Menezes and Paulo, K Monteiro (2006): "Corruption and Auction," *Journal of Mathematical Economics*,vol 42 , 97 – 108.

is the fixed bribe case. In practical cases the method of bribe payment is culturally determined.

One of the important outcome of the current model is that the efficiency does not get destructed due to corruption. The model assumes symmetry among the bidders and the bribery contract reduces the price to the highest bidder and there is no favoritism. Whosoever be the highest bidder, gets get the object at lower cost by paying the bribe. Bribery affects each bidder's equilibrium bidding strategy in a homogeneous manner. The model assumes that the seller is like a sleeping partner and he does not monitor the auctioneers activities. It may be that the cost of verifying auctioneer's activities are higher than any probable loss in revenue.

If an auctioneer proposes a proportional bribe then bidder bid is uniformly higher than their bid in SIPV model. It is always weakly dominant strategy for a bidder to accept the corruption agreement. The model allocates the object efficiently but a part of the revenue is transferred to the auctioneer. Like previous models of price manipulation here also the seller bear all the burden of corruption.

Now we move to the fixed bribe case. The current model shows that multiple equilibria are possible if auctioneer charges a fixed amount of bribe. Menezes and Monteiro demonstrated a heuristic derivation of equilibrium but they also illustrate an example where it was proved that a feasible demand of fixed bribe may cause a less damages for the seller. Unlike the proportional bribery case where any proportion is acceptable by the bidders, there is some restriction on the upper limit of bribe in this case. A fixed bribe, if it exceeds the no corruption level of expected revenue, a buyer is always denied to pay the bribe and therefore bids according to the SIPV model.

One interesting outcome of this model is that a Second Price Auction is corruption proof. It is meaningless for the highest bidder to buy information about second highest bid. Seller provide this information free of cost. So an auctioneer has no scope for practising corruption. Question is then why a bidder chooses a First Price mechanism. One possibility is that a truth revelation may fail in a second Price auction if third party exploits the private information [47]. We have already mentioned that a seller chooses a First price auction when bidders are risk averse as they bid aggressively. Bidding behavior in this model is similar and a seller's actual realization of price may be higher than what he would get in the SIPV model. This argument require expost analysis of the outcomes which we will discuss in the next chapter.

The current model provides many interesting results and in the later stages of our studies the Menezes - Monteiro model will be use as a benchmark model to initiate the other aspects of this study.[48].

A very recent study by Koc and Neilson (2008)[49] argues that bribery may be interim in nature. They assume that a corrupt auctioneer asks for a fixed bribe to each of the bidders and sell information about the highest bid. A

[47]Engelbrecht-Ws, R., Kahn, C.M., (1991): " Protecting the winner: Second-price versus oral auctions," *Economics Letters*, Vol 35 , 243 – 248

[48]We use the assumption of the model to analyze some more aspects including the principal - agent relation ship between seller and auctioneer.

[49]Koc, A.S. and Neilson, W.S. (2008) : "Interim bribery in auctions", *Economics Letters*, Vol 99 , 238 – 241

buyer may choose whether to pay a bribe or not. Suppose the highest bidder paid the bribe, he can revise his bid like in the previous models of corruption. But before exploring the other results of this model we need to make some comments. Firstly, the type of bribery agreement assumed here may not hold because bidders are risk neutral and never interested in buying a fragile insurance. Secondly, suppose we can divide buyers in two groups, one who accept to pay the bribe and others who are not. So a natural asymmetry would be crated and that results in separate bidding strategy for the two groups. In a way this model tries to relate the previous two models [50]. So even when there is no favoritism we may have different bidding functions. Koc and Neilson shows that all who are ready to pay the bribe bids their own valuation at optimum and those who are not ready to pay bid according to standard First price auction. In this case a bidder follows a cut off rule to choose whether he should go for bribery agreement or not. Similar to the previous model allocation is efficient and seller suffers the losses due to corruption.

Concluding Remarks

We have discussed the basic outlines of auction theory. The entire story is divided into two parts. The first part is the overview of fundamental concepts of auction models, provides us the basis of any research in this area. Any analysis, theoretical or empirical can not be done without knowing the outcomes of benchmark model. The second part is the review of some of the recent works in this field. We have broadly discussed two sort of price manipulation. Both types of corruption practices are strongly supported by

[50]model of Burguet and Perry and the other model by Menezes and Monteiro.

empirical evidences [51]. It is evident that the fundamental theory (or any of its extension) focuses on bidding behavior, efficiency in allocation and revenue of the seller. Vickery auction is considered as a scale to measure the outcomes of any other form of auction model. Even in models of corruption or collusion our interest is similar with basic theory. In addition to that we are also interested in knowing the spoils of corruption.

One interesting fact that we have observed invariably in corruption literature is that a seller suffers all the losses. It is by and large due to the limited role of seller or his passive behavior. We also observe how a seller can strategically respond against corrupt behavior of auctioneer.

Is there any way where the seller can take some action to safe guard his revenue from the bribery contract between auctioneer and bidders? There may be a number of ways to deal with this. An auctioneer makes profit by decreasing the price of object because at the first instance he has scope to do that. But another reason is the absence of a well defined contractual agreement between seller and an auctioneer. An auction with auctioneer is always corruption prone. Achieving SIPV level of expected revenue may not be possible and so one of the best strategies is to minimize the damages. We shall deal this problem following Menezes and Monteiro (2006) where a seller actively responds against corruption.

[51] We will review some of the empirical studies related to auction and corruption.

Chapter 3: First Price Auction With Corruption

Introduction

The central idea of this chapter is to discuss various aspect of a First Price Auction model affected by corruption. This study is basically motivated by the auction model with corruption described by Menezes and Monteiro (2006) [52]. Unlike the standard theory of auction, in this case there are three parties, a seller, a set of bidders and an auctioneer or a middle man who conduct the process of auction. The auctioneer is supposedly the corrupt party. The highest bidder can reduce his bid up to the second highest level with the help of auctioneer and they can share the gains. The outcomes of a First Price Auction are being analyzed under the presence of a corrupt auctioneer. The focus of this study to is to do a comparative analysis of outcomes with the SIPV model.

It is being shown that the presence of corruption adversely affects the seller's profit as well as it makes buyers bidding aggressively. Most interestingly, the winner is a bidder whose private valuation is highest and hence the auction allocates the object efficiently. A lack of well defined principal-agent relationship between the seller and the auctioneer is the basis of such kind of corruption.

The objective here is to find out what would be the necessary action a seller can take to stop such kind of price manipulation. The presence of a middleman or auctioneer is natural and a First Price Auction gives scope to

[52]Flavio, M. Menezes and Paulo, K Monteiro (2006) : *Corruption and Auction* , Journal of Mathematical Economics Vol 42 , 97-108

practice corruption. We analyze some actions that a seller can possibly take to safeguard his revenue from corruption. At the first place, a seller may offer a part of the revenue as an incentive to stop corruption. An incentive scheme may not be effective because of fragile enforcement mechanism. A seller may face further loss. Secondly we introduce a penalty scheme where a seller keeps eyes on auctioneer's movement. This method is a little complex and may have difficulty in implementation but has been found to work well.

Comparing the effectiveness of two schemes we found that a corruption practice is successful because it generates higher utility level for the winner. We will observe that a method which affects bidder's decision is more effective to stop corruption. This suggested method works without changing the auction format. We have added expost arguments related to the outcome of the auction. We postulate that the Second Price auction is an immune to the corruption. In the First price Auction, the gap between highest and second highest bids is the crucial factor for the derived outcomes.

We will study the above analysis in the third section of this chapter. In the first section we simply postulate the model of auction with corrupt auctioneer and derive the equilibrium bid. The second section is the extension of first where we add reservation price.

Basic Model

In this section we set up the First price Auction model under the presence of corruption described by Menezes and Montiero (2006). We will show that the existence of a monotonically increasing bid function affects the optimal response of all participants but their position as a winner or looser is same as it was in the SIPV model. We assume the the total gain of corruption divided between the auctioneer and the winner at a given proportion.

Suppose there are n risk neutral bidders with a valuation $v_i \in [0, 1]$ for the ith bidder. Each bidder is assumed to be symmetric and their valuation follow a distribution function $F(.)$ with corresponding density function $f(.)$. Let $Y = max_{i \neq 1} v_i$, is the maximum valuation of all other bidders except the first bidder. Suppose $G(.)$ is the distribution function of Y with corresponding density function $g(.)$. Clearly $G(x) = F(x)^{n-1}$.

A bribe function is a mapping from a set $X \subseteqq$ of R^2 to R_+. Where X contains all possible pair of highest and second highest bids. So the bribe function is the function of difference between first and second element of any arbitrary tuple of X.

We now move to find out the symmetric increasing equilibrium bid function. Let $b(.)$ is the required symmetric increasing equilibrium bid function. Let v be the valuation of highest bidder and y is the valuation of second highest bidder. Let auctioneer offer a bribe to the bidder with valuation v to revise his bid to the second highest level and then divide the gain in a $\sigma \in [0, 1]$. proportion. We assume that the value of σ is fixed by auctioneer though there may be an interim bargaining between the highest bidder and the auctioneer.

If a bidder accept auctioneer's offer, he has to pay an amount equal to $b(y) + \sigma(b(v) - b(y))$. By refusing the offer of bribe he simply pays $b(v)$. Note that if he refuses the bribe then his payment $b(v) = B(v)$, where $B(v)$ is the equilibrium bid for the SIPV model and it is given by

$$B(v) = v - \frac{\int_v^0 G(y)dy}{G(v)} \text{ ....}(A)$$

Clearly as long as $\sigma < 1$, corruption is always profitable and hence acceptable for the bidders and the expected payment is given by

$$\int_0^v [b(y) + \sigma(b(v) - b(y))]g(y)dy$$

Here corruption makes object cheaper to the bidder with highest valuation. Bidders internalize the effect of corruption to respond optimally and to extract the extra gain. However a bidder who was actually losing the game in SIPV model, will never win even in the presence of corruption The auction outcomes are efficient and allocates the good to the bidder who values it most. By choosing the bribery contract a bidder will either be exante better off or no worse off. So the gross expected payment must be equal to

$$\int_0^v g(y)dy$$

Since allocation is efficient under corruption, the revenue equivalence and consequently the following equality holds good. [53]

[53] For revenue equivalence see Riley, J. and Samuelson, J (1981): "Optimal Auctions,"

$\int_0^v [b(y) + \sigma(b(v) - b(y))]g(y)dy = \int_0^v g(y)dy$

$\Rightarrow (1-\sigma)\int_0^v b(y)g(y)dy + \sigma b(v)\int_0^v g(y)dy = \int_0^v yg(y)dy$

$\Rightarrow (1-\sigma)\int_0^v b(y)g(y)dy + \sigma b(v)G(v) = [y\int(y)dy]_0^v - \int_0^v G(y)$

$\Rightarrow (1-\sigma)\int_0^v b(y)g(y)dy + \sigma b(v)G(v) = vG(v) - \int_0^v G(y)$

Differentiating both the side w.r.t v , we get

$\Rightarrow (1-\sigma)b(v)G(v) + \sigma b(v)G(v) + \sigma b'(v)G(v) = G(v) + vg(v) - G(v)$

$\Rightarrow \sigma b'(v)G(v) + b(v)g(v) = vg(v)$

So above is the first order differential equation , we will now solve it to get an expression for equilibrium bid function.

Above differential equation can be written as

$\Rightarrow b'(v) + \frac{g(v)}{\sigma G(v)}b(v) = \frac{vg(v)}{\sigma G(v)}$

clearly, the integrating factor is $G(v)^{\frac{1}{\sigma}}$ and we can further simplify it as

$\Rightarrow G(v)^{\frac{1}{\sigma}}(b'(v) - 1) + (\frac{g(v)}{\sigma G(v)}G(v)^{\frac{1}{\sigma}})(b(v) - v) = -G(v)^{\frac{1}{\sigma}}$

American Economic Review. vol 71, 381-392.

$$\Rightarrow \frac{d}{dv}[G(v)^{\frac{1}{\sigma}}(b(v)-v)] = -G(v)^{\frac{1}{\sigma}}$$

$$\Rightarrow b(v) = v - \frac{\int_0^v G(y)^{\frac{1}{\sigma}}dy}{G(v)^{\frac{1}{\sigma}}} \quad ....(B)$$

The function $b(v)$ given by (B) is the symmetric increasing optimal bid function under the presence of corrupt auctioneer. The derivation of equilibrium bid is not similar as it is in the case of SIPV model. In the standard model the derivation is based on intuition . Here we are not worried about how to find the winner or a bidder with highest valuation. Any bidder who couldn't win in the no-corruption case, can not even win with corruption. The allocation is still efficient and the derivation uses the revenue equivalence. The selling mechanism is optimal according to Mayerson's definition which says that revenue equivalence require only the similar allocation rule [54] , it does depend upon the payment rule. Here revenue is equivalent to standard auction but from payment perspective. It has now two components. One is the seller's part and the other is the bribe. In a sense a bribe is a transfer of payment from seller to auctioneer. Any loss due to corruption is completely carried by the seller.

Though the revenue equivalence holds, the seller gets a revenue equal to the expected value of the bid made by the bidder with second highest valuation. A seller receipts the first component of $E[b(X_2)+\sigma(b(X_1)-b(X_2))]$ (Where X_1 and X_2 are the highest and the second highest valuations respectively), i.e. $E(b(X_2))$. Due to the corruption with no control over auctioneer's action, a seller predicts a lower revenue [55].

[54]Mayerson, R .(1981) : "Optimal Auction Design," Mathematics of Operation Research , Vol 6 , 58-73

[55]The internal mechanism of corruption over the bidding behavior does not necessarily

Another important implication of the bid function defined by (B) is that its form is analogous to the bid function for the case of risk averse bidders with the utility function given by $u(z) = z^{-\sigma}$ where $\sigma \in [0, 1]$. So the utility function follows constant relative risk aversion. Bidder of this kind bids uniformly more than a risk neutral bidders. The extra payment is supposed to be the risk premium. There is a similar scale up in bid due to the bribery agreement and hence

$\forall v \in [0, 1]$ $b(v) > B(v)$ whenever $\sigma \in [0, 1)$

In this case the reason behind over bidding is quite different. Here bidders are risk neutral and the offer of auctioneer make them paying aggressively. For a bidder accepting bribery agreement weakly dominates refusing. Bidders expect higher return because a bidder if he wins has to pay less than his willingness. The attraction of extra gain make them pay extra.

Suppose σ tends to zero, that implies bidder bids his own valuation and gets the entire gain of corruption. Whereas if σ is equal to one the the entire gain goes to the auctioneer. So whenever σ lies between zero and one, a buyer speculates higher gain. Probability of win increases with bid, a higher bid decreases the utility in same proportion due to risk neutrality. If we add any possible share of bid then the total expected return goes up.[56]

It is an important question here to discuss the that what should be the value

result a lower revenue for seller. We will analyze this issue later in the same chapter.

[56]By gain we mean the spoil due to corruption

of σ. There must be some interim agrement between bidder and seller to choose a suitable value. An auctioneer always try choose $\sigma = 1$, to extract the entire gain due to corruption. A bidder may not have direct control over the value of σ, but there is an indirect control through bid function. A lower value of σ say zero on the one hand, generates the maximum gap between the first and second highest bid. But it left minimum (zero) share of bribe for the auctioneer. So the auctioneer receives maximum expected bribe when $\sigma = 1$. Consider the following equality

$$E[b(X_2) + \sigma(b(X_1) - b(X_2))] = E(X_2)$$

The first term of this equality is the expected revenue of the seller and the second term is expected bribe of the auctioneer, Due to the revenue equivalence this sum up to the expected revenue of the SIPV model. All the terms in the left hand said of the above equality are the function of σ. But the right hand side term is independent of σ. We can write the expected bribe function

$$\tilde{B}(\sigma) = E(X_2) - E(b(X_2))$$

$$\frac{d}{d\sigma}\tilde{B}(\sigma) = -\frac{d}{d\sigma}E(b(X_2)) \geq 0$$

Since $\frac{d}{d\sigma}b(X_2) \leq 0$ and $E(b(X_2))$ is the sign preserving function of σ. So the expected bribe function $\tilde{B}(\sigma)$ is monotonically increasing in σ, therefore it attains it maximum at $\sigma = 1$.

We can view above results in the light of an example where the bidders valuation follows uniform distribution within the support $[0, 1]$.

$B(v) = \frac{n-1}{n} v$

and the expected revenue of the seller is $R = \frac{n-1}{n+1}$

Here $F(v) = v$ so we can apply (B) to derive equilibrium bid with corruption for this particular example.

$b(v) = v - \frac{1}{v^{\frac{n-1}{\frac{1}{\sigma}}}} \int_0^v y^{\frac{n-1}{\frac{1}{\sigma}}} dy$

and by simplifying above we have $b(v) = v(\frac{n-1}{n-1+\sigma})$

The first order statistics [57] of n valuation following same distribution is given by

$F_1(X) = F(X)^n$

and second order statistics is given by

$F_2(X) = F(X)^n + nF(X)^n - 1[1 - F(X)]$

we use the density function of this two to calculate the expected value of highest and second highest bids.

$E(b(X_1)) = \int_0^1 b(X_1) F_1'(X_1) dX_1$

[57] See Appendix C of Krishna, Vijay : Auction Theory , Academic Press , 2002

and by putting the respective values we get

$E(b(X_1) = \frac{n(n-1)}{(n+1)(n-1+\sigma)}$ and similarly

$E(b(X_2) = \frac{(n-1)^2}{(n+1)(n-1+\sigma)}$

So the expected bribe will be

$\tilde{B} = \sigma[\frac{n-1}{(n+1)(n-1+\sigma)}].$

Suppose $n = 2$, then $\tilde{B} = \frac{\sigma}{3(1+\sigma)}$, note that

$\frac{d\tilde{B}}{d\sigma} = \frac{1}{3(1+\sigma)^2} \geq 0.$

therefore, $\tilde{B}$ attains maximum at $\sigma = 1$.

<u>Reservation Price</u>

In this section we will try to analyze the role of reservation price on the outcomes of a First Price auction under the presence of a corrupt auctioneer. Our basic curiosity here is to observe the role of reservation price as a response against corruption. Suppose seller announce a reservation price r such that any bid below r can no longer be entertained. Consequently some bidder will be excluded if their valuations are below the reservation price. We will first analyze the bidding behavior of buyers who are still ready to submit there bid to the auctioneer.

The auctioneer offers a bribery contract to the remaining bidders and allocates the object efficiently and for those who are able to bid, the following holds for a given value of r.

$$\int_r^v [b(y) + \sigma(b(v) - b(y))]g(y)dy = \int_r^v g(y)dy$$

$$\Rightarrow (1-\sigma)\int_r^v b(y)g(y)dy + \sigma b(v)\int_r^v g(y)dy = [y\int(y)dy]_r^v - \int_r^v G(y)dy$$

$$\Rightarrow (1-\sigma)\int_r^v b(y)g(y)dy + \sigma b(v)(G(v) - G(r)) = [yG(y)]_r^v - \int_r^v G(y)dy$$

$$\Rightarrow (1-\sigma)\int_r^v b(y)g(y)dy + \sigma b(v)(G(v) - G(r)) = vG(v) - rG(r) - \int_r^v G(y)dy$$

Differentiating both the side w.r.t v and r is constant , we get

$$\Rightarrow (1-\sigma)b(v)g(v) + \sigma b'(v)(G(v) - G(r)) + \sigma b(v)g(v) = vg(v)$$

Since $g(r)$ is constant we assume $(G(v) - G(r)) = \hat{G(v)}$ and then a simplification of above expression gives the following differential equation.

$$\Rightarrow b'(v) + \frac{g(v)}{\sigma \hat{G(v)}}b(v) = \frac{vg(v)}{\sigma \hat{G(v)}}$$

Clearly the integrating factor will be $\hat{G(v)}^{\frac{1}{\sigma}}$ and we can make the following manipulation

$$\Rightarrow \hat{G(v)}^{\frac{1}{\sigma}}(b(v) - 1) + [\frac{g(v)}{\sigma \hat{G(v)}}\hat{G(v)}^{\frac{1}{\sigma}}](b(v) - v) = -\hat{G(v)}^{\frac{1}{\sigma}}$$

which immediately solves for $b(v)$

$$\Rightarrow b(v) = v - \frac{\int_r^v \hat{G(y)}^{\frac{1}{\sigma}} dy}{\hat{G(v)}^{\frac{1}{\sigma}}}$$

or $b_r(v) = v - \frac{\int_r^v (G(y)-G(r))^{\frac{1}{\sigma}} dy}{(G(v)-G(r))^{\frac{1}{\sigma}}}$ ....(C)

The expression by (C) is the symmetric increasing equilibrium under corruption with reservation price. There is not much changes in the outcome by introducing r , but the form of (C) draws our attention on two important features.First, like standard case here also a buyer with valuation r submits a bid equal to his valuation i.e. r, i.e.

$$b_r(r) = r$$

We can repeat the same example which we used earlier in section 1 with a reservation price r

$$b_r(v) = v - \frac{\int_r^v (y-r)^{\frac{1}{\sigma}} dy}{(v-r)^{\frac{1}{\sigma}}}$$

which can be simplified as

$$b_r(v) = \frac{v}{1+\sigma} + \frac{\sigma}{1+\sigma} r.$$

Suppose a case where all bidders have valuation below r except one bidder who has valuation exactly equal to r. In such a case any bribery agrement can not be accepted by the bidder. If he accepts the offer, then seller will be reported a bid lower than r by the auctioneer and the object remain unsold. Bribe offer can only be accepted if the bidders' valuations are strictly greater than r. So if all the other bidders has their valuation below r but there is only one bidder who has valuation strictly greater than r, the bidder will accept the bribery offer. In that case the total spoil will be equal to the difference between the bid of that bidder and the reservation price.

Secondly, suppose all the bidders has valuation above r, so the competition level will be same. So for any given value of r and σ, bidders become more aggressive compared to zero reservation price case.[58]. At the existing level of competition the spread of valuation become smaller and at given bid the probability of win decreases and the optimal response is to increase the bid.

If the reservation price excludes some bidders, implies an increase in probability of win at a given bid. A bidder can afford a low bid but he has to maintain it above reservation price. A lower number of bidders may or may not reduce the aggressiveness in bidding. It depends upon the probability of win and the value of reservation price.

Suppose a seller chooses a very high reservation price such that it excludes all the bidders except one. In such case a seller will receive only the announced reservation price regardless of the actual valuation of existing bidder. A reservation price may not be helpful against corruption. So a seller should choose a reservation price which maximizes the expected revenue of the seller.

Our objective is to maximize the total expected revenue of the seller to find the optimal value of r. We assume that in case if the object become unsold it fetches a utility equal to v_* to the seller it self.

[58]Because the second term in (B) is greater than the second term of (C) so $b_r(v) \geq b(v)$

Suppose there are n bidders in the competition. Suppose all the player has valuation below r, so none of the bidders can participate and the object remain unsold. In that case a seller receives $F^n(r)v_*$ as a expected revenue. If the bidders valuation allow them to submit a bid then corruption takes place and a seller receives expected value of second highest bid as expected revenue. Apart from these two cases there is an another possibility. Suppose there is a bidder whose valuation is strictly greater than r but the rest of the bidders have their valuations below r and can not participate. The corruption takes place and the seller receives only the reservation price. The probability of this event is $[1 - F(r)]F^{n-1}(r)$ and it is possible in n different ways. So the aggregate expected revenue of the seller has three parts and we can write following profit function.

$$\Pi = v_*F^n(r) + E(b(X_2)) + nr[1 - F(r)]F^{n-1}(r)$$

and we are interested to solve $\frac{d\Pi}{dr} = 0$ for r

and, $E(b(X_2)) = \int_r^1 b(t)[n(n-1)(1 - F(t))F^{n-2}(t)f(t)]dt$

where $n(n-1)(1-F(t))F^{n-2}(t)f(t)$ is the density function of second highest order statistics of valuations.

Note that $\frac{dE(b(X_2))}{dr} = -n(n-1)[(1 - F(r)F^{n-2}(r)f(r)]$ (Using the Leibnitz's rule)

$\therefore \frac{d\Pi}{dr} = nv_*F^{n-1}(r) - n(n-1)r[(1-F(r)F^{n-2}(r)f(r)] + nF^{n-1}(r)(1-F(r)) +$

$[(n-1)F^{n-2}(r) - nF^{n-1}(r)]nrf(r)$

We can simplify the right hand side by equating it equal to zero.

$\Rightarrow v_*F(r)f(r)-(n-1)(1-F(r))f(r)+(1-F(r))F(r)+r(n-1-nF(r))f(r) = 0$

$\Rightarrow v_*F(r)f(r)+rf(r)[n-1-nF(r)-(n-1)(1-F(r))]+(1-F(r))F(r) = 0$

$\Rightarrow v_*F(r)f(r) - rf(r)F(r) + F(r)(1-F(r)) = 0$

$\Rightarrow v_*f(r) - rf(r) + (1-F(r)) = 0$

So if the optimal r that maximizes Π is $\hat{r}$ then it is given by the following identity.

$\hat{r} = v_* + \frac{1-F(\hat{r})}{f(\hat{r})}$ ....(D)

The expression for optimal r in the corruption model is exactly what it is for the standard model.[59]. Announcing a reservation price is most common practice in auctions. The optimal reservation price depends upon seller's own valuation. Levin and Smith [60] argued that for more general class of auctions the optimal reservation price converges to the seller's own valuation.

[59]For the proof see Riley , J., AND W. Samuelson (1981): "Optimal Auctions", American Economic Review, vol 71, 381-392

[60]Levin, D. and Smith,J. (1996) "Optimal Reservation Price in Auction", The Economic Journal, vol 106, 12271-1283.

In corruption model, a seller's selection of r depends upon the probability distribution and on his own valuation. Optimal reservation price does not depends upon σ, simply because a bidder with valuation equal to reservation price never accepts bribery offer. Secondly, if all bidders has valuation below r , but there is one bidder who has a valuation greater than r win the object with probability one but accepts the bribery offer. A seller in that case receives only the reservation price regardless of what σ is.

Unlike the collusion model where reservation price can be used as a response against cartel [61], in corruption model it does not work well. A seller chooses an optimal reservation price in a similar manner in which he was choosing it for the SIPV models to achieve a maximum expected revenue (though affected by corruption). Suppose a seller chooses a very high value of reservation price and excludes almost all the bidders and left with only one bidder who can submit a bid. The seller can only receive the reservation price announced by him and can not stop the corruption. We can conclude that with reservation price an auction can only be corruption free if the highest valuation equal to the reservation price, which in itself is a rare case.

Incentive Scheme

In this section we will take attempt to show that in order to minimize losses due to corruption a seller may revise the contract with auctioneer. Suppose a seller has limited ability to trace out the possible illegal activities of auc-

[61] Grham,D. and Marshall, R.(1987), "Collusive Bidder Behavior at Single-Object Second-Price and English Auctions". The Journal of Political Economy, vol 95, 1217-1239.

tioneer and he simply offers a proportion of expected revenue. A seller, in presence of corruption speculates a lower expected revenue compared to the standard case. The objective of the seller is to improve the expected revenue as well as provide incentive to the auctioneer to stop corruption. We found that a seller may improve the revenue but this depends upon the value of σ and on the bidding behavior under the presence of corruption. However an incentive scheme can not guarantee the bringing down corruption in full and in extreme cases a situation of moral hazard is likely to occur.

Suppose, $\alpha =$ Proportion of share that would be transferred to the auctioneer. Clearly $\alpha \in [0, 1]$

$R =$ Expected revenue of the seller in the standard model. $= E(X_2)$

$\tilde{R} =$ Expected revenue of seller under corruption. $= E(b(X_2))$

$\tilde{B} =$ Expected bribe which is given by $= \sigma[E(b(X_1)) - E(b(X_2))]$

where X_1 and X_2 are the highest and second highest valuations respectively.

The incentive rule is simply given by following two conditions

$(1 - \alpha)R \geq \tilde{R}......(i)$

$\alpha R \geq \tilde{B}...........(ii)$

From (i) $\alpha \leq \frac{R-\tilde{R}}{R}$

From (ii) $\alpha \geq \frac{\tilde{B}}{R}$

Combining above two relation we will get

$$\frac{\tilde{B}}{R} \leq \alpha \leq \frac{R-\tilde{R}}{R}$$

For the existence of a suitable α we need to confirm that

$$\frac{R-\tilde{R}}{R} \geq \frac{\tilde{B}}{R}$$

or, $R - \tilde{R} \geq \tilde{B}$

This condition holds with equality because of the revenue equivalence. Clearly the only solution is

$$\alpha = \frac{R-\tilde{R}}{R}$$

Which shows that even if the auctioneer accepts the incentive honestly it does not helps the seller for the improvement in revenue. The incentive rule is a fragile structure because of lack of enforcement. An auctioneer may choose to be corrupt even if seller is providing an incentive which is as good as the expected bribe. An auctioneer can earn more dishonest gains by betraying the contract of simple incentive rule.

We can redefine the incentive scheme in the following manner. So consider

$\hat{B} = \frac{1-\sigma}{\sigma}[R - \tilde{R}]$

clearly, $\hat{B}$ can be interpreted as the gain of the winning bidder from the bribery contract. Under the assumption of basic model a seller can not monitor the auctioneer's activity and provides incentive to stop the corruption. Here the seller's objective is to recover the part of revenue taken by bidder under the bribery contract.

$\tilde{R} + \frac{1-\sigma}{\sigma}[R - \tilde{R}] \geq (1 - \alpha)R$

This condition implies that a seller's total revenue after improvement due to the incentive scheme must be as good as the net revenue.

$\alpha \geq (1 - \frac{\tilde{R}}{R})\tau$

where $\tau = 1 - \frac{1-\sigma}{\sigma}$

Total incentive must be as good as the expected bribe of the auctioneer. Which is given by

$\alpha R \geq +[R - \tilde{R}]$

$\Rightarrow \alpha \geq (1 - \frac{\tilde{R}}{R})$

This give us two simple conclusion. One is that the seller has to provide a sufficiently large amount of incentive. Clearly the seller's revenue improves but not up to the desired level. It must be strictly lower than the sum of revenue under corruption and the bidder's expected gain due to corruption. The incentive scheme is costless and a seller with limited ability improves upon. We can also conclude that the stability of an incentive scheme requires that the proportion of incentive must be higher than the proportion of bribe share. However the first condition on α explains its dependence on R and $\tilde{R}$. This indirectly shows its relation with the gap between highest and second highest valuation.

Penalty Scheme

Up to this we assumed that a seller acts like a sleeping partner and gives all independence to the auctioneer. As a result the auctioneer charges bribe and together with the highest bidder makes dishonest profits on the cost of seller's revenue. We have analyzed the outcomes of a First Price auction under corruption and try to add some adjustments so as to stop corruption. A seller is assumed to be neutral against corruption and even in the example of incentive, seller does not give a credible threat to the other parties. We now assume that a seller keep track on auctioneer's activity and announces a penalty which he can charge from auctioneer in case he is found taking bribe from the highest bidder.

We can formally arrange the structure of penalty scheme by assuming that if an auctioneer practices corruption, he may get caught by the seller with probability ρ. In that case a seller charges a penalty equal to P. Suppose b_1 and b_2 are the highest and the second highest bid then from the basic model of corruption we know that the bribe function is given by

$\tilde{B} = f(b_1 - b_2)$

in our case we assume $\tilde{B} = \sigma(b_1 - b_2)$ which is also equal to the share of bribe received by the auctioneer. We assume that a seller charges the penalty only from the auctioneer. A seller puts effort to control the activities of auctioneer only and not to the bidders. Monitoring bidders activity may not be feasible because there may be a huge number of bidders participating in the game. Also a bidder is always free to exit the game if he find that rules are not favorable for him. This assumption implies that a bidder participates in feasible bribery contract even if there is a penalty scheme. So if an auctioneer practices corruption under the penalty scheme, his expected bribe conditional on the equilibrium bids is given by

$$(1 - \rho)\sigma(b_1 - b_2) - \rho P$$

An auctioneer offer bribery contract to the bidder only if

$$(1 - \rho)\sigma(b_1 - b_2) - \rho P \geq 0$$

$$\Rightarrow (b_1 - b_2) \geq (\tfrac{\rho}{1-\rho})\tfrac{P}{\sigma}$$

Suppose $(\frac{\rho}{1-\rho})\frac{P}{\sigma} = \theta$ then we may say that if the highest bid is greater than the secon highest bid by θ , an auctioneer practices corruption. So θ is the criteria of being corrupt.

We are now going to examine the effect of penalty scheme on the bidding behavior. We know that if θ criteria satisfies then corruption takes place. A bidder decides whether to accept or to reject the bribery offer according to the minimum between $b(v)$ and $b(y) + \tilde{B}$

An auctioneer offers bribery contract if $(b_1 - b_2) \geq (\frac{\rho}{1-\rho})\frac{P}{\sigma}$ and the bidders accept it, if $b(y) + \tilde{B} \geq b(v)$.

Under penalty scheme, an auctioneer demands $(1-\rho)\sigma(b_1 - b_2)$ from the bidder as a bribe. Note that the amount of bribe demanded is such that it recovers the expected penalty.

So under corruption with penalty scheme bidder's payment is given by

$b(y) + (1-\rho)\sigma(b(v) - b(y))$.

Suppose an auctioneer offeres a feasible bribery contract to the bidders under the penalty scheme then the only change that will take place is that the effective rate of bribe share will get down but the rest of the game would be similar. So the allocation efficiency still holds and the following is immediate.
$\int_0^v [b(y) + (1-\rho)\sigma(b(v) - b(y))]g(y)dy = \int_0^v yg(y)dy$

For simplicity let $(1-\rho)\sigma = \hat{\sigma}$, clearly $\hat{\sigma} \in [0,1]$ and $\hat{\sigma} \leq \sigma$.

$\Rightarrow \int_0^v [b(y) + \hat{\sigma}(b(v) - b(y)]g(y)dy = \int_0^v yg(y)dy$

$\Rightarrow (1-\hat{\sigma}) \int_0^v b(y)g(y)dy + \hat{\sigma}b(v) \int_0^v g(y)dydy = \int_0^v yg(y)dy$

Differentiating both the side w.r.t to v , we get

$$\Rightarrow (1-\hat{\sigma})b(v)g(v) + \hat{\sigma}b(v)g(v) + \hat{\sigma}b'(v)G(v) = vg(v)$$

$$\Rightarrow \hat{\sigma}b'(v)G(v) + b(v)g(v) = vg(v)$$

Above is the require differential equation for $b(v)$ and then the integrating factor would be $G(v)^{\frac{1}{\hat{\sigma}}}$. We can solve it using a similar manipulation we did for the basic case of corruption and hence the equilibrium bid under corruption with penalty scheme is given by

$$\Rightarrow b_{\hat{\sigma}}(v) = v - \frac{\int_0^v G(y)^{\frac{1}{\hat{\sigma}}}dy}{G(v)^{\frac{1}{\hat{\sigma}}}} \;.......(E)$$

Clearly, $b_{\hat{\sigma}}(v) \geq b(v)$. In a penalty scheme a bidder become more aggressive. We can observe that the optimal bid function (E) is similar to the bid function given by (B). The only difference is that in (E) we have $\hat{\sigma}$ instead of σ. SInce $\hat{\sigma} \leq \sigma$ so at the same level of valuation a bidder submits higher bid.

An auctioneer demands the bribe after adjusting the probability of being caught. The amount of bribe should compensate at least the expected penalty. However a fear of being caught for corruption makes auctioneer charging a lower bribe. This provides scope for additional gain to the bidder. In Section 1 we have argued that over bidding is due to the extra gain created by corruption. In this case, the rate of bribe share gets down because the penalty scheme depresses the corruption[62].

[62]Actual proportion of bribe share is σ but the effective rate is $\hat{\sigma}$, which is lower than σ.

A bidder always internalizes the effect of penalty within the bid function . We assume that a seller never charges any penalty to the bidder involved in corruption. If the seller finds a corruption situation, the auctioneer will be penalized.The idea is that a seller recovers a part of the revenue from auctioneer, which is indirectly paid by bidder (as a bribe). The remaining loss get compensated from the overbidding.

We now illustrate a simple method to make the mechanism corruption proof. We know that whether corruption takes place or not, depends upon ρ and the gap between highest and second highest bid. Any speculation about the gap between these two values may not be concrete. To make the system corruption free all we require is that

$(b_1 - b_2) < \theta$, with the other constraint: $b_1 > b_2$.

Note that the first inequality holds if both b_1 and b_2 are below θ and we require to find a θ which satisfies this. Finding a θ which satisfies $b_1 < \theta$ and $b_2 < \theta$ is equivalent to finding a valuation such that the corresponding bid is higher than both the b_1 and b_2. Recall that the private valuations i.e. $v_i \in [0, 1]$ and we suggest the safest point is upper end of this range. We choose $b_{\hat{\sigma}}(1) = \theta$. As long as equilibrium bid functions are monotonically increasing, it will serve the purpose.

Once we choose $b_{\hat{\sigma}}(1) = \theta$, the following is immediate

$\frac{\rho}{1-\rho} \frac{P}{\sigma} = b_{\hat{\sigma}}(1)$

$$\Rightarrow \frac{\rho}{1-\rho} = \sigma \frac{b_{\tilde{\sigma}}(1)}{P}$$

$$\Rightarrow \frac{1}{\rho} - 1 = \frac{P}{\sigma b_{\tilde{\sigma}}(1)}$$

$$\Rightarrow \frac{1}{\rho} = \frac{\sigma b_{\tilde{\sigma}}(1)+P}{\sigma b_{\tilde{\sigma}}(1)}$$

$$\therefore \rho^* = \frac{\sigma b_{\tilde{\sigma}}(1)}{\sigma b_{\tilde{\sigma}}(1)+P}$$

Clearly for any $\rho \geq \rho^*$, the mechanism is corruption proof and equilibrium bid will be the same as it is in SIPV model. Note that $\rho^* \leq 1$ implies that a sufficient amount of threat works well. A seller can stop corruption by strategically choosing a value of ρ . If the corruption takes place, an auctioneer receives a bribe from the bidder which is at least as good as the expected penalty. As an alternative a seller may charge a very high penalty but that may not ensure the inverse of θ criteria.

Moreover we found that a possible threat of penalty from the seller may stops auctioneer from practicing corruption. An auctioneer receives a lower amount of bribe and a seller expects a revenue closer to the standard revenue. Bidders internalizes the effect of penalty threat in terms of a lower bribe payment in their equilibrium bid function and bid more aggressively.

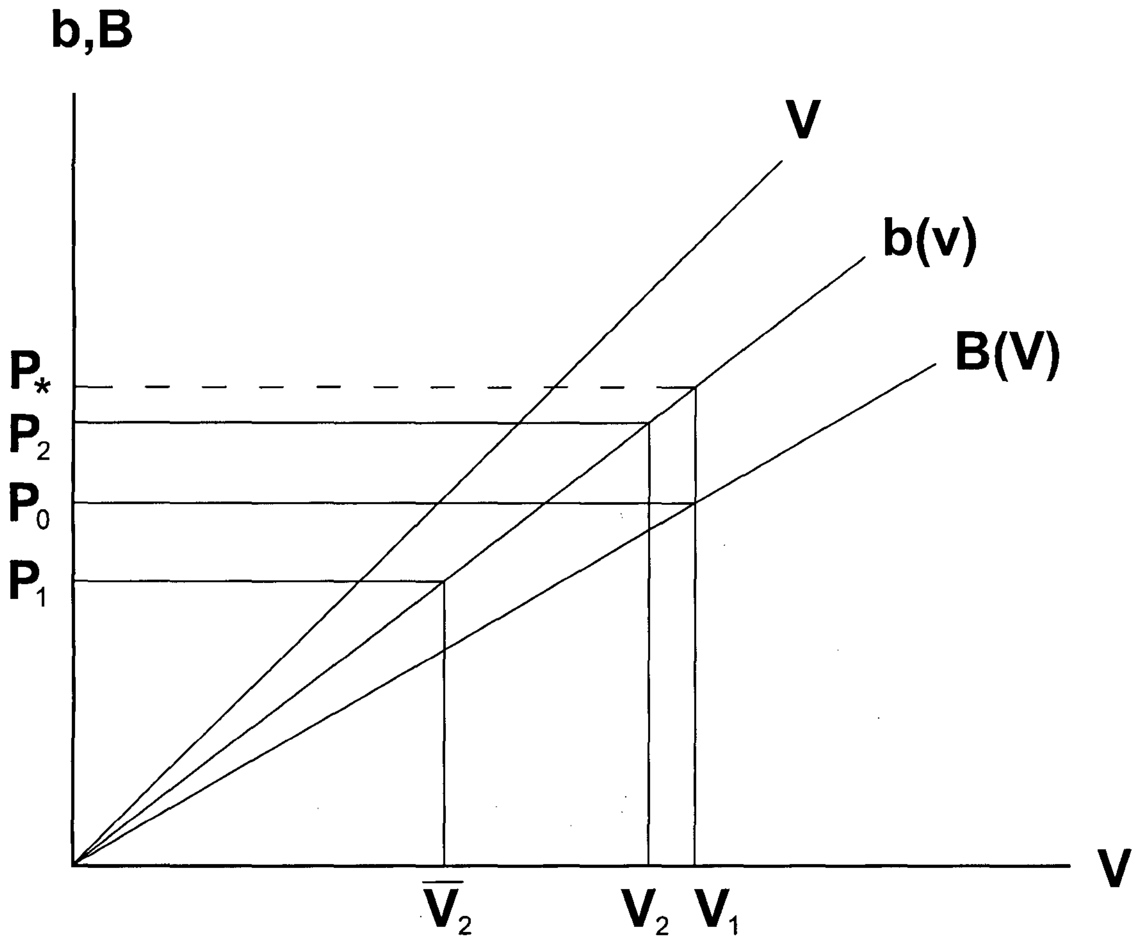
In the last two settings we found that in order to save the revenue from corruption a penalty scheme works well. A penalty may be more effective if we add a proper incentive scheme. But a penalty scheme may be costly for the seller. An immediate solution is to set a fine equal to the cost of state verification.

Corruption in Expost

A first price auction is always corruption prone but a second price auction is corruption proof because a bidder if wins, get the information of second price freely. A first best solution to stop corruption is always adopt second price auction, a seller then receives the standard revenue. But implementing a second price auction is not always feasible. At several points in this chapter we found that apart from value of σ the immune of the problem is the physical gap between highest and second highest bid.

We are here trying to argue that a seller may at times prefer corruption. Under corruption a seller's expost realization of price of the object may be higher than the price which he would received in the standard model i.e when there was no corruption. We refer the diagram given in the adjoining page to understand this argument. The figure simply shows that for a given value of σ, in expost the payment received by the seller may become more profitable. This profitability depends upon the gap between the highest and the second highest valuation.

We know that in the presence of corruption a seller gets bid corresponds to the second highest valuation. Note that a no corruption situation does not mean $\sigma = 0$, it requires non-existence of σ. In the diagram, $B(v)$ is the equilibrium bid for the standard first price auction which eventually the equilibrium bid function under corruption with $\sigma = 1$. similarly the ray through origin v is the optimal bid for a standard second price auction as well as for the corruption model with $\sigma = 0$. We assume these two to represent



b,B
V
b(v)
B(V)
P*
P2
P0
P1
V
V̄2
V2
V1

equilibrium bid function for standard First price and Second price auction respectively. The ray through origin denoted be $b(v)$ represents bid under corruption with an interim value of σ.

P_0 is the price that a seller receives in the standard first price auction if the winner's valuation is V_1. If corruption takes place and generates a bid function $b(v)$ then a seller receives P_1 if the second highest valuation is $\bar{V}_2$ and P_2 if second highest valuation is v_1. Clearly, $P_2 > P_0$ and $P_1 < P_0$. So in the expost whether a seller makes profit or lose due to corruption, depends upon the gap between highest and second highest valuation for a given level of σ. Suppose the gap is sufficiently small for a given value of σ then a seller may get a higher payment.

The reason is that a corruption results in aggressive bidding and payment depends upon the extent of aggressiveness. Since the optimal bid function is continuous and monotonically decreasing in the range of σ. Suppose the gap between the highest and the second highest bid is not very large. Given the assumption of independent private values and symmetry following is possible for a given value of σ.

$b(V_2) > B(V_1)$.

If above holds then a corruption is more profitable for the seller. Recall the same example of 2 bidders with uniform distribution. In that case this condition implies

$\frac{v_2}{1+\sigma} \geq \frac{v_1}{2}$

$\Rightarrow \sigma \leq 2(\frac{v_2}{v_1}) - 1$

Note that the feasibility of above condition requires that the term in the right hand side should take positive values i.e

$2(\frac{v_2}{v_1}) - 1 \geq 0$, which implies $v_1 \geq v_2 \geq \frac{v_1}{2}$. If the condition holds for this specific example then corruption in expost will be more profitable for the seller.

If a corruption is gainful for the seller then the burden of deadweight loss due to the corruption is beared by the winner. Note that the P_* is the optimal bid under corruption. The total spoil of corruption is $(P_* - P_2)$ when the second highest valuation is V_2 and it is equal to $(P_* - P_1)$ when the second highest valuation is V_1.

The above argument leads to two main conclusion. Firstly a seller's gain depends upon the gap and σ. For a fix size of gap, it is always possible to find a cut off for σ or for a given σ the optimal gap. Secondly a corruption always results in deadweight loss. In exante situation a seller is sole responsible for the loss. But in expost a buyer can be the bearer of deadweight loss depends upon the gap [63] and σ.

[63] By gap we mean the gap between the highest and the second highest bid

Concluding Remarks

This chapter contains the basic model of a First Price auction with a corrupt auctioneer. We explain a very simple case of corruption where motive of an auctioneer is to manipulate prices and making dishonest gain. In chapter 2 we have described some other form of price manipulation and in this chapter we observed the basic distinctions in terms of outcomes. The interesting fact about the corruption model is the allocation efficiency. A price manipulation via collusion also results in efficient allocation. We can treat our basic model as a collusion between the auctioneer and the bidder with highest valuation. The basic model does not reflect any sort of favoritism. A favoritism may causes inefficiency in allocation and discriminates bidding behavior.

The corruption model is based on the assumption similar to a SIPV model except that there is a third party in this model which is the auctioneer. It may be subject of interest to examine the outcome of auction after relaxing any or some of the assumption. For that matter assume that the bidders are risk averse. In no corruption case, a seller prefers First price auction but it may not be true under corruption. A utility maximizing bidder always accepts bribery contract and the seller's expected revenue become the expected value of the second highest bid which is again smaller than the standard expected revenue. In the context of basic corruption model it can be assumed that an auctioneer deamnds a fixed amount of bribe to the winner. Many works including Meneze and Monterio (2006) studied it extensively. This assumption has serious technical complexities but interestingly it provides criteria to accept or reject the bribery contract for each of the bidder separately.

One of the interesting results which we found in this chapter is the optimal reservation price. It eventually turns out that even in the presence of corruption revenue maximizing reservation price is same as it was in the SIPV model. Next we suggest an incentive scheme to stop corruption. Though the scheme is weak in nature, but gives a sufficient criteria to choose the incentive level. The incentive scheme is simple and costless. A moral hazard is likely to happen but a strategically chosen incentive scheme generates interim enforcement and improves the seller's expected revenue.

A suggested penalty scheme is more effective and its effects comes via bidding behavior. We have shown that how an optimal level of monitoring can rule out the corruption. Only problem is that a penalty scheme is complex in implantation. The basic difference between the incentive and the penalty scheme is that the effectiveness of later is more random than former. We have argued that a seller under corruption speculates a lower expected revenue. But the nature of corruption may lead to arrive at a situation where a seller can earn higher profit with positive probability.

Chapter 4: Measurement Issues

In the last two chapters we have studied auction theory in a general framework and the same under the presence of an auctioneer, particularly a corrupt one. This chapter attempts to illustrate some empirical evidences of corruption in an auction. The theoretical models are as usual based on certain assumptions. We turn our focus to the empirical issues i.e. how to construct measures to hypothesize and test the theoretical results of an auction model? Our aim here is to review the literature related to empirical research on auction theory. We also deal with the measurement issues in an auction. The objective of such an empirical research is to estimate private valuations, the underlying probability distribution and the bidding functions. Other than this one can try to explore preferences of agents e.g. buyer's attitude towards risk or the auctioneer's corruption practices.

Here at the first place, we follow the paper by Binmore and Klemprer (2002)[64], where they share their experience of British 3G spectrum auctions [65]. Their aim is to extract the crucial factors behind designing an auction. There are certain other factors that are neglected by theory but can be very important. The current authors divide this issue in two parts. First is the informational issue related to efficiency of auction. The second part is industrial -organization issue to ensure competition. An auction as big as British 3G auction involves huge amount of money. Any firm who gets a license might transfer the burden through prices to the consumers. Many firms got

[64]Binmore, K. and Klemprer, P. (2002): "The Biggest Auction Ever : The Sale Of The British 3G Telecom Licences," *Royal Economic Society*, Vol 112, $C74 - C96$

[65]In 1997 UK government's Radiocommunication Agency sold the third generation mobile phone licenses through an auction. The total valuation of auction was 23 million pounds.

the license from the British auction which motivated competition in telecom market. So prices depended only on the equilibrium in telecom market and the auction was not the cause for high prices. The issue of entry was solved by dividing the whole spectrum in five licenses of different sizes. This provides scopes for marginal bidder to remain in auction. They can bid for the suitable size. To reduce any possible dispute the auction rules should be legally sound and an incumbent has to have the full right to know all the rules of auction.

This example shows that a seller may have many other objectives than profit maximizing. Binmore and Klemperer shows that the profit maximization is not a very straight forward objective but it rather depends upon various constraints. We need to learn how to internalize these constraints in order to design auctions. The present chapter is broadly divided into two parts. The First part explores some case studies in auctions. Second part considers the role of econometrics as a tool to design an auction mechanism. It also deals with the preliminary idea of how to empirically measures the presence of corruption in an auction.

Some Case Studies

Designing auction for empirical studies is a rigorous task because it requires verification of basic assumptions. Like any other empirical work, construction of testable hypothesis would be the main concern. Preliminary econometric tools are rarely applicable and to estimate an auction model knowledge of nonparametric estimation is necessary [66]. For example Laffont *et al* (1995)[67] estimate a First price auction model. They obtain consistent estimates of parameters of underlying probability distribution function of private valuation using a simulated non linear least square method [68]. This paper uses a daily sales data of agriculture products for a given period of time. The data consist of 81 independent auctions.Structural model associated with equilibrium bid function of a First Price auction is being estimated. A first step of estimation is to determine the active number of bidders in the entire section. Laffont *et al* suggested a lack of fit criterion to estimate number of bidders. Once estimating the number of bidder the estimation of other factors was done by simulated non linear least square.

In a work by Lu and Perrigne (2006)[69] provides the technique of estimating First price auction when buyer's preferences exhibit risk aversion. Authors in this paper use timber auction data of United State Forest Service Department. The Department use both First and Second Price auction to sell its timber. The Data from these two auctions are used to estimate buyer's utility function by nonparametric method. The data set consists of information related to large number different auctions. Large sample ensures the robustness of non parametric estimation (of both utility and distribution function). Other than the winning bid, the data set also contain variables like number

[66]A non parametric method is a method of sampling which does not assume about the form of probability distribution. Estimation requires a theoretically robust simulation exercise.

[67]Laffont, J.J , Ossard, H. , Vuong, Q (1995): "Econometrics Of First-Price Auction," *Econometrica*,Vol 63, 953 – 980

[68]In the next section we will discuss these theoretical issues in detail.

[69]Lu, J and Isabelle, P (2006): "Estimating Risk Aversion From Ascending and Sealed-bid Auctions: The Case of Timber Auction Data," *MPRA working papers*, No. 948.

of bidders, appraisal value, volume of timber and etc. These variables together with non-parametrically estimated utility function are used to model the probability to choose a Second Price Auction over a First price Auction.

The above two example shows that in the benchmark model of auctions (with or without risk averse buyers) a nonparametric method of estimation is a necessary tool. Prior to estimation of any hypothesis on an auction, the estimation of underlying distribution is must. At this point we are saving this comment for future reference [70] .We are now going to explore examples of auctions with corruption.

A very recent work by Tran(2008) [71] uses a data of internal bribery of a trading firm. A firm is engaged with government transaction and keep records of bribery paid to the officials. Any trading agreement (not necessarily an auction) between any firm and the government is always very corruption prone. This work attempts to find out a way how such corruption can be reduced in government procurement through proper designing. This paper advocates that procurement over a simple trading may save revenues by corruption. The government of the country to which this firm belongs, mandates auction formats name best-value auction as a first policy change and then best -price auction. The hypothesis is being proved by author using the data taken from the undisclosed source. One benefit of the kind of data used here is that it measures corruption factor accurately because the firm itself maintains a record of bribery transactions. The proposed rule of auction says that a bidder has to give the technical specification of its product and not

[70] We will use a similar concept of estimation of an auction model with corrupt auctioneer

[71] Tran, A .(2008): "Can Procurement Auctions Reduce Corruption? Evidence from the Internal Records of a Bribe-Paying Firm," Working Paper -Harvard University.

only the price. Each component of bid gets a weight and then a winner get selected by picking the best combination of price and quality. The evaluation process worked well and depresses official's ability to distort the result and consequently there is a fall in corruption. A difference - in -difference [72] approach is being applied to measure the effectiveness of policies. All the auctions during the period when government policy suggested best-value format is considered as first treatment group and similarly auctions during the period of best-price format are considered as the second treatment group. Pre-policy period is the control group.

Wiharzida (2006)[73] applied a regression model on 1404 e - public procurement auction conducted by department of public works to test two hypotheses. One is that, an increase in number of bidder increases the competition and second it decreases equilibrium bid and bribe. The author used two models to test the hypothesis. The first model is a simple regression for each of the sub departments and the second is the pooled regression with fixed effects. The model consider two generated variables, percentage of cost efficiency and number of bidders.

Estimation of an Auction

The last two examples, one discussed that corruption practices depends upon the auction format.Other establishes the inverse coherence between corrup-

[72]The difference in difference estimator is defined as the difference in average outcome in the treatment group before and after treatment minus the difference in average outcome in the control group before and after treatment

[73]Wihardja, M.M., (2007): "Competition and Corruption in Public Procurement Auction:Practical Application of the E-Procurement in Indonesia," mimeo, Cornell University

tion and competition. Both the studies assumes (or have the data) the presence of a corrupt auctioneer. Theoretically an auctioneer's activities are unobservable. Having data on bribery agreements is not always possible. We are interested in setting up a hypothesis which tests the presence and extent of corruption in an auction.

We adopt a very simple approach and consider a First Price auction under the presence of corruption.We understood from the basic model [74] that bidders submit uniformly higher bid and the form of equilibrium bid allows us to infer that the probability of win in no-corruption case stochastically dominates the effective probability of win in corrupt auctioneer case. We use this fact as a hypothesis to test the presence of aggressiveness. Note that bidders are assumed to be risk neutral. One can use data on bid to identify buyers utility function and test whether the coefficient of risk aversion is significantly close to zero or not.

In general, the auction models are difficult to compute because the observed bid follow non linear functional system. Measuring the underlying distribution is the key necessity to model an auction and it requires nonparametric techniques of estimation.

We mention two methods,first is estimation of the empirical distribution of win and second is measuring the risk aversion and consequently bidder's preference whenever auction data is available. This part shows the basic steps of estimating an model using non parametric methods. We will follow the method introduced by Guerre *et.al.* [75]

[74] See section 1 of chapter 3

[75] Guerre, E. , Perrigne, I.. and Vuong, Q. (2000): "Optimal Nonparametric Estimation

The SIPV model for a First Price Auction with a reservation price r and private valuation distributed as $F(.)$ in the range $v_i \in [0, \bar{v}]$ with n symmetric bidders gives us following set of equation as the outcomes of the auction.

So the equilibrium bid function is given by

$$B(v_i) = v_i - \frac{1}{F^{n-1}(v_i)} \int_0^{v_i} F^{n-1}(x_i) dx_i$$

Clearly the optimal bid can be written as

$$\tilde{b} = \alpha(v_i, n, F(.))$$

Let b is the the equilibrium bid then the corresponding valuation $x = B^{-1}(b)$ so the the distribution of equilibrium bid will be $W(b) = F(B^{-1}(x))$ or $W(b) = F(x)$ with density function $h(b) = \frac{f(x)}{B'(x)}$. Note that $b \in [0, B(\bar{v})]$ and $W(b) = Pr[B(\bar{v}) \leq b]$. Now consider the differential equation of $B(.)$

$B'(v_i) = (n-1)(v_i - B(v_i))\frac{f(v_i)}{F(v_i)}$. We can now put private valuation as a function of bid and probability of win.

$$x = b + \frac{1}{n-1}\frac{h(b)}{H(b)}$$

.

The above expression can estimate the distribution of private valuation if $H(.)$ and $h(.)$ are known. But since these functions are unknown, so we need to estimate these prior to the estimation of $F(.)$. They can be estimated

of First-Price Auctions," *Econometrica*,Vol 68, 525 – 574.

from observed bids. Following Guerre *et. al.*, suppose we have data on bids for P homogeneous auctions and for a fixed set of bidders with size N then these distribution can be non-parametrically estimated by a kernel density [76] given by

$$\hat{H}(b) = \frac{1}{NP}\sum_{p=1}^{P}\sum_{n=1}^{N}(B_{pn} \leq b)$$

and $\hat{h}(b) = \frac{1}{NPt}\sum_{p=1}^{P}\sum_{n=1}^{N} K(\frac{b-B_{pn}}{t})$

where t is the bandwidth and $K(.)$ is the kernel. The function $\hat{h}(b)$ is therefore the non-parametric estimate of density function of the probability of win. These can be used to derive kernel for valuation function.

We now develop a simple idea of how to estimate buyer's utility function using data on auction. We adopt a method used by Lu, J. and Perrigne, I. (2006) where the authors use data on First Price auction and English Auction and describe a measure of degree of risk aversion and consider it as a tool to identify buyer's utility function. However we are interested in identification of utility function only and consider Guerra *et. al.*s technique for the estimation of distribution functions.

[76] A kernel $K(.)$ is a symmetric,twice differential weighted function, decreasing in absolute value of it's argument and satisfies $\int_{-\infty}^{\infty} K(t)dt = 1$. These function can be used to estimate the unknown probability distribution of a random variable. So if X is a random variable, its density at any x estimated by kernel density is given by
$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x-x_i}{h})$.
For a rigorous study of kernel density and non-parametric estimation see Pagan, A., and Ullah, A. (1999): "Nonparametric Econometrics, Themes In Modern Econometrics," *Cambridge University Press.*

We now move to demonstrate the method of identifying utility function. Suppose the bidders are risk averse and let $r(.)$ is the measure of absolute risk aversion. Then the following holds at equilibrium

$B'(x) = (n-1)r(x - B(x))\frac{f(x)}{F(x)}$ [77]

$h(b) = \frac{H(b)}{(n-1)r(x-B(x))}$ using the differential equation of $B(.)$

or, $h(b) = \frac{H(b)}{(n-1)r(x-b)}$

$\Rightarrow x = b + r^{-1}[\frac{1}{n-1}\frac{H(b)}{h(b)}]$

Suppose for any valuation v , $F(v) = \alpha$ so for the corresponding bid b , $H(b) = \alpha$ and we can use the expression for x to write

$r(v(\alpha) - b(\alpha)) = \frac{1}{n-1}\frac{\alpha}{h(H^{-1}(\alpha))}$

Note that if $v \in [0, \bar{v}]$ then $r(.) \in [0, \overline{v - b}]$. So we can non-parametrically estimate the measure of risk aversion using the underlying distribution of $H(.)$. With estimated $r(.)$, we can then identify utility function up to a scale as given by ,

$U(x) = Aexp[\int_{\underline{v-b}}^{x} \frac{1}{r(t)} dt]$

[77]This is the differential equation for First Price auction when bidders are risk averse, where n is the number of bidders. We follow Chapter 4 of the text book of auction theory by Krishna, V. and borrowed this differential equation directly from there.

We mention a two step procedure of identifying buyers utility function using the non - parametric estimate of risk aversion. However the derivation is heuristic, but it serves the purpose. In the First step we can estimate the underlying distribution function using the technique we have mentioned. Assuming this, we can next estimate the bidders risk aversion from the data on observed bids.

Under corruption the optimal bid function has one more argument, namely σ, the proportion of bribe.i.e.

$$\tilde{b} = \alpha(v_i, n, \sigma F(.))$$

Under corruption the differential equation of optimal bid function is given by

$$b'(v_i) = (n-1)(v_i - b(v_i))\frac{f(v_i)}{\sigma F(v_i)}$$

$$\therefore x = b + \frac{1}{n-1}\frac{h(b)}{\sigma H(b)}$$

We assume that σ is unknown and its effect is internalized into the distribution of valuation. We can use the same kernel density to estimate $H(.)$ and $h(.)$ and consequently distribution of valuation. Expression for x, clearly showing the inverse relation of x and σ. Due to the presence of σ estimated value of $H(.)$ estimates $F(.)$ differently.

We can evaluate the distribution of valuation separately for two cases. One without σ and the other with σ. It is clear that a similar kernel density of winning bid estimates the distribution differently. So the first testable hypothesis explores effects of this difference on the stochastic dominance of valuation distribution in no corruption case over the valuation distribution in corruption case. This confirms the phenomenon of overbidding. The second hypothesis is to test whether a measure of risk aversion is zero or not. We can use the utility identification method to estimate the measure of risk aversion. We can then test whether the estimated r is significantly zero or not. Suppose we have a data of bids submitted by a set of buyers for n similar auctions. Assume that the auction has a middle man who has conducted all the n auctions. We can apply the above techniques for the available data set to draw the inference on corruption. If the data set statistically exhibits overbidding and zero risk aversion, we can safely conclude about the existence of corruption.

Chapter 5: Conclusion

Our analysis began with some examples which show that price manipulation by the auctioneering authorities is a common phenomenon in auction. This helped us define the scope of our study, focussing on the effects of corruption in auction. We have reviewed the existing literature of auction to illustrate how the outcomes of an auction change when an auctioneer tries to seek dishonest gain.

We have described an auction model where collusion among buyers lead to decrease the price of the object that is put up for the auction. Collusive agreement in an auction implies that a subset of potential bidder share their private information in lieu of transfer of positive payoffs from one of the bidders who is in the collusion. Note that a collusion may not always gainful at least for two reasons. One, a profitable collusion requires that the winner of the object must belong to the set of collusive bidders. So a collusion must include a bidder who would also be the winner even without any collusion. The second argument is that a collusion may not always be stable as it provides scope of gain by unilateral deviation. One important observation from this model is that the most profitable collusion can be formed if it contains of two members, specifically the highest and the second highest bidder of the original game. The next two models of price manipulation which we have discussed in chapter 2 can be viewed in the light of this argument.

A collusion may also take place between a bidder and the auctioneer. The model of procurement by Burguet and Perry (2002) shows that such kind of collusion may result in aggressive bidding and lower expected revenue for the seller. An incumbent may be favored by the auctioneer[78] and manage to win the tender illegally, even if its costs are higher compared to the rival firms. A corruption in this model may result in inefficient allocation.

In the next model by Menezes and Monteiro (2006), we have seen that a bribery contract may take place between the highest bidder and the auctioneer. An auctioneer allows highest bidder to reduce his bid to the second highest level whenever the bidder pays the bribe. This model is different from the model of favoritism in terms of efficiency. This model shows that with enough scope of making private gains, an auctioneer practices corruption by providing the object to the highest bidder but at a lower price. Clearly, this model does not show any favoritism and allocates the object efficiently.

We have elaborately discussed the outcomes of this model in chapter 3. We found that a bribery agreement affect bidding behavior of each of the bidder and results in lower expected revenue for the seller. One interesting fact about this model is to determine the proportion of bribe share[79], there is no explicit bargaining between the bidder and the auctioneer. However the bidder can have control on this through bidding strategy.

Chapter 3 broadly discusses in reference to some techniques which a seller may adopt to respond against corruption in order to protect his expected revenue. A seller may announce a reservation price with the hope of discouraging corruption at least from the buyers side. This method work in a very special case where the highest bidder's valuation matches with reservation

[78]whenever he pays the bribe

[79]we denoted it by σ in chapter 3 and chapter 4

price.Question is that whether a seller should charge a very high reservation price? It will not be feasible because valuations are private information which is unknown to the seller. Even if it is possible, it does not ensure a "no-corruption" situation. A very high value of reservation price increases the chances that the object remain unsold. With these argument we may conclude that a seller with limited abilty[80] should choose a reservation price that maximizes the total expected revenue. This exercise is interesting because we have derived that the optimal reservation price is same as it would be in a SIPV model. Though a seller receives a lower expected revenue due to corruption but his own limitations make him choose reservation price in a similar manner.

With limited ability to monitor auctioneer's activity, a seller may improve his revenue by offering a part of his expected revenue to him. A seller's objective here is to retrieve the part of revenue taken by the bidder in the bribery contract. This method is fragile because it eventually cause a moral hazard problem in the absence of enforcement. An auctioneer may default by offering bribery contract along with incentive from the seller. However a seller may strategically choose the proportion of incentive i.e α which generate internal enforcement. The enforcement depends upon the size of the gap between the highest and second highest valuation (and bid) as well as the proportion of bribe share. A strategically chosen α may recover some revenue but not up to the desired level.

We have argued that an explicit contract is required between auctioneer

[80]The basic model of corruption assumes that a seller may not be able to monitor the auctioneer's activity. We denote this assumption by saying that a seller has limited ability

and seller . For this purpose we can relax the assumption of seller's limited ability of monitoring auctioneer's activities. We define a penalty scheme where a seller announces an amount of penalty chargeable to the auctioneer if he(seller) found that auctioneer practising corruption. We assume that a bidder is engaged in bribery contract is not not liable to pay the fine directly. A penalty scheme is supposed to stop corruption with positive probability. Bidders can be better off with such penalty scheme as the expected bribe payment is lower for a given valuation of the bidders. Consequently there is a rise in aggressiveness in bidding. Auction under penalty scheme can be corruption proof if the difference between the highest and the second highest bid is relatively smaller. We have shown that a mechanism can be corruption proof even if monitoring is not perfect. However a penalty scheme may be costly to the seller but we can argue that a seller adopts a penalty scheme whenever the expected recovery of revenue is higher than the cost of monitoring.

One of the interesting findings of this exercise is the effect of corruption on the seller's revenue in the expost. We constructed a very simple example to show that a seller may receive a price in presence of corruption which comparatively higher than that he received in the SIPV model. In a bribery contract a bidder bid aggressively as the contract gives scope for extra gains. Due to this a bidder intend to bid aggressively and put himself in a situation similar to winner's curse[81]. The expost gain of the seller depends upon the value of σ and the gap between the highest and second highest valuation

[81]A situation where a bidder, in order to ensure the win submits a bid higher than his own valuation. However in this case the winner's curse occurred in a weaker sense where a bidder losses his gain due to bribery.

(and consequently to the corresponding bids). We have shown that if this gap is sufficiently small then a seller may find the corruption is gainful in expost. Note that this expost profitability of corruption depends upon two basic assumptions. One is the assumption of private valuation and the second is the assumption of symmetry. Note that a corruption always results in deadweight loss. If in a situation where a seller is making expost profit then it is clearly the highest bidder who should have suffered the losses.

One of the interesting questions is whether the presence of corruption can be tested with available data on bids. We have presented a basic approach to answer this in chapter 4. We observe that the suggested approach uses the basic tools of econometrics of auction. The suggested method is a two stage process where in the first stage one has to test the tendency of overbidding by estimating the underlying distribution of bids and valuation. In the second stage we test for risk neutrality of the bidder. This concept can be modified with suitable knowledge of non parametric estimation for more general models of corruption in an auction.

In this dissertation we have discussed the effect of corruption in the benchmark model of auction. The analysis provided us with many interesting results which can be used for further analysis. We suggest the following avenues for future research.

(i) One important observation is the role of the value of total spoils i.e the the gap between the highest and second highest valuation and the corresponding bids. We have found that the effect of this "gap" in almost all part of this analysis. This provide us scope for further analysis where one can examine the optimality of the gap.

(ii) We argued that a suggested incentive scheme is fragile but costless to the seller. But a penalty scheme, in contrary is robust and costly to the seller. Designing a contract scheme which offsets this imbalance may be an interesting exercise.

(iii) Both the incentive and penalty schemes are explicit contracts since it require assumptions about the seller's potential of monitoring the activities of the auctioneer. Researchers may be interested in designing a corruption proof selling mechanism which does not make such assumption about seller's ability of monitoring.

(iv) We observed that the assumption of private valuation is crucial for the results that we have found. One can examine the effects of corruption when the values are affiliated.

(v) One can extend the basic model of corruption for the cases where there are multiple units for sale.

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