STUDIES ON UNDERSTANDING COMPLICATED MODELS

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C E R T I F I C A T E

The research work embodied in the Dissertation entitled "STUDIES ON UNDERSTANDING COMPLICATED MODELS" has been carried out in the School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi.

The work is original and has not been submitted in part or full for any other degree or diploma of any University.

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CONTENTS

1. INTRODUCTION

Modeling, with the advent of the digital computer, has become an important tool for understanding the complex social and economic systems. These models have been there in the past, but were predominantly mental models, which took into account only a few assumptions and interconnections.

These systems are characterised by multiple feed back loops and a non-linear behaviour of the parameters and a dynamic interaction of several parts of the system. The human mind can efficiently consider and understand only a few relationships at a time. As the complex models of social systems contain a large number of relationships and dynamic interactions, for a proper understanding of these systems, the aid of computer simulation and other computational techniques have to be taken (10).

Much of the modeling in this area has been undertaken at a regional level to aid the policy decision makers. It was the World 2 of Forrester and Club of Roma's World 3 model which attempted modeling at a global level. An overwhelming world attention has been focussed and much debate has been generated both for and against the underlying assumptions and conclusions about the W3 model - a particular world view which attempted to comprehend the entire system rather than just its single parts (9). It forecasts a world which our posterity would inherit , if proper policy decisions are not taken now, in which the industrial production has fallen to zero, the population, after crossing a sustainable limit, suffers a catastrophic decline due to hunger and malnutrition, and the air, sea and land are polluted beyond tolerable limits (9).

The World 3 model is a formal, mathematical model of a complex social system of the entire world. This model is a follow up of what is known as the World 2 model, which was developed at MIT under Prof. Jay W. Forrester. This model explored the nature and implication of physical growth in a finite world. The tools and techniques used are the systems analysis, which was developed by Jay W. Forrester at MIT, which was an outgrowth of more than 30 years of research. The system dynamics represents the complex multiple feed-back loops and relations pictorially and mathematically in terms quickly understood by everyone.

The W3 model analyses the implications, in a formal way using computer simulation, of the incessant quest for growth in, what they suppose to be, the finite world.

The world model was built specifically to investigate five major trends of global concern - accelerating industrialisation, rapid population growth, widespread malnutrition, depletion of non-renewable resources and a deteriorating environment. The World 3 model seeks to understand the causes of these trends, their inter-relationships and their implications as much as one hundred years in the future.

Accordingly the World 3 model consists of five interconnected sub models or sub sectors that were considered to be the key variables, they are -

1. Capital,
2. Non-renewable resources,
3. Persistent pollution,
4. Population, and
5. Agricultural sector.

A tentative job sector has been included to take into account the possible effects of extreme labor scarcity.

The W3 model assumes that there is a limit to material and population growth in the finite world. The underlying assumptions about the limits are the following (10).

1. The amount of potentially arable land that can be developed into actually cultivated land through the investment of capital is finite. As the stock of potentially arable land is diminished, the marginal cost of land development, measured in terms of capital and energy increases.

2. There is a limit to the amount of food that can be produced from each hectare of arable aland each year. This limit can be approached by investment in agricultural inputs such as fertilisers, pesticides and tractors. Eventually, however, there are diminishing returns to these inputs. The land yield limit can be decreased by very high levels of pollution or over-intensive cultivation, and it can be restored to its original value by investiment in land maintenance.

3. The stock of non-renewable resources in the earth is finite. The absolute limit of available resources is the entire mineral content of the earth's crust. However, long before that limit is reached, the marginal cost - in capital and energy - of extracting and processing each unit of resource will rise to prohibitive levels.

4. There is a limit to the rate at which environmental pollutions can be rendered harmless by

natural assimilating processes. Pollution levels can be kept below that limit by reducing the toxicity or quantity of industrial and agricultural emissions. However, pollution control requires investiment, which is subject to diminishing returns. Moreover, the rate of natural pollution absorption can be lowered by the pollutants themselves if they reach levels that interfere with the environment's assimilation mechanisms.

Basing on these underlying assumptions and taking the available data computer simulations were done and the Club of Rome reached the following conclusions (10) -

1. If the present growth trends in world population, industrialisation, pollution, food production and resource depletion continue unchanged, the limits to growth on the planet will be reached sometime within the next one hundred years. The most probable result will be a rather sudden and uncontrollable decline in both population and industrial capacity.

2. It is possible to alter these growth trends and to establish a condition of ecological and economic stability that is sustainable far into the future. The state of global equilibrium could be designed so that the basic material needs of each person on earth are

satisfied and each person has an equal opportunity to realise his individual human potential.

3. If the people of the world decide to strive for this second outcome rather than the first, the sooner they begin working to attain it, the greater will be their chances of success.

The philosophy and the assumptions, the equations representing the system and conclusions of the world 3 model have been published in technical reports of the club of Rome - The limits to Growth (9) and Dynamics of growth in a finite world (10). Since the publication of these reports several articles have been written, which discussed the global issues brought up by the club of Rome. The major objective of most of these studies was to examine the philosophy and assumptions underlying the modeling as also the conclusion drawn and policy recommendations made. It is possible that the conclusions reached with the help of a model may or may not be acceptable, inspite of the fact that the model incorporates powerful techniques of problem solving. Only a thorough study of the inner workings of the model can help in answering the question why the model behaves the way it does. Efforts to analyse the model's innerworkings to obtain a better understanding of the behaviour of the model, though important, have been almost meagre.

Understanding of the innerworkings of complex models like the world 3, with all the multiple feedback loops and dynamic interractions between various components of the model is very essential to make the exercise of modeling meaningful (5). For a good understanding of the innerworkings of any complex model, a thorough analysis is needed.

Among the few who investigated the innerworkings of the complex global models, Dutch project group, "Global Dynamics" is an important one. The major thrust of the studies of this group has been to show that a thorough analysis of the behavior of the model can lead to an understanding of its working. Thus facilitating one to draw better conclusions, than are otherwise usually done. The methodology of analysis of World 3 has been discussed by Thissen in a series of papers (15, 16, 17 & 18) and he has given broad guidelines or lessons for understanding complicated models such as the World 3 model.

The publication of urban dynamics (7) has ushered in an era of formal models of economic and social systems to aid the policy makers at all levels.

Even the regional models, that have been developed after Forrester, are highly complex with dynamic elements and numerous multiple feed-back loops. Hence, in order to comprehend the already existing models or develop a new model and derive maximum benefit out of it, a thorough understanding of the inner workings and the underlying structure is essential.

2. SOCIAL SYSTEM MODELS

Simulation models of economic and social systems consist of four well defined elements. These are components, variables, parameters and functional relationships (12).

Components of the models differ considerably depending on the purpose and system being simulated. For example in the club of Rome's World 3, the five sectors viz. capital, non-renewable resources, agricultural, population and persistent pollution are the components. Again each sector is simulated with several components. In the economic systems it depends on whether the system being simulated is an economy, an industry, a firm or some component thereof. Macro-economic models, to a large extent, have been simulated with major sectors such as the household business and government sectors as components.

The variables in the simulation models of economic and social systems are used to relate one component to another. These variables are further classified into exogenous variables, status variables and endogenous variables.

Exogenous variables are the independent or input variables of the model, which are assumed to have been predetermined and given independently of the system being modeled.

These variables act upon the system but are not acted on by the system. Exogenous Variables are of two kinds - controllable and uncontrollable. Controllable variables are those Variables or parameters that can be manipulated or controlled by the decision makers or policy makers of the system while non-controllable variables are those generated by the environment in which the modeled system exists and not by the system itself or its decision makers.

Status Variables describe the state of system or one of its components either at the beginning of a time period, at the end of a time period or during a time period. The Status Variables interact with both the exogenous and endogenous variables of the system according to the assumed funcional relationships of the system. The value of a status variable during a particular time period may depend not only on the values of one or more exogenous variables but also on the values of certain output variables in preceeding periods. Whenever a component takes its input from a portion of its own output from a previous period, a feed-back loop is said to occur. Such complex interactive feed-back loops are common in economic and social system models.

Endogenous Variables, on the other hand, are the dependent or output variables of the system and generated from the interaction of the system's exogenous and status variables according to the system's operating characteristics. For example, in the agricultural sector of the World 3 model, food F, persistent pollution generated due to agricultural output PP AO are the endogenous variables.

The classification of variables as exogenous variables, status variables and endogenous variables is done according to the purpose of the research.

The functional relationships describe the interaction of the variables and components in a model. These are identified and operating characteristics and are used to generate the behaviour of the system. Identities are either definitions or statements about the components of the model. For example in the agricultural sector of the World 3, land fertility is defined as the land yield without any modern inputs. An operating characteristic is a mathematical equation, which relates the system's endogenous and status variables to its exogenous variables. These equations usually are either a set of simultaneous equations or differential equations.

In view of the complex nature of the social and economic systems, with all the dynamic elements and the numerous feed-back loops, simulation is used to study these systems for the following important reasons (12).

1. Simulation makes it possible to study and experiment with the complex internal interactions of a given system, whether it be a firm, an industry, an economy, a social system or some subsystem of one of these.

2. Through simulation one can study the effects of certain informational, organisational and environmental changes on the operation of a system. This could be done by making changes in the components, variables, parameters or functional relationships of the system and observing the effect of these changes on the behaviour of the system.

3. In complex economic and social systems only a few among the many variables, describing the system, contribute to the actual behaviour of the model.

Simulation can be used to determine which of the variables are more important and how these interact with each other.

4. Simulation can be used to try out new policies and decision rules for operating a system before running the risk of experimenting on the real system.

5. Dynamic systems can be studied, using simulation, either in real time, compressed time or expanded time.

The foregoing discussion is about various social systems models and economic models undertaken at regional levels as opposed to the world level as in World 2 and World 3 and also the related work done in the field.

Most of these models are based on the guidelines of Urban Dynamics (7), which are developed to aid the policy makers. Some of the important models among these have been considered here.

In the early seventies a multi-disciplinary team at Michigan State University has developed a regional model to study the problems of agricultural sector development in Northern Nigeria (8). The chief motivation of the program has been to explore the usefulness of simulation as a tool for aiding policy makers of Nigeria, who are concerned with meeting the nutritional needs of the population, the generation of foreign exchange earnings and raw material inputs for industrialisation and other important questions relating to long term development of the agricultural sector and its interaction with non-agricultural development.

This model includes important decisions regarding allocation of resources to development programs in various regions of the country, interregional trade, the establishment of tariff and tax policies and the allocation of development resources to various public activities like research, agricultural education, and extension. At the macro-level the impact of agricultural policies upon the rest of the economy and vice-versa have been considered. And at a more micro-level, important policy decisions with regard to allocation of development resources to particular commodities and to particular development programs within regions of a country have been taken into account.

On the same lines as this model, the Southern Nigerian agricultural economy has also been modeled (1).

This model comprises of five major production enterprises viz. cocoa, palmproducts, rubber, annual non-food crops and staple foods. In this project a sensitivity analysis and a comparison with the real world data has been done as a part of the validation process.

Policy runs to test the simulated consequences and real world implications for the economy of policy alternatives such as marketing, board pricing, taxes and production campaigns are undertaken.

Making use of the generalised urban dynamics model of Forrester, a test model for Harris County Tex, an area of dynamic growth, has been constructed (13). The model was intialised with data from the year 1950. The results were seen to be closely tuned to the County's major statistical variables thus being helpful to policy makers. The time span of the model has been 200 years from 1950 onwards.

The most successful non-large scale econometric models have been those used for forecasting and policy analysis. The principal among these are the Wharton Quarterly econometric model of the US economy, the Brookings model, the Michigan model etc. (2).

All these models are moderately large systems of simulataneous equations. The equation system is dynamic and contains non-linearities which are essential for a realistic representation of the economy.

An economic model for simulating the economic behavior of the state of Oregon has been modeled (5). This is a continuous and dynamic macro-economic model, which views Oregon's economy as a small, open entity in a much larger US economy. The economy of Oregon, in this model, has been considered to be comprising of four interconnected submodels viz. 1) The wage structure and consumer demand submodel, 2) Natural resource submodel, 3) Capital formation submodel and 4) Dynamic production submodel.

In another model, an interactive system has been designed to study the spatial aspects of urban growth in Santa Clara county, San Jose, Calif. (4).

The objective of this modeling exercise has been to aid evaluation of local government policies to ensure that San Jose's future growth will proceed in an orderly, planned manner to achieve a balanced composition of industrial, commercial, residential and public uses which preserve and advance the quality of the existing environment.

This model includes regional forecasting models and the industrial location model. The regional forecasting models comprise of two interdependent models - an econometric model and a demographic model which are used to generate regional forecasts of employment and households and a projection of future population by age and sex and converts this population into labor force and households. The industrial location model projects industrial activity.

The study by James et al gives a method of optimising social system dynamics models (3). Forrester's world model has been taken as an example for demonstrating the methods. Parameter optimisation techniques and variational calculus have been used for optimising. The influence of these on the behavior of the world model has also been examined. The study brings out that the aforementioned optimising techniques generate additional information and insights regarding model behavior and hence contribute to model validity and utility. The paper further shows that analytical methodology can produce supplementary information, in the context of a specified value system. And this can be used in understanding and controlling a dynamic model of a complex social system.

A similar study of optimising of social systems has also been undertaken by Roger F Naill (11).

The Forrester's urban model is exceedingly complex and hence consideration of the features becomes very tedious and comprehension difficult. Also the model appears highly stable and insensitive to parameter changes. This latter fact of the model leads one to suspect that considerable reduction in the complexity of the model can be accomplished, without changing its behavior (14). Considering these facts, a greatly simplified version of Forrester's model of an urban area has been described by Michael Stonebraker (14).

This simplified model contains only 9 states and 81 equations as against 20 states and 150 equations in the original Forrester's model and leads to the following conclusions -

1. Deleting the delays and accelerators in Forrestor's model does not substantially alter the dynamics of response.

2. The same inferences can be drawn from both Forrester's model and also its greatly simplified version. Hence the additional detail in this instance is unnecessary.

Study of sensitivity analysis of social systems simulation has been done by Andew et al and a new measure of sensitivity for social system simulation models has been considered. In this project the sensitivity of a system dynamics model of energy boom town has been considered with New Mexico as an example (6).

Apart from the more important social system and economic models mentioned here, numerous, but less important models on a smaller scale have been developed by various teams and organisations on the guidelines provided by Forrester in urban dynamics.

3. REFORMULATED WORLD 3 MODEL

In this chapter we discuss briefly the various sectors of the World 3 as reformulated by Thissen along with the interactions between the different sectors. The World model consists of five interconnected sub modles which describe what are considered to be the key variables in the capital, non-renewable resources, persistent pollution and agriculture sectors. A tentative job sector, to take into account the possible labour shortages is also considered. A standard or reference run is considered. It is that in which many of the co-efficients were set to such values that a good agreement was obtained between the model's results and what is known about reality between 1900 and 1970.

The behavior of the standard run is as shown in the fig. 3-1. The list of all symbols used and their meaning are given in the appendix.

Most of the Variables, initially display growth. The capital and population reach a peak in the middle of the 21st century, which is followed by a decline that is mainly caused due to scarcity of resources. This decline continues beyond the year 2100.

In the investigations carried out by Thissen, as capital and resources sectors interact heavily with each

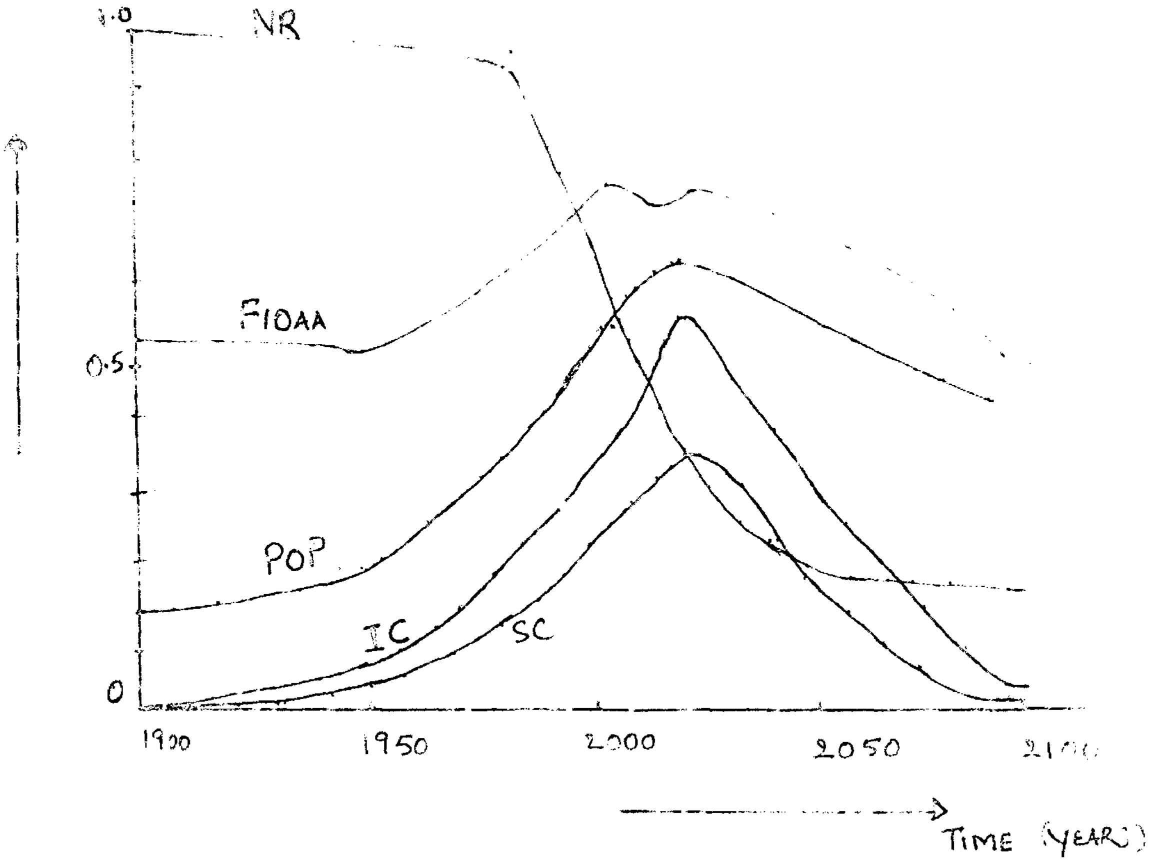


Fig 3.1

STANDARD-RUN BEHAVIOR OF WORLD3 MODEL

SCALE:

NR = 10^{12} IC = $1.5\ 10^{13}$

FIOAA = 0.20 SC = 1.10^{13}

POP = 10^{10}

other under standard run and similar conditions they are considered as a single sector (18).

Hence the model, in Thissen's analysis, is decomposed into a division of four sectors:-

1. Capital and resource,
2. Agriculture,
3. Pollution, and
4. Population.

The job sector has been completely left out of consideration since the job sector only influences the model behavior under a few very specific circumstances.

The interaction between the four submodels or sectors of model is of the pattern shown in the fig. 3.2 (18).

In the model the capital and resource sector is the most important one and affect all other sectors. This is done via industrial output IO. The population sector is affected by the capital and resources sector via service output so. The influences exerted on the other model components by the population sector is via the size of total population POP. Similarly the influences exerted by the agriculture sector on the capital sector and the pollution sector are via total

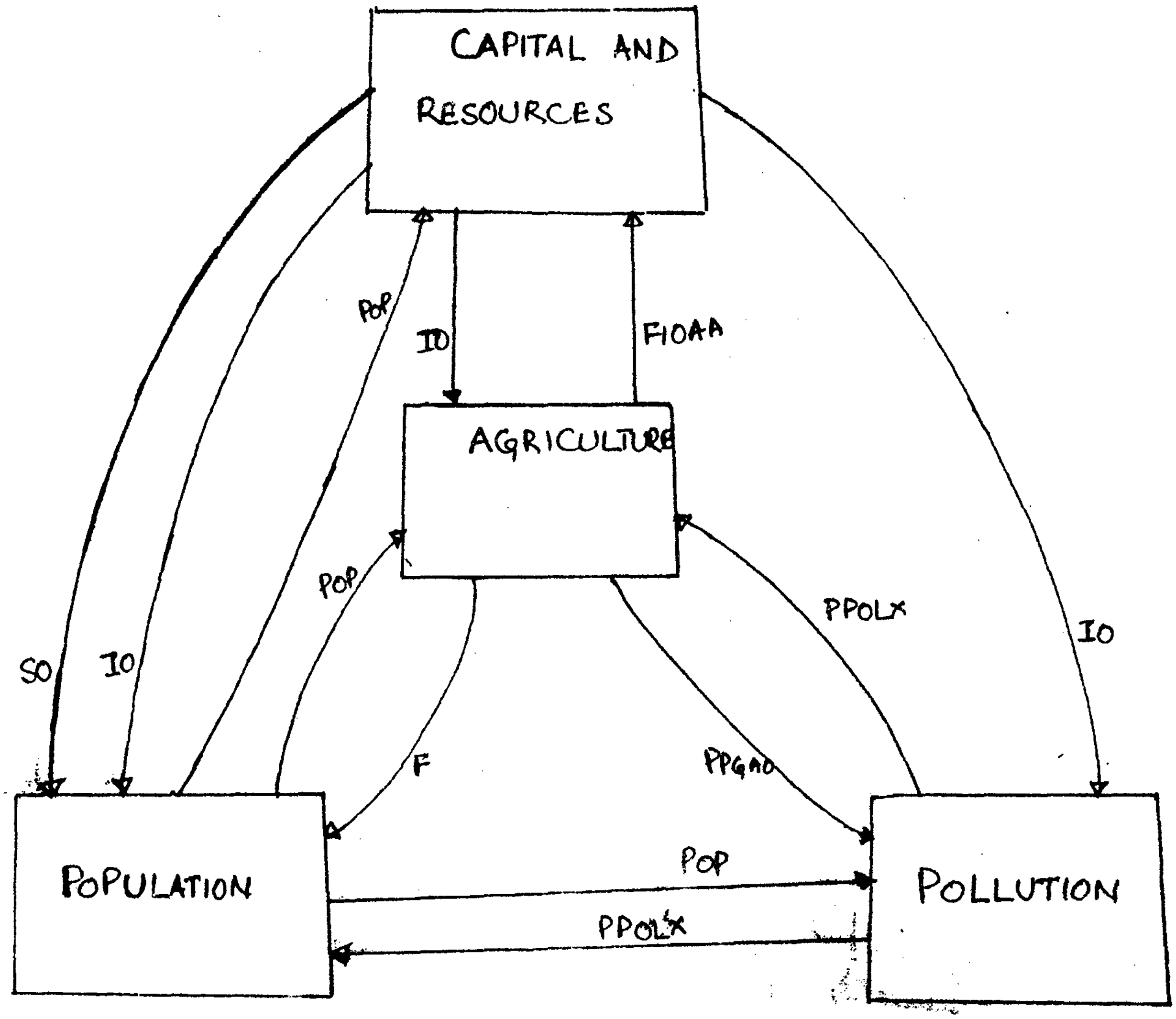

CAPITAL AND
RESOURCES
POP
IO
FIOAA
AGRICULTURE
POP
PPOLX
SO
IO
IO
F
PPGAO
POPULATION
POP
POLLUTION
PPOLX

Fig 3.2

food production F, and the persistent pollution generated by agricultural output PPGAO. The persistent pollution sector affects the population and agricultural sectors via PPLOX, the index of persistent pollution is the persistent pollution relative to 1970. A list of all symbols and their meanings is given in the appendix.

OUTLINE OF THE VARIOUS SECTORS

1. CAPITAL AND RESOURCE SECTOR

The capital and resource sector is the most important sector of the entire model. This sector imposes its behavior of growth followed by decline on all other sectors and the assumptions that lead to the overall behavior of the model are contained in this single sector (15). A list of the equations of the submodel, which are written in state space notation are given below:

Equations of capital and resource sector of World 3
State equations:

$\dot{IC}$ = FIOAI * IO - IC/ALIC (• means derivative with respect to time.)

$\dot{SC}$ = FIOAS * IO - SC/ALSC

$\dot{NR}$ = -NRUF + PCRUM * POP

Initial Conditions:

IC(1900) = $2.1 * 10^{11}$ (dollors)

SC(1900) = $1.44 * 10^{11}$ (dollors)

NR(1900) = $1.0 * 10^{12}$ (dollors)

Coupling Equations:

FIOAI = 1-FIOAC-FIOAA-FIOAS=U-FIOAS

IO = IC * CUF * (1-FCAOR)/ICOR

IOPC = IO/POP

SO = SC * CUF/SCOR

SOPC = SO/POP

NRFR = NR/NRI

Table functions:

ISOPC = f_{is} (IOPC)

FIOAS = f_{fi} (SOPC/ISOPC)

PCRUM = f_{pc} (IOPC)

FCAOR = f_{fc} (NRFR)

Constants:

ALIC = 14 years

ALSC = 20 years

FIOAC = 0.43

ICOR = 3 years

NRI = $1.0 * 10^{12}$ resource units

NRUF = 1

SCOR = 1 years

Input Variables from other sectors:

POP (population sector)

FIOAA from agricultural sector

CUF from job sector.

In the capital and resource submodel the total amount of Industrial Capital (IC) is adjusted for investments and depreciation at each time interval. The industrial capital output ratio (ICOR), the capital utilization fraction (CUF) and the co-efficient (1-FCAOR) taking into account possible resource shortages, the industrial capital are calculated from the value of industrial capital. And the amount of output produced is divided into four parts, each being allocated to different purposes. A fraction FIOAS for investments in service capital, a constant fraction FIOAC for consumption, a fraction FIOAA for investments in agriculture and the remaining part (FIOAI) for investments in industrial capital. Both FIOAA and FIOAS are calculated in such a way that the model will behave in accordance with the development pattern, which gives the ratio between industrial, service, and agricultural output as a function of income per head.

The depreciation of industiral and service capital is modeled in the form of an exponential decay mechanism as represented in the fig. 3.2. The rate at which non-renewable resources (NR) are depleted is taken to be proportional to the product of the number of people (POP) and the usage rate per head (PCRUM), which in turn, is a function of industrial output per head (IOPC).

The complete scheme of the equations of this sector is given in the fig. 3.3 below, with the three exogenous variables POP, CUF and FIOAA represented in U which equals (1-AIOAC-FIOAA) are underlined.

2. AGRICULTURAL SECTOR

In the agricultural sector the global production of food in response to economic development, population, growth and persistent pollution is described (16). The equations of the sector, described in state space notation, are those given below.

Equations of the agricultural sector

State equations:

1. $\dot{AL} = LDR - LRUI - LER$
2. $\dot{PAL} = -LDR$
3. $\dot{UIL} = LRUI$

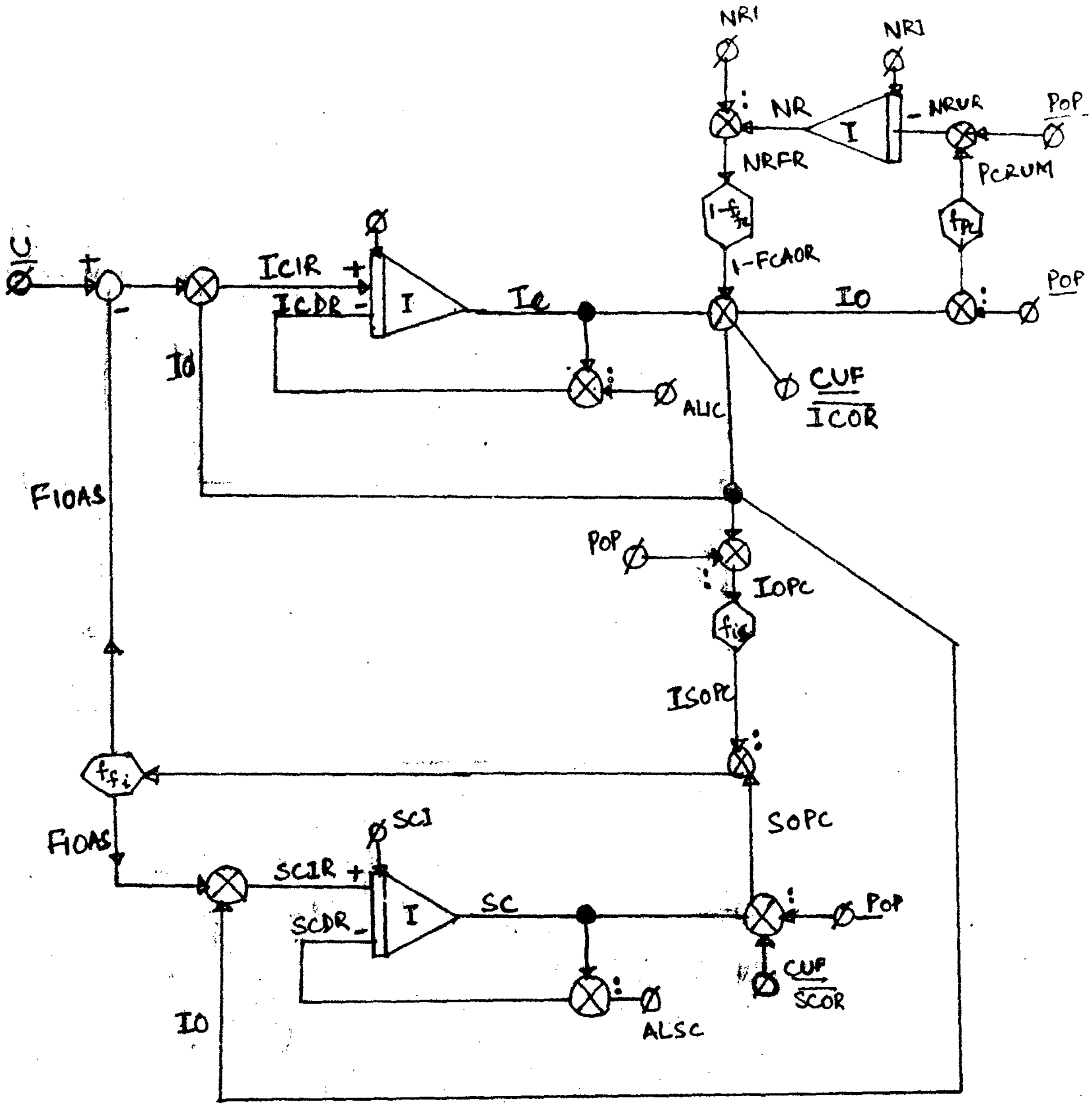
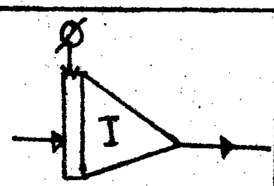
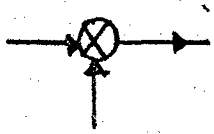
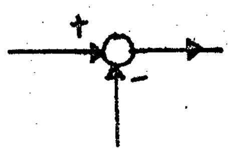


FIG 3:3.

ANALOG FLOW DIAGRAM OF COMPLETE SET OF SECTOR EQUATIONS.

	INTEGRATION WITH RESPECT TO TIME
	MULTIPLICATION, OR, IF A COLON (:) IS ADDED, DIVISION.
	ADDITION OR SUBTRACTION
	(table)- FUNCTION.

EXPLANATION OF THE SYMBOLS USED IN FIG 3.3.

4. $\dot{LFERT}$ = (600-LFERT)LFRT-LRERT/LFDR

5. $\dot{AI}$ = (CAI-AI)/2

6. $\dot{PFR}$ = (FR-PFR)/2

Coupling equations:

LDR = FIALD * TAI/DCPH

FIALD = f_{108}(MPLD/MPAI)

MPLD = LY/(DCPH*0.07)

LY = LFERT*LYMC = LFERT*f_{102} (AIPH)*f_{106}(IO)

AIPH = AI*(1-FALM)/AL

FALM = f_{126}(PFR)

DCPH = f_{97}(PAL/PALT)= f_{97}(PAL/3.2x10^9)

MPAI = 2*LY*MLYMC/LYMC = 2*LY*f_{111} (A1PH)/f_{102}(AIPH)

TAI = FIOAA * IO

FIOAA = f_{94}(FPC/IFPC)

FPC = F/POP

F = LY*AL*0.63

IFPC = f_{90}(IOPC) = f_{90}(IO/POP)

LRUI = MAx {(POP*f_{117}(IO/POP) -UIL)/10) }

LER = AL/ALI = AL/(6000*f_{114}(LY/600))

LFRT = f_{125}(FALM)

LFDR = f_{122} PPOLX

CAI = TAI*(1-FIALD)

FR = FPC/230

Exogenous inputs:

IO from capital sector

POP from population sector

PPOLX from persistent pollution sector

Initial conditions:

AL (1900) = 0.9×10^9 (hectares)

PAL (1900) = 2.3×10^9 (hectares)

UIL (1900) = 8.2×10^6 (hectares)

LFERT (1900) = 600 (vegetable equivalent kilogram/hectare)

AI (1900) = 5×10^9 dollars/year

PFR (1900) = 1

The structure of the sector with only the major couplings is as shown in the fig. 3.4.

In the agricultural sector Industrial Output IO and population POP affect the allocation part of the sector and also determine the land removal for urban-industrial use, affecting the area of arable land. Persistent pollution influences land fertility via the index of persistent pollution PPOLX.

Food production F, acting on the population sector and the fraction of industrial output allocated

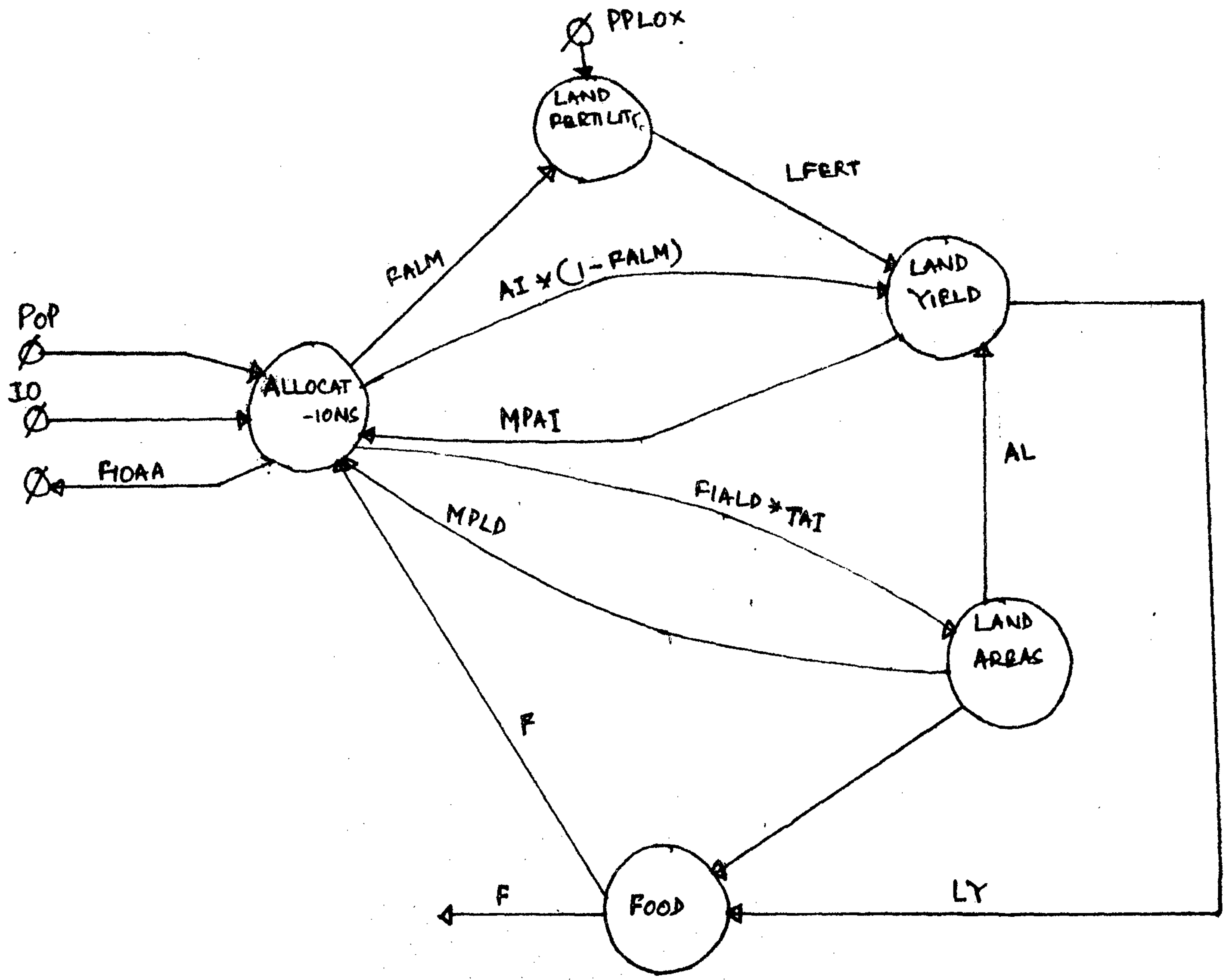

PPLOX
LAND FERTILITY
LFERT
FALM
AI * (1 - FALM)
LAND YIELD
PoP
IO
FIOAA
ALLOCAT -IONS
MPAI
AL
FIALD * TAI
MPLD
LAND AREAS
F
F
FOOD
LY

FIG 3.4

to agriculture (FIOAA), which directly influence the rate of growth of industrial capital are the two most important output variables of the sector. Persistent pollution is affected via the persistent pollution generated by agricultural output PPGAO.

The allocation decisions part plays a crucial role in the agricultural sector. FIOAA is computed from the ratio between indicated food per capita IFPC, which is a function of industrial output per capita IOPC, and food per capita. Total agricultural inputs TAI given by the product FIOAA and IO are allocated to three different purposes as shown in the fig. 3.5. The allocations are to:-

1) The development of new land;
2) Land maintenance
3) Increase in land yield by means of equipment, fertilizer, pesticides etc.

The following scheme has been adopted in deciding the allocations. The fraction of inputs allocated to land development FIALD is computed from the ratio between the marginal productivities of agricultural inputs MPAI and of land development MPLD. MPAI is a function of the marginal increase in land yield apart from other things and MPLD depends on the actual land yield and on development costs per hectare. A lag between CAI (equal to (1-FIALD) *TAI)&AI exists which has a

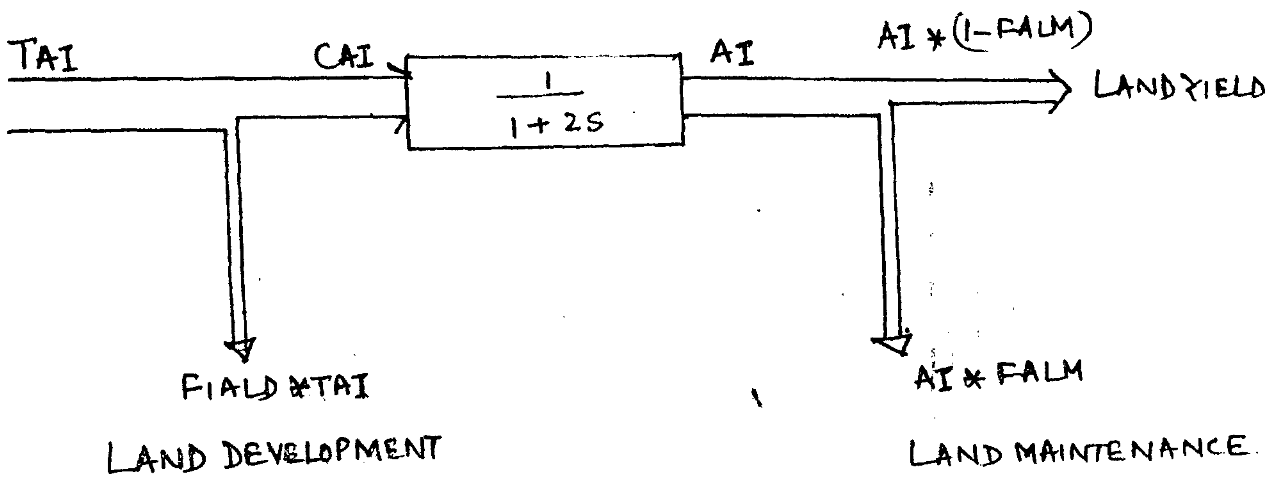

TAI
CAI
1
1 + 2S
AI
AI * (1-FALM)
LANDYIELD
FIALD *TAI
LAND DEVELOPMENT
AI * FALM
LAND MAINTENANCE

Fig 3.5

gain equal to one and a time constant of two years. AI is divided into two parts with one fraction FALM, depending on the percieved food ratio PFR, being allocated to land maintenance and the remaining part to the increase in land yield.

Land yield is the product of land fertility LFERT (defined as the land yield without any modern inputs), the land multiplier from air pollution LYMAP (a function of IO) and the land yield multiplier from capital LYMC, which is a function of AI per hectare (equal to AI*(1-FALM)/AL).

Total food production F is the product of arable land AL, of land yield LY and of two constant factors taking into account the land fraction harvested (0.7) and the effects of processing losses (0.9).

The rate of decrease of land fertility LFERT, which is modeled as a state variable depends on the persistent pollution index PPLOX and LFERT itself. Land fertility regeneration rate LFRT is determined by the fraction of inputs allocated to land maintenance FALM.

The use of land is described by the potentially arable land PAL, arable land AL, and urban-industrial land, which are the state variables. The rate of change of these state variables depends on the land development rate LDR (a function on FIALD * TAI and of development costs per hectare DCPH), on the land erosion rate LER which determines the average land life (depending on AL and on land yield LY) and on the land removal for urban industrial use (a function of IO, POP and UIL).

3. POPULATION

The population sector fulfills the following functions -

1) Calculates the total population size, and
2) represents the demographic response of the population, through the birth rate and the death rate, to the changing resource supply.

The population sector is coupled with all the other subsystems and affects them through the variable POP, the total number of people. And the IO industrial output, service output SO, food production F and the persistent pollution ratio PPLOX influence the population sector (17).

The population sector is distinguished into three basic components which are coupled as shown in the fig.3.6.

In the subsystem I total fertility TF and life expectancy LE are calculated from PPLOX and POP and from

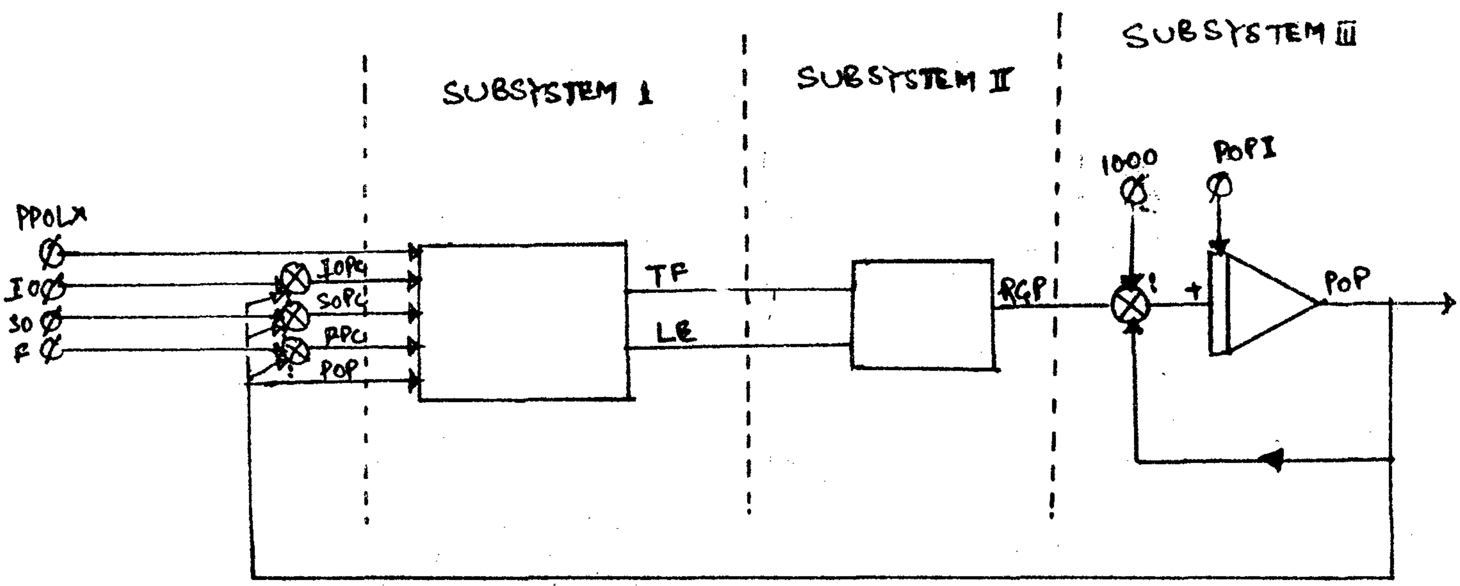


FIG 3.6

BLOCK DIAGRAM OF WORLD 3 POPULATION SECTOR WITHOUT AGE DISTRIBUTION.

the percapita values of IO, SO and F. The subsystem consists of lags, which are linear first-or-third-order systems with a static gain equal to 1, and non-linear algebraic relationships. The algebraic relations consist of arithmetic operations viz. multiplications, divisions, etc. and table functions.

In the subsystem II the net birth rate NBR, which is the net rate of increase in total population per 1000 people per year, is calculated from the values of LE and TF. And the subsystem III represents the calculation of the number of people POP according to the equation

$$\frac{d\ POP}{dt} = (NBR/1000) * POP$$

The feedback of POP to subsystem I completes the overall scheme. The per capita values of IO, SC and F act as the inputs to sector.

A model with four age groups has been used for simulations.

The equations of World 3 POP subsystem are the following -

<u>State equation</u>

POP = B-D

Initial condition

POPI = POP (1900) = 1.6×10^9

Coupling equations

D = POP / LE

LE = $28 * f_{25}$ (EHSPC)* f_{20}(FPC/230)

*f_{29}(PPOLX) * LMC

LMC = 1-f_{27}(IOPC) * f_{26}(POP)

SOPC = SO/POP

FPC = F/POP

IOPC = IO/POP

B = POP * 0.21 * TF/30

TF = MIN(MTF,MTF*(1-FCE)+DTF*FCE)

MTF = 12 * f_{34}(LE)

FCE = f_{45}(FCFPC)

PLE = DLINF3(LE,20)

<u>INPUT Variables</u>

IO Capital Sector

SO Capital Sector

F Agricultural Sector

PPOLX Persistent pollution sector.

4. PERSISTENT POLLUTION SECTOR

The persistent pollution sector in World 3 describes the generation, appearance, presence and assimilation of those pollutants that may be transported all over the world and that may be harmful to the global biosphere for some significant amount of time after their generation. The aggregate effects of the many different pollutants which may be generated are described by one index of pollution in the model (16).

In the persistent pollution sector there are two pollution generation factors. One is the persistent pollution generation from industrial output (PPGIO), which is contributed by the industrial activity. The other is the persistent pollution generation from agricultural output (PPGAO) which is due to the agricultural activity. PPGIO and PPGAO are proportional respectively, to the usage rate of resources and to net agricultural inputs. Each of the two sources produce about half of the total pollution generation PPGR (= PPGIO + PPGAO) for 1970. PPAPR persistent pollution appearance rate is formed as a lagged effect of PPGR with an overall time constant of 20 years. The total amount of harmful pollution present in the biosphere (PPOL) is calculated by integration of the difference between PPAPR and the persistent pollution assimilation rate PPASR.

$$\dot{PPOL} = PPAPR - PPASR$$

PPASR is a function of PPOL itself and of the assimilation of half-life AHL.

$$PPASR = PPOL/(AHL*1.4)$$

AHL increases with PPOLX according to

$$AHL = \begin{cases} 1.5+0.06*(PPLOX-1) & \text{if } PPLOX \quad 1 \\ 1.5 & \text{if } PPLOX \quad 1 \end{cases}$$

PPOLX is the only variable of this sector which affects other sectors. The population sector is effected via the impact on life expectancy and the agricultural sector via the influence on land fertility degradation. For an understanding of behavior and the underlying structure of the various sectors and the complete model of World 3 Thissen has adopted a set of simplifications and strategies.

4. SIMPLIFICATIONS AND STRATEGIES FOR UNDERSTANDING COMPLICATED MODELS

For a better understanding of complicated models various simplifications and strategies have to be used. In this chapter we discuss the various simplifications and strategies that have been adopted in the analysis and understanding of the World 3 model taking examples from all the sectors.

The sectors have been analysed for studying both standard run behavior and non standard run behavior. Standard-run behavior is that in which the values of variables have been set to such values as to reproduce the reality known for the period 1900 to 1970 (18).

For analysing the standard-run behavior the following important strategies apart from others have been used in exploring the relations actually playing an influential part during the sector's standard run simulation.

a) Observation of the range of behavior of variables.

b) Freezing of input variables.

c) Sensitivity analysis.

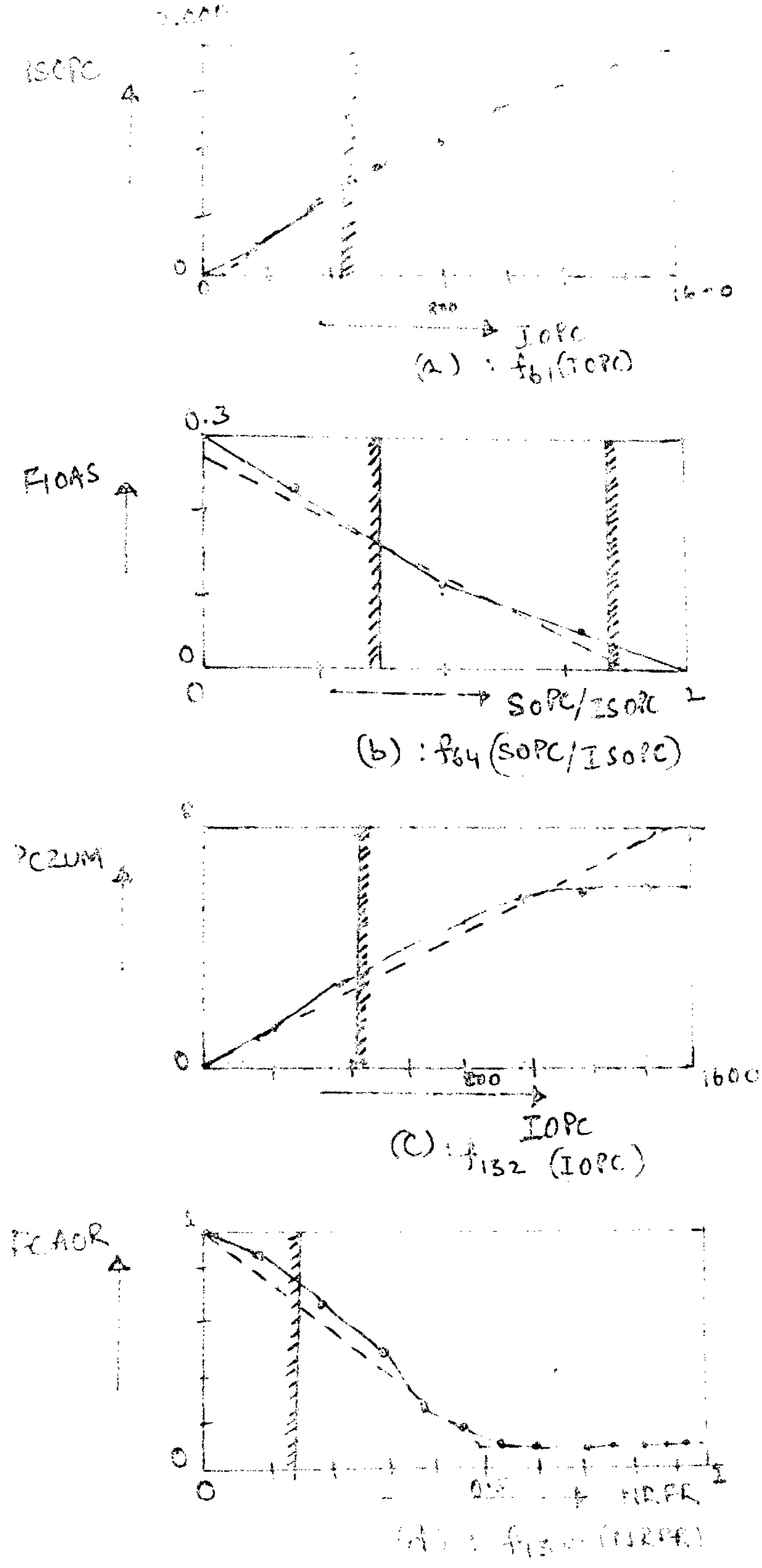
d) Simplification of sets of equation.

1. Observation of the range of behavior of variables

It was found that each specific type of behavior of World 3 is primarily determined by only a fraction of all the assumptions and equations. And further it was found that a number of variables do not influence the behavior considerably and hence may be replaced by a constant value without affecting the overall outcome.

This aspect of a number of variables which do not affect the behavior can be seen in all the sectors. In the capital and resources sector of World 3 CUF which was found to differ from 1 only slighly during the last part of the run (from 2070 to 2100) and is exactly equal to 1 during the preceding part is assumed to be equal to 1 for standard run and similar conditions. This allows eliminating one of the three input variables of the sector (15).

An examination of what part of the table functions are used during calculations and what their shapes are as shown in figures 4.1 a,b,c,d were also found to introdunce a lot of simplification in the analysis. For values of IOPC below 400 the function ISOPC and PCRUM were approximated by expressions which are linear in and proportional to IOPC.


ISOPC
0
IOPC
(a) : f61(IOPC)
0.3
FIOAS
0
SOPC/ISOPC
(b) : f64 (SOPC/ISOPC)
PC2UM
0
800
1600
IOPC
(c) : f132 (IOPC)
FCAOR
0
NRFR

$$PCRUM \quad apc * IOPC, \; apc = 0.0053$$

and

$$ISOPC \quad a_{is} * IOPC , \; a_{is} = 1.67$$

This approximation was made without introducing any difference in the model outcome, while leading to the hypothesis that the size of population has no very pronounced effect as long as only the nearly linear interval of both table functions is used as shown in the fig. 4.1. This also leads to the conclusion that the only variable playing a part is FIOAA. Apart from these conclusions the subsystem's flow diagram was also simplified to a considerable extent.

In the agricultural sector, during the whole standard-run simulation, on account of the fact that IO does not exceed ten times its 1970 value, land yield multiplier from air pollution LYMAP being constant was set equal to 1.

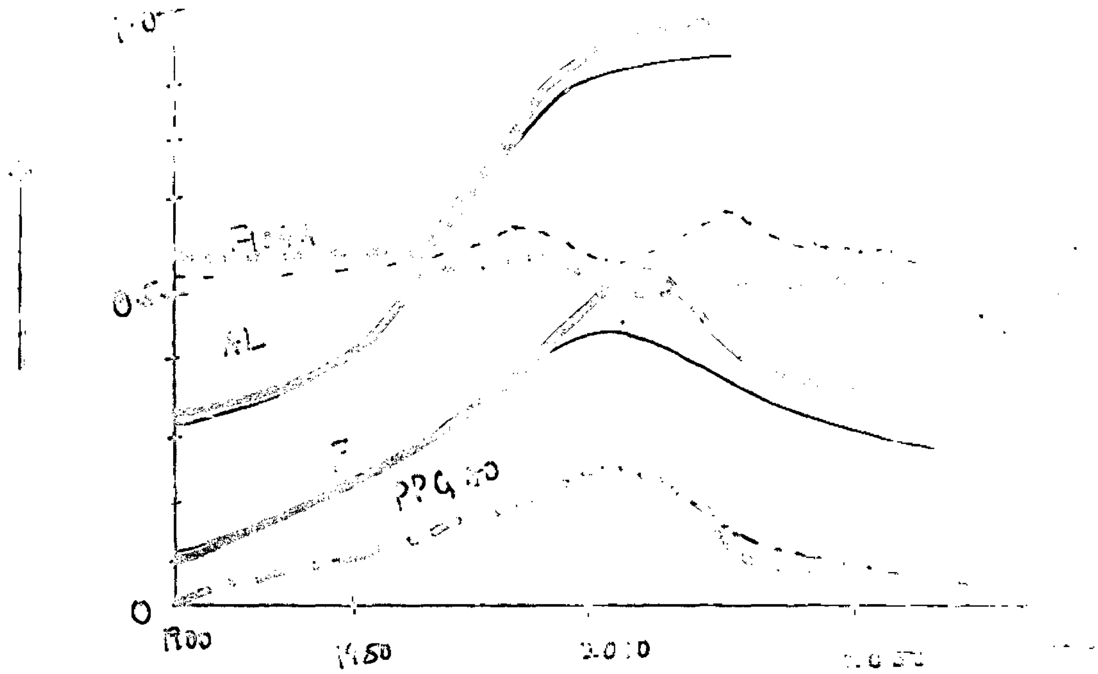
Land erosion rate was neglected, by setting LER to zero. This was found to have a very light affect on the behavior of AL and virtually no influence on the sector's output variables for standard-run and similar conditions (16).

Land removal for urban industrial use LRUI was set to zero, omitting all influences of the urban industrial land subsector as shown in the fig. 4.2, on the other model variables. This assumption was also found to have introduced no significant changes in the sector's overall standard-run behavior. LFERT was like-wise set to a constant value (570) without affecting the overall results. These can be seen in the figure.

In the population sector the values of MTF, was set equal to 8.5 and MPLE = 1.30 and SFSN=1.06, without affecting the behavior of population. Also, since the influences of fertility control effectiveness FCE and of crowding are fairly weak, the values of FCE and LMC were substituted by the constant values 0.90 and 0.96 respectively. However, substitution of these values, in addition to the constant values of MTF, PLE and SFSN was found to have an affect on POP in a more marked manner (17). These affects are as shown in the fig. 4.3.

Freezing of input variables

Freezing the values of state variables from a certain point in time onwards is an important tool in enhancing the comprehensibility of a complex model.



Scale LFERT : 0 - 500

PPGAO : 0 - 5 * PPOLAO

FIOAA : 0 - 0. .0 , AL . 0 - 2 . 10

F : 0 - 5 10^{12}

THIN LINE : STANDARD RUN

HEAVY : LYMAP = 1, LER = LRUI = 0, FIALD . 0. ,

LFERT = 570 AND FALM . 0.05.

FIG 4.

Standard-run behavior (thin line) compared to behavior if effects of air pollution, land erosion and land removal for urban-industrial use are completely ignored, and if fractions of investments allocated to land development and land maintenance, and land fertility are kept constant (heavy line).

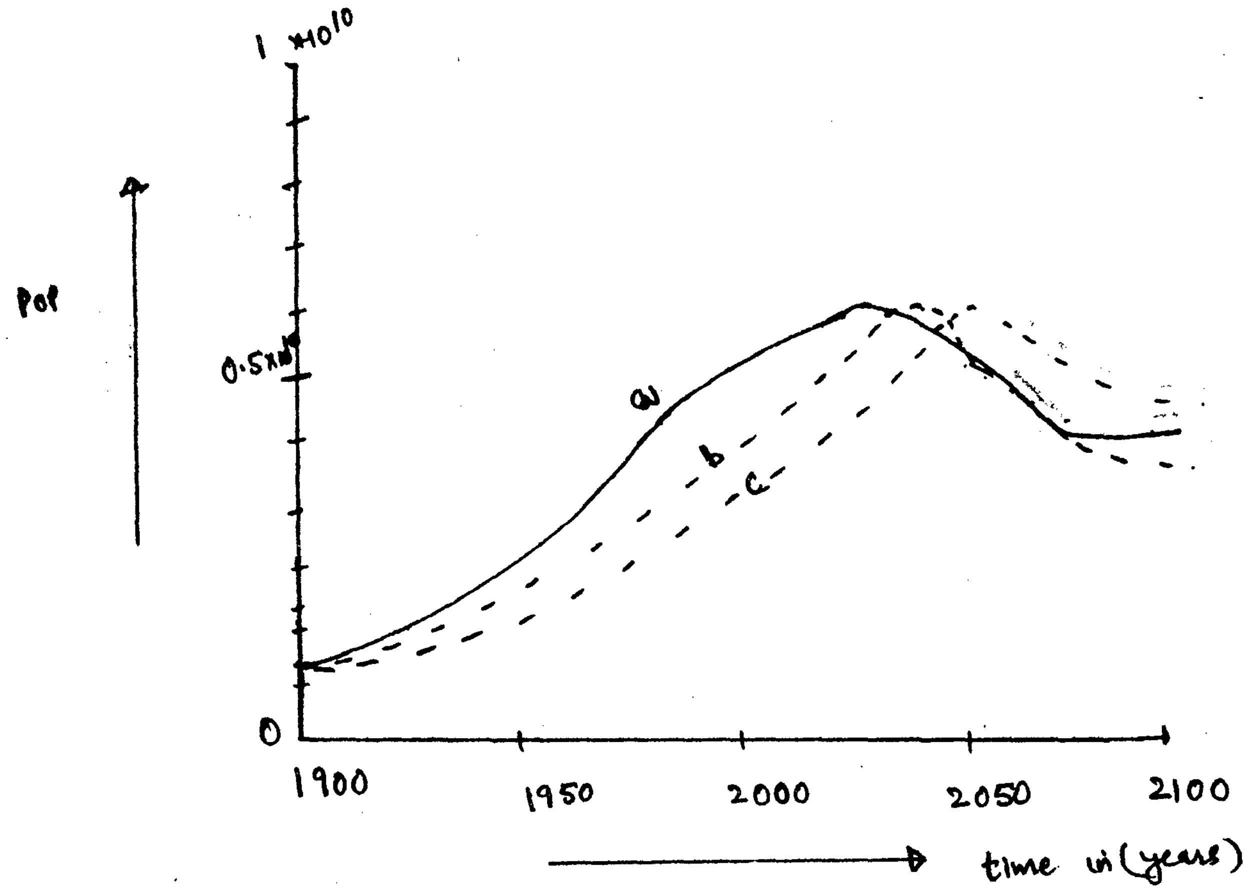


Fig 4.3.

The result of freezing the value of a state variable is to cut off all the dynamic influences acting via the variable under consideration. The significance of these influences can be brought about by comparing the resulting behavior due to freezing of the variable with that of the original model, using this technique information regarding the heirarchy of state variables may also be acquired. This technique has been applied in a demonstration of the model's three level hierarchy for standard-run conditions.

In the analysis of World 3 by Thissen, the examples of employing freezing of input variables to enhance the comprehensibility of the model can be seen in the various sectors.

Input variables, in the agricultural sector, have been frozen in the analysis showing that they affect the model outcome very significantly.

Freezing of FIALD at 0.19, which otherwise normally varies between 0.003 and 0.26, was found to affect the standard run behavior in a minimal way (16).

The freezing of FALM, the fraction of inputs allocated to land maintenance at 0.06, which otherwise varies between 0.04 and 0.07, was found to have no affect on the standard run behavior.

Similarly PPLOX, being frozen at a value less than 10 was shown to cause no considerable changes in behavior. And the behavior of F, AL and pollution generation from agricultural output PPGAO was not affected by freezing all influences of population POP on the agricultural sector.

On the contrary, the growth and decline mode of F was eliminated by freezing all influences of IO on the agricultural subsystem. However this was found not to affect the behavior of FIOAA. All these results can be seen in the fig. 4.4.

For finding out which out of the many relations essentially determine standard-run behavior, in the simulations made to study the population sector, freezing of variables and function, apart from other modifications, was used.

A conclusion that pollution does not influence population in any manner is arrived at, freezing PPLOX from 1970 onwards. This is because PPLOX remains lower than 12 in a standard run and hence life time multiplier from persistent pollution, LMP deviates less than 2 percent from unity (16).

It was found that the rate of population growth and decline reduces on account of freezing IO and SO or F from 1970 onwards.

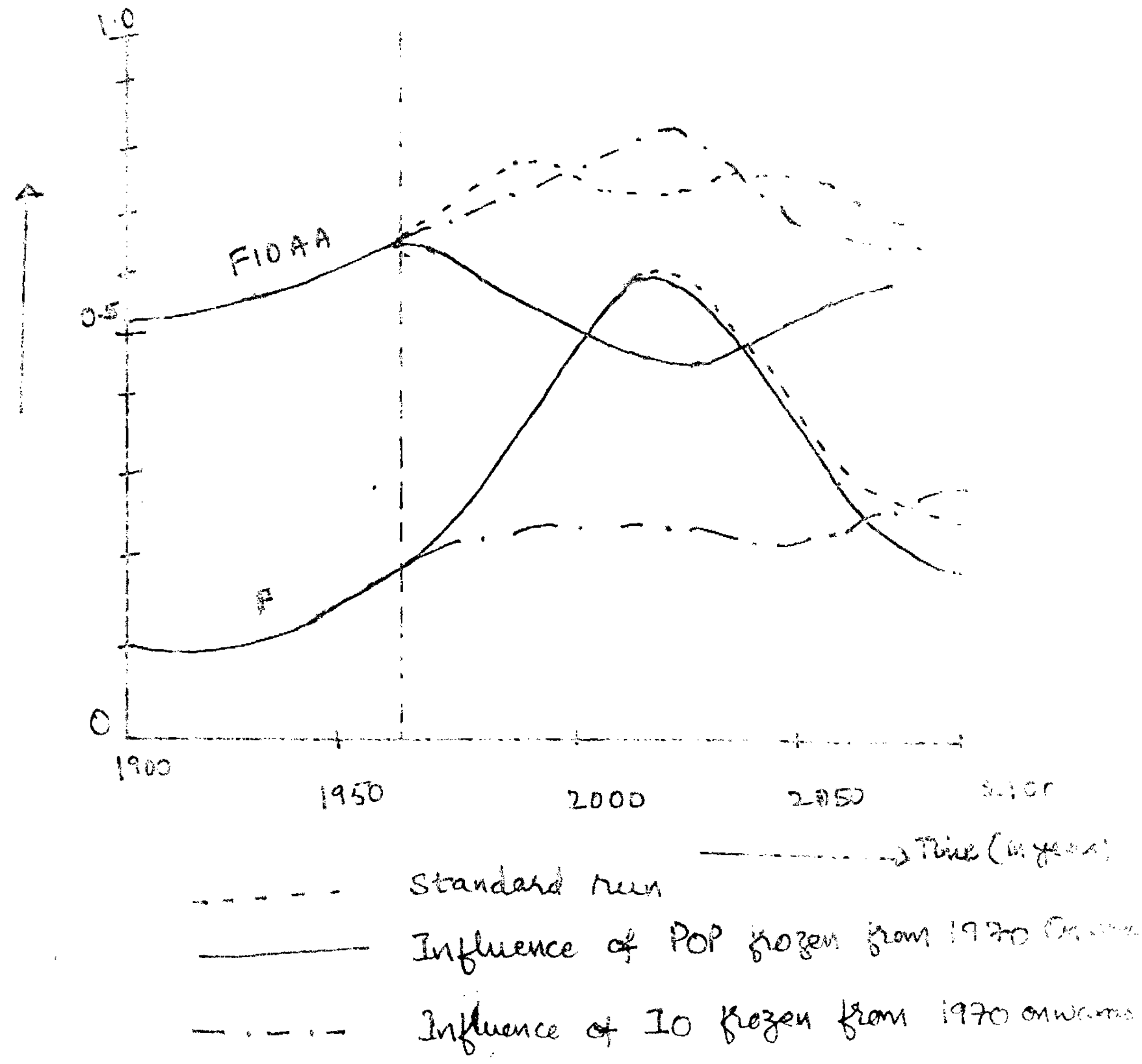

1.0
0.5
0
FIOAA
F
1900
1950
2000
2050
Standard run
Influence of POP frozen from 1970
Influence of IO frozen from 1970 onwards

Fig 4.4.

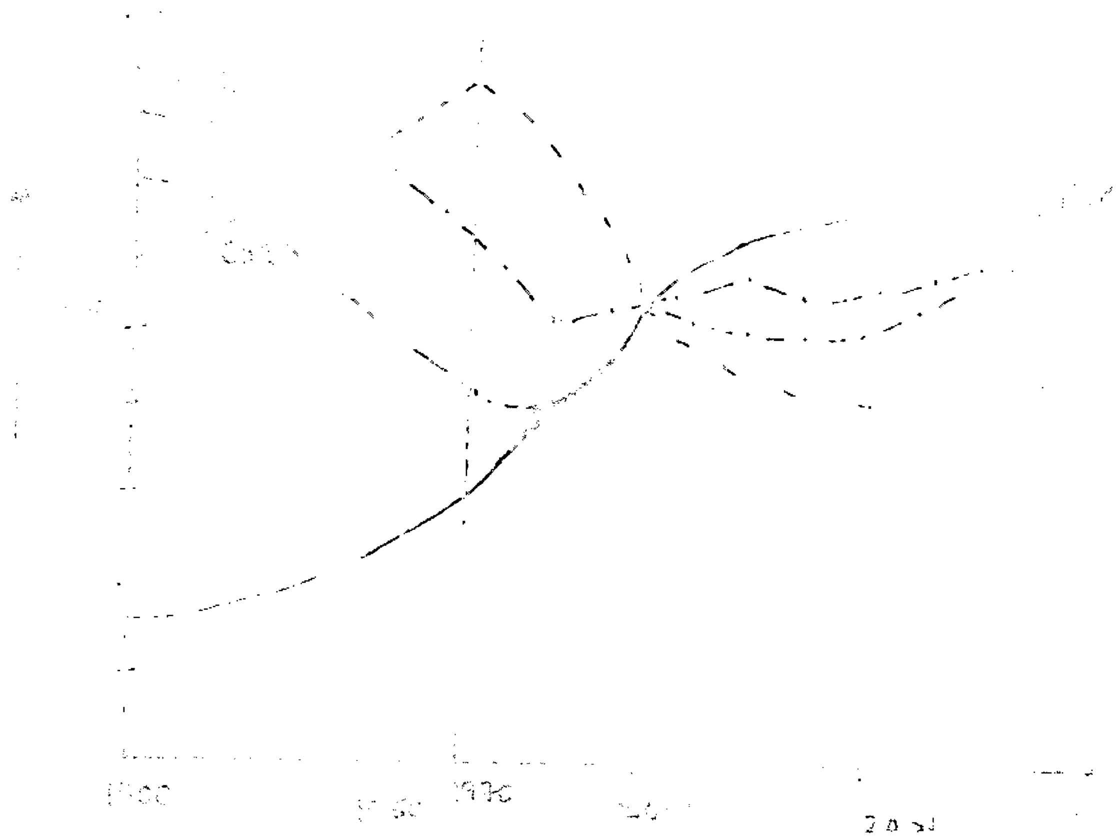
Once all the four input variables were frozen from 1970 onwards, it was seen that the decline in population was fully eliminated. From this observation three conclusions were arrived at -
that the

a) growth and decline are not determined endogenously but are impressed on the sector by the behavior of the inputs IO, SO, and F;
b) Since, in the final situation, food supply is near the subsistence level, the conditions under which population equilibrium is reached are unattractive; and
c) The time constant (larger than 100 years) with which equilibrium is approached, indicates that the sector responds slowly to changes in the inputs.

All the affects of freezing variables from 1970 onwards can be seen in the fig. 4.5.

Sensitivity analysis

Sensitivity tests are used to gain an understanding of the uncertainity of the model's results, without having the need to perform extensive calculations. This can be achieved by selecting extreme cases i.e. choosing a set of representative variations in which the affects reinforce each other.



SCALE

POP = 0 – 10^{10} [illegible]

CBR – 0 – 50   [illegible]

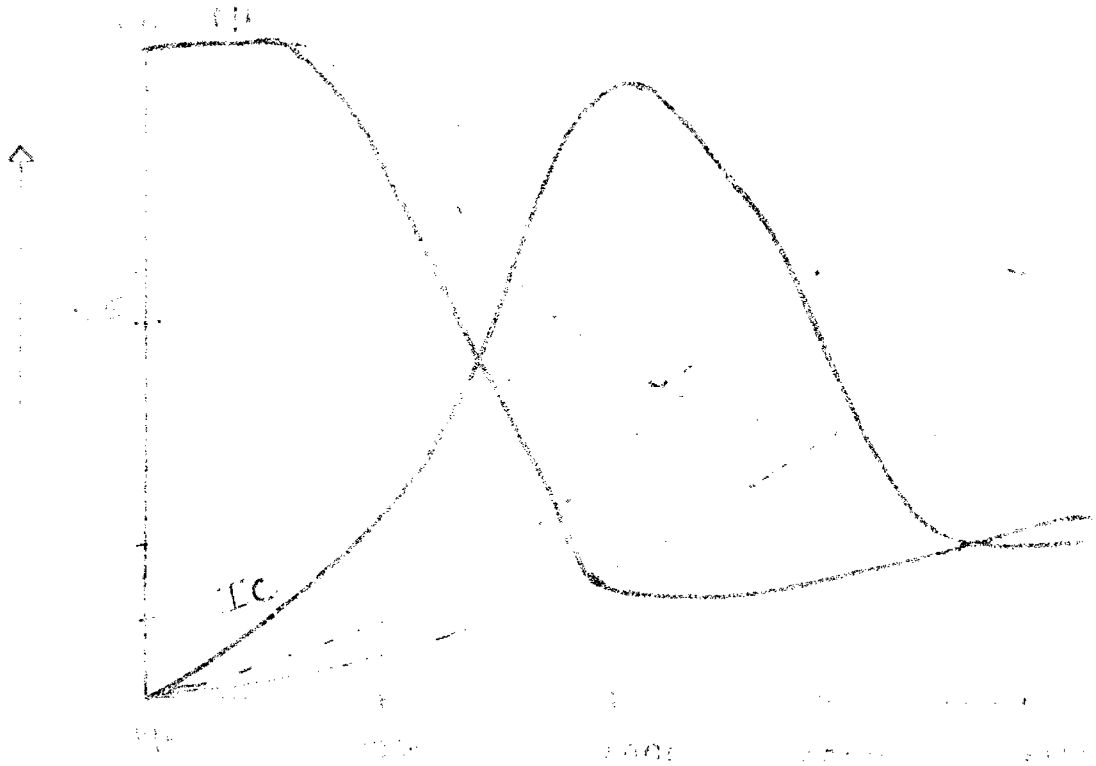
Fig 4.5

The sensitivity tests have been used in the capital and service sector to determine whether the effects of simultaneous changes in several parameters will reinforce or counteract each other and hence to predict what will be the outcome. This was done by gaining insight into the effects of separate variation of each of the parameters.

After identifying a list of parameter groups, which have been reproduced below, the results arrived at by performing sensitivity analysis are the following:

1. 1-FIOAA-FIOAC
2. b_{fi}
3. CUF
4. (1-FCAOR)/ICOR
5. ALIC
6. af_i/a_{is} * SCOR
7. ALSC
8. a_{pc} * NRUF

That the model's outcome was very sensitive to the value of the parameters of group 1 and to a similar degree for the groups (1-FCAOR)/ICOR, CUF and ALIC. On the other hand the variations in other parameters was found/to show no effect on the behavior of the model (16). This can be seen in the fig. 4.6.



SCALE VALUES

4. Simplification of sets of equations

For an easy and better understanding of the model, simplifying sets of equations is an important aid.

In all the sectors the differential equations representing the state equations have been replaced by difference equations. This substitution introduces a lot of simplification in the calculation used in the understanding of the behavior of the phenomenon under consideration.

Also different parameters can be grouped together which behave in the same way. This strategy helps us to easily determine the combined effect which is important to the behavior. And also leads to more simpler expressions using which a systematic analysis can be carried out. This strategy has been fruitfully utilised in the capital and resource sector and eight parameter groups have been isolated.

1. 1-FIOAA-FIOAC
2. b_{fi}
3. CUF
4. (1-FCAOR)/ICOR
5. ALIC
6. $a_{fb}/(a_{is}$*SCOR)
7. ALSC
8. a_{pc} * NRUF

A more condensed sets of equations, in which the overall effect of the combined operations becomes more clearer, can be obtained by combining a number of functional operators like multiplications, divisions table functions and other functions to form single function. This strategy has been put to an advantageous use in the derivation of pollution absorption to substantially make the understanding easier. Finally sets of equations are reformulated, *i.e.* rewritten in a more familiar form, which need not necessarily be simpler than the original sets of equations to make model understanding easier. This strategy has been exploited in all the sectors of the model.

For example in the land distribution subsector of the agricultural sector the original equations

$$\dot{AL} = LDR - LER - LRUI \qquad - \quad (1)$$

$$\dot{PAL} = - LDR \qquad - \quad (2)$$

and

$$\dot{UIL} = LRUI = (UILR - UIL)/10 \qquad - \quad (3)$$

are replaced by one equation

$$AL(t) = p * l \left\{ q * \int_{1900}^{t} FIALD(\xi) * TAI(\xi) \, d\xi + r \right. \qquad - \quad (4)$$

to describe the behavior of the underlying mechanisms and also to gain a better insight into the working of the subsector easily.

5. VERIFICATION OF THE SIMPLIFICATIONS

The simplifications made to gain an easy understanding and better insight into the working of the underlying mechanisms of the agricultural sector have been verified and their results discussed.

The original equations in the Meadows' World 3 model of the agricultural sector, for arable land potentially arable land and land development, are the following (16).

$$\dot{AL} = LDR - LER - LRUI \qquad (1)$$

$$\dot{PAL} = - LDR \qquad (2)$$

$$\dot{UIL} = LRUI = (UILR-UIL)/10 \qquad (3)$$

The behavior of arable land is characterised by a rapid growth in the beginning which then almost remains constant. This being so because, initially the potentially arable land available for development is high and the development cost per hectare are also low. After a time, however, the availability of easily convertible land becomes scarce and the development costs per hectare rise exorbitantly. This results in larger amounts of agricultural inputs being diverted to land maintenance to maintain or increase the food production. The behavior of arable land is that of diminishing marginal returns of investments in land development.

The standard run behavior of AL in the Medow's World 3 model of the agricultural sector can be seen in the fig. 5.1.

In the analysis of the agricultural sector it has been shown that the behavior of AL is of diminishing marginal returns of investments in land development using just one equation instead of the three differential equations as given in the original model, introducing simplifications.

The derivation of the equation is as follows:

The three differential equations are reduced to one, ignoring the effects of LRUI and LER i.e. setting their values to zero. This makes the equations (1) and (2) dependent on each other resulting in the equation

$$\dot{AL} = LDR = -\dot{PAL} \qquad (4)$$

ALI and PALI are the initial values of AL and PAL respectively in 1900, so that

$$AL(t) - ALI = \int_{1900}^{t} LDR(\tau)\, d\tau$$

$$= -PAL(t) + PALI \qquad (5)$$

but potentially arable land at any instant of time is given by

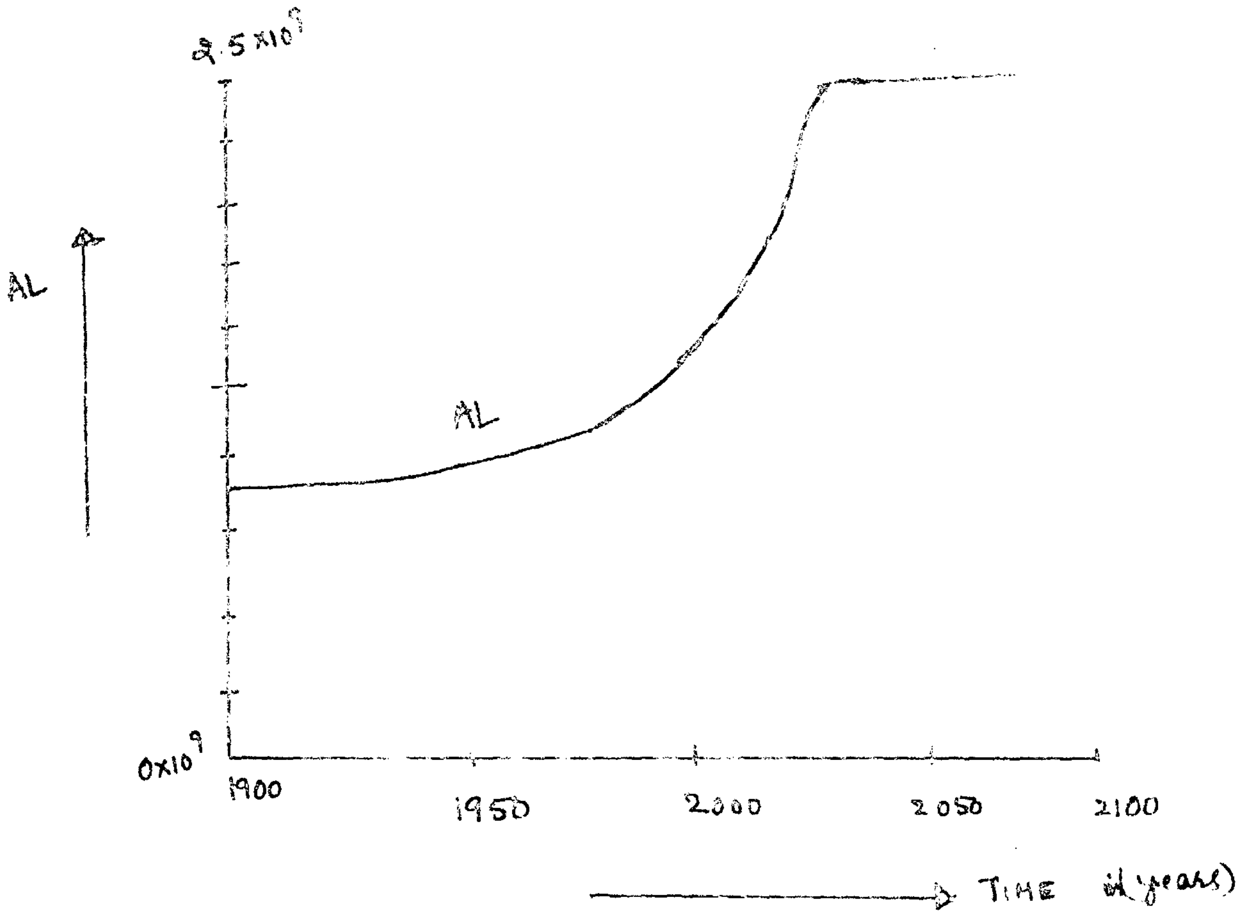


Fig. 5.1

$$PAL(t) = PALI + ALI - AL(t)$$

therefore the equation (4) becomes

$$PAL(t) = PALI + ALI - AL(t)$$

$$= PALT - AL(t) \qquad (6)$$

(because PALI + ALI = PALT
total potentially arable land)

Land development rate is given by

$$LDR = FIALD * TAI/DCPH \qquad (7)$$

and

DCPH is given by

$$DCPH = a * \exp(-b * PAL) \qquad (8)$$

Where a and be are positive and constant.

Substitution of equations (6), (7), (8) result in

$$\dot{AL} = (FIALD*TAI/a)*\exp\{b*(PALT-AL)\} \qquad (9)$$

substituting I for

FIALD * TAI in equation (9)

we get

$$\dot{AL} = \left(\frac{1}{a}\right) * \exp\{b* (PALT-AL)\} \qquad (10)$$

Substituting C for (1/a) * exp b*PALT :

$$\dot{AL} = C*I*\exp\{-b * AL\} \qquad (11)$$

rearranging and integrating with respect to time results in

$$(1/b * \int d(\exp\ b*AL) = C * \int I\ dt \qquad (12)$$

integrating from time = 1900 to t and rearranging, we get

$$\exp\left\{b^*AL(t)\right\} = \exp\left\{b^*ALI\right\} + b^*C^* \int_{1900}^{t} I(\tau)\, d\tau \qquad (13)$$

substituting $P = 1/b$

$q = b^*C$ and $r = \exp(b^*ALI)$

which are positive and constant

yields the equation

$$AL(t) = P^*\ln\left\{q^* \int_{1900}^{t} FIALD(\tau) * TAI(\tau)\, d\tau + r\right\} \qquad (14)$$

This equation, derived by Thissen, aids in an easy explanation of the underlying mechanisms of the behavior of AL. In addition to other things this equation also shows that AL is determined by FIALD and TAI alone and also the logarithmic function showing the effect of diminishing marginal returns on investments in land development.

This equation which make analysis of the underlying mechanisms of the behavior of AL is verified and we got the following results.

The evaluation of arable landusing the equation (14), by employing numerical integration methods, yielded results which describe the behavior of arable land (AL) in the agricultural sector in a clear and easy manner.

The effect of diminishing marginal returns on investments in land development is also clearly brought out. As the correct values of the constants P,Q,R in equation (14) are not available, only approximate values of these are substituted. Due to this fact; the exact values of arable land (AL), at a particular instant of time is not obtained, but the nature of the behavior of AL is clearly portrayed by the results we obtained. This is shown in the fig. 5.2.

2. An important feature of the land fertility subsector of the agricultural sector in World 3 model is that the land fertility LFERT will always be lower than or equal to ILF inherent land fertility. And also LFERT will be lower for larger values LFDR and/or LFRT (16).

The original land fertility equation is a differential equation

$$LFERT = (ILF-LFERT)/LFRT - LFERT * LFDR \qquad (1)$$

A rearrangement of the equation, and considering that FALM and PPLOX are constant in the steady state, a value for the LFERT in a steady state is obtained,

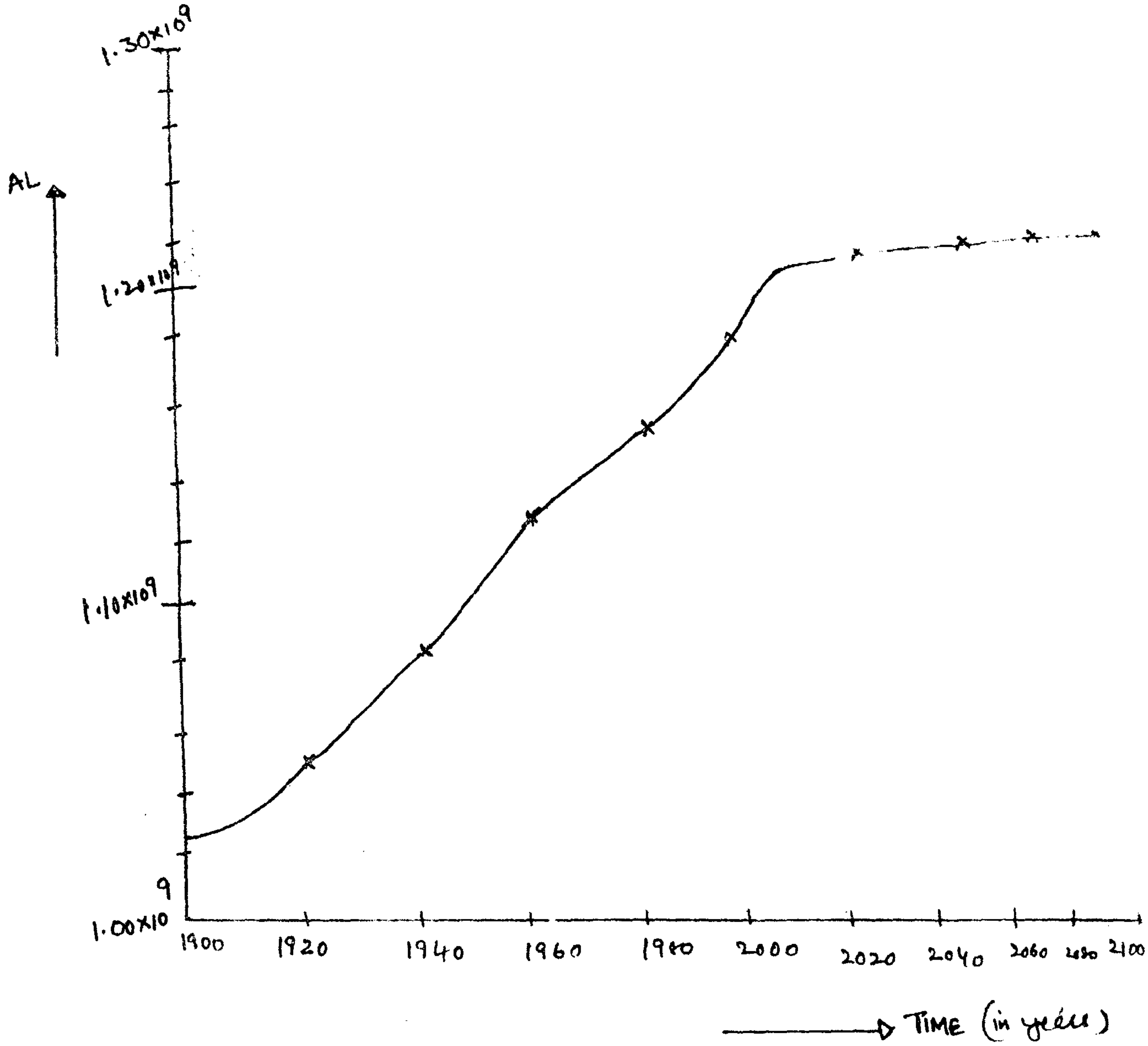

1.30x10^9
AL
1.20x10^9
1.10x10^9
9
1.00x10
1900
1920
1940
1960
1980
2000
2020
2040
2060
2100
TIME (in years)

FIG 5.2

which is given by

$$LFERT_{st} = ILF/(1+LFDR * LFRT) \qquad (2)$$

with a characteristic time constant

$$LFERT = LFRT/(1+LFDR*LFRT) \qquad (3)$$

The equation 2, which produced results not different from the origihal behavior of the standard run has been tested for different value combinations of LFDR and LFRT. This test yielded the following information. The value of ILF is a constant and is equal to 600 vegetable-equivalent kilograms/hectare-year.

The land fertility is always less than inherent land fertility, which is 600 vegetable-equivalent kilograms/hectare-year. For low values of (LFRT) land fertility regeneration time and (LFDR) land fertility degradation rate, the LFERT (land fertility) is sufficiently high. Increasing either of the two, keeping the other low reduces the land fertility. And for simultaneous high values of LFRT and LFDR the LFERT is extremely low of the order of 10 vegetable equivalent kilograms/hectare-year.

CONCLUSION

Formal mathematical models of socio-economic systems have become an important tool to aid the policy decision makers. These systems are very complex due to the inherent structure of multiple feedback loops and dynamic interractions. As a result of this, apart from the exercise of modeling, it becomes important to examine the underlying structure and the behavior of the model so as to make it more useful for the policy decision makers.

In this dissertation we made an effort to study the various simplifications and strategies adopted to understand the complicated models. We have taken Thissen's work on Club of Rome's World 3 model as our basis. A discussion of various socio-economic systems, which are sufficiently complicated, though not on the same level as that of the World 3, has been made.

We have examined the various sectors of the Club of Rome's World 3 model and the reformulation of it by Thissen. The simplifications and strategies adopted in the agricultural sector to uncover the underlying mechanisms and the behavior of the system have been verified and results are presented.

The results show that indeed the simplifications and strategies made do bring out the characteristic behavior of the phenomenon under consideration and the underlying mechanisms in a lucid and easily understood manner. The dependence of land fertility on land fertility degradation rate and the land fertility regeneration time has been shown. In the land distribution subsector of the agricultural sector it could easily be shown that the investments in land development have an effect of diminishing marginal returns. The behavior of arable land is also brought out clearly.

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APPENDIX

LIST OF SYMBOLS USED

a_i : constant in approximation of table functions; index i refers to the corresponding equation number.

AHL : Assimilation half-life in years.

AI : Agricultural inputs (dollars/year)

AIOPC : Average industrial output per capita (dollars/person year)

AIPH : Agricultural inputs per hectare (dollars/hectare-year)

AL : Arable land (hectares)

ALI : Arable land initial (hectares)

ALIC : Average life-time of industrial capital(years)

ALL : Average life of land (years)

ALSC : Average life time of service capital (years)

B : Births per year (persons/year)

b_i : constant in approximations of table functions; index i refers to the corresponding equation number.

CAI : Current agricultural inputs (dollars/year)

CBR : Crude birth rate (births/1000 persons - year)

CDR : Crude death rate (Deaths/1000 persons-year)

CMPLE : Compensatory multiplier from percieved life expectancy (Dimensionless).

CUF : Capital utilisation fraction (dimensionless).

D : Deaths per year (persons/year).

DCFS : Desired completed family size (dimensionless)

DCFSN : Desired completed family size normal (dimensionless).

DCPH : Development costs per hectare (dollars/hectare).

DTF : Desired total fertility (dimensionless).

EHSPC : Effective health services per capital (dollars/person-year).

F : Food (vegetable - equivalent kilograms/year).

f_i : table function.

FALM : Fraction of inputs allocated to land maintenance (dimensionless).

FCAOR : Fraction of capital allocated to obtaining resources (dimensionless).

FCE : Fertility control effectiveness (dimensionless)

FIALD : Fraction of industrial output allocated to land development (dimensionless).

FIOAA : Fraction of industrial output allocated to agriculture (dimensionless).

FIOAC : Fraction of industrial output allocated to consumption (dimensionless).

FIOAI : Fraction of industrial output allocated to industry (dimensionless).

FIOARC : Fraction of industrial output allocated to resource conservation (dimensionless).

FIOAS : Fraction of inputs allocated to services (dimensionless).

FPC : Food per capita (vegetable - equivalent kilograms/person-year).

FR : Food ratio (dimensionless).

IC : Industrial capital (dollars).

ICI : Industrial capital initial (dollars).

ICOR : Industrial capital output ratio (years).

IFPC : Indicated food per capita (vegetable-equivalent kilograms/person-year).

ILF : Inherent land fertility (vegetable-equivalent kilograms/hectare year).

IO : Industrial output (dollars/year).

IOPC : Industrial output per capita (dollars/person-year).

ISOPC : Indicated service output per capita (dollars/person-year).

LDR : Land development rate (hectars/year)

LE : Life expectancy (years).

LER : Land erosion rate (hectares/year).

LFDR : Land fertility degradation rate (year)

LFERT : Land fertility (vegetable-equivalent kilograms/hectare year).

$LRERT_{st}$: Steady state value of LFERT.

LFRT : Land fertility regeneration rate (years)

LMC : Life time multiplier from crowding (dimesionless).

LMP : Life time multiplier from persistent pollution (dimensionless).

LRUI : Land removal for urban industrial use (hectares/year).

LY : Land yield (vegetable equivalent kilograms/hectare-year).

LYMAP : Land yield multiplier from air pollution (dimensionless).

LYMC : Land yield multiplier from capital (dimensionless)

MLYMC : Marginal land yield multiplier from capital (hectares/dollar)

MPAI : Marginal productivity of agricultural inputs (vegetable-equvalent kilograms/dollar).

MPLD : Marginal productivity of land development (vegetable equivalent kilograms/dollor)

MTF : Net birth rate (1/year)

NR	:	Non-renewable resources (resource units).
NBR	:	Net birth rate (1/year).
NRFR	:	Nonrenewable resource fraction remaining (dimensionless).
NRI	:	Nonrenewable reource initial (resource units).
NRUR	:	Non-renewable resource usage rate (resource units/year).
PAL	:	Potentially arable land (hectares).
PALI	:	Potentially arable land initial (hectares)
PALT	:	Potentially arable land total (hectares).
PCRUM	:	Per capita resource usage multiplier (resource units/person-years).
PFR	:	Percieved food ratio (dimensionless).
PLE	:	Percieved life expectancy (year).
POP	:	Population (persons)
POPE	:	Population equilibrium value (persons).
PPAPR	:	Persistent pollution appearance rate (pollution units/year).
PPASR	:	Persistent pollution assimilation rate (pollution units/year).
PPGAO	:	Persistent pollution generated by agricultural output (pollution units/year).
PPFIO	:	Persistent pollution generated by industrial output (pollution units per year).

PPOL : Persistent pollution (pollution units).

PPOLX : Index of persistent pollution(dimensionless).

SC : Service capital (dollars).

SCDR : Service capital depreciation rate (dollors/year).

SCI : Service capital initial (dollors).

SCIR : Service capital investment rate (dollors/year).

SCOR : Service capital output rates (1/year).

SFPC : Subsistence food per capita (vegetable-equivalent kilograms/persons year).

SFSN : Social family size norm (dimensionless)

SO : Service output (dollors/year).

SOPC : Service output per capita (dollors/person-year).

t : time (years).

TAI : Total agricultural inputs (dollars/year).

TF : Total fertility (dimensionless).

U : 1 - FIOAC - FIOAA.

UIL : Urban-industrial land (hectares).

UILPC : Urban industrial land per capita (hectares/person).

: lag-time, time constant.