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Low Frequency Beam Driven Instabilities in Plasmas

By

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This dissertation entitled "**Low Frequency Beam Driven Instabilities in Plasmas**" embodies the work carried out at the School of Environmental Sciences, Jawaharlal Nehru University, New Delhi. This work is original and has not been submitted in full or in part for any degree or diploma of any university.

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CHAPTER-I

INTRODUCTION

The plasma is basically an assembly of charged particles which exhibit collective behaviour. The term "plasma" was first introduced to describe ionized gases by Tonks and Langmuir in 1929 while studying electrical discharges. They observed oscillations at plasma frequency $\omega_{pe} = \left(\frac{4\pi \eta_e e^2}{m_e}\right)^{\frac{1}{2}}$, where η_e is the electron number density, e, m_e are the electron charge and mass respectively.

Due to the collective behaviour of charged particles, a plasma displays interesting electromagnetic and electrostatic characteristics. A variety of waves in various high and low frequency ranges characteristic of electron and ion oscillations respectively, can exist in a plasma. The multiplicity of modes increases with the inclusion of external magentic field (i.e. magnetized plasma). The frequencies which characterize various modes are (i) Electron plasma frequency, (ii) Electron cyclotron frequency ($\omega_{ce} = \frac{eB_o}{m_e c}$) where B_o is the external magnetic field, (iii) Ion plasma frequency $\omega_{pi} = \left(\frac{4\pi \eta_i e^2}{m_i}\right)^{\frac{1}{2}}$ where m_i is the mass and η_i is the density of ion, and (iv) ion cyclotron frequency $\omega_{ci} = \frac{eB_o}{m_i c}$. Given in the next page is a summary of various types of modes which can exist in a plasma.

High frequency regime

Unmagnetized plasma

Mode	Nature of mode
Plasma oscillations	Electrostatic
Light waves	Electromagnetic

Magnetized plasma

Mode	Nature of mode
Upper hybrid oscillations	Electrostatic
Ordinary wave	Electromagnetic
Extraordinary wave	Electromagentic
Whistler mode	Electromagentic
L wave	Electromagenic

Low frequency regime

Unmagnetized plasma

Mode	Nature of mode
Ion acoustic wave	Electrostatic
Ion oscillation	Electrostatic

Magnetized plasma

Mode	Nature of mode
Lower hybrid	Electrostatic
Electrostatic ion cyclotron	Electrostatic
Alfven wave	Electromagnetic
Magnetosonic wave	Electromagnetic

A system consisting of a plasma (usually magnetized) penetrated by particle beam is frequently encountered in laboratory experiments as well as ionosphere and magnetospheric environment. In view of the existance of high and low frequency waves in the auroral zone (Hasegawa 1975, Christiansen et.al. 1981, upstream waves in the solar wind and the relations of radiowave brusts and electron beam of solar origin), the injection of particle beams into natural as well as laboratory plasmas for diagnostic and heating purposes and the coupling of relativistic beams with slow wave structures/or guiding systems for generation of high power microwaves (Walsh et al. 1976; Tripathi 1984; Jain et al. 1986) in mm and sub mm ranges, particle beam driven instabilities have attracted a great deal of attentions over last couple of decades.

Indivisual charged particles passing through a plasma loose energy at the rate of approximately 10^{-3} to 10^{-5} ev/cm due to Cherenkov effect, Doppler effect and polarization phenomena. Beam particles, may, however excite oscillations and waves in a plasma and consequently may loose energy at a far higher rate (10^3 - 10^4 ev/cm).

Different types of interactions can take place between charged particle beam and plasma (or guiding system). They are classified as cherenkov, normal Doppler effect (fast cyclotron interaction) and anomalous Doppler effect (slow cyclotron interactions). Cherenkov interaction in a plasma is characterized by the resonance condition $\frac{\omega}{k} = v_p = v_o$ where v_p is the phase velocity of the wave with frequency ω and wave number k, v_o is the particle (beam) velocity.

In case of normal Doppler effect the resonance condition is $\omega = k_{\parallel} v_o + \omega_{ce}$, where v_o is the beam velocity along the magnetic field, $k_{\parallel}$ is the wave number parallal to the magnetic field and ω_{ce} is the cyclotron frequency of the beam particle. The resonance condition implies that the Doppler shiefted frequency of the wave equals the electron cyclotron frequency of the beam particle. The normal Doppler effect may excite an instability when the beam electron possess transverse energy. Radiation emitted when there is a greater concentration of particles in the phase region where particle looses energy to the electromagnetic or electrostatic field than in the phase region where particles absorb electromagnetic energy. Excitation of waves by these resonance particles is called fast cyclotron excitation.

Instabilities can also be excited by the anomalous Doppler effect (resonance condition $\omega = k_{\parallel} v_o - \omega_{ce}$) when the velocity of the radiating particle exceeds the phase velocity v_p of the wave in plasma (or in wave guide system). The excitation of waves by these resonance particles is called slow cyclotron excitation. The instabilities caused by above mentioned interactions may be viewed as due to the couplings of plasma modes with the slow space charge, fast and slow cyclotron waves on the beam. In addition to these three excitation mechanism plasma modes can also be excited by beams which are anisotropic ($T_{\perp} > T_{\parallel}$; where $T_{\perp}$ and $T_{\parallel}$ are related to thermal spreads in beam velocities perpendicular and parallel to magnetic field direction) in nature (Hasegawa 1975).

Initially the studies of the beam plasma instabilities were limited to the linear regime (i.e. $\Phi << kT_e$ where Φ is the wave potential, T_e is the electron temperature). Beam plasma instabilities in the linear regime have been discussed in a number of reviews (Briggs 1964, Mikhailovski 1974, Akhiezer et al 1975; Allen and Phelps 1977; F.F. Cap 1978). The electrostatic waves and their excitation by parallal beams in magnetized plasma have been studied both theoretically and experimentally by many workers (Apel 1969, Seidl 1970, Tataronis et al. 1970 a,b; Self et al. 1971; Puri et al 1973, 1975). In the high frequency regime ($\omega \sim \omega_{pe}, \omega_{ce}$), for a cold ($k_{\perp} \rho_e \ll 1$, where $k_{\perp}$ is the wave number in direction perpendicular to the magnetic field and ρ_e is the electron larmore radious) magnetized beam plasma configuration electron plasma mode and electron cyclotron mode instablities and their excitation mechanisms were investigated by Malmberg et al. (1969), Apel (1969) and Cabral (1976). These modes were found to be unstable as a result of cherenkov and anomalous Doppler effect interactions. The subsequent works of Idehara et al. (1969) and Seidl (1970) included finite temperature effects. The finite plasma and beam electron temperature in general reduced the growth of instabilities, the reduction in growth rate being more marked in case of the latter. For the hot beam plasma configurations (the condition $k \rho_e \gtrsim 1$ easily met in this beam produced plasmas) Self et al. (1971), Mizuno and Tanaka (1974), Jain and Christiansen (1981, 1984) have observed electron cyclotron harmonic wave instabilities (also known as electron Bernstein mode instabilities) in various cyclotron harmonic bands ($\eta\omega_{ce} \lesssim \omega \lesssim (\eta+1)\, \omega_{ce}$; $\eta=1,2,3,$etc.) These instabilities become unstable due to cherenkov and/or slow cyclotron interactions. The results obtained from the theoretical analysis (Self, Jain, Mizuno) involving the computations of the dispersion relations were in good agreement with the experimental observations.

In the beam plasma experiments high frequency instabilities have often been found to be modulated with low frequency ($\omega \sim \omega_{pi}$, ω_{ci}) waves (Apel, 1969; Jain, 1981). This has initiated considerable interest in the study of low frequency instabilities as they play a very important role in the eventual saturation of the high frequency instabilities (Liu and Tripathi, 1986; Yu and Shukla, 1977; Wong and Quon, 1975; Jain, 1981). In the low frequency regime known electrostatic modes are ioncyclotron, ion acoustic, lower hybrid and ion plasma oscillations. The low frequency modes can be excited by different coupling mechanisms between plasma and beam waves. Low frequency modes ($\omega \sim \omega_{pi}, \omega_{ci}$) studied by Vermeer et al. (1967, 1970), Bhatnagar et al. (1972), Nyack et al. (1974) were driven unstable via couplings with slow space charge and slow cyclotron mode waves on the beam. Yamada and Owens (1977) invoked a cross field drift ($\frac{\vec{E} \times \vec{B}_o}{B_o^2}$, where $\vec{E}$ is the radial electric field) excitation mechanism to account for instabilities, in their experiments, around ion plasma and lower hybrid frequencies. The same mechanism has been used to explain observed spectra of ioncyclotron waves ($\omega \sim nf_{ci}$; $n=1,2,3,4$) by Wall et al. (1981) and Jain (1981). Ion cyclotron waves are also known to be excited due to longitudinal currents drawn through the plasma (Hendel, 1976; Satyanarayana, 1986).

In beam plasma experiments, in certain parameter regimes, the saturation of the high frequency instabilities and the ultimate relaxation of the system depends on the low frequency waves. Therefore, it is imperative to know whether the low frequency waves observed in a beam plasma experiment are driven via some nonlinear process like parametric decay and modulational instability (Tripathi et al. 1977; Tripathi, Liu, Gregobi, 1979; Liu and Tripathi, 1980; Gregobi and Liu, 1980; Sharma et al. 1982; Liu et al. 1984; Zakharov, 1972, 1975; Yu and

Shukla, 1977) or are infact linearly excited. It was for these reasons, extensive experimental investigations were carried out by Jain and Christiansen (Private communication) on the excitation of low frequency instabilities in a beam plasma system (the details of which are given in Chapter II) they found that at extremely low pressures $p \sim 4 \times 10^{-6}$ Torr, waves with $\omega \gtrsim \omega_0$ ($\simeq 10\,\omega_{ci}$) were unstable and the spectra had a very strong harmonic content ($\omega_l \simeq l\omega_o$; $l = 1,2,3,4,$). At large values of beam current, in addition to the harmonics of the fundamental wave instability, subharmonics ($\omega_m \simeq \frac{\omega_o}{m}$; $m=2,3,$) also appeared. Distinct thresholds were observed for 1/2 and 1/3rd subharmonic wave instabilities.

In this thesis an attempt has been made to explain the experimentally observed low frequency instability and its subharmonic generation. Chapter II gives a brief account of the experimental set up used in obtaining the experimental results by Jain and Christiansen (Thesis, 1981). In Chapter III dispersion relation for electrostatic waves in a magnetized beam plasma system is derived using fluid and kinetic treatment. From the dispersion relation an expression for the growth of instability is deduced. A nonlinear treatment describing the subharmonic generations is presented in Chapter IV. The results and conclusions are discussed in Chapter V.

CHAPTER II

EXPERIMENTAL SET UP

The experimental observations pertaining to the linear and nonlinear evolution of the low frequency instabilities, for which the theoretical analysis is presented in subsequent chapters, were obtained by V.K. Jain using the beam plasma apparatus at the University of Sussex (U.K.) during 1982-83. A very brief account of the experimental set up is given below (for details see Ph.D. thesis of Jain, 1981).

2.1 Description of Apparatus

The plasma vessel is a cylindrical pyrex tube of length 150 cm and diameter 10 cm. A rotary pump and a diffusion pump are used to create vacum $\sim 4 \times 10^{-6}$ Torr. Argon gas can be fed into the system through one end. Pressure in the system is monitored by Pirani and ionization gauges. The vessel is immersed in uniform magnetic field, which is variable up to a maximum of 700 gauses.

Detection of waves in the beam plasma system is done by straight wire probes. An axial probe which can move forward and backward is available to determine the axial evoluation of the instabilities. Six side ports are available along the vessel for

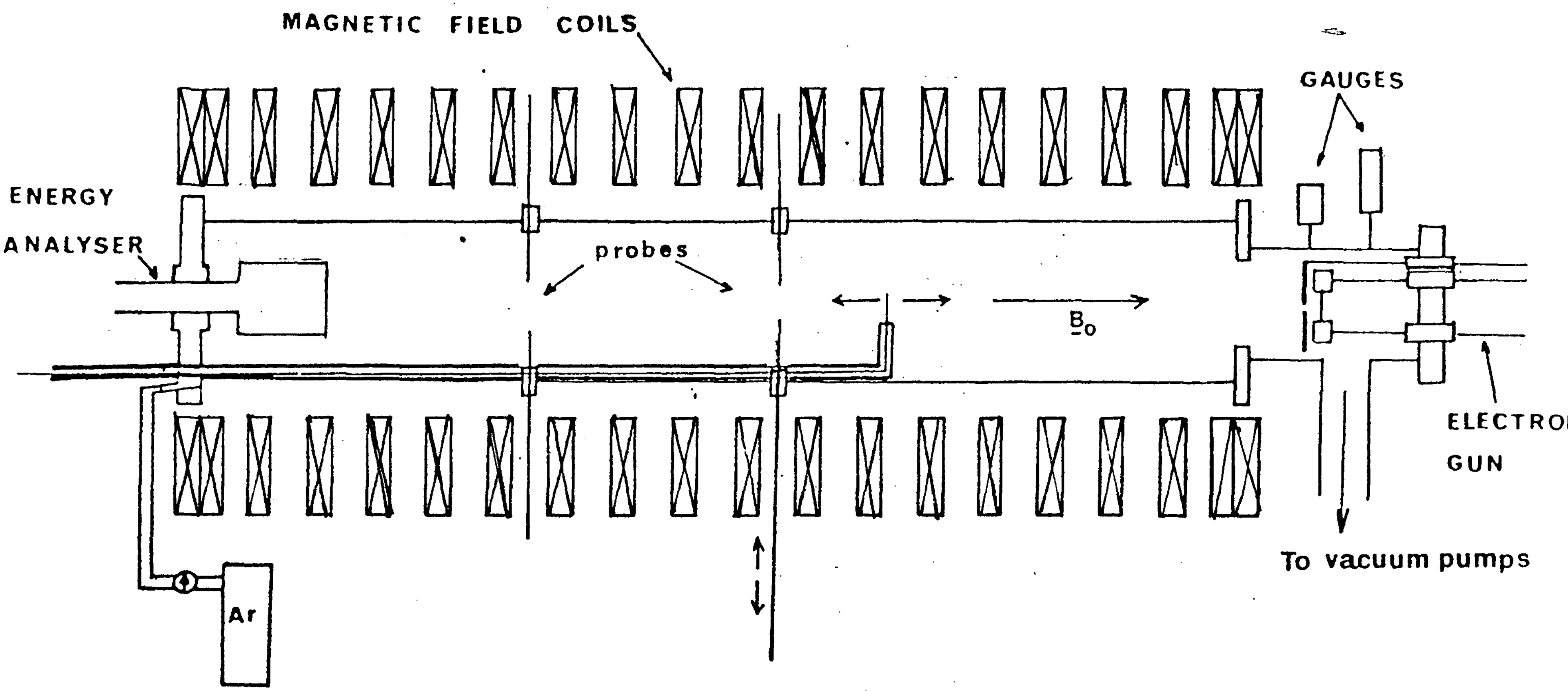


FIG.2·1 SCHEMATIC DIAGRAM OF THE APPARATUS radial measurements. A radial probe 1 is mounted vertically in the system. At the same axial distance, a continuously movable radial probe '2' is inserted through a side port at right angle with respect to verticle probe '1'. Two more L shaped probe placed in horizontal plane containing the magnetic field axis are used for launching and detection of Bernstein mode waves.

2.2 Electron Beam Source

The schematic diagram of the electron gun used is shown in Fig. 2.2. The gun is capable of producing a solid cylendrical beam of 0.75 cm diameter with energies variable in the range 0-300 ev and current of few mA. A few tens of amperes of current is passed through the filament made of tungsten wire (D = .025 cm). Emitted electron from the heated filament are accelerated by biasing the accelerating electrode positive with respect to the filament. In fact the accelerating electrode is kept at ground potential. Accelerated electrons propagating along magnetic field lines produce plasma through collisions with the neutral argon gas atoms and are collected at the end plate. At typical working pressure $\sim 4 \times 10^{-6}$ Torr, the plasma electron density is in the range $\sim 10^{6}-10^{7}/cm^{3}$ and electron temperature $\lesssim 0.25$ ev.

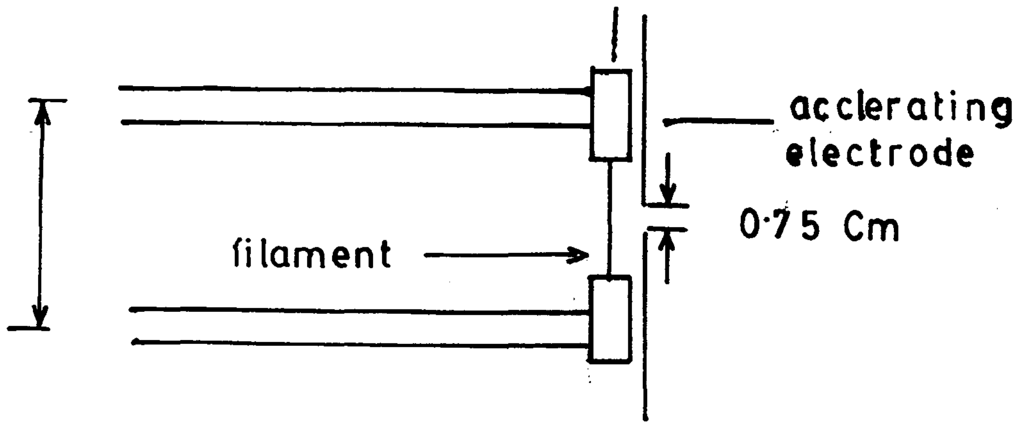


Fig 2'2 schematic diagram of electron beam

2.3 DIAGNOSTIC TECHNIQUES

To determine the plasma density and electron temperature usually single Langmuir probe method is used. In the beam region, the probe current measurement will be affected by the presence of the beam electrons, especially when the plasma density is low so that beam is offset due to ion current. Other method which gives reliable density measurements involves the use of the cut off of the perpendicularly propagating electrostatic waves, i.e., Bernstin mode waves.

Measurement of phase velocity of wave can be made from the measurement of wavelength obtained from the interferometer technique. The frequency spectra of the excited waves have been obtained by a Hemlett Packard spectrum analyzer which consist of a 141T display unit, a 8555A RF unit, 8553 B RF unit and a 8552 A IF unit. The band width of the spectrum can be varied between 3-300 KHz on the RF unit and the scanning width is variable within 2 KHz - 100 MHz range. Using zero scan facility on the spectrum analyser, it can be used as a tuned filter. Amptitude variations of a certain frequency signal can be plotted as a function of distance on a X-Y recorder. Beam energy is measured by retarding type energy analyser.

CHAPTER III

DISPERSION RELATION OF ELECTROSTATIC WAVES AND EXCITATION OF ION PLASMA WAVE INSTABILITIES

3.1 Introduction

Electrostatic mode waves ($\vec{E} = -\vec{\nabla} \Phi$) can be excited by beam plasma interactions. The wave grows due to the transfer of energy from beam particles to the wave. Such a situation can arise in the case of a non-Maxwellian distribution of particles in the plasma as shown in Fig. 3.1 which corresponds to the passage of a beam of charged particles moving with drift velocity $\vec{V}_0$ through a Maxwellian plasma.

It can be seen that a wave moving with velocity (ω/k) sees more electrons moving slightly faster than the wave that it sees moving slightly slower than the wave. Consequently, the wave obtains more energy from the particles than the particles from the wave, and therefore, wave grows. A wave growing spatially/or temporally is termed as instability, the Propagation characteristics of which can be studied by solving the dispersion relation which relates the frequency and the wave number of the

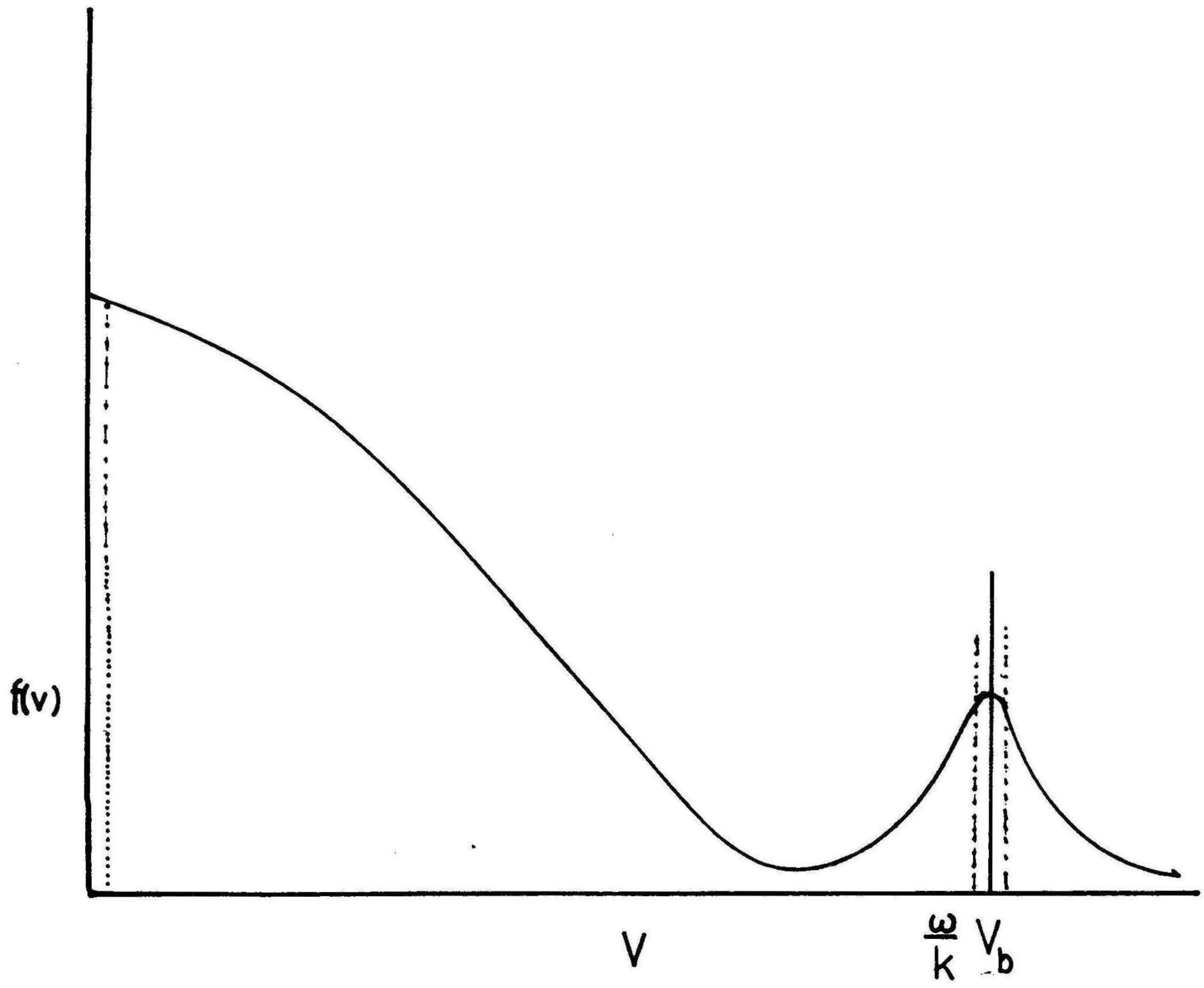


Fig 3·1 Electron velocity distribution function vs velocity in a Beam plasma system. Dotted lines correponds a cold plasma cold beam case.

instability as a function of parameter of the system. The dispersion relation can be derived using either fluid or kinetic theory. The former is adequate for describing a system with cold particles (electrons + ions). However, for electrons and ions with some finite temperatures, the kinetic theory has to be used as fluid theory is not able to account for effects such as Landau damping.

In sections (3.2) and (3.3) derivations of electrostatic wave dispersion relations based on fluid and kinitic treatments are given.

3.2 DISPERSION RELATION FOR ELECTROSTATIC WAVES IN A MAGNETIZED PLASMA

We consider uniform cold plasma immersed in an uniform external magnetic field $\vec{B} = B_0 \hat{z}$ which is composed of ions and electrons having mass m_j, charge e_j, number density n_j and velocity $\vec{V}_j$, where suffix j denotes the species. The electrons of density n_{oe} are assumed to drift with uniform speed $\vec{V}_o$ in the $\hat{x}$ direction through an uniform background of stationary ions.

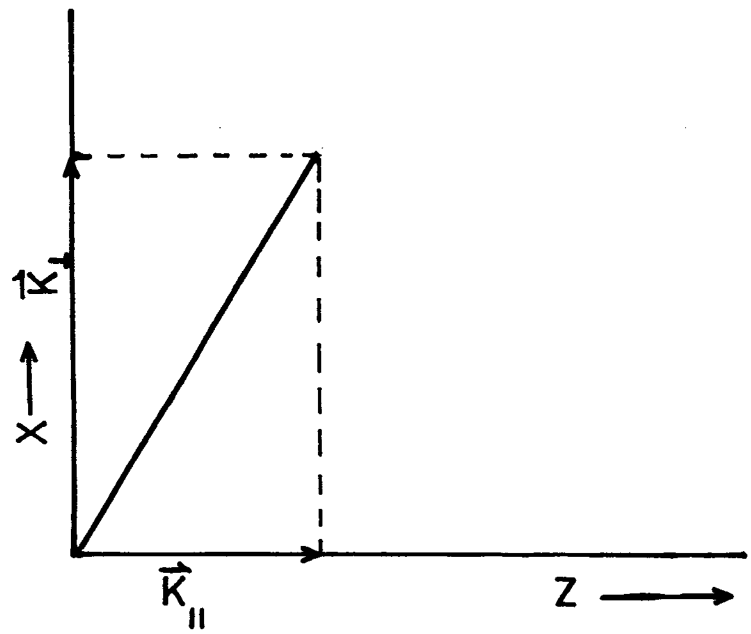


Fig 3·2

The system can be described by a set of momentum and continuity equations along with poissons equations.

Equation of motion,

$$m_j \left[\frac{\partial \vec{V}_j}{\partial t} + (\vec{V}_j \cdot \vec{\nabla})\, \vec{V}_j\right] = e_j\left[\vec{E} + \frac{\vec{V}_j \times \vec{B}}{c}\right] \quad \ldots\ldots\ldots \quad 3.1$$

Continuity equation

$$\frac{\partial n_j}{\partial t} + \vec{\nabla} \cdot (n_j \vec{V}_j) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad 3.2$$

and poissons equation $\vec{\nabla} \cdot \vec{E} = 4\pi e\,(n_i - n_e)$ 3.3

For electrostatic instabilities the electric field is derivable from a scalar potential, i.e. $\vec{\nabla} \times \vec{E} = 0$ and the fluctuating magnetic field is absent. To solve the above set of equations we first linearize them (Chen, 1974). We assume the amplitude of oscillations is small and term involving higher powers of amplitude can be neglected. Let us first separate the dependent variables into two parts; an equillibrium part indicated by a subscript '0' and a perturbation part indicated by a subscript '1'.

$$n_j = n_{oj} + n_{j1}; \quad \vec{V}_j = \vec{V}_{oj} + \vec{V}_{j1}; \quad E = \vec{E}_o + \vec{E}_1$$

$$n_{j1} << n_{oj} \qquad \ldots \qquad 3.4$$

In equillibrium we have,

$$n_{oe} = n_{oi} = n_o \text{ (constant in time and space)}$$

$$\vec{V}_{oi} = 0 = \vec{E}_o$$

$$\vec{V}_{oe} = \hat{x}\, \vec{V}_o \qquad \ldots \qquad 3.5$$

The linearized momentum, continuity and poissons equations can then be written as

$$m_i \frac{\partial \vec{V}_{i1}}{\partial t} = e\vec{E}_1 + \frac{e}{c} \vec{V}_{i1} \times \vec{B} \qquad \ldots \qquad 3.6$$

$$\frac{\partial n_{i1}}{\partial t} + n_o \vec{\nabla} \cdot \vec{V}_{i1} = 0 \qquad \ldots \qquad 3.7$$

$$m_e \left[\frac{\partial \vec{V}_{e1}}{\partial t} + V_o \frac{\partial}{\partial x} \vec{V}_{e1}\right] = -e\vec{E}_1 - \frac{e}{c} \vec{V}_{e1} \times \vec{B} \quad \ldots . \quad 3.8$$

$$\frac{\partial n_{e1}}{\partial t} + \vec{\nabla} \cdot (n_o \vec{V}_{e1} + \hat{x} V_o n_{e1}) = 0 \quad \ldots \ldots \quad 3.9$$

$$\vec{\nabla} \vec{E}_1 = 4 \pi e (n_{i1} - n_{e1}) \quad \ldots \ldots \quad 3.10$$

We will assume that the oscillating quantities behave sinosoidally:

$$n_1 = n_1 e^{i(k_\perp x + k_\parallel Z - \omega t)}$$

$$V_1 = V_1 \; e^{i(k_\perp x + k_\parallel Z - \omega t)} \quad \ldots \ldots \quad 3.11$$

$$E_1 = E_1 \; e^{i(k_\perp x + k_\parallel Z - \omega t)}$$

where $k_\parallel$ and $k_\perp$ are respectively parallel and perpendicular wave vector and ω is the angular frequency, which may be complex ($\omega = \omega_r + i\omega_i$). Positive ω_i denotes growth.

Writting in component form we get from 3.6 and 3.7,

$$V_{i1x} = \frac{ie}{m_i \omega} E_x$$

$$V_{i1y} = \frac{e}{m_i} \frac{\omega_{ci}}{(\omega^2 - \omega_{ci}^2)} E_x \quad \ldots \ldots \quad 3.12$$

$$V_{i1z} = \frac{ie}{m_i \omega} E_z$$

and $$n_{i1} = \frac{n_{oi}}{\omega}\left[\frac{ie\omega k_1 E_x}{m_i(\omega^2-\omega_{ci}^2)} + \frac{iek_{11}}{m_i\omega}E_z\right]$$

where, $\omega_{ci} = \frac{eB_o}{m_i c}$ is the ion cyclotron frequency.

From 3.8 and 3.9, we get,

$$V_{e1x} = \frac{-ie(\omega - k_1V_o)\,E_x}{m_e[(\omega - k_1V_o)^2-\omega_{ce}^2]}$$

$$V_{e1y} = \frac{e}{m_e}\frac{\omega_{ce}\,E_x}{[(\omega - k_1V_o)^2-\omega_{ce}^2]} \qquad 3.13$$

$$V_{e1z} = -\frac{ieE_z}{m_e(\omega - k_1V_o)}$$

and $$n_{e1} = \frac{n_{oe}}{(\omega - k_1V_o)}[\,k_1V_{e1x} + k_{11}V_{e1z}]$$

where, ω_{ce} is the electron cyclotron frequency.

Substituting n_{e1} and n_{i1} from (3.12) and (3.13) in (3.10) we obtain the following dispersion relation :

$$1 + \varepsilon_{pe} + \varepsilon_{pi} = 0$$

where,

$$\varepsilon_{pe} = - \frac{\omega_{pe}}{(\omega - k_{\perp}V_o)^2} \frac{k_{\parallel}^2}{\kappa^2} - \frac{\omega_{pe}^2}{[(\omega - k_{\perp}V_o)^2 - \omega_{ce}^2]} \frac{k_{\perp}^2}{k^2} \qquad 3.14$$

$$\varepsilon_{pi} = - \frac{\omega_{pi}^2}{(\omega - k_{\perp}V_o)^2} \frac{\kappa_{\parallel}^2}{k^2} - \frac{\omega^2}{(\omega^2 - \omega_{ci}^2} \frac{k_{\perp}^2}{k^2}$$

ε_{pe} and ε_{pi} are called the electron and ion susceptibilities (Seidl, 1970).

The above dispersion equation describes the propagation of waves in a system which consists of background stationary ions and an electrons component drifting accross the magnetic field with velocity $\vec{V}_o$. If in addition a beam of electrons with $\vec{V} = V_b\hat{z} + V_o\,\hat{x}$ also exists in the plasma, it can be shown that the dispersion relation modified to include the contribution due to the third component and we have

$$1 + \varepsilon_{pe} + \varepsilon_{pi} + \varepsilon_b = 0 \qquad \ldots\ldots\ldots \qquad 3.15$$

where,
$$\varepsilon_b = - \frac{\omega_{pb}^2\, k_{\parallel}^2}{\left\{ \begin{matrix} \omega - k_{\parallel} V_b \\ -k_{\perp}V_o \end{matrix} \right\}^2 k_{\parallel}^2} - \frac{\omega_{pb}^2}{\left[\left\{ \begin{matrix} \omega - k_{\parallel} V_b \\ -k_{\perp}V_o \end{matrix} \right\}^2 - \omega_{ce}^2 \right]} \frac{k_{\perp}^2}{k^2} \qquad 3.16$$

3.3 DISPERSION RELATION FOR ELECTROSTATIC WAVES IN A MAGNETIZED PLASMA BY KINETIC THEORY

The dispersion relation can be derived by using Maxwells equation and equivalent dielectric tensor which characterize the electromagnetic propertis of plasma. The equivalent dielectric tensor can be obtained using Vlasov and Maxwells equations which can be written as :

$$\frac{\partial f_\alpha}{\partial t} + \vec{V} \cdot \frac{\partial f_\alpha}{\partial \vec{r}} + \frac{q_\alpha}{m_\alpha} (\vec{E} + \frac{\vec{V} \times \vec{B}}{C}) \cdot \frac{\partial f_\alpha}{\partial \vec{V}} = 0 \qquad \ldots\ldots \quad 3.17$$

$$\vec{\nabla} \times \vec{E} = - \frac{1}{C} \frac{\partial \vec{B}}{\partial t} \qquad \ldots\ldots \quad 3.18$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{C} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{C} \vec{J} \qquad \ldots\ldots \quad 3.19$$

where $f_\alpha = f_\alpha(\vec{r}, \vec{v}, t)$ is the distribution function of particles of species α, m_α and q_α are respectively mass and charge of particles of species α.

We consider a uniform cold plasma of infinite extent immersed in an external magnetic field $\vec{B}_o$, and we assume that Collisional effects are negligible. We further assume that the amplitude of oscillation is small and term involving higher powers of amplitude factor can be neglected. $\vec{E}_1$, $\vec{B}_1$, $\vec{J}_1$, $f_{\alpha 1}$ are small perturbations which are assumed to vary as, $\exp[i(\vec{k}.\vec{r}-\omega t)]$, where $\vec{k}$ is the wave vector and ω is the angular frequency which in general can be complex.

In the linear approximation perturbed current density $\vec{J}_1$ is related to $\vec{E}_1$,

$$\vec{J}_1 = \bar{\bar{\sigma}}(\omega, \vec{k}) \cdot \vec{E}_1 \qquad \ldots\ldots \qquad 3.20$$

where $\bar{\bar{\sigma}}(\omega,\vec{k})$ is the conductivity tensor.

From equations (3.18) and (3.19) we have

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + \frac{\omega^2}{C^2} \bar{\bar{\varepsilon}} \cdot \vec{E} = 0 \qquad \ldots\ldots \qquad 3.21$$

where $\bar{\bar{\varepsilon}}$ is a dielectric tensor and is given by

$$\bar{\bar{\varepsilon}} = 1 + \frac{4\pi}{i\omega} \bar{\bar{\sigma}} \qquad \ldots\ldots \qquad 3.22$$

Taking divergence of equation (3.21) and using the relation $\vec{E}_1 = - i \vec{k} \Phi$, where Φ is the potential, we get,

$$D(\omega,\vec{k}) = k_i \ \varepsilon_{ij} \ k_j = 0 \qquad 3.23$$

The above equation is the dispersion relation for electrostatic waves in a plasma. It is clear from equation (3.23) that the knowledge of $\bar{\bar{\varepsilon}}$ is central to determining the dispersion relation. To evaluate $\bar{\bar{\varepsilon}}$ we write

$$f_\alpha = f_{\alpha o} + f_{\alpha 1} \quad , \quad \vec{E} = \vec{E}_1 \; ; \; \vec{B} = \vec{B}_o$$ in equation (3.17),

and the linearized Vlasov equation becomes

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{q_\alpha}{m_\alpha}\left(\frac{\vec{v}\times\vec{B}_o}{C}\right)\cdot\frac{\partial}{\partial \vec{v}}\right] f_{\alpha 1} = -\frac{q_\alpha}{m_\alpha}\left[\frac{\partial f_{\alpha o}}{\partial \vec{v}}\right] \times$$

$$\left[\left(1-\frac{\vec{k}\cdot\vec{v}}{\omega}\right)\bar{\bar{I}} + \frac{\vec{k}\cdot\vec{v}}{\omega}\right]\cdot\vec{E} \qquad 3.24$$

where $\bar{\bar{I}}$ is a unit tensor.

The above differential equation can be solved by the method of integrating over unperturbed orbits (Krall and Trivelpiece, 1973). We define

$$\frac{d\vec{r}\,'}{dt'} = \vec{v}\,', \quad \frac{d\vec{v}\,'}{dt'} = \frac{q_\alpha}{m_\alpha}\,\frac{\vec{v}\,'\times\vec{B}_o}{C}$$

with boundary conditions $\vec{r}\,'(t'=t) = \vec{r}$

$\vec{v}\,'(t'=t) = \vec{v}$

where $\vec{r}$, $\vec{v}$ are position and velocity coordinates in $(\vec{r}, \vec{v})$ phase space. The differential operators on the left hand side of equation (3.24) can be replaced by a time derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{d\vec{r}}{dt} \cdot \frac{\partial}{\partial \vec{r}} + \frac{d\vec{v}}{dt} \cdot \frac{\partial}{\partial \vec{v}}$$

writing, $\vec{E}_1 = \delta\vec{E} \exp\,[i(\vec{k}.\vec{r} - \omega t)]$

$f_{\alpha 1} = \delta f_\alpha \exp\,[i(\vec{k}.\vec{r} - \omega t)]$ in Eq.(3.24) and integrating along the unperturbed trajectory we get,

$$\delta f_\alpha \exp[i(\vec{k}.\vec{r} - \omega t)] = -\frac{q_\alpha}{m_\alpha} \int_{-\infty}^{t} \left(\frac{\partial f_{\alpha o}(\vec{v})}{\partial \vec{v}'} \right)\left[\left(1 - \frac{\vec{k}.\vec{v}'}{\omega}\right) \bar{\bar{I}} + \frac{\vec{k}.\vec{v}}{\omega} \right] \cdot \delta\vec{E} \exp[i(\vec{k}.\vec{r}' - \omega t')]\, dt'$$

Substituting $\vec{k}.(\vec{r}-\vec{r}') - \omega\zeta = \Psi(\zeta)$, where $\zeta = t-t'$, we get,

$$\delta f_\alpha = -\frac{q_\alpha}{m_\alpha} \int_{o}^{\infty} \left[\frac{\partial f_{\alpha o}(\vec{v}')}{\partial \vec{v}'} \right]\left[\left(1 - \frac{\vec{k}.\vec{v}'}{\omega}\right) \bar{\bar{I}} + \frac{\vec{k}.\vec{v}'}{\omega} \right] \delta\vec{E} \exp[-i\,\Psi(\zeta)]\, d\zeta \qquad \text{...... } 3.25$$

Now we choose $\vec{V}_{11}$ in the direction of $\vec{B}_o$ and $\vec{V}_\perp$ in the x-y plane,

$$\vec{V} = V_\perp \cos\theta\, \hat{x} + V_\perp \sin\theta\, \hat{y} + V_{11}\, \hat{z} \qquad 3.26$$

and at t', $\vec{V}'$ can be written as

$$\vec{V}' = V_\perp \cos(\omega_{c\alpha}\zeta + \theta)\, \hat{x} + V_\perp \sin(\omega_{c\alpha}\zeta + \theta)\, \hat{y} + V_{11}\, \hat{z} \qquad 3.27$$

As a result of perturbed distribution function δf_α the perturbed current density may be expressed as,

$$\vec{J}_1 = -\sum_\alpha q_\alpha n_\alpha \int \vec{v}\ \delta\ f_\alpha\ d\vec{v}$$

using Eqs. (3.20), (3.22) and (3.25) we obtain the following expression for dielectric tensor,

$$\varepsilon\ (\omega,\vec{k}) = 1 + \sum_\alpha \left(\frac{\omega_\alpha^2}{i\omega}\right) \int_0^{2\pi} d\theta \int_0^\infty v_1 dv_1 \int_{-\infty}^\infty dv_{11} \int_0^\infty d\zeta\ v\left[\frac{\partial f}{\partial \vec{v}'}\right] \left[1-\frac{\vec{k}.\vec{v}'}{\omega}\right] \bar{\bar{I}}$$

$$+ \frac{k.v'}{\omega}]\ \exp\ [-i\ \Psi(\zeta)]\ .\ .\ .\ .\ .\ 3.28$$

where, $\omega_\alpha^2 = \left(\frac{4\pi n_\alpha q_\alpha^2}{m_\alpha}\right)$

Partial integration of above equation with respect to ζ yields

$$\varepsilon(\omega,\vec{k}) = \left(1-\frac{\omega_p^2}{\omega^2}\right) \bar{\bar{I}} + \sum_\alpha \frac{\omega_\alpha^2}{\omega^2} \int_0^{2\pi} d\theta \int_0^\infty v_1 dv_1 \int_{-\infty}^\infty dv_{11} \int_0^\infty \vec{v}\Big(\frac{\partial}{\partial\zeta}\ \frac{\partial f_\alpha(\vec{v}')}{\partial\vec{v}'}$$

$$- ik\frac{\partial f(\vec{v}')}{\partial\vec{v}'}\ \vec{v}'\Big)\ \exp[-i\ \Psi\ (\zeta)]\ d\zeta\ \ .\ .\ .\ .\ .\ 3.29$$

To carry out the above integration, we choose wave vector $\vec{k}$ as

$\vec{k} = k_\perp\ \hat{x} + k_{11}\hat{z}$ 3.30

$$\frac{\partial f_\alpha(\vec{v}')}{\partial\vec{v}'} = \frac{\partial f}{\partial v_1}\left[Cos(\omega_{c\alpha}+\theta)\hat{x} + Sin(\omega_{c\alpha}+\theta)\hat{y}\right] + \frac{\partial f_\alpha}{\partial v_{11}}\hat{z} \qquad 3.31$$

and $\exp[-i\Psi(\zeta)] = \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} J_n\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right) J_{n'}\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right)$

$$\cdot \exp[-i\{n(\omega_{c\alpha}\zeta+\theta) - n'\theta + k_{\parallel}v_{\parallel}\zeta - \omega\zeta\}] \quad 3.32$$

Substituting (3.31) and (3.32) in (3.29), we get (see Ichimarv, 1973),

$$\varepsilon(\omega,\vec{k}) = \left(1-\frac{\omega_p^2}{\omega^2}\right)\bar{\bar{I}} - \sum_\alpha \frac{\omega_\alpha^2}{\omega^2} \sum_{n=-\infty}^{\infty} \int d\vec{v} \left(\frac{n\omega_{c\alpha}}{v_\perp}\frac{\partial f_{\alpha o}}{\partial v_\perp} + k_{\parallel}\frac{\partial f_{\alpha o}}{\partial v_{\parallel}}\right) \cdot$$

$$\frac{A_\alpha}{(n\omega_{c\alpha}+k_{\parallel}v_{\parallel}-\omega)} \quad \cdots\cdots\cdots \quad 3.33$$

where,

$$A = \begin{vmatrix} n^2\omega_{c\alpha}^2 J^2 & iv_\perp\left(\frac{n\omega_{c\alpha}}{k}\right)J_n J_{n'} & V_{\parallel}\left(\frac{n\omega_{c\alpha}}{k_\perp}\right)J_n^2 \\ -iv_\perp\left(\frac{n\omega_{C\alpha}}{\kappa_\perp}\right)J_n J_n' & v_\perp^2(J_n')^2 & -iv_{\parallel}v_\perp J_n J_n' \\ v_{\parallel}\left(\frac{n\omega_{c\alpha}}{k_\perp}\right)J_n^2 & iv_{\parallel}v_\perp J_n J_n' & v_{\parallel}^2 \; J_n^2 \end{vmatrix}$$

$$J_n = J_n\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right), \qquad J_n' = \frac{dJ_n(k_\perp v_\perp/\omega_{c\alpha})}{d(k_\perp v_\perp/\omega_{c\alpha})} \quad \cdots\cdots\cdots\cdots \quad 3.34$$

Substituting $\varepsilon(\omega,\vec{k})$ in Eq.(3.23) we get the dispersion relation for electrostatic waves in a magnetized plasma

$$D(\omega,\vec{k}) = 1 + \sum_\infty \frac{\omega_\alpha^2}{k^2} \sum_{n=-\infty}^{\infty} \int dv\left(\frac{n\omega_{c\alpha}}{v_\perp}\frac{\partial f_{\alpha o}}{\partial v_\perp} + k_{\parallel}\frac{\partial f_{\alpha o}}{\partial v_{\parallel}}\right)$$

$$J_n^2\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right) / (\omega - k_{\parallel}v_{\parallel} - n\omega_{c\alpha}) \quad \cdots\cdots \quad 3.35$$

To evaluate the integral in Eq.(3.35) one has to know the unperturbed distribution function $f_{\alpha o}$ for particles. We assume an isotropic Maxwellian velocity distribution for plasma ions

$$f_{pi} = n_{pi}(1/\pi v_{ti}^2)^{3/2} \exp[-\{v_{\parallel}^2 + v_{\perp}^2)/v_{ti}^2]$$

$$\text{where} \quad v_{ti} = \left|\frac{2T_i}{m_i}\right|^{\frac{1}{2}} \quad \ldots \ldots \quad 3.36$$

and an isotropic drifted Maxwellian velocity distribution for electrons

$$f_{pe} = n_{pe}\left(\frac{1}{\pi v_{te}^2}\right)^{3/2} \exp\left[-\{v_{\parallel}^2 + (v_{\perp}-v_o)^2\}/v_{te}^2\right] \qquad 3.37$$

where n_{pi} and n_{pe} are the plasma ion and plasma electron densities respectively. V_{ti} and V_{te} are respectively the thermal velocity of plasma ions and electrons. V_o is the drift velocity of electrons perpendicular to the magnetic field. For beam electron the distribution function may be taken of the form

$$f_{pb} = n_b\left(\frac{1}{\pi v_{tb}^2}\right)^{3/2} \exp\left[-\{(v_{\parallel}-v_b)^2] + (v_{\perp}-v_o)^2]\}/v_{tb}^2\right] \qquad 3.38$$

The distribution functions described by Eqs. (3.36), (3.37) and (3.38) characterize a beam-plasma system consisting of plasma ions, plasma electrons having transverse drift velocity and beam electrons having parallel as well as transverse drift velocity components.

Upon substitution of (3.36), (3.37) and (3.38) in (3.35) and using the following definite integral

$$\int_0^\infty e^{-p^2t^2} J_n(at) J_n(bt)\, t\, dt = \frac{1}{2p^2} e^{-(a^2+b^2)/4p^2} I_n(ab/2p^2) \quad ,\ldots. \quad 3.39$$

we get,

$$D(\omega,\vec{k}) = 1 + \varepsilon_{pi} + \varepsilon_{pe} + \varepsilon_{pb} \qquad 3.40$$

where, $$\varepsilon_{pi} = \left(\frac{2\omega_{pi}^2}{k^2 v_{ti}^2}\right) \{1 + \frac{\omega}{|k_{\parallel}| v_{ti}} \sum_{n=-\infty}^{\infty} Z \; \frac{(\omega - n\omega_{ci})}{|k_{\parallel}| v_{ti}} \exp(-\lambda_i) \; I_n(\lambda_i) \qquad 3.41$$

$$\varepsilon_{pe} = \left(\frac{2\omega_{pe}^2}{k^2 v_{te}^2}\right) \{1 + \frac{\omega - k_{\perp} v_o}{|k_{\parallel}| v_{te}} \sum_{n=-\infty}^{\infty} Z \; \left(\frac{\omega - k_{\perp} v_o - n\omega_{ce}}{|k_{\parallel}| v_{te}}\right) \exp(-\lambda_e) \; I_n(\lambda_e) \ldots 3.42$$

$$\varepsilon_{pb} = \left(\frac{2\omega_{pb}^2}{k^2 v_{tb}^2}\right) \{1 + \frac{\omega - k_{\parallel} v_b - k_{\perp} v_o}{|k_{\parallel}| \; v_{tb}} \sum_{n=-\infty}^{\infty} \left(\frac{\omega - k_{\parallel} v_b - k_{\perp} v_o - n\omega_{ce}}{|k_{\parallel}| \; v_{tb}}\right) \exp(-\lambda_b) I_n(\lambda_b) \qquad 3.43$$

with

$$\lambda_i = \frac{k_{\perp}^2 v_{ti}^2}{2\omega_{ci}^2}$$

$$\lambda_e = \frac{k_\perp^2 v_{te}^2}{2\omega_{ce}^2}$$

$$\lambda_b = \frac{k_\perp^2 v_{tb}^2}{2\omega_{ce}^2}$$

where, ω_{ci}, ω_{ce} are ion cyclotron and electron cyclotron frequencies respectively. ω_{pi}, ω_{pe} are respectively the ion and electron plasma frequencies, and

$$Z(\xi) = 2i \exp(-\xi^2) \int_{-\infty}^{i\xi} \exp(-t^2)dt \quad . \quad . \quad . \quad . \qquad 3.44$$

is the plasma dispersion function (Fried and Conte, 1961), ϵ_{pi}, ϵ_{pe}, ϵ_{pb} are called plasma ion, plasma electron and beam electron susceptibility respectively.

The plasma characterized by finite but not too large values of ξ_n and the condition $\frac{k_\perp^2 v_{te,i,b}^2}{2\omega_{ce,i,b}^2} \ll 1$ is often termed as warm plasma. For such a case, Eqs. (3.41), (3.42), (3.43) respectively reduces to,

$$\epsilon_{pi} = \frac{2\omega_{pi}^2}{k^2 v_{ti}^2} \{ 1 + \xi_{pi} Z(\xi_{pi})\} \qquad 3.45$$

$$\epsilon_{pe} = \frac{2\omega_{pe}^2}{k^2 v_{te}^2} \{ 1 + \xi_{pe} Z(\xi_{pe})\} \qquad 3.46$$

$$\varepsilon_{pb} = \frac{2\omega_{pb}^2}{k^2 v_{tb}^2} \{1 + \xi_{pb} Z(\xi_{pb})\} \qquad 3.47$$

where

$$\xi_{pi} = \left(\frac{\omega}{k_{11} v_{ti}}\right) ; \quad \xi_{pe} = \left(\frac{\omega - k_1 v_o}{k_{11} v_{te}}\right)$$

and

$$\xi_{pb} = \left(\frac{\omega - k_{11} v_b - k_1 v_o}{k_{11} v_{tb}}\right) \qquad 3.48$$

For cold beam and cold plasma ($v_{ti} \simeq v_{te} \simeq v_{tb} \simeq 0$), and using asymptotic expansion of $Z(\xi)$,

$$Z(\xi) = -\xi^{-1}\left(1 + \frac{1}{2\xi^2} + \ldots\ldots\right); \quad \xi \longrightarrow \infty \qquad \text{Eq.(3.40)}$$

retaining the terms upto $n = 0, \pm 1$ terms only, simplifies to,

$$D(\omega,\vec{k}) = 1 - \frac{\omega_{pi}^2}{\omega^2}\frac{k_{11}^2}{k^2} - \frac{\omega_{pi}}{\omega^2 - \omega_{ci}^2}\frac{k_1^2}{k^2} - \frac{\omega_{pe}^2}{(\omega - k_1 v_o)^2}\frac{k_{11}^2}{k^2}$$

$$- \frac{\omega_{pe}^2}{[(\omega - k_1 v_o)^2 - \omega_{ce}^2]}\frac{k_1^2}{k^2} - \frac{\omega_{pb}^2 k_{11}^2}{(\omega - k_{11} v_b - k_1 v_o)^2 k^2}$$

$$- \frac{\omega_{pb}^2 \quad k_1^2}{[(\omega - k_{11} v_b - k_1 v_o)^2 - \omega_{ce}^2] k^2} = 0 \qquad 3.49$$

3.4 THE DISPERSION CHARACTERISTICS OF A COLD PLASMA PENETRATED BY A COLD BEAM WITH CROSS FIELD TRANSVERSE DRIFT

It is seen from Eqs.(3.15) and (3.49) that for the case of cold beam plasma, both fluid and kinetic approaches yield the same dispersion equation. According to our experimental results (presented in Chapter V) for $\omega_{pb} < \omega_{pe} \ll \omega_{ce}$, the observed instabilities have frequency and wave numbers such that $\omega \sim \omega_{pi} \gg \omega_{ci}$ and $k_{11} \ll k_1$. Under these parameter conditions the last three terms of Eq.(3.49) can be neglected and after some simplication we get,

$$D(\omega, \vec{k}) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2 \; k_{11}^2}{(\omega - k_1 v_o)^2 k^2} = 0 \qquad 3.50$$

This equation is equivalent to a fouth order equation for with real co-efficients. In the absence of beam the dispersion relation for cold plasma reduces to that of ion plasma waves.

$$f(\omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} = 0 \qquad (3.51)$$

The solution of above equation is $\omega = \omega_{pi}$ and is shown in Fig. (3.3). It can be seen that only those wave modes which have $\omega = \omega_{pi}$ can exist. Such wave modes have zero group velocity can have any phase velocity depending on the value of k.

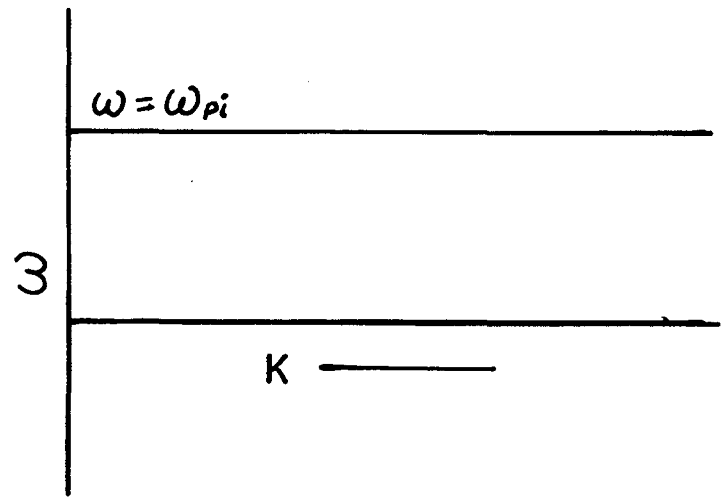


Fig 3·3 Dispersion characteristic of an ion plasma wave in a cold plasma.

We shall now discuss the dispersion relation given by equation (3.50) in the context of stability analysis (Hasegawa, 1975). Dispersion relation for drifting electrons can be obtained by putting $\omega_{pi} = 0$ in equation (3.50),

$$1 - \frac{\omega_{pe}^2 k_{11}^2/k^2}{(\omega - k_1 v_o)^2} = 0$$

or, $$\omega - k_1 V_o = \pm \frac{\omega_{pe} k_{11}}{k} \qquad 3.52$$

The above equation shows that the drifting electron beam component supports two wave modes. Depending on the phase velocity of these waves in relation to the beam velocity, these waves are termed as fast and slow space charge waves (Fig. 3.4).

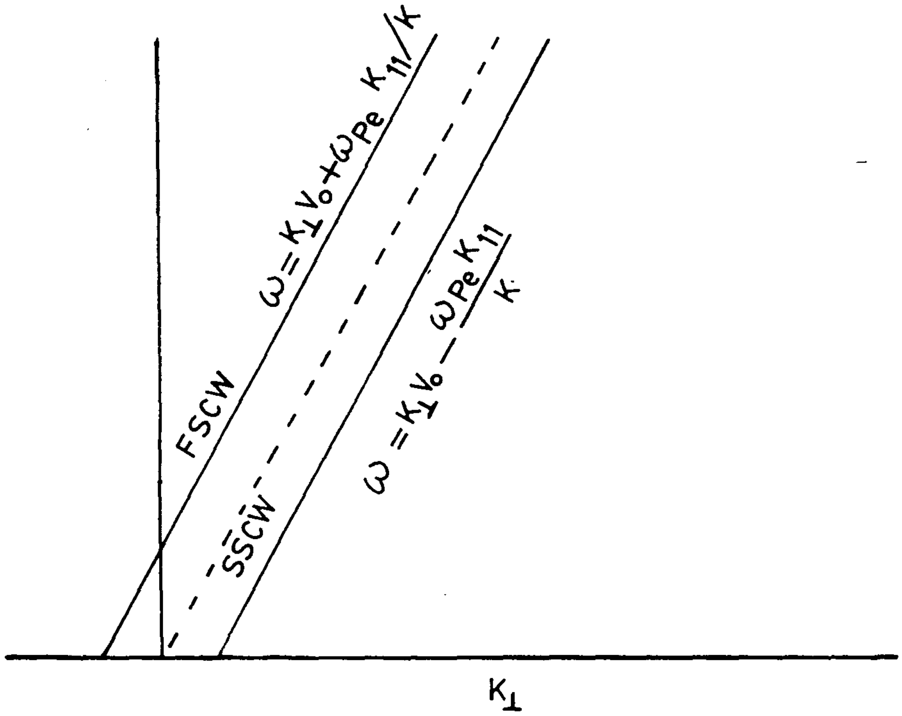


Fig 3·4 DISPERSION CHARACTERISTICS OF SLOW (SSCW) AND FAST (FSCW) SPACE CHARGE WAVES ON THE BEAM

The slow and fast space charge waves which correspond to the lower (-ve) and upper (+ve) signs in Eq.(3.52) carry negative and positive energies respectively and are, therefore, also known as negative and positive energy waves on the drifting beam on electrons. In a plasma which consists of a beam of electrons streaming through stationary ions, the dispersion relation for which is given by (3.50), the coupling of negative energy waves on the beam electrons with the positive energy waves in the plasma can result in an instability, when a negative energy wave interacts with a positive energy plasma wave which is able to extract energy from the negative energy wave, both waves grow at the expense of energy of the beam. The coupling of beam and plasma waves is shown in Fig. (3.5).

Solution of (3.50) can be obtained either for complex ω $(= \omega_r + i\omega_i)$ or complex k_1 $(k_{1r} + ik_{1i})$. A root of equation (3.50) with $\omega_i > 0$ or $k_{1i} < 0$ represents growth. ω_i, k_{1i} are known as spatial and temporal growth rates respectively.

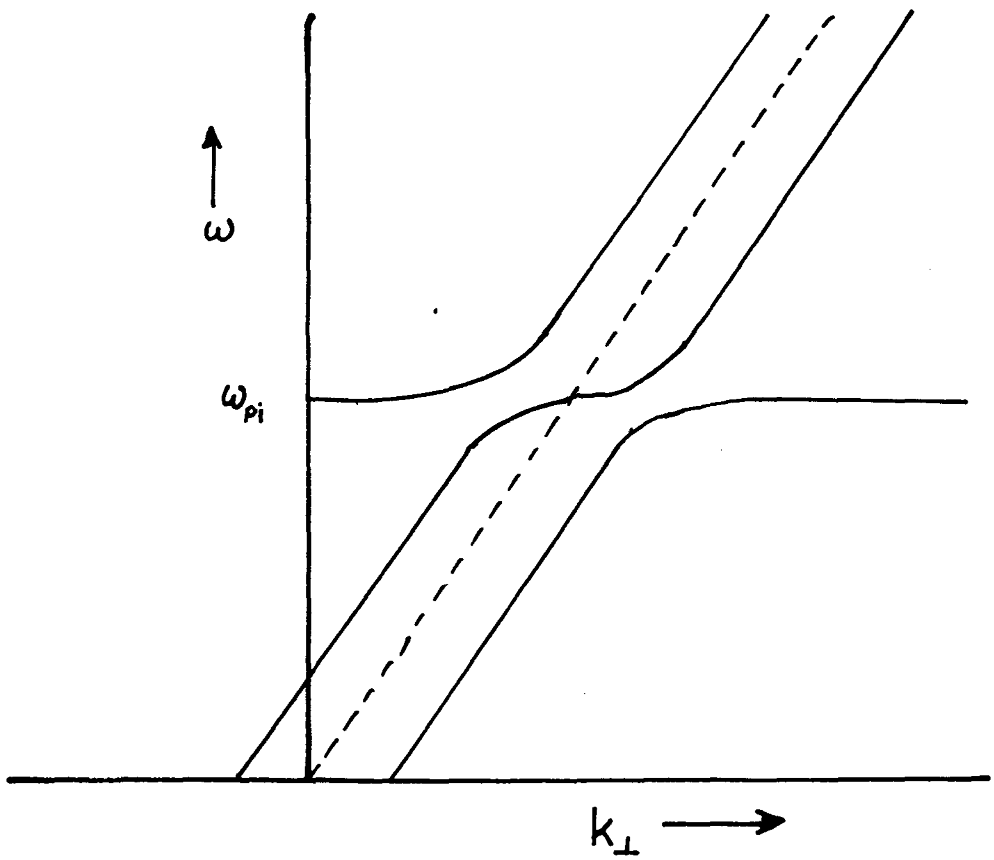


Fig 3·5 Coupled plasma and beam mode branches.

3.5 CALCULATION OF GROWTH RATE

The growth rate for the ion plasma instability can be obtained analytically by two different methods a) Exact method, (b) perturbation method. In the first Eq.(3.50) is solved exactly as in the case of two stream instability type situation (Akhiezer, 1975). In the other method the electron beam term is assumed to be a perturbation. The expression for the growth rate is obtained by evaluating the effect of this perturbation on the real part of the dispersion of ion plasma waves.

a) Exact Method: We recall Eqn. (3.50) which is

$$D(\omega, \vec{k}) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2 k_{11}^2/k^2}{(\omega - k_1 v_o)^2} = 0$$

With $x = \frac{\omega}{\omega_{pi}}$; $x-y = \frac{\omega}{g} - \frac{k_1 v_o}{g}$; where $g = \frac{\omega_{pe} k_{11}}{k}$, the above equation can be written as

$$\frac{1}{x^2} + \frac{1}{(x-y)^2} = 1$$

or $$y = x \pm x \frac{(x^2-1)^{\frac{1}{2}}}{(1 - x^2)} = x \mp \frac{1}{(x^2-1)^{\frac{1}{2}}}$$

Therefore, $\frac{\omega}{\omega_{pi}} - \frac{\omega k}{\omega_{pe} k_{11}} + \frac{k k_1 v_o}{\omega_{pe} k_{11}} = \frac{\omega}{\omega_{pi}} \mp \frac{\omega/\omega_{pi}}{[(\omega/\omega_{pi})^2 - 1]^{\frac{1}{2}}}$

or $\quad \omega = k_1 V_o \pm \frac{\omega(\omega_{pe} k_{11})/(\omega_{pi} k)}{[(\omega/\omega_{pi})^2 - 1]^{\frac{1}{2}}}$

For $\omega < \omega_{pi}$ above equation gives complex ω the imaginary part of which can be written as

$$\frac{\omega_i}{\omega} = \frac{(m_i n_e/m_e n_i)^{\frac{1}{2}} k_{11}/k}{[1 - (\omega/\omega_{pi})^2]^{\frac{1}{2}}} \qquad 3.53$$

The above equation gives the growth rate of ion plasma wave instability for $0 < \omega < \omega_{pi}$.

b) Perturbation Method: We write Eq.(3.50) in the form

$$f(\omega)(\omega - k_1 V_o)^2 = \omega_{pe}^2 k_{11}^2 / k^2$$

where $\quad f(\omega) = 1 - \frac{\omega_{pi}^2}{\omega^2}$

The first two terms on the left hand side of the above equation are coupled via the term on the right handside. The instability arises as a result of the coupling of ion plasma wave ($\omega = \omega_{pi}$)

with the beam mode waves ($\omega = k_1 V_o$). The resonance interaction between the two takes place at the point in (ω,k) space characterized by $\omega \simeq \omega_{pi} \simeq k_1 V_o$ i.e., where plasma and beam dispersion terms are simultaneously zero. Unperturbed (without beam) dispersion relation is given by $f(\omega_r, k) = 1 - \frac{\omega_{pi}^2}{\omega^2} = 0$. To calculate growth rate we will assume the third term in Eqn.(3.50) is a small perturbation. We will look at solutions close to $\omega_r \simeq k_1 V_o$ the resonance condition. Expanding $f(\omega)$ around $\omega = \omega_r$ we get,

$$f(\omega) = f(\omega_r) + (\omega - \omega_r) \left. \frac{\partial f}{\partial \omega_r} \right|_{\omega = \omega_r}$$

or

$$f(\omega) = f(\omega_r) + i\omega_i \left. \frac{\partial f}{\partial \omega_r} \right|_{\omega = \omega_r}$$

Since $f(\omega_r) = o$, we have

$$i\omega_i \left. \frac{\partial f}{\partial \omega_r} \right|_{\omega = \omega_r} = \frac{\omega_{pe}^2 k_{11}^2/k^2}{(\omega - k_1 v_o)^2}$$

Therefore,

$$\omega_i = \left| \frac{\omega_{pe}^2 k_{11}^2/k^2}{\partial f/\partial \omega_r \Big|_{\omega=\omega_r}} \right|^{1/3} \cos\frac{(2n+1)}{6}\pi \qquad 3.54$$

where, n = 0, 1, 2, ...

CHAPTER IV

4.1 INTRODUCTION

In most branches of physics, including plasma physics, until the development of specific analytic techniques, and before computers of the sufficient size and power were available, the hope of obtaining agreement between real situations and their mathematical models was restricted to linear phenomena.

However, plasma is basically a nonlinear medium. When plasma instabilities are excited due to some free energy feature and they grow to large amplitudes and a number of interesting nonlinear phenomena appear which modify the plasma particles. According to linear theory the amplitude of plasma wave instability increases exponentially until few e-folding occur. This duration of linear growth cannot normally be ascertained since one usually does not know when to start looking, but instead one observes the wave only after they have grown to a large, steady state amplitude. The fact that the waves are no longer growing means that the linear theory is no longer valid and some nonlinear effect is preventing the waves from growing to catastrophic proportions in the plasma. This has led to much interest in the intensive experimental and theoretical investigations of various possible nonlinear mechanisms which cause the amplitude saturation and various associated effects.

During last couple of decades particle traping, 3-wave decay, 4-wave parametric process, filamentation and modulational instabilities, cavitational effects and anharmonic effects (Zakarov, 1972, 1975; Bobin et al. 1973; Porkolab, 1974, 1977; Porkolab et al. 1977; Tripathi et al. 1979; Liu et al. 1980, 1984; Gregobi and Liu, 1980; Stevens and Getty, 1980) have been suggested as possible saturation mechanisms. Each of the above mentioned nonlinear process may be dominant in some parameter regime. In particle trapping (Drummond et al. 1970; O'Neil et al. 1971; Matsiborko et al. 1972; Nyack and Christiansen, 1974; Parkar and Throop, 1979), if the instability is sufficiently monochromatic, i.e. the wave with maximum growth rate has completely outgrown the unstable waves at neighbourhood frequencies, the beam electrons become trapped in the potential troughs of the wave. This limits the growth of the instability and thereafter wave amplitude oscillates with the bounce frequency which is determined by the wave electric field. Parametric effects such as 3-wave decay, modulational and filamentation instabilities are mainly associated with large amplitude high frequency electron plasma wave and electron cyclotron harmonic wave instabilities. Other nonlinear mechanisms which have been proposed to cause amplitude saturation is the anharmonic effect (Hsuan, 1968), wave particle scattering (Dupree, 1968), mode-mode coupling (Stix, 1969). Theoretical models based on anharmonic oscillator have also been frequently used to understand the nonlinear saturation of plasma instabilities via mode-mode coupling both at low and high frequencies (characteristic of ion and electron motions) in plasmas (Hsuan, 1968; Keen & Fletcher, 1970; Deneef and Lashinsky, 1973).

Using mode-mode coupling approach the evolution of plasma instabilities in some cases can be described by van der pol oscillator (Minorsky, 1962). Stix (1969) employed van der pol equation to explain the saturation of collisional drift waves in the plasma.

Using hydrodynamic equations for the ions and electrons Shutko (1970) derived a van der pol equation, which describes stationary ion acoustic waves. It has been shown that if the ion thermal velocity is high, in which case higher harmonics of a given mode are damped in the linear approximation, the plasma supports harmonic of ion acoustic wave that propagate essentially accross the magnetic field. The amplitude of this wave is proportional to the square root of the linear growth rate for a given mode while the frequency is shifted by an amount proportional to the growth rate. In case of small ion thermal velocity the van der pol equation allows solutions of the form of a stationary sawtooth wave.

In many experimental studies the theory of van der pol oscillator has been invoked to understand the suppression of self excited or externally driven instabilities (see for example Nakamura 1971, Tsuru et al. 1973, Tsuru, 1976; Itakura, 1976; Michelson et al. 1979). Nakamura (1970) and Nakamura et al. (1970) observed that beam driven electrostatic instabilities could be suppressed by modulating the beam density at frequencies close to the instability frequency. Suppression was also noticed when the modulation frequency was equal to that of another mode in the plasma (Nakamura and Kojima, 1970). It was shown that the instabilities obeyed van der pol equation with a forced oscillation term. Feedback stabilization of a drift instability (Keen and Aldridge, 1969; Keen, 1970), remote feedback stabilization of the ion sound instability (Keen and Fletcher, 1970) and asynchronous quenching (Keen and Fletcher, 1970) have observed when waves in plasma are launched by external means.

The experimental results on these nonlinear processes have successfully been interpreted by van der pol equation which is derived from fluid equations where in the continuity equation a source term which depends on the large amplitude instability is added. While studying the suppression of a two stream type instability in a beam plasma system by external AC electric field, subharmonic generation was noticed by Obiki et al. (1968). The instability under study was an ion acoustic wave (60 Hz).

When a modulating signal is applied to the beam, the ion acoustic oscillation is affected in various ways depending on the amplitude and frequency of applied signal. The modulating frequency was kept constant while the modulating voltage was varied from 0 to 20 v. Above a certain threshold value of modulating voltage half harmonics were observed. Okiki et al. (1968) did not give any physical explanation but conjectured that the observed phenomena may be explained by Malthew-Hill equation. This led to further investigations on subharmonic generation of instabilities (Nakamura et al. 1970; Keen and Fletcher, 1972).

In a positive column glow discharge the nonlinear behaviour of large amplitude ionization waves (Ohe and Takeda, 1973, 1980) and ion acoustic waves (Keen and Fletcher, 1972) was extensively studied and it was shown that above a certain threshold values of the pump wave amplitude subharmonic generation takes place which was satisfactorily explained by van der pol equation. Recent experiments of Yamamoto and Tsukishima (1978) show the simultaneous existence of 1/3 and 2/3 harmonics of ion acoustic wave in an argon positive column. When the amplitude of the pump ion acoustic wave with frequency f_o and wave number k_o exceeded a certain critical value two more ion acoustic waves with frequencies ($f_o/3$, $2f_o/3$) and wave numbers ($k_o/3$, $2k_o/3$) were observed.

While studying the electron beam excited instabilities in a beam plasma system in the low frequency regime ($\omega \sim \omega_{ci}, \omega_{pi}$), Jain observed subharmonics of the fundamental wave. Unlike the work of other researchers the subharmonics in his experiment were observed with naturally occurring high amplitude instabilities without modulating the beam or using externally launched waves. The fundamental pump wave is the initial instability at $\omega \sim \omega_{pi}$ and the excited subharmonics were observed at $\omega_{pi}/2$, $\omega_{pi}/3$, $2\omega_{pi}/3$...., where ω_{pi} is the ion plasma frequency.

In section 4.2, we derive van der pol equation for the electron density perturbation starting from momentum and continuity equation. A theoretical analysis of the subharmonic generation is presented in section 4.3.

4.2 Half Harmonic Generation

The plasma stability is investigated using two fluid equations which describe the motion of the electrons and ions. For the sake of mathematical simplicity, certain condition have been used.

Ion equation of motion is

$$m_i \frac{d\vec{v}_i}{dt} + m_i \nu \vec{v}_i = - e \vec{\nabla} \Phi + \frac{e}{c} \vec{v}_i \times \vec{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.1$$

where $\vec{v}_i$ is the ion velocity, $\vec{B}$ is the magnetic field, ν is the ion neutral collision frequency.

The continuity equation for ion can be written as

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (n_i \vec{v}_i) = S_i(n, T_e, E) \quad . \quad . \quad . \quad . \qquad 4.2$$

Here S_i is a source term which arises because of the presence of large amplitude fluctuations in the plasma, creating electron heating, local ionization etc. (Keen and Fletcher, 1972). S_i is assumed to be a function of local density n, electron temperature T_e and local electric field $\vec{E}$. The density n is taken of the form $n_i = n_o + n_1$, where n_o is a steady state average value and n_1 is a perturbed oscillating quantity which is a time varying function of $\vec{r}$.

The electric field from thermodynamic consideration (Hsuan et al. 1967) can be written as

$$\vec{E}(n, T_e) = \vec{E}_o + \left(\frac{\partial E}{\partial n}\right)_o n_1 + \tfrac{1}{2}\left(\frac{\partial^2 E}{\partial n^2}\right)_o n_1^2 + \frac{1}{6}\left(\frac{\partial^3 E}{\partial n^3}\right)_o n_1^3 + \ldots.$$

$$= \vec{E}_o + a n_1 + b n_1^2 + c n_1^3 + \ldots \ldots \ldots \ldots \qquad 4.3$$

The electric field $\vec{E}$ has been expanded in a power series and the subscript 0 refers to the value of $\vec{E}$ at $\vec{E}_o$. The source term can be written in terms of the change of electric field

$$\delta E = (E - E_o) \quad \text{as}$$

$$S_e = \frac{\partial S_i}{\partial E} \delta E$$

$$= \frac{\partial S_i}{\partial E} (an_1 + bn_1^2 + cn_1^3 + \ldots\ldots\ldots)$$

$$= \alpha n_1 - \beta n_1^2 - \gamma n_1^3 \qquad 4.4$$

where α β γ are small positive constants β and γ are responsible for nonlinear saturation of instability amplitude. The negative sign infront of β and γ have been substituted from the experimental relationship between E and n_1.

The instability of interest in our case is associated with ion plasma wave. Spatial variations of the form $\exp(\vec{k}.\vec{r}-\omega t)$ are considered, where $\vec{k}$ is the wave vector associated with this wave and ω is the frequency. Ion density n_i can be written as (Jain, Tripathi, Sharma; in Press)

$$n_i = - \frac{\omega_{pi}^2}{\omega^2} \frac{\nabla^2 \Phi}{4 \pi e} \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.5$$

By eliminating $\vec{v}_i$, Φ from (4.1), (4.2), (4.5) and using (4.4), a differential equation for the time varying portion of the density, after neglecting smaller order terms is obtained,

$$\frac{d^2 n_1}{dt^2} + \frac{dn_1}{dt} \{ (\nu - \alpha) + 2\beta n_1 + 3\gamma n_1^2 \} + \omega^2 n_1 = 0 \qquad 4.6$$

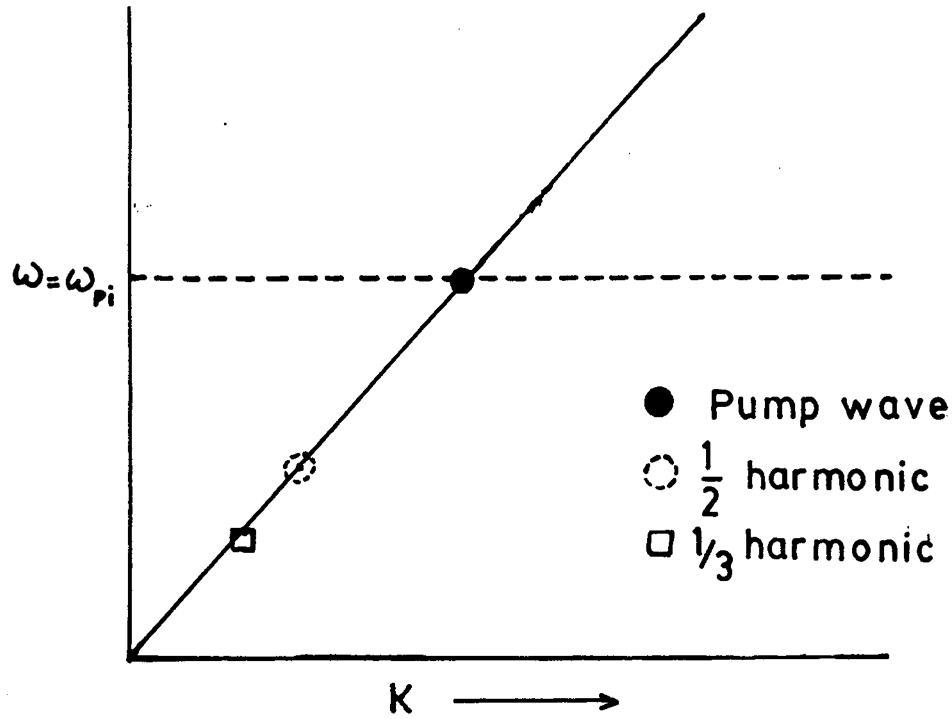
In the low frequency regime quasineutrality condition $n_i \simeq n_e$ holds. We can write the equation for the time varying portion of the density perturbation of electron as,

$$\frac{d^2 n_1}{dt^2} + \frac{dn_1}{dt} \{ (\nu - \alpha) + 2\beta n_1 + 3\gamma n_1^2 \} + \omega^2 n_1 = 0 \qquad 4.7$$

where ω satisfies the linear plasma electron transverse drift beam dispersion $(\omega - k_1 V_o)^2 = \frac{\omega_{pe}^2 k_{11}^2}{k^2}$. In the treatment presented here we assume that the initial instability at $\omega \sim \omega_{pi}$ in the beam plasma system acts as a forcing function at the transverse beam dispersion branch (Fig. 4.1), whose perturbed electron density acts like a van der pol oscillator.

Therefore, equation (4.7) can be written as with the forcing term

$$\frac{d^2 n_1}{dt^2} + \omega^2 n_1 + \{ (\nu - \alpha) + 2\beta n_1 + 3\gamma n_1^2 \} \frac{dn_1}{dt} = \omega_o^2 A \sin \omega t \quad . . . \quad 4.8$$



Fig(4.1) Proposed scheme for nonlinear subharmonic generation.

This is a van der pol equation with forcing term. If $A \neq 0$ and $\omega_o = 2\omega$, 3ω, 4ω, etc. then one has the manifestation of the subharmonic resonance of order 1/2, 1/3, 1/4, etc. Since in our case, ω follows the dispersion $(\omega - k_1 V_o)^2 = \frac{\omega_{pe}^2 k_{11}^2}{k^2}$ condition $\omega_o = 2\omega$, 3ω, 4ω etc. are always satisfied. Therefore, we may expect subharmonic generation. However, in our theoretical calculations we will keep ω as constant and ω_o as variable, therefore, equation 4.8 can be rewritten as

$$\frac{d^2 n_1}{dt^2} + \omega_o^2 n_1 + \{ (\nu - \alpha) + 2\beta n_1 + 3\gamma n_1^2 \} \frac{dn_1}{dt} = \omega^2 A \sin \omega t \quad \ldots\ldots 4.9$$

4.3 Forcing Oscillation at Second Harmonic $\omega = 2\omega_o$

Substitute $\omega_o t = \tau$ and $n_1 = \frac{\omega_o x}{\beta}$ equation 4.9 reduces to,

$$\frac{d^2x}{d\tau^2} + x + \left\{ \frac{(\nu-\alpha)}{\omega_o} + 2x + \frac{3\gamma\omega_o}{\beta^2} \right\} \frac{dx}{d\tau} = \frac{4A\beta}{\omega_o} \text{Sin } 2\tau \quad . \quad . \quad . \qquad 4.10$$

The above equation can be put in the form,

$$\frac{d^2x}{d\tau^2} + x + f(x, \frac{dx}{d\tau}) = \lambda \text{ Sin } 2\tau . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.11$$

where $f(x,\frac{dx}{d\tau}) = (g + 2x + \eta x^2) \frac{dx}{d\tau}$ 4.12a

and λ , g, η are given as $\lambda = \frac{4A\beta}{\omega_o}$; $g = \frac{(\nu-\alpha)}{\omega_o}$; $\eta = \frac{3\gamma\omega_o}{\beta^2}$ 4.12b

By adopting a trial solution of the form

$$x = a \text{ Sin } \tau - b \text{ Cos}\tau - \frac{\lambda}{3} \text{ Sin } 2\tau$$

$$= X \text{ Sin } (\tau - \psi) - \frac{\lambda}{3} \text{Sin } 2\tau \qquad 4.13$$

where ψ is a phase angle and X is the amplitude of the oscillations produced at frequency ω_0 ; and substituting in Eq. 4.12a we get,

$$b \left[g + \eta \left(\frac{a^2}{4} + \frac{b^2}{4} + \frac{\lambda^2}{18}\right)\right] = -a\lambda/3 \qquad 4.14$$

$$a \left[g + \eta \left(\frac{a^2}{4} + \frac{b^2}{4} + \frac{\lambda^2}{18}\right)\right] = -b\lambda/3 \qquad 4.15$$

with $a^2 + b^2 = X^2$ and $X^2 + \frac{2\lambda^2}{9} = Y^2$ we get,

$$Y^4 + \frac{8g}{\eta} Y^2 + \frac{16}{\eta^2}\left(g^2 - \frac{\lambda^2}{9}\right) = 0$$

or $$Y^4 + Y^2 G + H(\lambda) = 0 \quad . \quad . \quad . \quad . \quad . \quad 4.16$$

where $$G = \frac{8g}{\eta} \quad ; \quad H(\lambda) = \frac{16}{\eta^2}\left(g^2 - \frac{\lambda^2}{9} \right) \quad . \quad . \quad . \quad . \quad 4.17$$

Therefore

$$X^2 = -\frac{2\lambda^2}{9} - \frac{G}{2} + \frac{1}{2}\left[G^2 - 4H \right]^{\frac{1}{2}} \quad 4.18$$

For subharmonics to exist $X^2 > 0$. The first term on right hand side is always negative, therefore second bracketed term on right hand side should be positive and greater than the first two terms. It can be shown that there is a minimum value of λ required for oscillation to set in. Again there is an upper limit of λ above which oscillations will not occur. The upper limit and lower limit of drive amplitude can be found

as $$\lambda = \frac{\beta^2}{\gamma\omega_o}\left[1 \pm \left(1 - \frac{6g\gamma\omega_o}{\beta^2}\right)^{\frac{1}{2}}\right] \quad 4.19$$

Therefore, for 1/2 harmonic oscillations to exist we should have

$$\frac{\beta^2}{\gamma\omega_o}\left(1 - \left[1 - \frac{6g\gamma\omega_o}{\beta^2}\right]^{\frac{1}{2}}\right) < \lambda < \frac{\beta^2}{\gamma\omega_o}\left(1 + \left[1 + \frac{6g\gamma\omega_o}{\beta^2}\right]^{\frac{1}{2}}\right) \quad 4.20$$

The value of $\lambda = \lambda_o$ for which maximum amplitude occurs, can be found from equation 4.18 by taking $\frac{dX}{d\lambda} = 0$, which gives $\lambda_o = \frac{\beta^2}{\gamma\omega_o}$

CHAPTER V

RESULTS AND DISCUSSION

5.1 Linear Excitation

In the experiment the magnetic field was varied in the range 100-500 gauss, consequently ω_{ce} takes on values from 280-1400 MHz. Ion cyclotron frequency (ω_{ci}) is determined by the relation $\omega_{ci} = \omega_{ce} \frac{m_e}{m_i}$ where $\frac{m_e}{m_i} = \frac{1}{40 \times 1836}$ for Argon ions. The typical operating pressure was 4×10^{-6} Torr. From the measurements of radial edge electric field (E_r) the value of azimuthal drift velocity was calculated and was found to be $\sim 8.4 \times 10^8$ cm/sec.

Typical beam density profiles are shown in fig.(5.1). Four different sets of spectra with varying magnetic field strength and beam current are shown in figs. (5.2 - 5.5). In each case it can be noticed that the unstable wave is approximately at ion plasma frequency. In each spectrum it is seen that the instability frequency has increased as the beam current is increased. This is probably due to the fact that the instability occurs at ion plasma frequency and as the beam current increases more ions are produced and therefore plasma frequency increases. k_{11} of the instabilities is measured by probes using interferometer technique. The value of the perpendicular wave number can be inferred in the following

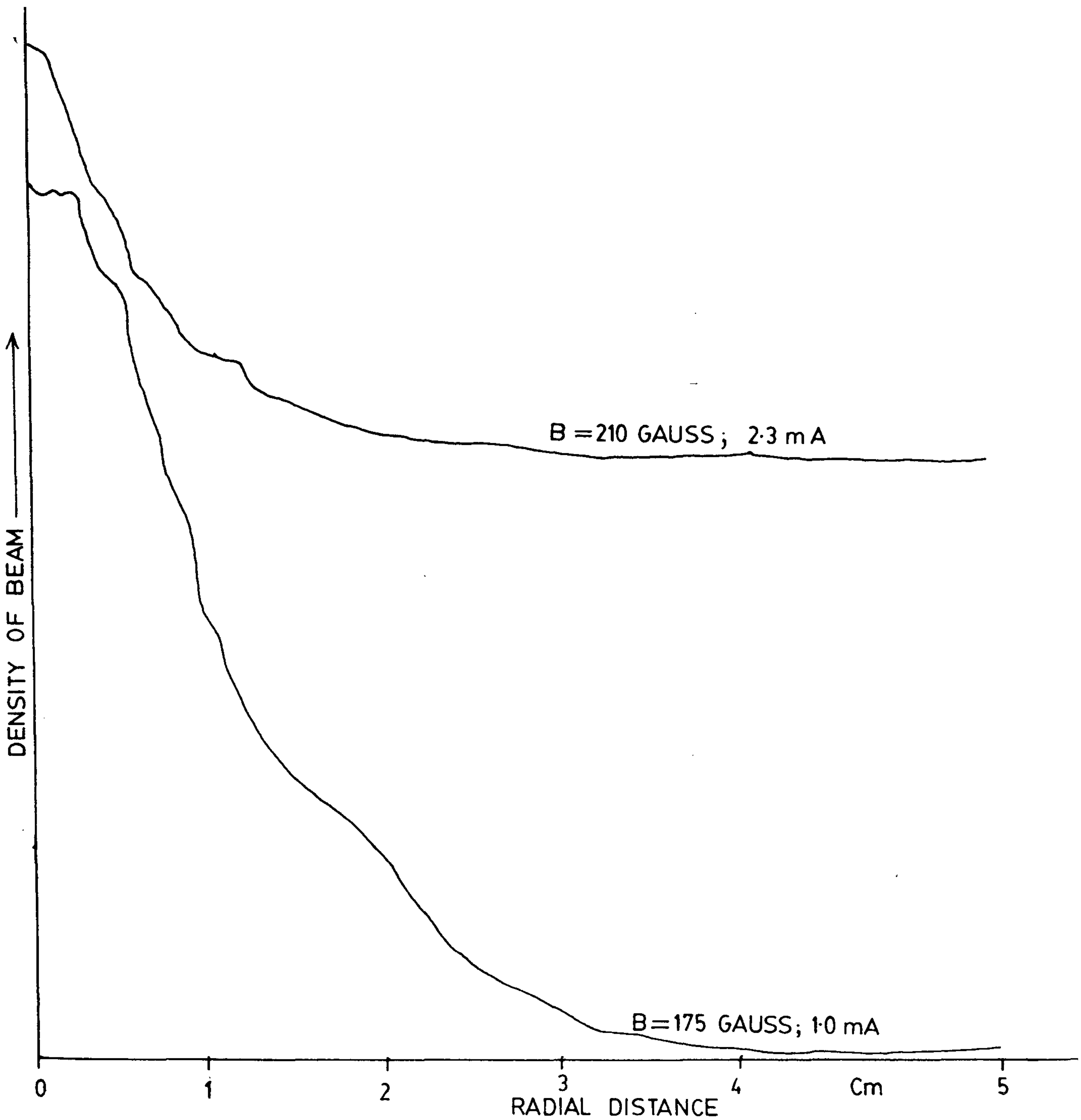


Fig: 5·1 · TYPICAL BEAM DENSITY PROFILES manner. Assuming that one complete azimuthal wavelength fits in the circumference of the beam we obtain $k_1 \simeq k_\theta = 3.3 \ \mathrm{cm}^{-1}$, where the beam radius a is taken to be $\simeq 0.3$ cm. It is clear from the experimental estimates of the parameters of the system that the following approximations can be made,

$$\omega_{pe} \ll \omega_{ce} , \qquad \omega_{pi} \gg \omega_{ci}$$

$$\frac{k_1^2 v_t^2}{2\omega_{ce}^2} \ll 1 , \quad \frac{k_1^2 v_{ti}^2}{2\omega_{ci}^2} \ll 1 , \quad \frac{k_1^2 v_{tb}^2}{2\omega_{ce}^2} \ll 1 .$$

It is interesting to note that

$$k_{11} \ll k_1 , \qquad \omega \sim \omega_{pi} \simeq k_1 v_o$$

Under these parameter conditions the system can be modelled as a cold beam cold plasma system. The analysis for such a configuration has been presented in Chapter III. Recalling the dispersion relation Eq. (3.52) it is evident that unstable waves exist with maximum growth rate around $\omega_o \simeq \omega_{pi} \simeq k_1 v_o$ which is in agreement with the experimental observations. Increasing the beam current ω_{pe} increases and therefore the growth rate increases which explains the behaviour of the amplitude of the instability with variation in the beam current. Therefore it may be concluded that the observed instabilities are

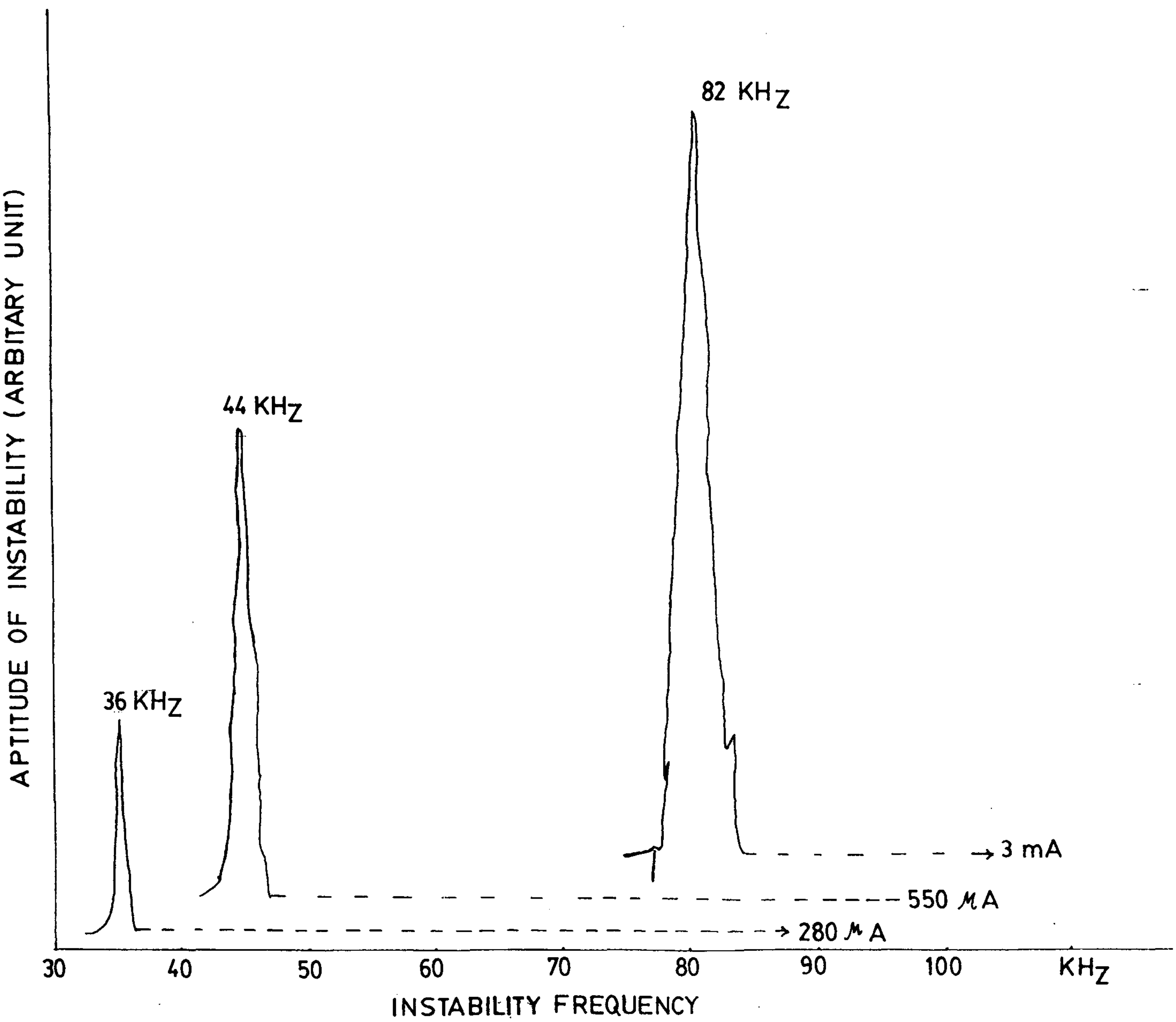


Fig. 5·2 INSTABILITY SPECTRA WITH BEAM CURRENT AS PARAMETER. THE OTHER PARAMETERS ARE B=105 GAUSS; BEAM ENERGY 300 ev; PRESSURE 4×10^{-6} Torr

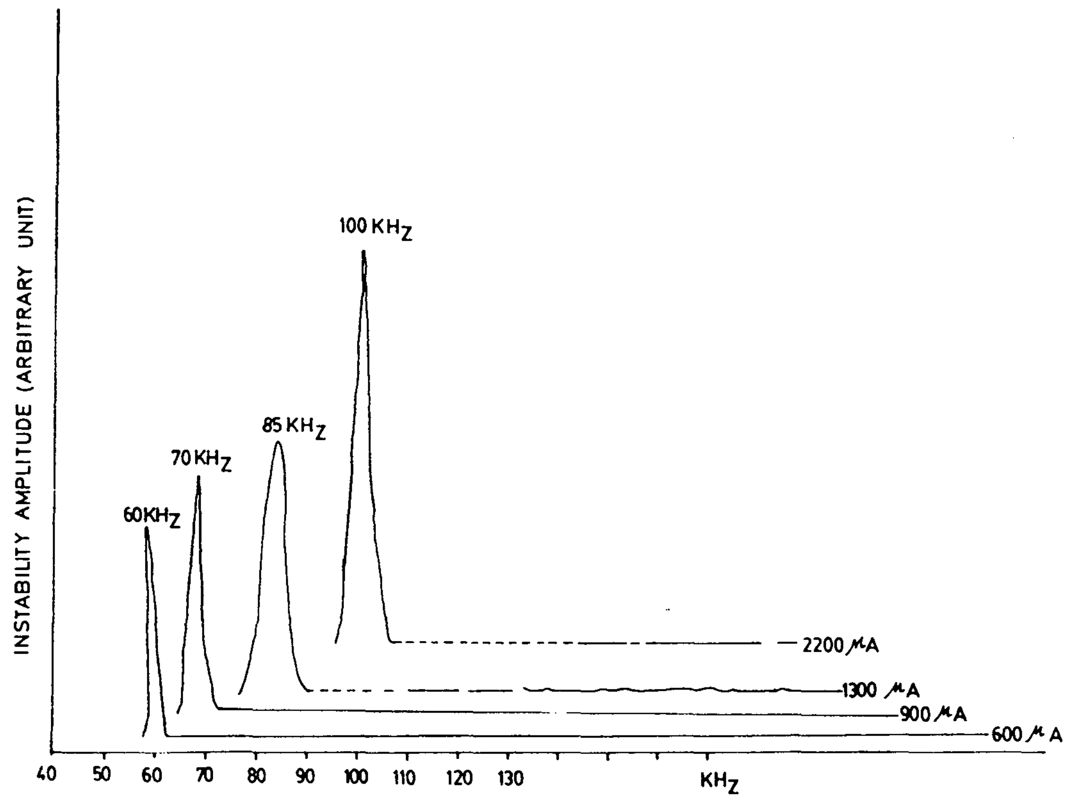


Fig: 5·3 INSTABILITY SPECTRA WITH BEAM CURRENT AS PARAMETER. THE OTHER PARAMETERS ARE B = 210 GAUSS. BEAM ENERGY 300 ev; PRESSURE 4×10^{-6} Torr.

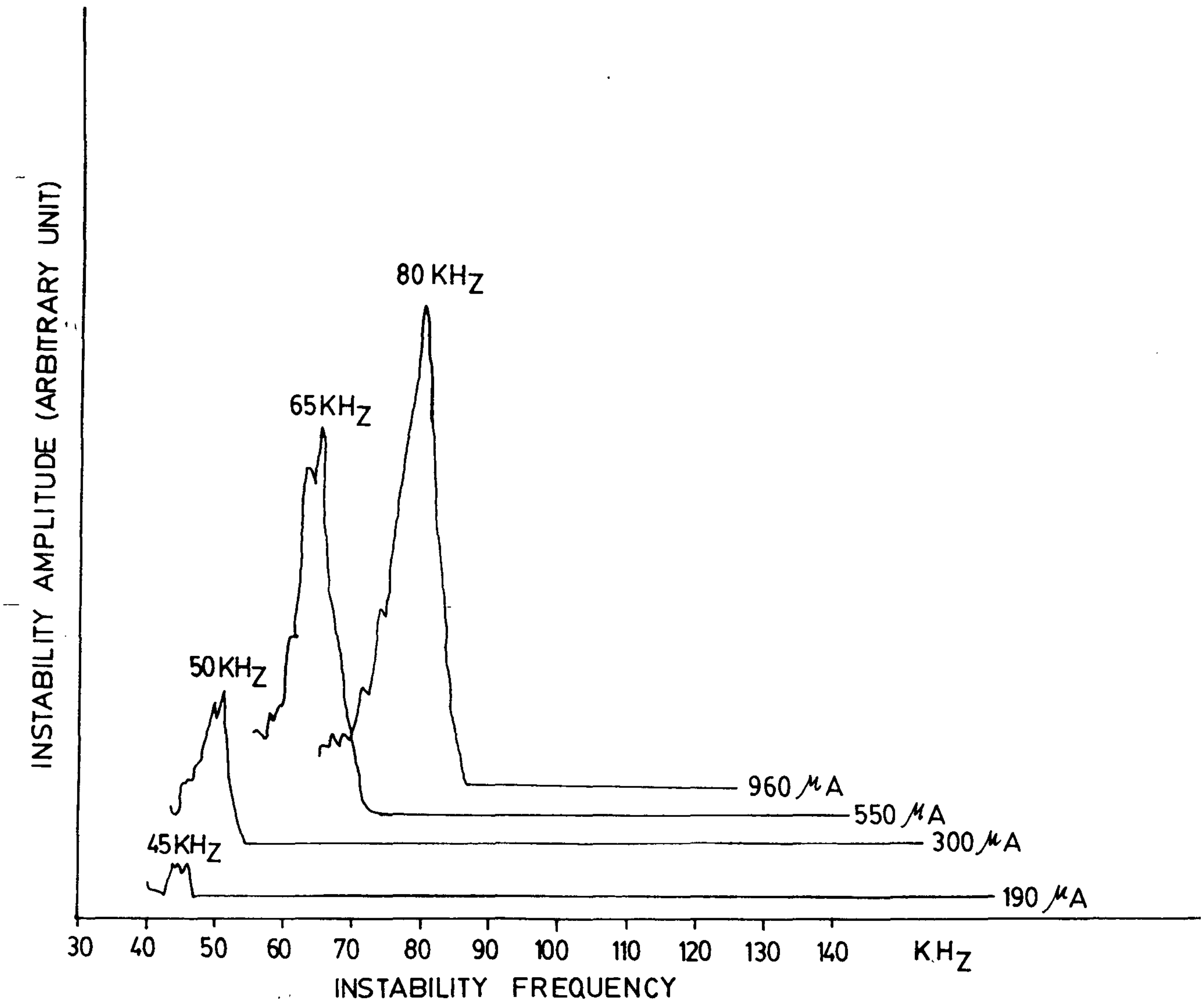


Fig: 5·4. INSTABILITY SPECTRA WITH BEAM CURRENT AS PARAMETER. THE OTHER PARAMETERS ARE B=385 GAUSS; BEAM ENERGY 300 ev; PRESSURE 4×10^{-6} Torr.

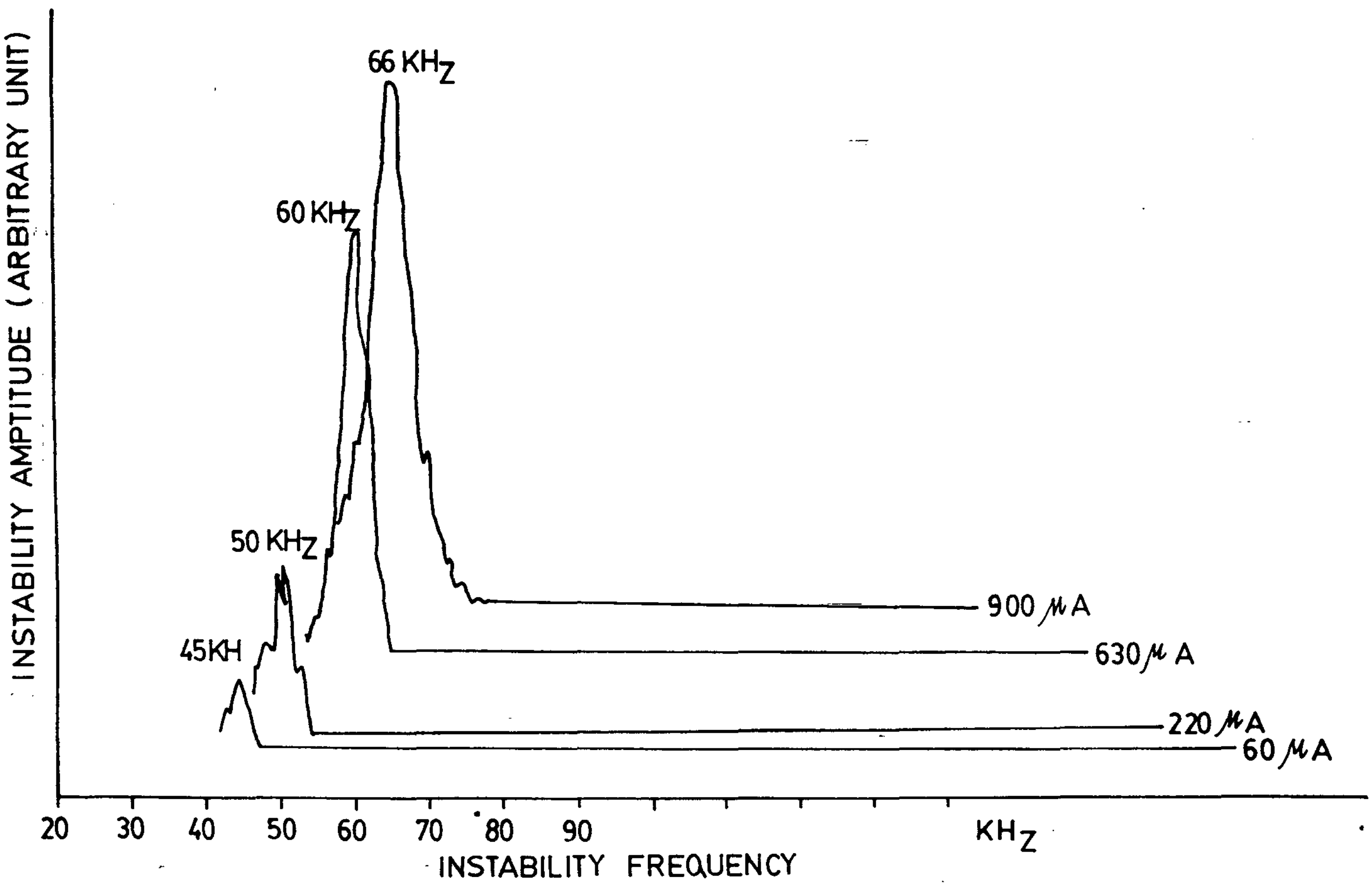


Fig: 5·5 INSTABILITY SPECTRA WITH BEAM CURRENT AS PARAMETER. THE OTHER PARAMETERS ARE B=490 GAUSS. BEAM ENERGY 300 ev; PRESSURE 4×10^{-6} Torr.

associated with the ion plasma mode waves which are excited as a result of two stream type interaction involving the plasma ions (stationary) and azimuthally drifting plasma electrons which may be considered as an electron beam with transverse drift velocity.

Since $\rho_i \gg [\frac{1}{\Phi}\frac{d\Phi}{dr}]^{-1}$ the plasma ions do not experience the effect of radial electric field which is generated as a result of charge non-neutralization. The possibility of these instabilities being associated with other low frequency mode waves (ion cyclotron, ion acoustic) is discounted for the reasons given below. According to our experimental observations $\omega_{ci} \ll \omega_{inst}$ Therefore the unstable modes are not ion cyclotron waves. For the ion acoustic waves ($\omega \sim kC_s$; where $C_s = [T_e/m_i]^{\frac{1}{2}}$ is the ion acoustic speed) the phase velocity of the mode is approximately equal to the ion acoustic speed and also $C_s \ll v_b$. Therefore the Cerenkov interaction (i.e. $\frac{\omega}{k} = v_b$) is ruled out. The slow cyclotron interaction ($\omega - k_{11} v_b + \omega_{ce} = 0$) however could give rise to an instability in the ion acoustic mode (Nyack and Christiansen, 1975). For such an interaction to occur the following relations should be satisfied

$$\omega \simeq k\, C_s \qquad 5.1$$

and $$\omega - k_{11}\, v_b = -\omega_{ce} \qquad 5.2$$

which yield $k_{11} \simeq \frac{\omega_{ce}}{v_b}$; $C_s \ll v_b$

This implies finite parallal wavelengths $\lambda_{11} = \frac{2\pi}{k_{11}} \simeq 1.5$ cm for typical experimental values of $\frac{\omega_{ce}}{2\pi}$ = 560 MHz and v_b = 8.4×10^8 cm/sec. However, from the measurement of k_{11} it was established that $\lambda_{11} \simeq 2L$ = 300 cm, where L is the length of the system. This shows that the instabilities could not be ion acoustic modes.

5.2 Nonlinear Excitation

Experiments were performed for different magnetic field values (100-500) gauss. In each case the beam current was increased in steps. For B = 210 gauss observed frequency spectrum of plasma instabilities is shown in fig (5.7). It is seen that observed subharmonic is predominantly 1/2 harmonic. Initially fundamental instability (ω_o) was noticed with harmonics ($\omega_m = m\omega_o$; m = 2, 3,...). As the beam current is increased above a certain thresold value of beam current subharmonics are noticed. Above thresold further increasing the beam current the amplitude of subharmonic increases first and then decreases. Similar behaviour of the instabilities is observed at different values of magnetic field, frequency spectra for which are shown in fig. (5.6) and (5.8). In each case subharmonics are noticed above a critical thresold value of beam current. It is noticed that at high magnetic field strengths subharmonic generation is not as prominent as in the case of low magnetic fields. In fact for very high magnetic field values no subharmonics are observed.

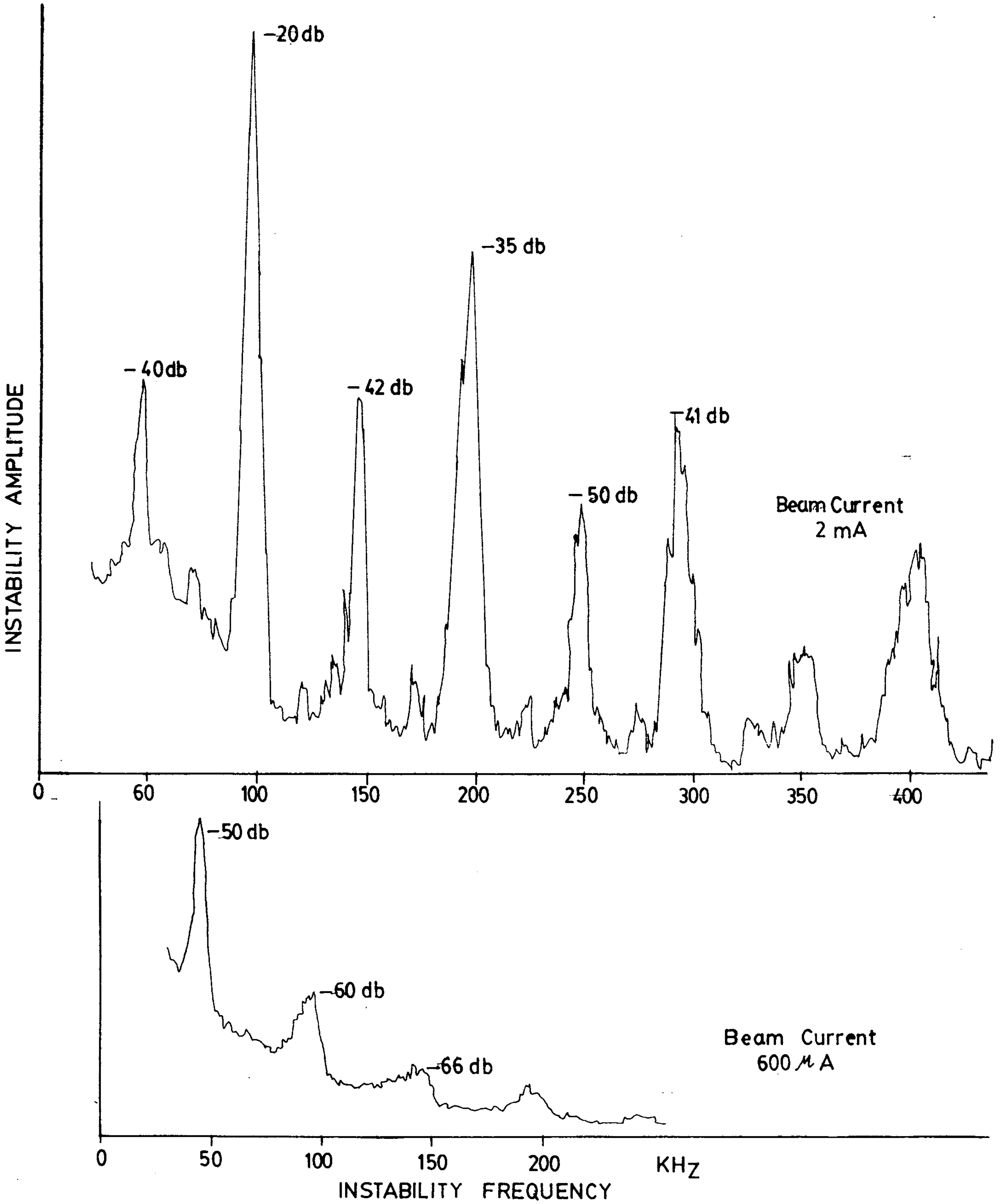


Fig 5·6 INSTABILITY SPECTRA SHOWING EXISTANCE OF ½ HARMONIC GENERATION. PARAMETER OF THE SYSTEM ARE B=105 GAUSS; $P=4\times10^{6}$ Torr; $E_b=300$ ev.

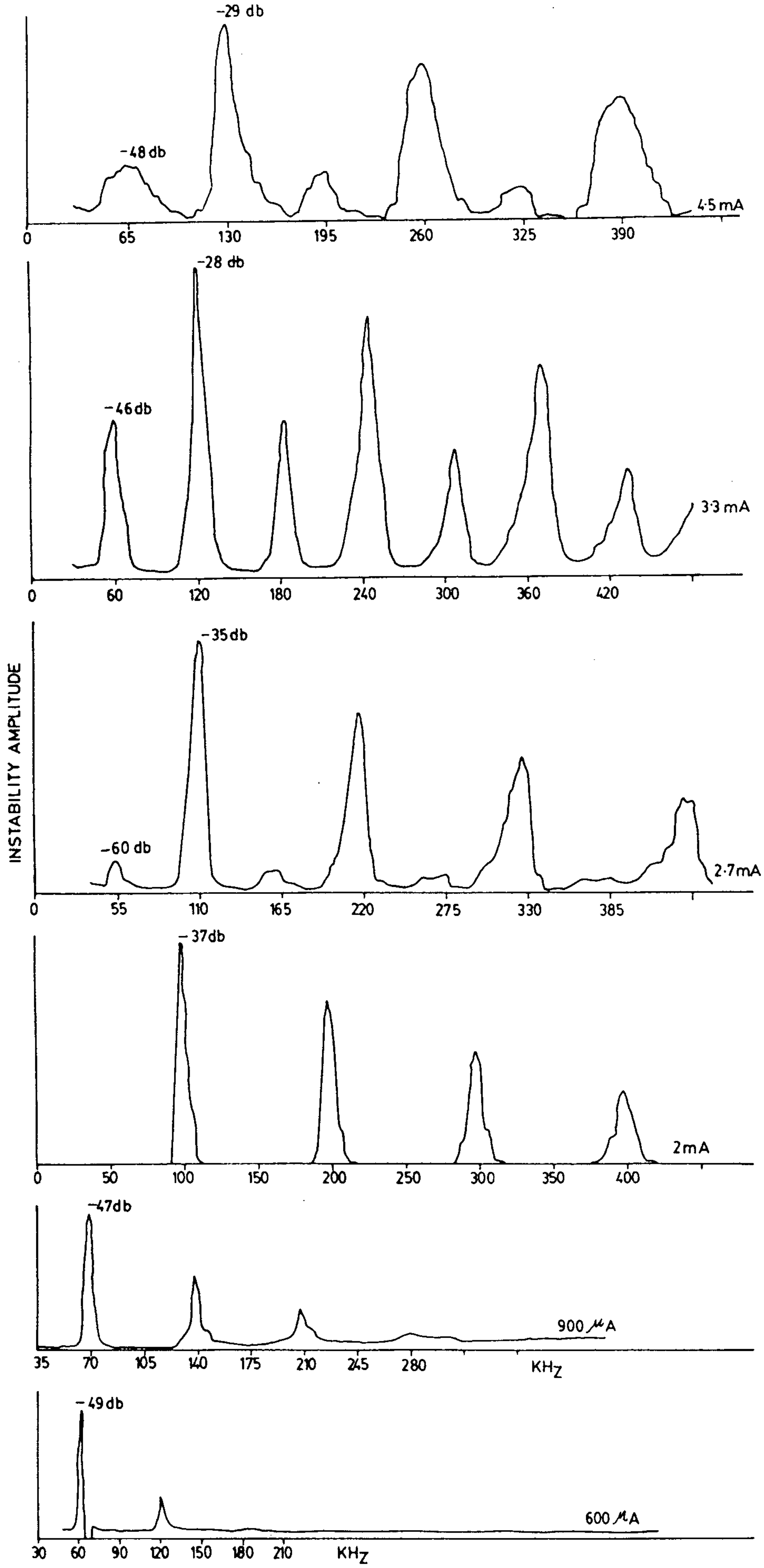


Fig: 5·7 INSTABILITY SPECTRA SHOWING EXISTANCE OF 1/2 HARMONIC GENERATION. PARAMETER OF THE SYSTEM ARE B = 210 GAUSS $P = 3 \times 10^{-6}$ TORR ; $E_b = 300$ ev.

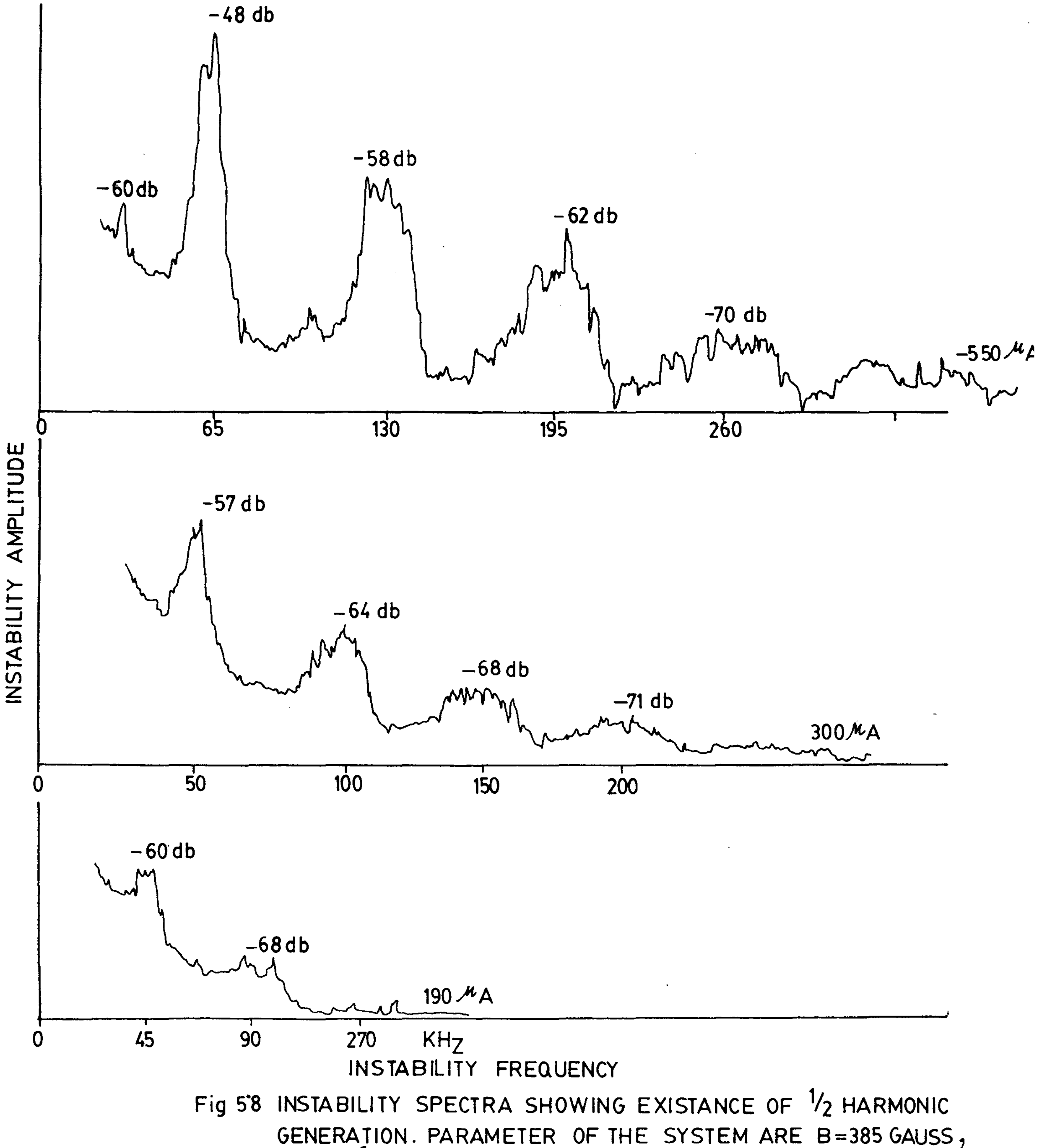


Fig 5.8 INSTABILITY SPECTRA SHOWING EXISTANCE OF 1/2 HARMONIC GENERATION. PARAMETER OF THE SYSTEM ARE B=385 GAUSS, $P = 4 \times 10^{-6}$ Torr. $E_b = 300$ ev.

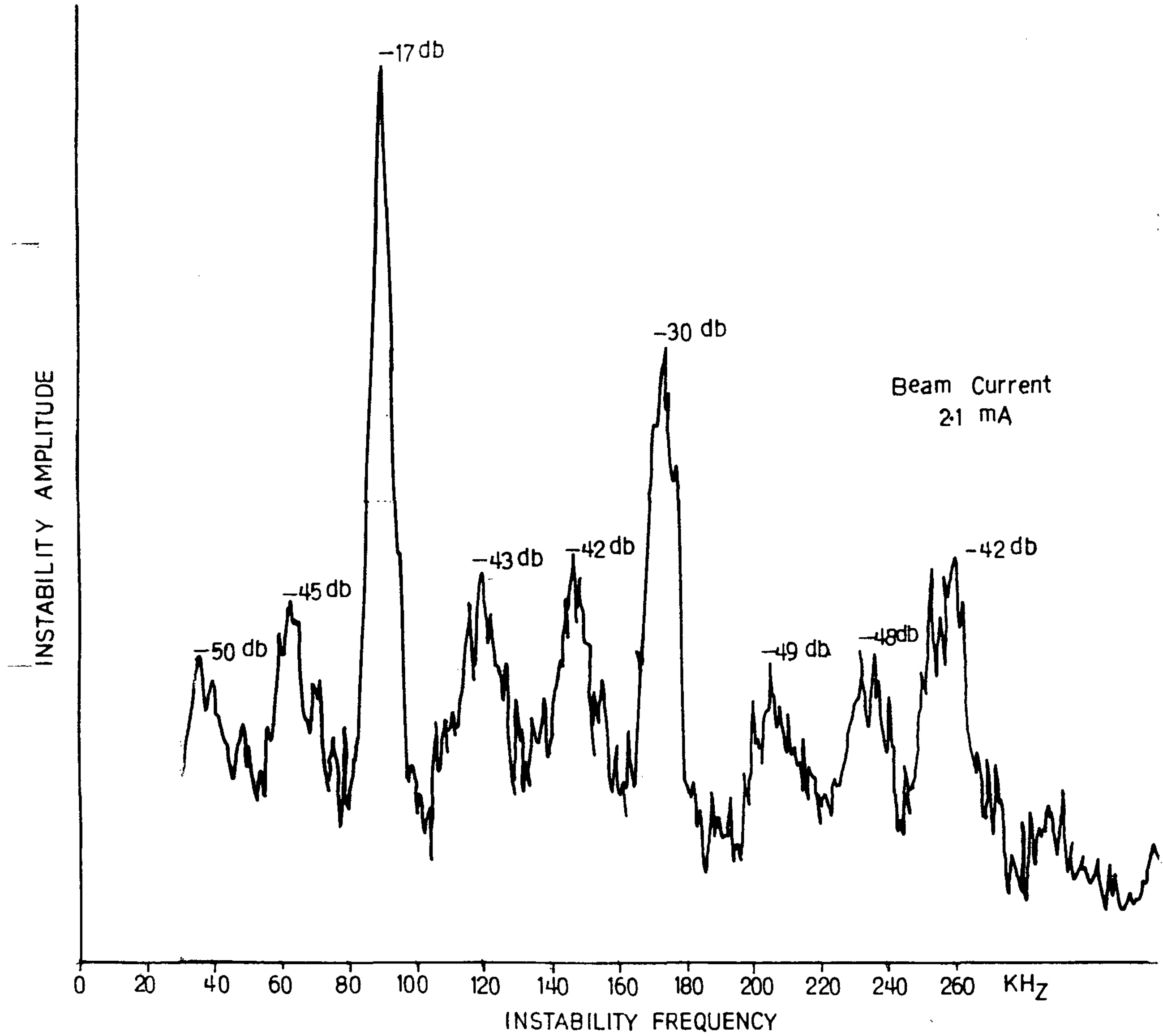
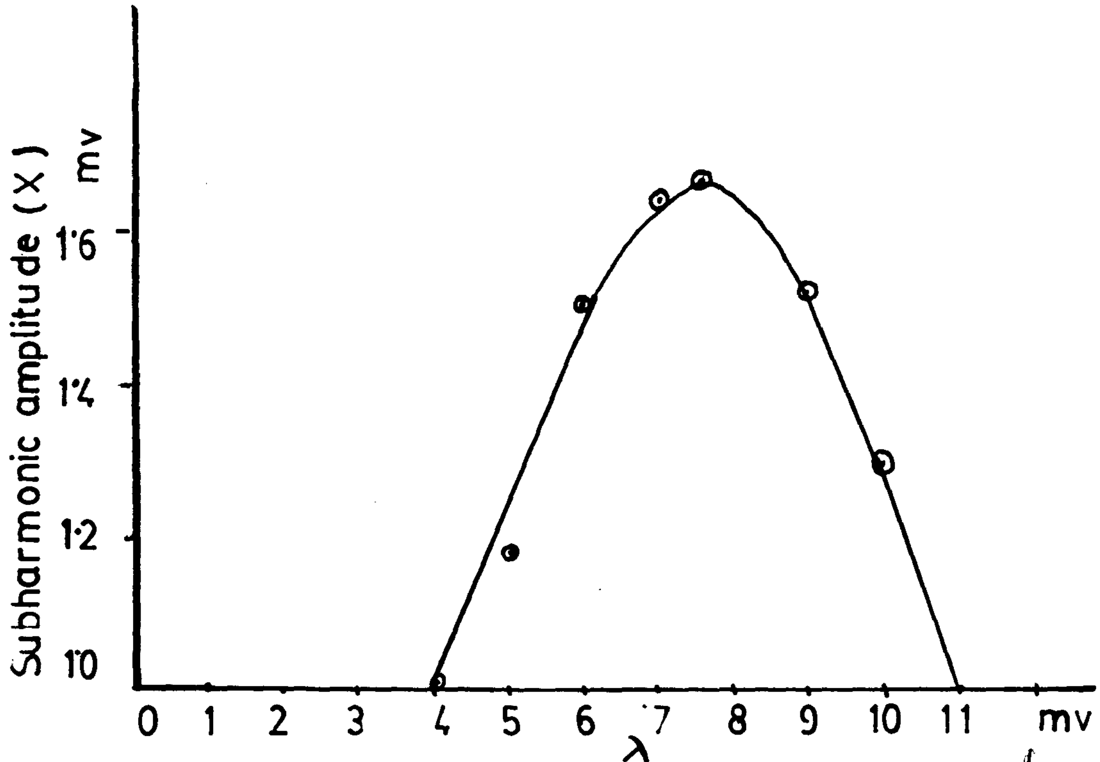


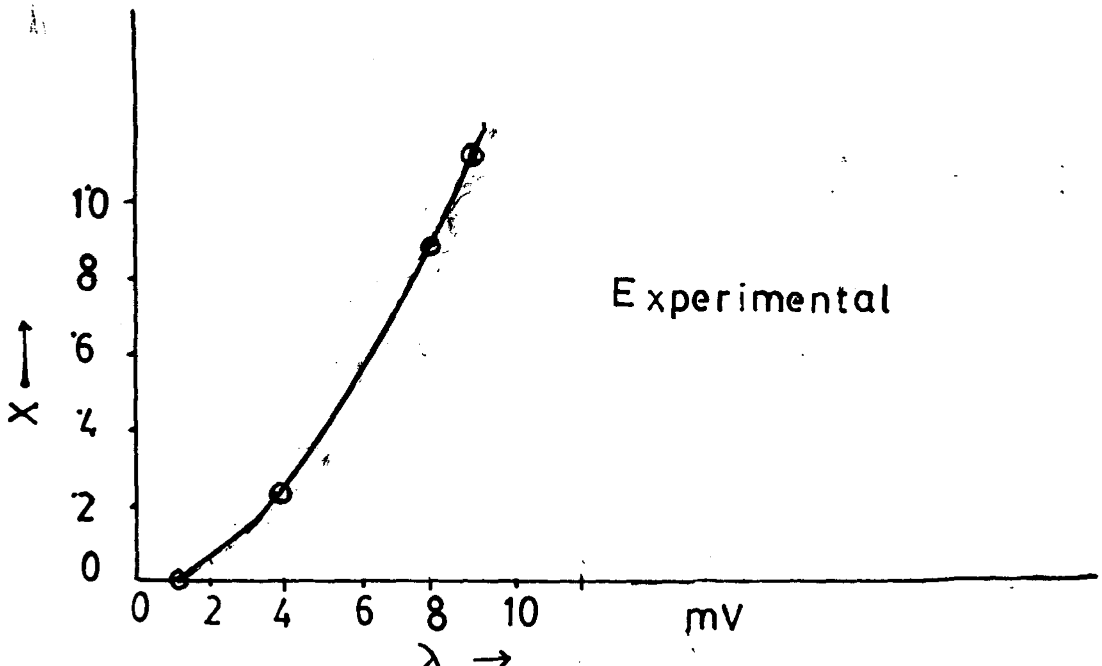
Fig: 5·9 INSTABILITY SPECTRA SHOWING EXISTANCE OF 1/3 rd AND 2/3 HARMONIC GENERATION. PARAMETER OF THE SYSTEM ARE B = 105 GAUSS; $P = 4\times10^{-5}$ Torr; E_b=300 ev.

The mechanism of 1/2 harmonic excitation can be considered as follows: first a linear instability arises at ion plasma frequency (ω_{pi}) due to interaction of cross field transverse beam with ions of the plasma. As the amplitude of this instability increases nonlinearity becomes important and this initial instability then acts as a forcing function on the beam branch, as is schematically shown in Fig. (4.1). However, beam branch gives rise to forced oscillations as in the case of van der pol oscillator under the influence of initial instability which acts as pump wave. The forced oscillations are then excited as subharmonics.

We recall Eqn.(4.18) from Chapter IV, which gives the relationship of 1/2 harmonic amplitude squard (X^2) at ω with the amplitude of pump wave. The observed variation of 1/2 harmonic amplitude as a function of the amplitude of initial instability is shown in Fig. (5.10). The experimentally observed variation is in reasonable agreement with the one deduced from theoretical analysis. It is seen in Fig. (5.10) that the amplitude of the 1/2 harmonic increases with increasing amplitude (λ) of the pump wave. It should then decreases at large λ. With further increase of λ, 1/2 harmonic should disappear at some value of λ . However, this could not be varified from the experimental results since the fundamental instability with sufficiently large λ can not be excited.



Theoretical prediction of subharmonic amplitude (X) as a function of drive amplitude (λ).



Experimental variation of subharmonic amplitude (X) as function of drive amplitude (λ).

Fig: 5·10

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