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SKELETONIZATION OF DIGITAL PICTURES
A THINNING ALGORITHM

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C E R T I F I C A T E

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This work is original and has not been submitted
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A C K N O W L E D G E M E N T

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CHAPTER I

INTRODUCTION

The information that is contained in an image is conveyed to us through the variations in its brightness and colour. When we perceive it, we attach a meaning to it with the help of previously sensed images. The determination of meaning is subjective. When a person is studying the sky for different cloud formations, any object like a bird which enters his field of vision is considered as "noise". When an image is given to a computer to be processed, the variations in brightness and colour are sensed, quantized and encoded. This is done in a systematic manner without regard to the meaning or information contained in the image. The abstracting of meaning is done later, and this is referred to as "pattern recognition". Pattern recognition is defined as the extraction of features of interest of the input data from a background, whose details are irrelevant, and assigning them into specific classes.

Automation of pattern recognition is useful in situations where the number of patterns to be recognised is very large or the required speed of recognition is too

viewing the patterns etc. Some of the areas where this is useful are recognition of chromosomes, blood cells, finger prints, faces, speech, electrocardiograms, electroencephalograms, seismic signals, bubble or spark chamber events, defects in printed circuits, targets in aerial photographs, echo cardiography and so on, Fu⁽⁵⁾ gives the areas where syntactic pattern recognition is useful. In (2), Raeside and Chu give the application of pattern recognition in automated echocardiogram analysis.

Pattern recognition problems deal with the theory and practical techniques for using machines to perform recognition tasks. Let us consider for example the problem of automatic mail sorting where a machine has to recognize alphanumeric characters. The machine has to recognize the characters and assign them to 37 classes, that is, the 26 letters of the alphabet, 10 numerals and 1 rejection class which contains the characters which cannot be accepted as belonging to any of the 36 classes.

Before the patterns are fed into the machine they have to be converted into a form that is acceptable by the machine. One method of doing this is to convert the pattern to a binary representation. A grid is placed

over the pattern, and if more than half of a square is shaded, it is marked 1, otherwise 0. So we get a pattern (fig. 1) of 1's and 0's, the 1's giving the figure and the 0's the background. In some cases, the pattern can be represented by using the structural relationship. In the figure (2.a) shown, primitive elements a, b, c are defined, and the figure is represented by the string of symbols aaabbbccc. Another example fig. (2.b) is shown and the square wave is represented by the string abac. A structural pattern is described by the inter-connection of the primitive elements.

The representation of patterns by strings is possible in cases where the structure is such that there is a simple relationship between the primitives. But this is not adequate in more complex situations where a tree representation will be more suitable. If there is a hierarchical ordering scheme this leads to a tree structure. An example fig. (3.a,b) is shown where an image and its tree representation are given. The tree representation is obtained using the relation 'contains'. The image contains regions a, b, and c. Region a contains d. Region b contains e and f. e in turn contains h and i,

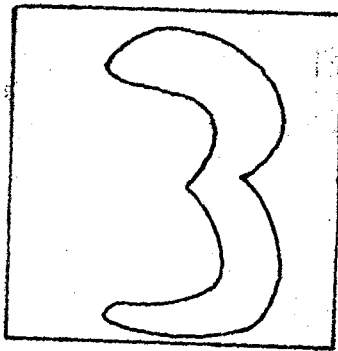


FIG 1 a CHARACTER

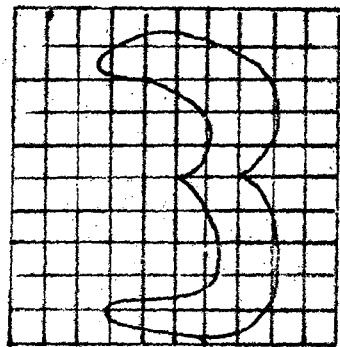


FIG 1 b MEASURING GRID

```
0000000000
0001111100
0000011100
0000001100
0000011100
0000011100
0000001100
0000001110
0000001110
0001111100
```

FIG 1 c BINARY PATTERN

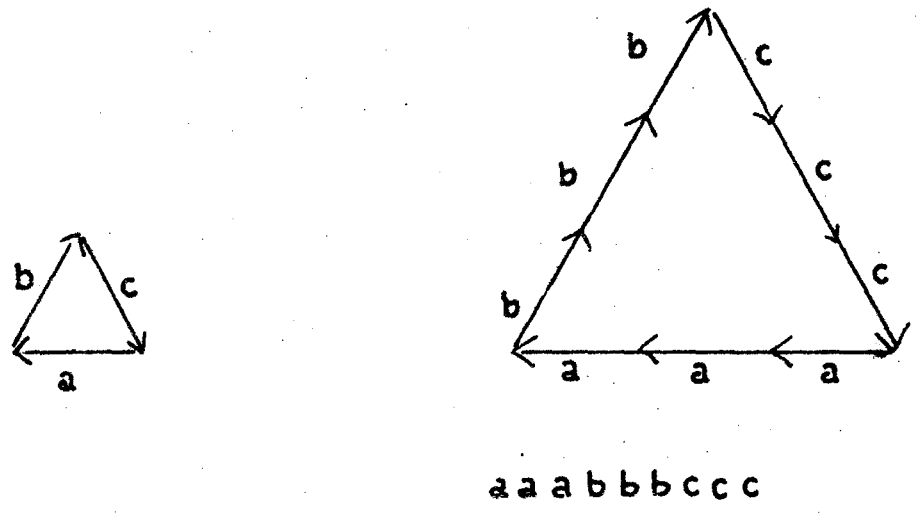


FIG 2a

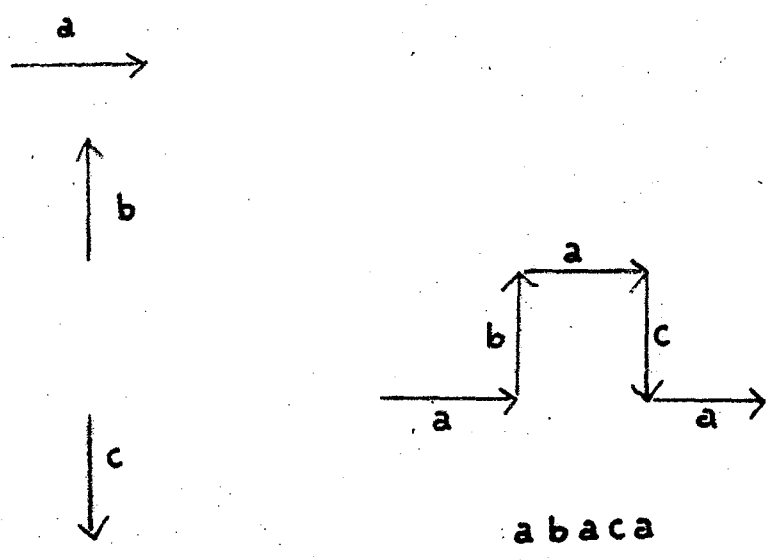


FIG 2b STRING REPRESENTATION

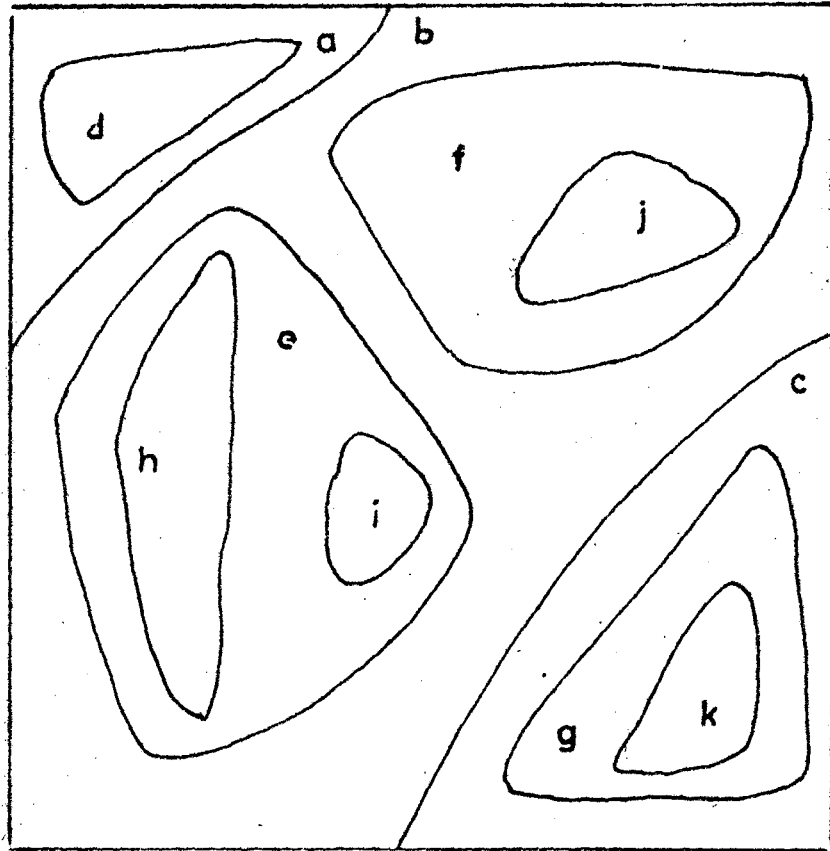


FIG 3a IMAGE

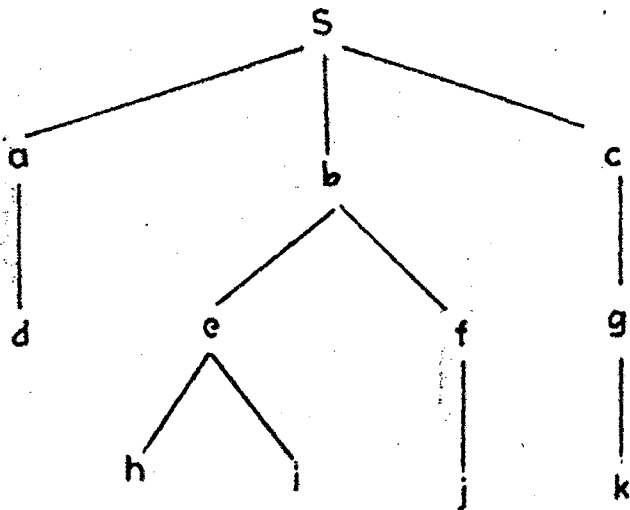


FIG 3b TREE REPRESENTATION

and f contains j. Region c contains g which contains k. Another example fig. (4.a,b) is shown where the scene contains objects and background. The objects are three cubes C1, C2 and C3. The cube C1 contains the rectangular structures S1, S2 and S3, C2 contains S4, S5 and S6, and C3 contains S7, S8 and S9. The background is a table. The tree structure with the relation 'contains' is shown. In both these representations the image was split into its primitive components and was thus a top-down interpretation of the tree. A bottom-up interpretation can also be done as for example, the relation 'part of' can be used. S1, S2, S3 are part of cube C1, and table is part of background, and so on.

A more general representation is a web, which is a graph whose nodes are labelled. The figure (5.1, a,b,c) shows an object which is approximated by polygons, and the graph representation is also shown. Another example (5.2 a,b) shows an image and the web, using the relation 'is connected to'. S1, S2 and S3 are connected and so on.

So we have seen three representation schemes which are strings, trees and webs. The choice of any particular representation depends on the problem at hand.

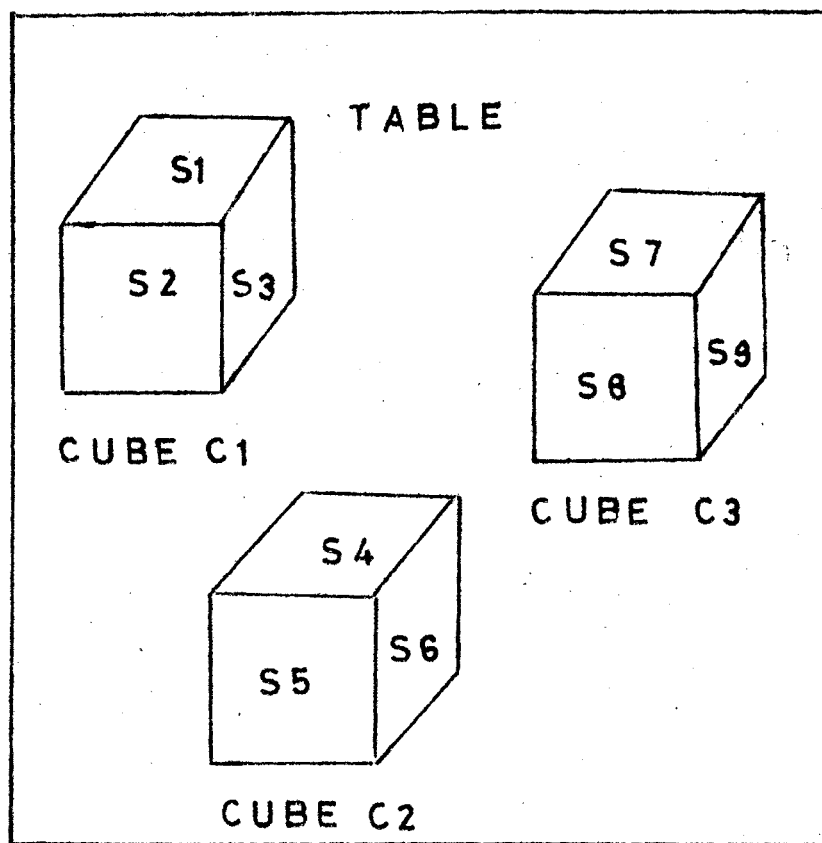


FIG 4a SCENE

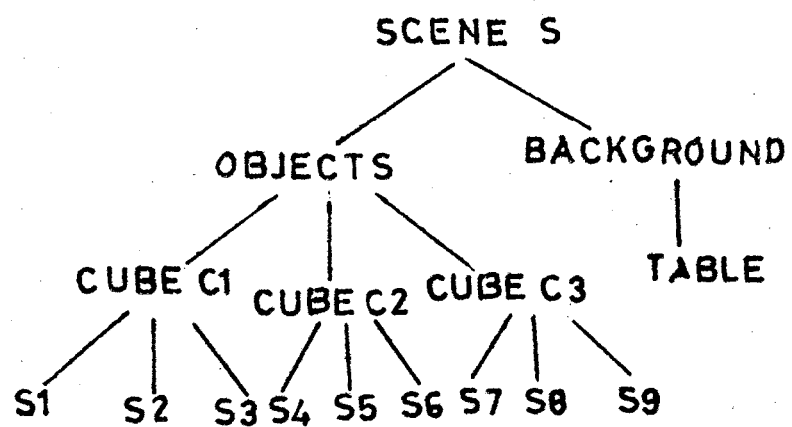
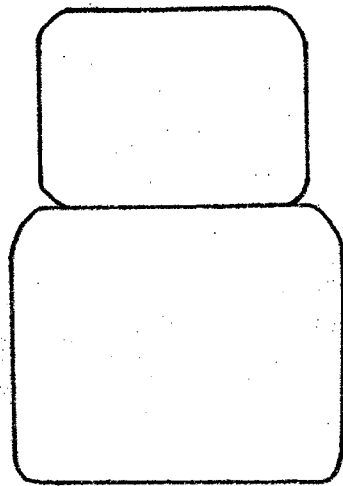
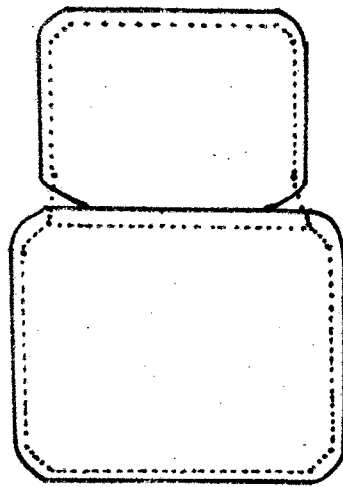


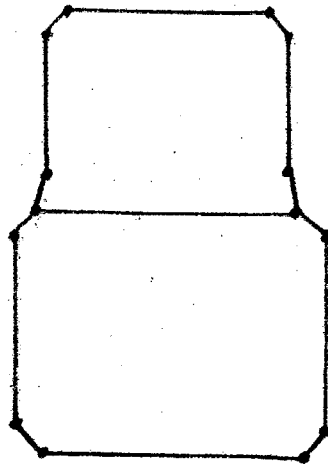
FIG 4b TREE REPRESENTATION



a



b



c

FIG 5.1 GRAPH REPRESENTATION

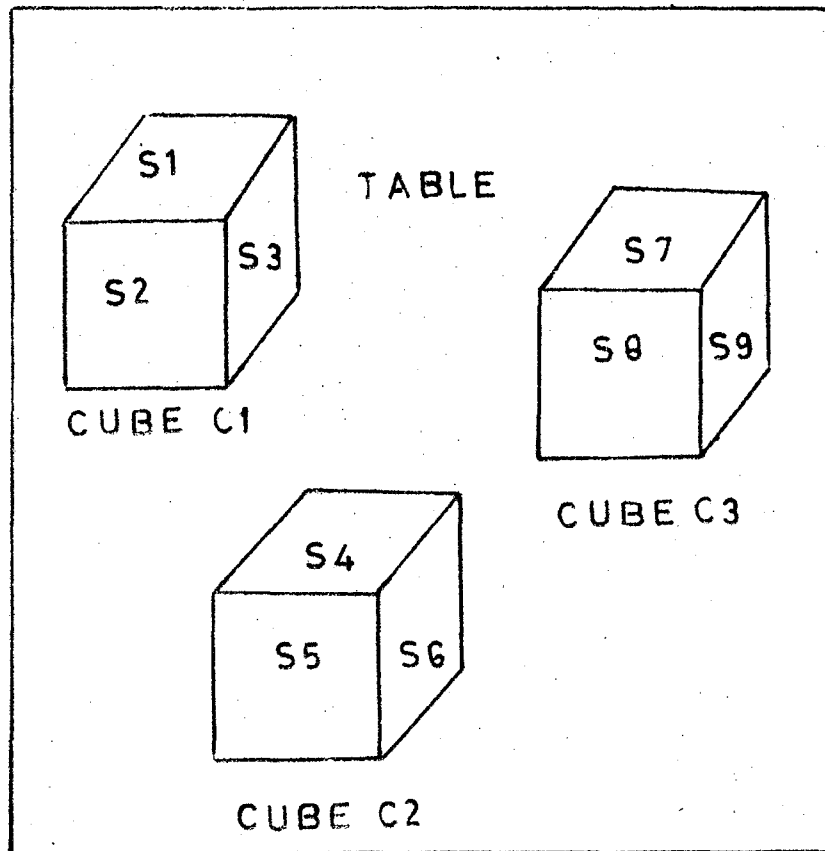


FIG 5.2 a SCENE

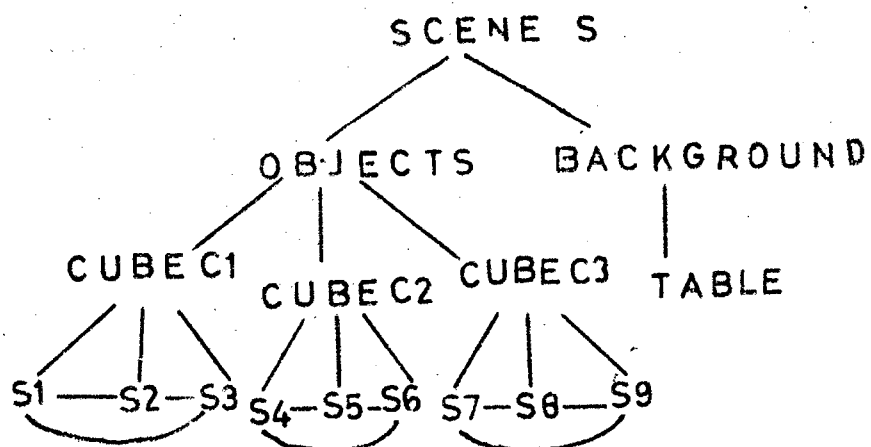


FIG 5.2 b GRAPH REPRESENTATION

There are two approaches to pattern recognition system design. They are (1) the decision-theoretic approach and (2) the syntactic approach. There are many similarities between the two models. Statistical classification procedures can be involved in the extraction of "primitives" in the syntactic model, and the association of patterns with generative grammars is equivalent to the classification of patterns into categories.

DECISION - THEORETIC APPROACH

The decision-theoretic approach uses decision functions for classifying pattern vectors. The problem consists of assigning an unknown to one of several categories. With each category is associated a list of attributes and with each attribute a set of attribute values. The space of attribute values is divided into mutually disjoint regions and each region is assigned to one of the categories. Now the given attribute, on the basis of its attribute values is assigned to one region in the property space. Fu⁽⁴⁾, Nilsson⁽¹⁴⁾, Patrick⁽¹⁵⁾, Chen⁽¹⁾ and Tou and Gonzalez⁽²³⁾ are among the workers in this field. The decision - theoretic

approach to pattern recognition plays a very important role in applied as well as theoretical work in this field.

SYNTACTIC APPROACH

When patterns can be meaningfully represented in vector form, the decision-theoretic approach is suited. But in some applications, the structure of the pattern plays an important role in the classification process, and in these situations the decision-theoretic approach has serious drawbacks because it does not have the formalism to handle pattern structures and their relationships.

In the structured approach the process of recognition involves not only assigning a pattern to a particular class, but also describes certain aspects of a pattern which make it ineligible for assignment to another class. In this, a complex pattern is described using smaller sub-patterns, and a set of rules. Each pattern is described in terms of simpler sub-patterns and each of these simpler sub-patterns is described in terms of even simpler sub-patterns. The simplest sub-patterns chosen are the 'pattern primitives'. The language which gives the structural description of a pattern in terms of the

pattern primitives and their composition operations is the pattern description language, and the rules which determine the composition of the pattern primitives into patterns is the grammar.

Figure 6.b shows the decomposition of two chromosome structures in terms of the primitives defined in (a). By going along the boundary of the chromosome in the clockwise direction, we can detect and encode these primitives in the form of a string of qualifiers. The string abchabdbabcbabdb represents the sub-median chromosome and the string ebabcbab represents the telocentric chromosome.

Suppose that each primitive is a symbol in some grammar, where the grammar gives a set of rules of syntax for generating sentences from the symbols. The strings representing the chromosomes could be sentences. So we can have grammars G_1 and G_2 which generate languages $L(G_1)$ and $L(G_2)$, $L(G_1)$ would consist of sentences representing the sub-median chromosomes and $L(G_2)$ represent the telocentric chromosomes. If an input sentence representing a pattern is given it is first found out in which language it forms a valid sentence. So if it belongs to $L(G_1)$ we say that it represents a sub-median



FIG 6a PRIMITIVES

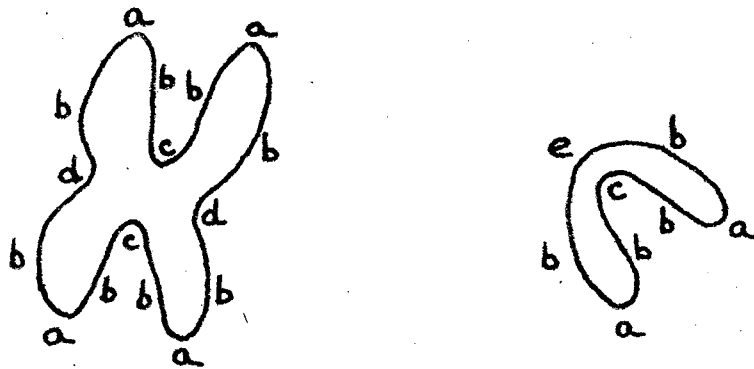


FIG 6b CODED SUBMEDIAN AND
TELOCENTRIC CHROMOSOMES

(FROM LEDLEY [1964])

chromosome, and if it belongs to $L(G_2)$ it represents a telocentric chromosome. If the sentence belongs to both languages then a unique decision cannot be made, and if it does not belong to either, it is put into the rejection class.

If there are more than two pattern classes then there will be more grammars, with at least one grammar for each class. If the sentence belongs to only $L(G_i)$ it is assigned to the i^{th} class. If it belongs to two languages, then no unique decision is made, and if it is invalid it belongs to the rejection class.

If the pattern has a tree or web representation, the recognition process is done using tree or web grammars.

Recently the tree systems approach to pattern recognition has been gaining popularity. A tree structure is used to represent the pattern in terms of its primitives and relations. If the pattern can be represented by a tree structure, then the tree grammar is constructed and tree automata can be used to classify the unknown patterns, according to the pattern grammar. This has been found to be an effective method in the classification of bubble

chamber events, finger print pattern recognition and so on.

Pictorial pattern recognition has very important applications in cytology. Some of the areas in hematology and exfoliative cytology are the automatic detection of abnormal Pap smears and automation of differential white blood cell count. In cytogenetics, the application is in identifying and measuring individual chromosomes that are found in the image of a mitotic cell. This includes separation of the chromosome spread into connected components, separating overlapping chromosomes, and making size and shape measurements on individual chromosomes. A normal human cell has 46 chromosomes, 44 of these are autosomes which account for the 22 homologous pairs of chromosomes. The remaining 2 chromosomes are the sex chromosomes X - X in a female and X - Y in a male. The study of aberrations in chromosomes is often of great value in diagnosis. One may like to know whether particular kinds of aberrations are in any way an indication of certain congenital malfunction or disease. In order to answer this question, samples of tissues are collected from several individuals and studied.

The work done in this dissertation gives a thinning algorithm which has been applied on chromosomes to obtain their skeletons. Skeletonization is one of the methods of representing a figure in automated pattern recognition. For most of the advanced pattern recognition techniques, a subdivision or segmentation of the picture is necessary. The segmentation may be a division of the picture into regions corresponding to the objects, or parts of objects or background. After the division of the picture into regions, these regions are represented and described in a number of ways. Some of these representations preserve all the information about the regions, while other representations intentionally lose information, preserving only the information that is of interest. These descriptions provide features or properties that are directly of use for classification purposes, or indirectly as a step toward scene description. One of the methods of representing regions is by thinning or shrinking them into skeletons. Another approach to segmentation is based on detecting edges or curves in the picture. These linear features may be the boundaries or parts of boundaries of the regions of interest or they

may be themselves features of interest. After they are found they are represented and described in a variety of ways.

Chapter 2 gives a general and brief idea of the background of image recognition. Various thinning algorithms are discussed in chapter 3, and in chapter 4, a thinning algorithm is presented. The algorithm works quite well and preserves the connectivity of the given objects.

CHAPTER II

IMAGE RECOGNITION

Image recognition involves matching or template matching, segmentation and property measurement.

MATCHING

One of the basic techniques for image recognition is matching. In order to detect the presence of a particular pattern in an image, the image is matched with a standardised version of the pattern, called a template. The picture is searched for an object by applying the template at each location, and the extent to which it matches or does not match the object is calculated. This measure is compared with a pre-determined threshold to find if the object is found at a particular location. Template matching is computationally costly. So a sub-template can be used, and when the degree of match of the sub-template exceeds a pre-determined value, then the rest of the template can be used. In most picture analysis systems, the shapes are likely to be distorted, and a hierarchical template matching method can be adopted. The object is split into smaller patterns, and templates for

these portions are applied to the picture. Since the areas are small, the distortion also will be less and reasonable matches to the sub-templates can be got. Then a combination of sub-template matches in approximately the correct relative positions is found. These positions are defined according to the direction of the whole object template.

SEGMENTATION

Image segmentation separates a picture into objects and background. Histogram techniques, region-based techniques and edge-based techniques are some of the methods of segmentation. Combinations of these methods are also used. The objects in a picture can be detected by thresholding the picture's grey level, choosing the threshold value so that the object points have a value above the threshold and the background points have a lower than threshold value. If $f(x,y)$ is the grey level at point (x,y) , then by thresholding the picture at a threshold t , we get a two-valued picture $f_t(x,y)$ where

$$f_t(x,y) = 1 \text{ if } f(x,y) \geq t$$

$$= 0 \text{ otherwise}$$

In order to choose the threshold, the frequency with which the different grey levels occur in the picture is noted. Suppose a grey level s occurs with a frequency $f(s)$ in the picture. $f(s)$ is called the grey level histogram of the picture. If a picture contains two populations of points, object points and background points, the picture's histogram will be bimodal, that is, it will have two peaks, one corresponding to each population. We can choose a threshold which corresponds to the bottom of the valley between the two peaks, in order to distinguish between the object and background.

A method of variable thresholding has been suggested for image segmentation. The entire image is divided into windows and a grey level histogram is computed for each window. Thresholds are selected for the windows that have bimodal histograms and these thresholds are then interpolated or extrapolated to get a variable threshold for the entire image. This method is useful in cases where there is a large variation in grey scale from one part of the image to another. Chow and Kaneko have successfully used this method to detect the heart region on chest X-rays.



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Another approach to segmentation is the region-based method. Here a seed location is taken, and neighbouring points are joined to it. The neighbouring points, if they have the same property as that of the growing seed, are joined. Otherwise when there are no neighbouring points which can be joined to it, a new seed point is chosen. This is continued till every point in the picture is attached to a region. Region detection methods are of two types. In the merging method, there are a large number of small regions in the picture which are then merged to form larger regions. In the splitting method, the picture is divided into smaller and smaller regions till some conditions are satisfied. A combination of these two is the split and merge method. An abstract formulation of the problem is given by Horowitz (9).

Let X be the domain of a picture. Let $f(x,y)$ be the brightness function defined on X . A logical predicate P is defined on subsets S of X as follows:

$$P(S) = \begin{cases} \text{true if there exists a constant 'a'} \\ \text{such that } |f(x,y) - a| \leq \epsilon \\ \text{for all points } (x,y) \in S, \\ \text{false otherwise,} \end{cases}$$

where ϵ is a prescribed error tolerance.

A segmentation of X is a partition of X into subsets S_i , $i = 1, \dots, m$, for some m such that

$$a) \quad X = \bigcup_{i=1}^m S_i,$$

$$b) \quad S_i \cap S_j = \emptyset \quad \text{for all } i \neq j$$

$$c) \quad P(S_i) = \text{true for all } i$$

$$d) \quad P(S_i \cup S_j) = \text{false for all } i \neq j$$

provided S_i and S_j are adjacent in X .

A merging scheme starts with a partition satisfying (c) and proceeds to fulfil (d); a splitting scheme starts with a partition satisfying (d) and proceeds to fulfil (c). A split and merge procedure begins with an arbitrary partition satisfying neither condition and produces a partition satisfying both.

Most methods of edge detection make use of symmetrical edge detection operators. Differences of averages are computed over neighbourhoods of a particular

size, and thus edges are detected. Ahuja et al. (13) give a method of edge detection which is not based on symmetrical edge detection operators. The picture is divided into regions R each having an approximately constant grey level. (x,y) is a point in one of the constant regions R and $N_r(x,y)$ is a disk of radius r centred at (x,y) . It is required to find the largest $r = r(x,y)$ so that $N_r(x,y)$ is entirely contained in R . A simple statistical test, a multimodality test is applied to find if $N_r(x,y)$ is contained in a single constant region, or overlaps several of the regions. The maximal $N_r(x,y)$'s are used as averaging neighbourhoods. If P is a point where two such $N_r(x,y)$'s touch, the difference of averages over the $N_r(x,y)$'s is taken as the edge value at P . Thus edge values along region edges are high and they are low or zero elsewhere.

There are many ways in which these processes can interact. The relaxation method is one such. Let A_1, \dots, A_n be objects in a scene, each of which belongs to one of the classes C_1, \dots, C_m . An initial set of probabilistic guesses is given, as to the class

assignments of the objects. That is for every $i = 1, \dots, n$, and $\ell = 1, \dots, m$, there is a number $P_i(\ell)$ which is an estimate of the probability that object A_i is in class C_ℓ . These estimates are iteratively updated until hopefully each A_i is almost certain to belong to a single class C_ℓ .

Sometimes isolated points from each region are given, and these are referred to as incompletely specified regions. A segmentation of such an image is also necessary sometimes, and for this, the region boundaries must first be constructed. The authors in (17) present a method for achieving this.

PROPERTIES

Studies in the field of perception have shown that there is a lot of redundant information in images. There is the example of a "sleeping cat" given by Attneave. The points of curvature on the figure are marked. The curved lines are removed, and these points of curvature are joined by straight lines in an appropriate manner. The necessary information is preserved, and we still recognize the cat. Here, the features of high information

value are the points of curvature. We can also consider the case of cartoons, where the few lines successfully communicate a great deal including emotions. Thus, when an image is described, we do not need to preserve all the details. Only the relevant information need be described. After the regions of the picture are extracted, the image is described in terms of its properties. The shape of a region is described in terms of properties like connectedness, convexity, compactness, angles, sides, and so on. The description of the region may involve its boundary or its 'skeleton'.

CHAPTER III

THINNING ALGORITHMS

In this chapter, a review of thinning algorithms is attempted.

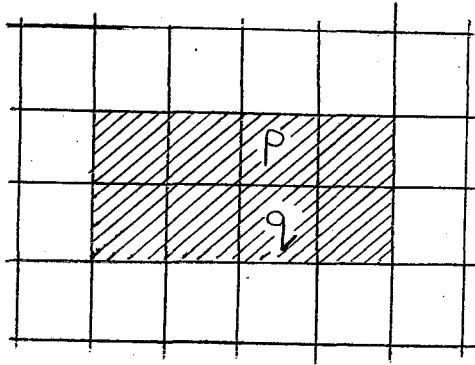
Hilditch⁽¹¹⁾ presents an algorithm, which has the advantage that it operates effectively on pictures whose range of grey levels is greater than two. She lists the following as requirements that should be satisfied by the skeleton:

Thinness: Points which lie on the edge of the picture are successively eroded away until there are only lines which are one point wide.

Position: The thin lines should lie along the centres of the line-like parts of the object. Each time the picture is scanned, the removal of points is symmetric and parallel so that only points which lie on the edge of the initial subset are removed. For the next scan, a picture which is one layer thinner than the previous is given.

Connectivity : Each point is tested to see if its removal will alter the connectivity of the picture. If

it does, then it is not removed even if it lies on the edge. When this is done in parallel, there is a problem when it comes to points which lie so that they are two points wide, for the simultaneous removal of both would affect the connectivity.



Neither the removal of p , nor that of q , will alter connectivity, but the simultaneous removal of both will alter it. Therefore, when a point is removed, it is ascertained whether its removal alters connectivity, and if any of its neighbours has been removed then the removal of these points together should not alter the connectivity.

Stability : When the final thin structure is reached, this fact should be recognized and the process of removal of points should stop.

The algorithm requires several passes. At the beginning, a subset Q of some subpicture $f_0 | P$ is given. Q is defined as those points of P for which f_0 takes one of a set of values, say I , and all other points of P take one of a set of values N . In particular the picture usually takes the form of a characteristic function with points in Q having value one and all other points zero. Throughout a pass through the points of the picture, a record of the initial subset Q must be maintained. So a third set of values R is kept for points which have been removed. $R \cap I = \emptyset$ and $R \cap N = \emptyset$. Each point $q \in P$ is tested, and the point is set to a new value $f_1(q)$. $f_1(q) \in R$ if the point is removed, and $f_1(q) \in I$ or $f_1(q) \in N$ depending on whether the point was originally in Q or not. When one pass is complete, all points with values in R must be reset to have values in N . A point P will be removed, that is, it will be set to a new value $f_1(p) \in R$ if and only if it satisfies the following conditions:

1. It belongs to Q and its removal is allowed.

$$f(p) \in I$$

2. It satisfies the edge condition.

$\mu(p) \geq 1$ where

$$\mu(p) = a_1 + a_3 + a_5 + a_7$$

and $a_i = 1$ if $f(n_i) \in N$,

$a_i = 0$ otherwise.

n_1, n_3, n_5 and n_7 are the axially adjacent neighbours

$$\begin{array}{ccc} n_4 & n_3 & n_2 \\ n_5 & p & n_1 \\ n_6 & n_7 & n_8 \end{array}$$

3. It is not the tip of a thin line, that is, it has more than one neighbour which belongs to Q , i.e.

if
$$V(p) = \sum_{i=1}^8 (1 - a_i), \quad a_i \text{ defined as above,}$$

then the 'not tip' condition is $V(p) \geq 2$.

4. It is not the last remaining point of a small 'circular' subset which means that at least one neighbour of the point in Q should not have been removed.

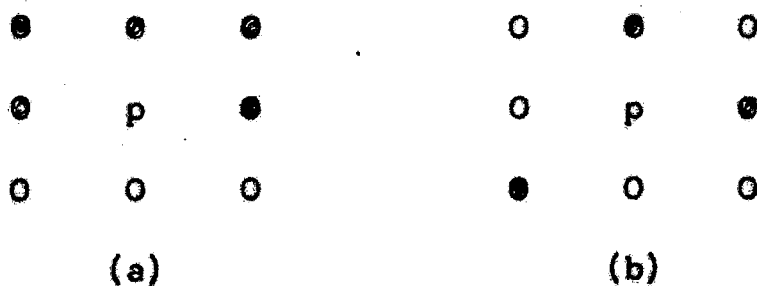
$$w(p) = \sum_{i=1}^8 C_i \quad \text{where}$$

$$C_i = 1 \text{ if } f(n_i) \in I$$

$$C_i = 0 \text{ otherwise}$$

The condition is $w(p) \geq 1$

5. Its removal does not alter connectivity. This will be true if the set consisting of the neighbours of p contains one connected component. The number of components is found by calculating the number of times a 'bug' taking a walk around p by way of its neighbours would have to cross from outside to inside the subset. This number is called the 'crossing number' of p with respect to the subset. This number is equal to the number of components, but is zero in the case where the bug does not have to leave the subset at all. The 'bug' must go from one axially adjacent neighbour of p to the next axially adjacent one, bypassing the diagonal if by so doing it avoids leaving the subset. In (a) the crossing number at p is one and in (b) it is two:



The crossing number $X(p)$ is calculated as follows

$$X(p) = \sum_{i=1}^4 b_i \quad (\text{where } b_i = 1 \text{ if } f(n_{2i-1}) \in N$$

and either $f(n_{2i}) \in IUR$ or

$$f(n_{2i+1}) \in IUR$$

and $b_i = 0$ otherwise)

The connectivity condition is

$$X(p) = 1$$

6. Its removal together with any one of its neighbours does not alter the connectivity of Q . If neighbour n_1 of p has been removed so that $f(n_1) \in R$, then if the value of this neighbour is temporarily altered so that $f(n_1) \in N$, the crossing number at p , $X_1(p)$ should still be one.

$$f(n_1) \notin R \text{ or } X_1(p) = 1 \quad (i = 1, \dots, 8)$$

By redefining I and R , the algorithm can be used for pictures with a range of density values.

Golay (8) has suggested a two dimensional parallel computer based on a hexagonal array of modules.

Each module has 6 equally distant neighbours unlike the square array where there are 4 immediate neighbours at unit distance and 4 secondary neighbours at $\sqrt{2}$ distance.

A	B	C
D	P	E
F	G	H

Square module array

B	C	
A	P	D
F	E	

Hexagonal module array

A module can be either in state 1 or in state 0, and given the state of any one module in a hexagonal group, and the states of its 6 nearest neighbours, it is possible by a simple operation to either change or not to change the state of this module as a function of its state and of the states of its six nearest neighbours.

There are $2^6 = 64$ possible combinations of 0's and 1's forming the immediate neighbourhood or surround of a module. This can be divided into two sets. In the first set, there is at most a single group of adjacent 1's and in the second set there are more than one group of adjacent 1's. The weights indicate how many distinct

PATTERN	$\begin{matrix} 0 & & 0 \\ 0 & + & 0 \\ & & 0 \end{matrix}$	$\begin{matrix} 0 & & 0 \\ 1 & + & 0 \\ 0 & & 0 \end{matrix}$	$\begin{matrix} 1 & & 0 \\ 1 & + & 0 \\ 0 & & 0 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 1 \\ 0 & & 0 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 1 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 1 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 1 \\ 1 & & 1 \end{matrix}$	
INDEX	0	1	2	3	4	5	6	
WEIGHT	1	6	6	6	6	6	1	$\Sigma=32$

PATTERN	$\begin{matrix} 0 & & 1 \\ 1 & + & 1 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 1 & & 0 \\ 1 & + & 1 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 1 & & 0 \\ 1 & + & 0 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 1 \\ 0 & & 1 \end{matrix}$	$\begin{matrix} 0 & & 0 \\ 1 & + & 1 \\ 1 & & 0 \end{matrix}$	$\begin{matrix} 0 & & 0 \\ 1 & + & 1 \\ 0 & & 0 \end{matrix}$	$\begin{matrix} 1 & & 1 \\ 1 & + & 0 \\ 1 & & 1 \end{matrix}$	
INDEX	7	8	9	10	11	12	13	
WEIGHT	2	6	6	6	6	3	3	$\Sigma=32$

TABLE 1

similar patterns can be formed by rotating the pattern shown. Table 1 shows the sum of the weights is 64 which is the total number of possible patterns. The patterns 8 and 9 are the only asymmetric patterns.

The two sets of patterns are separated by the concept of connectivity. In the first set, the addition or deletion of a 1 at the centre does not alter the connectivity of the 1's of the surround, whereas in the second set it is affected.

All operations in two-dimensional parallel computers must be static in that two neighbouring modules A and B should never be operated upon simultaneously. This could introduce the logical difficulty that the computer would not be able to decide, say whether to use the old state of A when finding the new surround of B and then the new state of B when finding the surround of A or vice versa. This difficulty is overcome by giving an ordering sequence to A and B. So the hexagonal subfield is divided into three subfields, no two modules of the same subfield ever being adjacent, and then operating simultaneously on all the modules of each subfield, taking one sub-field after the other.

	3	1	2	3	1	2	
1	2	3	1	2	3	1	
	3	1	2	3	1	2	
1	2	3	1	2	3	1	
	3	1	2	3	1	2	

Subdivision of a hexagonal field of modules into 3 subfields of non-neighbouring modules.

The shrinking operation is defined by

$$M^3_{1-3, n} \left[k = \overline{a_{i(k)}} \cdot k \right]$$

$i(k)$, the subscript of a is the index of the surround of any element of the field k which is to be operated on. The first subscript of M indicates that whenever the index is 1, 2 or 3 as defined in Table 1, we have

$$a_{i(k)} = 1 \quad \text{and} \quad \overline{a_{i(k)}} = 0,$$

whereas whenever the element of the k field is surrounded by a pattern of index 0, or 4, 5,, 13, we have

$$a_{i(k)} = 0 \quad \text{and} \quad \overline{a_{i(k)}} = 1$$

The second subscript n indicates the number of times the complete operation is performed. The superscript indicates that each complete operation consists of three partial operations performed successively on the three subfields of k . Within the bracket, the k before the $=$ sign refers to the next state of a module, whereas after the $=$ sign, it refers to its former state. When the operation is repeated until no further changes occur in k , that is until $a_i(k) = 0$ for all modules, which is indicated by making $n = \infty$, we get the thinned figure.

k is used alternately to denote the field or to represent the 0 or 1 state of any module of the field, as no ambiguity results from this usage.

In (19), Rosenfeld and Pfaltz present an algorithm for determining the 'distance skeleton' which they define. Here, the advantage is that the 'flesh' can be fully determined from the 'skeleton'. But the skeleton obtained here is not invariant under rotation.

The 'distance' between two points is first defined. Let P and Q be two distinct points of a digitized picture, and let $d^*(P, Q)$ be the smallest positive integer such that there exists a sequence of

points $P = P_0, P_1, \dots, P_n = Q$ with P_i
 a neighbour of P_{i-1} , $1 \leq i \leq n$.

This d^* is called the distance from P to Q .
 If $P = Q$, the distance between them is defined as zero.
 The distance from a point P to a given subset Q of the
 picture is defined as the smallest of the distances
 from P to Q .

First a distance transform of the digitized
 picture is constructed. For this, each element has an
 integer value equal to its distance from the set of 0's,
 assuming that this set is non empty. So the 0's are
 unchanged, since they are at zero distance from them-
 selves. The 1's which are horizontal or vertical
 neighbours of zeros also remain unchanged. The 1's which
 are horizontal or vertical neighbours of such 1's become
 2's and so on. The transform is done using the two
 sequentially applied local operations as follows:

$$\begin{aligned} \text{Let } f_1(a_{i,j}) &= 0 && \text{if } a_{i,j} = 0 \\ &= \min(a_{i-1,j+1}, a_{i,j-1+1}) \\ &&& \text{if } (i,j) \neq (1,1) \text{ and} \\ &&& a_{i,j} = 1 \\ &= \min && \text{if } (i,j) = (1,1) \text{ and} \\ &&& a_{1,1} = 1 \end{aligned}$$

$$f_2(a_{1,j}) = \min(a_{1,j}, a_{i+1,j+1}, a_{i,j+1+1})$$

Since no two points of the picture can be distance $m + n$ apart, we know that $a_{1,1}$ is at a distance less than $m + n$ from the set of O's if this set is non empty; thus the final value of $a_{1,1}$ will be the value assigned to it by f_2 .

If $P = (p_{i,j})$ is any picture, $P' = (p'_{i,j})$ will be called a partial picture of P if $p'_{i,j} = p_{i,j}$ or 0 for all i, j .

If $T = (t_{i,j})$ is a distance transform picture, the partial picture $T^* = (t^*_{i,j})$ defined by $t^*_{i,j} = t_{i,j}$ if none of $t_{i-1,j}$, $t_{i+1,j}$, $t_{i,j-1}$, $t_{i,j+1}$ is $t_{i,j} + 1$; 0, otherwise, will be called the 'skeleton' of T . That is, T^* is the set of local maxima of the distance transform.

This skeleton is the smallest subset of the transform picture from which the entire transform picture can be reconstructed by reversing the distance measuring

process.

Two rectangles and their skeletons are shown.

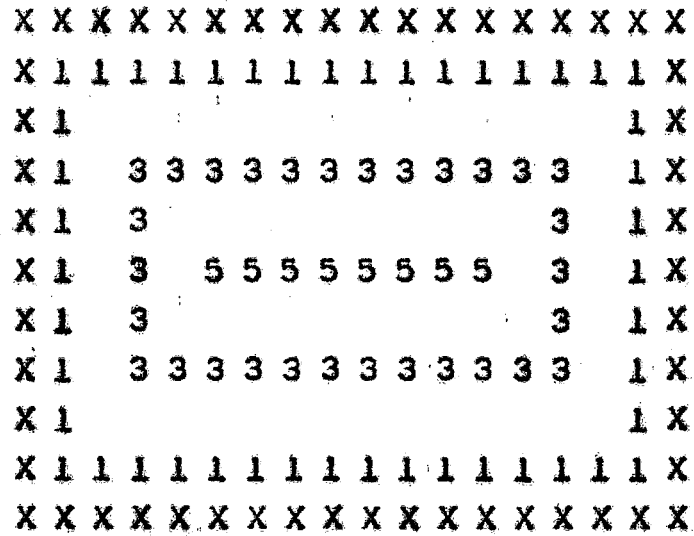


Fig. 7.a DISTANCE TRANSFORM

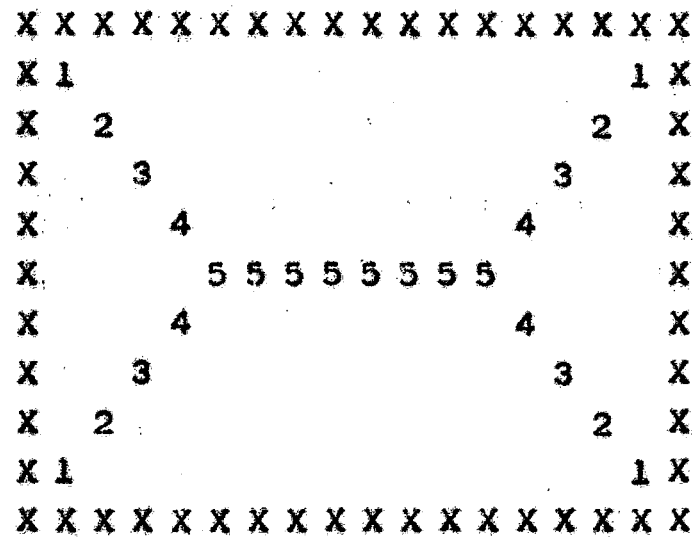


Fig. 7.b SKELETON

FIG 8a DISTANCE TRANSFORM

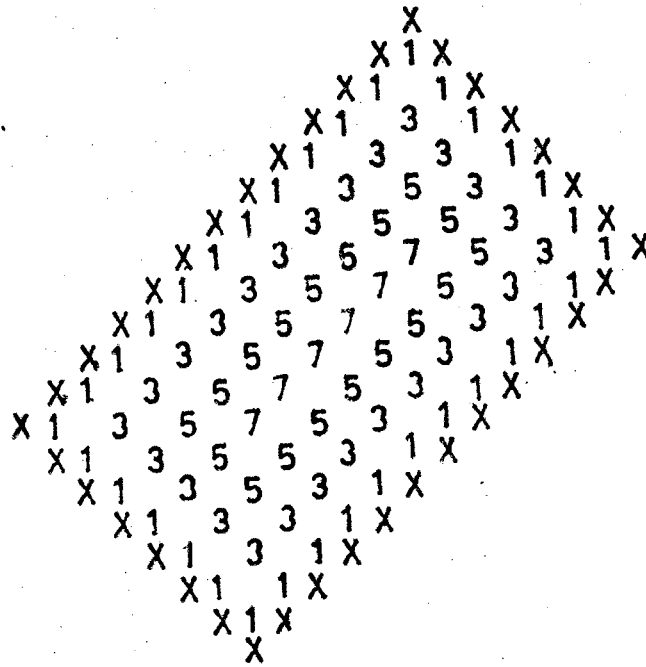
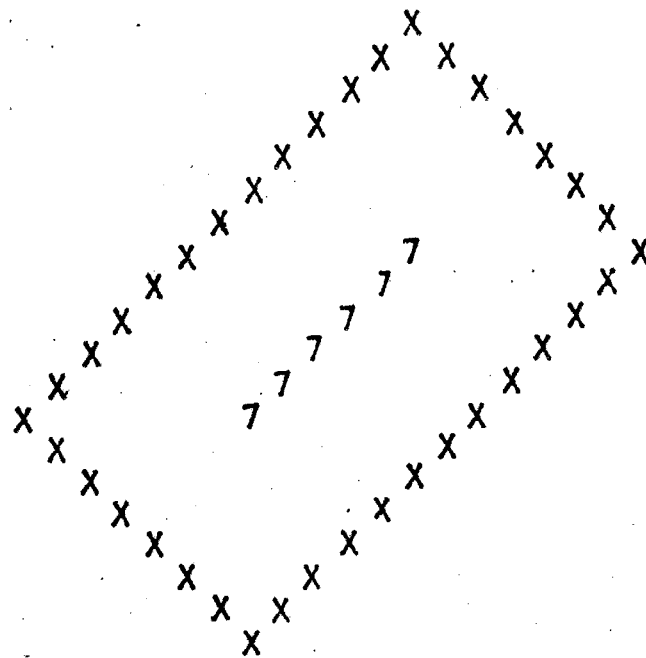


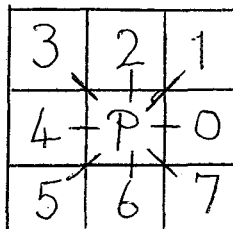
FIG 8b SKELETON



In (21), the authors give a method for obtaining the medial line of a figure. Contour points are defined, and at each iteration all the contour points are removed except those which might belong to the medial line, which are called final points. The final point conditions are also given. Each cycle is divided into four sub-cycles.

The original picture is initially stored in a matrix and in another matrix initially cleared, the final points detected at each sub-cycle are stored. A third matrix C_i contains the set of contour points eliminated at the i^{th} sub-cycle ($i = 0, 1, 2, 3$). In order to avoid elimination of the contour points which are also final points, all the contour points are first deleted from the picture and then the final points are added.

Povlidis (16) has described a thinning algorithm which goes along the contours of regions, peeling off the contour and leaving only multiple pixels. In the figure showing the relative positions of a pixel and its eight



neighbours, the ones in the even directions are direct or D-neighbours and the ones in the odd directions are indirect or I-neighbours. The contour of a set of pixels Q is defined as the set of pixels in Q which have at least one D neighbour not in Q.

The C-neighbours of a pixel on a contour are defined as the previous and next pixels found during contour tracing.

The author has defined multiple and skeletal pixels. A pixel is said to be multiple if one or more of the following conditions hold:

- a) It is traversed more than once during tracing, i.e. at the completion of the process its value is greater than 2. (When a pixel is found to belong to the contour, its value is incremented).
- b) It has no neighbours in the interior of the region (i.e. no neighbour with value 1).
- c) It has at least one D-neighbour which belongs to the contour but which is not one of its C-neighbours.

A skeletal pixel is one for which one of the following conditions is true:

- a) It is a multiple pixel
- b) It has a D- or L- neighbour which has been identified as a skeletal pixel during an earlier tracing.
- c) (Optional) Its two C-neighbours form an angle of 90° .

S denotes the skeleton, Q the set to be thinned, B(Q) its contour, L(Q) the set of pixels of B(Q) which are not multiple, M(Q) the set of multiple pixels of B(Q) and K(Q) the set of all pixels in B(Q) which have a neighbour in S, or (optionally) satisfy condition (c) of the definition of a skeletal pixel.

Initially S is set equal to \emptyset . The algorithm finds B(Q) by a contour tracing method. L(Q), M(Q) and K(Q) are found by retracing B(Q) while checking conditions of a multiple pixel and condition (c) of a skeletal pixel. Then S is set equal to $S \cup M(Q) \cup K(Q)$ and Q is set equal to $Q - B(Q)$. The process is repeated as long as Q is not empty.

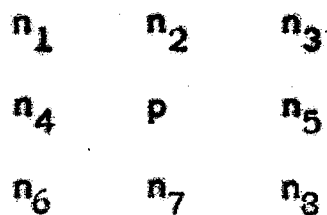
The skeletons obtained using this algorithm are not symmetric, but this defect is not serious if the initial pictures are not thick. The author says that thinning algorithms should be applied only to figures which are rather thin to begin with and so this serves the purpose. Moreover, a reconstruction algorithm is also given, which gives an approximate reconstruction of the original shape from the skeleton.

CHAPTER IV

A THINNING ALGORITHM

When black and white pictures are considered, a picture is a finite two dimensional array of points, say a square array of size $N \times N$ points, each point having a value black/white. Every such assignment of values to the N^2 points constitutes a picture.

The concept of connectivity is fundamental to many pattern transformations and is based on nearest neighbours logic. Each point is considered as having eight neighbours, as shown



Following Rosenfeld and Pfaltz (1966), a subset of a digitized picture is connected, if for any two points p and q of the picture, there exists a sequence of points

$$p = p_0, p_1, \dots, p_{n-1}, p_n = q$$

such that p_i is a neighbour of p_{i-1} , $1 \leq i \leq n$.

The algorithm that we have developed scans each 3 x 3 window in the digital picture and examines if the central point in the window can be removed or not. It is removed, that is, changed from 1 to some other number say 2, if certain conditions hold. At the end of each scan or pass, all the 2's are changed into 0's.

We use Rosenfeld's⁽⁶⁾ definition of a border point. Let S be a subset of a digital picture. If P is a point of S , and the neighbourhood of P is

a	b	c
d	p	e
f	g	h

then we call p a north border point if $b = 0$; an east border point if $e = 0$, a west border point if $d = 0$, and a south border point if $g = 0$.

If the following conditions are found

x	x	x	x	0	y	x	x	0	0	y	y
0	1	0	x	1	y	x	1	y	x	1	y
y	y	y	x	0	y	0	y	y	x	x	0

where at least one x and one y are non-zero, then the central point is preserved and the next 3×3 window is processed.

If the above four conditions are not found, then the central point in the window is examined to find if it has at least two 1's as its neighbours. Each of these 1's should have at least two 1's in their respective neighbourhoods also. This preserves the connectivity of the picture. If this condition holds good, the point is removed, that is, changed to some other number, say 2.

In the first scan of the process, north border points or west border points are removed if they satisfy the above conditions. In the next scan, south border points or east border points that satisfy these conditions are removed. After each scan through the whole picture, all the 2's which indicate the points removed are set equal to zero. The process of removal is repeated till there are no more points to be removed. We are finally left with a thin version of the original picture.

This algorithm was applied to some pictures of chromosomes, and it was found to work well. The algorithm

is presented here.

The picture is digitized, and the input is a matrix $A(m,n)$ of 1's and 0's. An indicator S points out whether it is the first or second sub-cycle. There is another indicator F which shows if the point is reached when there are no more points that can be removed.

The Algorithm:

- Step 1 : Read $A(I,J)$, $J = 1,N$, $I = 1,M$
- Step 2 : Set $S = 1$, $I = 1$, $J = 1$
- Step 3 : Is $A(I,J) = 1$? If 'No' go to Step 10.
- Step 4 : Is $S = 1$? If 'Yes' go to Step 6.
- Step 5 : Is it a south or east border point ?
If 'No' go to Step 10. If 'Yes' go to Step 7.
- Step 6 : Is it a north or west border point ?
If 'No' go to Step 10.
- Step 7 : Does it have at least two 1's as its neighbours ? If 'No' go to Step 10.

- Step 8 : Does each of its neighbouring 1's have at least two 1's for their neighbours ? If 'No' go to Step 10.
- Step 9 : Set $A(I, J) = 2$
- Step 10 : Is $J = N$? If 'No' then set $J = J + 1$ and go to Step 3.
- Step 11 : Is $I = M$? If 'No' then set $I = I + 1$, $J = 1$, and go to Step 3.
- Step 12 : Print $A(I, J)$, $J = 1, N$, $I = 1, M$
Set $I = 1$, $J = 1$, $F = 0$.
- Step 13 : Is $A(I, J) = 2$? If 'No' then set $A(I, J) = A(I, J)$ and go to Step 15.
- Step 14 : Set $A(I, J) = 0$; $F = 1$
- Step 15 : Is $J = N$? If 'No', set $J = J + 1$ and go to Step 13.
- Step 16 : Is $I = M$? If 'No' set $I = I + 1$; $J = 1$, and go to Step 13.
- Step 17 : Is $S = 1$? If 'Yes', then set $S = 2$ and go to Step 19.

Step 18 : Set $S = 1$

Step 19 : Is $F = 1$? If 'Yes' then set
 $I = 1, J = 1$ and go to Step 3.

Step 20 : Stop.

Two of the pictures of chromosomes and the successive stages of the thinning process are shown here.

EXAMPLE 1

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 2 1 0 0 0 0 0
0 0 0 0 0 0 1 2 0 0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 0 1 1 2 0 0 0 0 1 1 2 0 0 0 0
0 0 0 0 0 2 2 1 0 0 0 0 1 1 1 1 2 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 2 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 2 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 2 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 2 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 2 0 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 2 0 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 2 0 0 0 0 0
0 0 0 0 0 0 0 1 1 1 2 2 2 1 0 0 0 0 0
0 0 0 0 0 0 1 1 1 2 0 0 0 0 1 2 0 0 0 0
0 0 0 0 0 0 1 1 2 0 0 0 0 0 2 1 0 0 0 0
0 0 0 0 0 0 1 1 2 0 0 0 0 0 0 1 2 0 0 0
0 0 0 0 0 1 1 2 0 0 0 0 0 0 1 1 2 0 0 0
0 0 0 0 0 1 1 1 1 0 0 0 0 0 2 1 2 0 0 0
0 0 0 0 2 1 1 1 2 0 0 0 0 0 0 2 1 0 0 0
0 0 0 0 0 2 2 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

EXAMPLE 2

CHAPTER V

CONCLUSIONS

Chromosome analysis involves the classification of the various chromosomes into groups depending on size and position of the centromere.

The reasons for introducing automation for chromosome analysis are obvious from the nature of tasks involved. The objects are viewed through a microscope, and the carefulness with which the first slide is handled and the carelessness with which the 10th slide is handled varies from person to person. The number of cells that can be dealt with by a technician skilled in the field is limited. Factors like fatigue affect the judgement of the worker who does the analysis manually. Since these can be overcome by a machine, and the machine makes decisions based on measurements, the analysis is done on a quantitative basis. Also due to the ability of the machine to store large amounts of data, it can identify very minute differences which may not be possible for a human to do.

The process of automatic analysis involves feeding into the computer a digitized picture of the chromosomes.

Segmentation methods are used to extract the regions of interest. These regions are represented so that it facilitates further analysis. Then measurements are made, based on which decisions are taken.

The work done in this dissertation gives a method of representing the regions of interest which have been obtained after segmentation. The method represents the region by its skeleton. The algorithm presented here works quite well and the skeletons of a few chromosome images have been obtained. There was no loss of connectivity. The algorithm is conceptually very simple, and its simplicity is a big advantage. The chromosome pictures that are used here have been taken from Hilditch's work (9).

It was found that if there is excessive noise, so that two arms of a chromosome appear to merge, then the algorithm does not seem to succeed. An example is shown where the digitized image of the chromosome has a connection between two arms due to noise. In this case, a noise cleaning operation can be applied first and the thinning algorithm can be applied later.

Character recognition was one of the first industrial applications of pattern recognition. Attempts can be made

to use the algorithm presented in this dissertation on characters, particularly handwritten ones. Mechanical or electronic scanning is used to convert an optical image into electrical signals. This output is then sampled and digitized. In order to facilitate the feature extraction and recognition logic, filtering is done by applying line thinning algorithms which reduce the character to its skeleton.

Perhaps the algorithm may also be used for multigrey level pictures, but this has not been attempted in this dissertation.

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