TECHNICAL CHANGE, GROWTH AND ENVIRONMENT: AN ECONOMIC ANALYSIS

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DECLARATION

I declare that the dissertation entitled "Technical Change, Growth and Environment: An Economic Analysis" submitted by me for the award of the degree of Master of Philosophy of Jawaharlal Nehru University is my own work. The thesis has not been submitted for any other degree of this university or any other university.

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CERTIFICATE

We recommend that this dissertation be placed before the examiners for evaluation.

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CONTENTS

Chapter 1: Introduction

1.1 Background

Growth theory due to its multi-dimensional aspect has generated considerable research interest. In the sphere of theoretical and empirical macro-economics, it inarguably stands to be one of the most fundamental and sought after subject. Ever since the idea of endogenous growth was discovered, research in growth theory has been revived, combining it with factors such as learning-by-doing, increasing product variety or quality improvement, all of which contribute to sustaining long-run growth of the economy.

In particular, the endogenous growth literature due to technological change driven by research and development (R&D) has given a new edge to growth theory. Romer (1990) sets innovation leading to expansion in the variety of products driving the growth process whereas the Schumpeterian approach of Aghion and Howitt (1992) describes growth through a process of creative destruction, as new improved product quality replaces the existing ones. A large volume of literature based on R&D models of endogenous growth followed these two seminal works. Nonetheless, all the innovation based growth models pre-dominantly stand on the premise that the innovations in the intermediate sectors take place with the same average frequency. But in reality, some sectors are inherently more innovative than others. An important reason for this difference among sectors has to do with their size. Other things being equal, since a successful innovator has a larger market in a larger sector (in terms of the size of the sector employing that technology), it is more profitable to innovate here. As a result, technical change tends to be directed or biased towards larger sectors than smaller ones. This makes us to think of the bias of technological progress and the factors that drive it. Acemoglu (1998) and Acemoglu (2002) develop the idea of the bias of technology and call it the **directed technical change.** This has been an important development paradigm, with widespread application not only in macroeconomics, but also in the other areas of research relating to labour and development economics. One such area where the role of technology is of surmount importance is the study of economic growth while incorporating the constraints imposed by the environmental and natural resources. Before embarking on this specific enquiry, we allude to the linkages between the growth and the environment in general.

In the recent decades, growth theory has been coupled with the growing concern over environmental degradation and exhaustion of natural resources. In the context of the environmental and resource concerns, two aspects of growth theory have received considerable attention: one that addresses the relationship between economic growth and natural resources depletion, and the other dealing with the relationship between economic growth and environmental pollution. Both have focussed on the need to understand whether the growth process is sustainable in the long run and whether it is compatible with environmental protection or optimal resource extraction. As for the first, Dasgupta and Heal (1974), Arrow and Fisher (1974) and Brock (1973) underline the harm due to over-extraction of resources and indicate the process of optimal depletion of natural resources when it is essential for the economy. On the second issue, the inter-linkage between pollution and growth, Elbasha and Roe (1996), Gradus and Smulders (1993), Bovenberg and Smulders (1995) and Mohtadi (1996) provide a good exposition of environment externalities in the form of pollution on growth.

Survey articles by Xepapadeas (2005), Brock and Taylor (2005) and Ricci (2007) provide an impressive review of existing theoretical and empirical research in the area of growth and environment. These span research that utilizes both exogenous and endogenous growth models, while also providing the directions for future research. The coverage of endogenous growth models relying on innovation and purposeful R&D emphasises the role of technical change in pollution-oriented growth models. Technology change impacts prospects for sustainable growth. The papers by Smulders and Nooij (2003), Andre and Smulders (2004) and Grimaud and Rouge (2005) too emphasize the role of technology in endogenous growth models with pollution externalities. However, most of this research assumes unbiased technological progress. As mentioned earlier, observably, the pace of innovation occurs at different rates in different sectors of the economy, there are some specific economic factors that drive research in one sector vis-a-vis the other, thus determining the direction of technical change. Models of directed technical change are now at the forefront of research in endogenous growth theory.

Notably, this idea of directed technical change is not new; it dates back to Hicks (1932). Kiley (1999) reintroduced it in the context of skilled and unskilled labour and it has been formalised in the context of modern growth theory by Acemoglu (1998, 2002 and 2003). Utilizing the endogenous direction of technical change in the context of pollution externalities and studying its implications for the sustainability of economic growth is of great relevance in view of the rising concern over environmental implications of growth or the effect of environmental constraints on long-run growth prospects of an economy. In the wake of this idea as the direction of technical change can be influenced, it becomes very critical for sustainable development to channelize these technological developments towards cleaner and less polluting sector. Grimaud and Rouge (2008) and Acemoglu, Aghion, Bursztyn and Hemous (2010) (henceforth referred as Acemoglu et.al.) use this idea of equilibrium bias of technological change in an growth-environment setting and formalise the conditions and policy directions under which growth can be sustainable. Both assume a two sector R&D model, one of which is clean (or non-polluting) and the other is polluting. The analysis provides prescriptions on optimal policies to prevent an environmental disaster and also to move towards clean research than dirty ones under alternative production structures varying by the elasticity of substitution among its inputs.

Considering the above issue in the context of environmental pollution, this dissertation proposes to examine how environmental dimension can be embodied in the existing endogenous growth models to have sustainable growth, when the direction of technical change is endogenously determined. It differs from earlier research in two specific aspects: first, it characterises the structure completely not just from the producer's side but also integrating the consumer's behaviour to arrive at a complete picture of the economy under the constraints that arise out of growing concern for the environment. Also, the direction of technical change is jointly determined by the producers and the consumers. Secondly, it characterises the social optimal outcome and the policies that could help arriving at the desired outcome and compares these with the decentralized market outcome. Besides, this it provides a more holistic view of the economy in the long run, as it incorporates in the framework both pollution and the natural resources and characterises the economy in their presence.

The key results derived by us are:

(i) In both the decentralised market equilibrium and the social planner's equilibrium, a steady state is ensured by only dirty sector technical progress. Therefore, the long run growth is purely resource augmenting technical change (which is the dirty sector technological change) with the stagnant capital-augmenting (clean) technical change.

(ii) A social planner's equilibrium always ensures a sustainable growth path which the decentralised market equilibrium may or may guarantee.

1.2 Structure of the dissertation

The dissertation is organised as follows. In the next chapter that follows, we provide a detailed review of the existing theoretical literature relevant to our research based on which we define the key questions for which we intend to find answers through this research.

In chapter 3, we build the structure of the model, and characterise the behaviour of the economy along a balanced growth path at the decentralised market equilibrium. The idea of directed technical change is discussed at length, in both the steady state and the out of steady state.

In Chapter 4, we characterise the social planner's solution for the economy described in chapter 3. We then compare the social optimum with the private (market) outcome.

Chapter 5 of the dissertation concludes the work and provides some policy implications of the above. It also identifies some areas of future research.

Chapter 2: Review of Literature

Since our work relates to a very new idea of directed technical change in growth theory and it is being applied in the context of environmental pollution, we would present the literature under two broad themes. The first is to do with the literature on the role of innovation and the R&D in the models of growth, leading up to the discussion on the models of directed technical change. Under the other theme, we review the existing literature that links endogenous growth and the environment with thrust on the role of technology in the growth process in the presence of environmental constraints. Finally, these two strands of literature are combined to review recent work related to the direction of technological change and its application to study the role of environment in economic growth and its sustainability.

2.1 Endogenous Growth and Technology

Models of endogenous technological progress were introduced in Romer (1986 and 1990), and then subsequently, analyzed by, among others by Grossman and Helpman (1991) in an open economy context and Aghion and Howitt (1992) in the context of quality ladders. The remarkable feature of these models is the fact that R&D spending and investments are shaped by profit incentives, which, in turn, determine the rate at which the technological progress of the economy evolves over time. Technological differences across countries are likely to be important explanations in accounting for their income differences. Thus, it is very important to understand the sources of technology differences while studying the mechanics of economic growth. These models emphasize the importance of profits in shaping technology choices. Their departure from the neo-classical models of growth is that they take into account the role of monopoly power and patent length on the equilibrium growth rate. The lab-equipment model is another version of such a case, which appears in Rivera-Batiz and Romer (1991). The paper specifically focuses on an open economy situation, and thus develops extensions under different degrees of economic integration due to knowledge spill-over. Gancia and Zilibotti (2005) provide an excellent survey of many of the models related to endogenous technological progress. The baseline model of Schumpeterian growth presented is based on the work by Aghion and Howitt (1992). It introduces the basic Schumpeterian model of economic growth to emphasize the importance of competition among firms - both in the innovation process and in the product market. In the Schumpeterian growth framework, it introduces a process of creative destruction, under which new products or machines replace older models and new firms replace incumbent producers. It features process innovations which lead to quality improvements in the baseline model. A critical take away from the Schumpeterian models is that growth comes with potential conflict of interest. The process of creative destruction destroys the monopoly rents of previous incumbents. Aghion and Howitt (1998) provide an excellent survey of many Schumpeterian models of economic growth and numerous extensions.

Models of directed technological change are closely related to the literature on purposeful or induced innovation. Hicks, in The Theory of Wages (1932) argues that, "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind–directed to economizing the use of a factor which has become relatively expensive." Works of Kennedy (1964) brought the concept of "innovation possibilities frontier" that determines the factor distribution of income rather than the shape of the neo-classical production function determining it. This paper is relevant to the directed technical change literature as he extends it such that the economy would move to equilibrium with constant factor shares. In Habakkuk's (1962) argument, the central determinant of technological progress is the search for labour saving inventions. The analysis in Acemoglu (1998) and the subsequent work in this area, unlike earlier work, builds on the explicit micro-foundations of the endogenous technological change models. The basic models of directed technological change differ from the endogenous technological change models because, along with determining the rate of aggregate technological change, they endogenize the direction and bias of technological change also towards some specific sectors in the economy. Using the idea of directed technical change, Acemoglu (2003) gives a sound economic model of revisiting the neoclassical result which states that, in the long run, growth is purely through labour-augmenting technical change and with a constant share of labour in the GDP. Acemoglu (2007) introduces the alternative concept of weak absolute bias and strong absolute bias, which instead of looking at the relative marginal product of a factor, looks at the marginal product of that factor. Under fairly weak assumptions, endogenous changes in technology that are induced by an increase in the relative supply of a factor, is relatively biased towards that factor. Consequently, any increase in the ratio of skilled to unskilled workers or in the capital-labour ratio will have major implications in terms of endogenous changes in technology, and hence, the bias of technical change. This would for that matter change the relative productivity of these factors. The more surprising result is of the strong equilibrium bias, which states that, (relative) demand curves can slope up implying that the demand for a factor increases with an increase in factor prices, contrary to the basic producer theory. In particular, if the elasticity of substitution between factors is sufficiently high, a greater relative supply of a factor causes sufficiently strong induced technological change to make the resulting relative price of the more abundant factor to increase. In other words, the long run (endogenous-technology) relative demand curve becomes upward-sloping. Acemoglu and Zilibotti (2001) discuss the implications of directed technological change for cross-country income differences. Acemoglu and Linn (2004) had empirically shown the relevance of directed technical change in a pharmaceutical industry where innovations are systematically directed towards drugs used by wealthier customers indicating a significant effect of market size on the innovation flow for each category of drugs. Gancia and Bonfiglioli (2008) use directed technical change in the context of North-South trade where they show that, under weak protection of IPRs in poor countries, market integration shifts technical change in favour of rich countries that amplify the international wage differences; this is also established empirically by them. Very recently, models of directed technical change have found use in the context of environment, a discussion on which ensues.

2.2 Growth and Environment

Starting with the literature on growth and environment, some early studies analyze the relationship between economic development and the environment by correlating the level of incomes to the indicators of environment quality (air and water pollution etc.) (see for example, Grossman and Krueger (1991, 1995) cross-country analyses in this respect) to study the so-called environmental Kuznets curve hypothesis. Mohtadi (1996) and Smulders and Gradus (1996) discuss the design of environmental policy to arrive at a socially optimal path using an optimal tax on production and a subsidy on pollution abatement or both or a combination of quantity control and tax/subsidy schemes depending on the framework that they consider. Seminal contributions have also been made by Gradus and Smulders (1993) and Bovenberg and Smulders (1995). Gradus and Smulders (1993) describes the effect of a shift in society's preferences towards a larger concern for a cleaner environment on the long run rate of growth. They discuss it in the context of three prototype growth models. The effect on the long run growth rate of this shift in preferences is lower in AK type models due to crowding out of investment compared to models with human capital (Lucas type), where it is ambiguous as it depends on whether pollution affects the human capital. The assumption regarding the production technology and the relationship between pollution, production and abatement is very critical to their result. Bovenberg and Smulders (1995) extend the Lucas and Rebelo type models in a different way. Theirs is a two-sector growth model. One sector produces the final good which can either be consumed or invested for capital accumulation while the other sector generates a public good in the form of knowledge in pollution reduction. The elasticity of substitution between environmental services and consumption in the utility function and between environmental services and man-made factors of production in the production function must be unitary for the balanced growth. A constant environmental quality is feasible and optimal only if this substitution effect offsets exactly the income effect due to growth in productivity. Elbasha and Roe (1996) develop an endogenous growth model to examine the interaction between trade, economic growth, and the environment. Whether trade improves or retards growth depends on the relation between the factor intensities of exportable, importable, and R&D and also the relative abundance of the factor that R&D uses more intensively. Also, depending on the elasticities of supply for the two traded goods, the terms-of-trade effect on growth, and pollution intensities, trade positively or negatively affects the environment and welfare. With a higher degree of monopoly power in the innovation sector and stronger environmental externalities, the market growth rate is higher than the optimal one. In a recent study, Fullerton and Kim (2008) consider a holistic approach by taking a combination of various elements, viz., public R&D spending affecting abatement knowledge, distortionary taxes affecting capital formation, pollution taxes affecting environmental degradation etc. and their implications for endogenous growth; something that earlier studies considered separately but not together. Bretschger (1998) is an important study in the class of endogenous growth models which examines the role of accumulation and substitution of man-made inputs for natural resources to achieve long term sustainable growth. He identifies increasing prices for environment and economy's sectoral changes as the main driving forces to bring the substitution of natural inputs into effect needed for sustainable growth. Though an ever declining growth rate is witnessed in one-sector models, with unitary elasticity of substitution between capital and natural resources which is non-sustainable, the same leads to a positive growth rate in the long run in the multi-sector models. His argument of achieving long-term growth is through sectoral changes exploiting the allocative forces of the market economy. The survey articles by Xepapadeas (2005), Brock and Taylor (2005) and Ricci (2007) provide a comprehensive reviews of existing theoretical and empirical research in this area on growth and environment spanning the current research and provide a direction for future research. All three of these establish the link between the process of economic growth and the state of the environment. The focus is on how the development of clean technologies and role of abatement activities in emissions reduction impacts the sustainability of economic growth. They identify future research efforts towards a theory of induced innovation where both relative prices and pollution regulations determine the pace and direction of improvements in abatement technology. The direction of technical change models as the very basic idea is that technical change can be biased towards some specific sectors in the economy. On that premise it becomes important that this bias can be used to redirect technical change to cleaner sectors vis-a-vis the dirty sector or vice-versa.

The direction of technical change has become quite important with analysis of growth in the presence of environmental constraints. As we have discussed earlier, the theory of directed technical concludes that the elasticity of substitution between the inputs is very critical in determining the direction of the technical bias. Smulders and Nooij (2003) develop a growth model in which energy is an essential input and endogenous technical change drives long-run growth. Innovation is modelled as a purposeful activity in form of a rational investment behaviour driven by profit maximisation. Energy-related innovation is much worse than other types of innovation if the appropriability captured by the share of returns to innovation accrue to the inventing firm. There is a fall in growth in response to energy conservation if energy share is already close to its steady state level. Andre and Smulders (2004) utilize an endogenous growth model where resource owners endogenously determine the extraction path and firms endogenously determine the rate and direction of technological change. With resources becoming increasingly scarce, technical change shifts towards energy-saving technological change. But this comes at the cost of total factor productivity growth, which drives long run per capita income growth. Grimaud and Rouge (2008) and Acemoglu, Aghion, Bursztyn and Hemous (2010) (referred as Acemoglu et.al.) utilizes the idea of equilibrium bias of technological change and formalises the different conditions and policy directions under which growth can be sustainable.

Grimaud and Rouge (2008) study the effects of an economic policy in an endogenous growth general equilibrium framework. The framework produces a consumption good which requires two resource inputs: non-renewable resource, which is polluting and a labour resource, which is non-polluting. The use of polluting non-renewable resource emits pollution in the atmosphere which affects welfare. Each resource is associated with a specific research sector. The paper provides a full welfare analysis and the equilibrium paths in a decentralized economy. The effects of three associated economic policy tools: a tax on the polluting resource and two research subsidies are seen. There are three fundamental distortions at the decentralized equilibrium: the environmental externality from the polluting resource in the form of pollution and two externalities arising from the research in two sectors as the entire value of innovations cannot be extracted from the users of innovations by the innovators. The following are the main results of the paper. Firstly, it derives the existence of a stable unique feasible steady-state corresponding to the optimum which is the case with no pollution, or equivalently pollution does not affect welfare. Secondly, comparing the social optimum and the 'laissez-faire'/decentralized regimes, it is shown that the latter type of economy uses the non-renewable resource too fast, and thus generates excessively high pollution in the early stages. Also, the overall research effort or the equilibrium quantity of research is sub-optimal. Moreover, in the early stages, R&D in grey (dirty) research is high compared to that in green (clean) research. But, it is important to note that after a certain period this situation in grey research is reversed. The length of this period is inversely correlated to the distortion in the innovation market, that is, the gap between the price paid by users of innovation and their marginal willingness to pay. The direction of technical change, measured here as the difference between the growth rates of 'green' and 'grey' resource stocks (referring to Acemoglu (2002)), is too 'grey-oriented' and hence non-optimal. Thirdly, the decentralized equilibrium growth is sub-optimal, which means that early generations consume too much to the detriment of the future generations. Finally, the effects of the two economic policies are determined. The introduction of an R&D policy promotes both types of research effort (green and grey) as a result of which both the quantity and the quality of research increase. However, the direction of technical change remains unaltered. But, following this, the flow of resource extraction (and thus of pollution) are also unchanged and so are the dynamics of the environment. The optimal climate policy entails levying a decreasing tax on fossil fuels. This works as it will hold back the pace of extraction and hence slow down polluting emissions. A simple intuition is that the price of the resource (including the tax) becomes relatively higher today. Furthermore, it explains that with increased growth rates of resource extraction and green knowledge, this policy fosters output growth. There are another set of results that relate to the impact of the optimal climate policy on the overall R&D effort and the direction of technical change. It is shown that the quantity of research is not changed. However, the quality of research is changed: there is redirection of effort from 'grey' to 'green' research. Alternatively, a decreasing environmental tax spurs technical change in the 'desired' direction. Furthermore, the impact of the climate policy on the ratio of green and grey resources' marginal productivities is studied which is referred to as the bias of technical change (following Acemoglu (2002)). It is shown that, in the short-term, the environmental policy is grey-biased, and in the long-term, it is green-biased.

Acemoglu et.al. (2010) too confirms some of the results of Grimaud and Rouge (2008) which are in the same direction where both use the idea of directed technical change. Acemoglu et.al. provides the results in a more generalised production structure whereas Grimaud and Rouge (2008) uses it in a specific framework where the inputs in production are complements. Also, Acemoglu et.al. very well integrates it with the literature in this area by explicitly bringing out the market size effect and the equilibrium bias of technical change. They also clearly outline the role of elasticity of substitution in the process of endogenous and biased technical change. The analysis of the case of an environmental disaster is an interesting exposition in Acemoglu et.al. Moreover, Acemoglu et.al goes beyond mere theorisation and attempt numerical simulations which seem to support their findings. Their main results are as follows. Innovation always occurs in the dirty sector in a decentralised equilibrium, if the two inputs are gross substitutes. But in case the inputs are gross complements, innovation initially occurs in the clean sector, but eventually occurs in both the sectors. Throughout it is assumed that the dirty sector is sufficiently advanced initially, so that gross substitutability implies stronger productivity and market size effects which dominate the price effect, and hence make innovation in the dirty sector more profitable. On the other hand, with gross complementarities, direct productivity effect gets dominated and hence, innovation is directed towards the more backward sector which is the clean sector. As a result, the laissez-faire economy always leads to an environmental disaster since the production of dirty input is always growing in this setting. Secondly, with two inputs being strong substitutes and with a sufficiently high carrying capacity of the environment, a temporary subsidy to clean research will prevent an environmental disaster which cannot prevent the disaster when the two inputs are complements or weak substitute. In case of strong substitution between inputs, the dirty input will not be growing in the long run and thus an environmental disaster is avoided. The market size effect dominates in case of strong substitutes but in case of weak substitutes the price effect dominates, which leads to increasing the production of dirty input even when the dirty sector technology level does not improve and is stagnant. Therefore, when inputs are complements or weak substitutes, a temporary subsidy cannot avoid an environmental disaster. Two complementary inputs would result in innovation in both the sectors thus making a disaster for certain. Thirdly, a social optimum for this economy can be obtained by correcting for the three kinds of distortions in the economy. First, the environmental externality in the form of pollution exerted by dirty input producers. Second, the R&D knowledge externalities as scientists do not internalize the effect of their research on future productivity. The third kind of distortion exists due to the monopoly pricing of machines. These distortions have also been discussed in Grimaud and Rouge (2008) which discusses that the social planner can implement the social optimum through a tax on the use of the dirty input (a "carbon" tax), a subsidy to clean innovation, and a subsidy for the use of all machines (all proceeds from taxes / subsidies being redistributed / financed lump-sum).

Another paper worthy of mention over here is Maria and Valente (2008). It uses the framework of Acemoglu (2003) to analyse an economy under the idea of directed technical change when there is a natural resource owned by the agents in the economy used in the production. They show that a steady state exists under a decentralised equilibrium for an economy in the presence of a non-renewable natural resource used in production.

2.3 Research Theme

My work involves characterising an economy where consumers are adversely affected by polluting emissions entering into the environment. This feature is different from the Maria and Valente (2008) as they look at the environment only from the perspective of a non-renewable natural resource. Instead, in our paper we take into account as well as model environmental pollution and a non-tradable renewable natural resource which is used in production. The resource also provides an added utility to the consumers in the form of its existence value (in addition to its use value in production). A key difference of my work as compared for the other three papers (Acemoglu et.al. (2010), Grimaud and Rouge(2008) and Maria & Valente (2008)) is that I characterise the social planner's equilibrium and derive that all the policies towards redirecting R&D towards clean sector are effective in the short run, but in the long run growth is sustainable in the steady state only through the dirty sector technological progress. The above result also holds for a market economy too. Further, I show that the market equilibrium is sub-optimal and may imply a situation of unsustainable growth if ξ *and* η (i.e. the rate of regeneration of the non-tradable non-renewable natural resource and the rate of pollution decay) is sufficiently low and $\phi > \psi$ (i.e. the intensity of the disutility from pollution is dominates the intensity of the existence value of the presence of natural resource stock in the utility). But a case of sustainable growth is possible vice-versa. On the other hand, the social planner's equilibrium always provides for a sustainable growth scenario as the planner optimally chooses the usage of the polluting natural resource. A major departure of my results from Acemoglu (2010) and Grimaud and Rouge (2008) are that, they show R&D flowing into both the clean and dirty sectors in the long run, which stands in contradiction to my work as I show the existence of as sustainable growth path with all research into dirty sector in the long run steady state equilibrium. While they predict an environmental disaster with the growth of the dirty technological sector, I show a sustainable growth path is possible in the long run with the presence of polluting natural resource which is used in production and also has an existence value to the consumers.

These are the main lines along which literature on growth and environment is developed in the context of endogenous growth models and directed technical change. In the next chapter, I build the structure of my model and then go on to present my propositions and results.

Chapter 3: Market equilibrium under environmental constraints

In this chapter we characterise the decentralised market outcome for the economy. This is the equilibrium of the economy when decisions are market determined such that the consumers' maximise their utility and producers' maximise their profits in a market setting. Integrating the behaviour from the two we characterise the behaviour of the economy.

3.1 The Structure of the economy

The economy consists of identical individuals distributed over the interval (0, 1). They own the natural resources in the economy. These individuals are owners of the production firms and also act as scientists who perform R&D activity. The utility function of the representative individual is given by

$$U = \int_0^\infty \frac{\left[C(t)(Z(t))^{-\emptyset}H(t)^{\psi}\right]^{1-\theta}}{1-\theta} e^{-\rho t}.dt \ \ where\ \theta \neq 1, \emptyset, \psi > 0, \rho > 0. \ \ ....(3.1)$$

$C(t)$ is the consumption of the individual at time point t. This is a constant relative risk aversion (CRRA) preference. θ is the elasticity of marginal utility of C. It can also be interpreted as a measure of risk behaviour of the consumers so that the consumers are risk-averse or risk-loving accordingly as $\theta \lesseqgtr 1$. $Z(t)$ is the stock of pollution at time point t. Pollution generates disutility for the consumers and $\emptyset$ is the intensity of this disutility. $H(t)$ is the natural resource which has an existence values i.e. the presence of the stock of this natural resource gives utility to the consumer and this intensity is ψ. $\emptyset \lesseqgtr \psi$ implies that the consumers get lower or same or higher disutility from the stock of pollution than an equivalent amount of stock of natural resources due to the existence value. U gives the life time discounted utility of the consumers with ρ as the constant rate of time preference. The budget constraint of the consumer is given by:

$$Y \leq C + I \quad where \quad Y = r.K + \tau.D + w.s + \Pi ......(3.2)$$

The above implies that the total income in the economy is no more than what is spent. So, the income is either consumed or invested (I). r is the interest rate. K is the capital, τ is the price of the natural resource, D is the use of natural resource in a period, w is the wage for the scientists, s is the employment of scientists and Π is the profits from production. It is also assumed that there is no depreciation of capital[1]. Therefore, $\dot{K} = I$. Thus, briefly, the income of the scientists is derived from their wage income from R&D, rental income from the extraction of natural resource, interest income from capital and from the production profits.

Now we characterise the production side of the economy. The economy produces a final good which is used for consumption and investment. The final good Y is produced in a competitive market according to the following constant elasticity of substitution (CES) production function.

$$Y = \left[Y_c^{\frac{\sigma}{\sigma-1}} + Y_d^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} \quad where \quad 0 \leq \sigma < \infty. \dots\dots\dots (3.3)$$

The value of the elasticity of substitution is very critical to our analysis (as we will see later) as contingent upon this the R&D, the returns to R&D are being shaped and thus R&D gets directed to specific sectors.

$Y_c, and\ Y_d$ are producible clean and dirty intermediate inputs. The clean and dirty inputs are defined in the sense that the production of the dirty input uses the natural resource and generates pollution which the clean input does not require. σ is the elasticity of substitution between the two intermediate inputs in the production of Y. σ is a constant and for different values of it we can get different production functions. For $\sigma = 0$, it is a Leontief production structure with inputs being perfect complements. For $\sigma = 1$, we get the Cobb-Douglas form and for $\sigma = \infty$, it reduces to the case of perfect substitutability between the two inputs. Hence, our production function is a generalised one.

1. Introducing depreciation does not change the results of the model qualitatively. We ignore it just for simplicity.

The intermediate inputs Y_c, and Y_d are produced competitively from the following constant elasticity of substitution (CES) production function which is capital intensive and resource intensive respectively with constant elasticity of $\frac{1}{1-\beta}$.

$$Y_c = \left[\int_0^m (y_j^c)^\beta . dj\right]^{\frac{1}{\beta}} \quad and \quad Y_d = \left[\int_0^n (y_j^d)^\beta . dj\right]^{\frac{1}{\beta}} \quad where \quad \beta \in (0,1). \dots\dots (3.4)$$

Since $\beta \in (0,1), \frac{1}{1-\beta} > 0$ and thus the inputs are gross substitutes. Two inputs are gross substitutes if the increase in the factor price of one increases the demand for the other. Each of Y_c, and Y_d are produced using a continuum of m variety of y_j^c and n variety of y_j^d machines respectively. This implies that these are two different sets of intermediate machines. An increase in m and n respectively implies capital-augmenting and resource-augmenting technical change. The latter tantamount to pollution-augmenting technical change.

Intermediate machines are produced by monopolists, who hold relevant patent for the production of their machines. They are produced by a linear production function

$$y_j^c = k_j \ , \ j \in (0,m) \qquad and \qquad y_j^d = d_j \ , j \in (0,n), \qquad \dots\dots\dots (3.5)$$

where k_j and d_j are the capital and natural resource used in the production of the machine j. The aggregate use of capital and natural resource is thus denoted by

$$\int_0^m k_j . dj = K \qquad and \qquad \int_0^n d_j . dj = D. \ \dots\dots (3.6)$$

The amount of natural resource is supplied by the extracting sector where we assume that there is zero extraction cost. The natural resource is renewable and non-tradable. Hence the following constraint is binding

$$\dot{H} = \xi H - D \quad where \ \xi > 0, \qquad \dots\dots\dots (3.7)$$

where $H(t)$ indicates the stock of the natural resource. The above constraint gives the equation of motion for the stock of natural resources. ξ is the rate of regeneration of this resource and $D(t)$ is the extraction every period. So, the difference of the two gives the change in the stock of the resource. ξ is very critical for our analysis as we will see later that a sufficiently low or high value of it leads us to whether the growth would be sustainable or not. The use of this natural resource generates pollution which is denoted by *Z(t)* and its dynamics is governed by

$$\dot{Z} = \alpha D - \eta Z \quad where\ \eta > 0\ and\ \alpha > 0. \quad \dots\dots\dots (3.8)$$

α is the pollution generated for every unit of natural resource used and with a higher stock of pollution in the environment, the pollution generated is higher for the same amount of natural resource used. η is the rate at which the pollution gets decayed through a natural process. Like ξ , η is important too (as we will see later) in determining whether the growths is sustainable or not. So, it is the self-decomposition process in the environment.

Now, we characterize the R&D sector. The innovation possibilities frontier provides the technological possibilities for transforming resources into blue-prints for new varieties of capital-intensive and resource intensive intermediates. These blue-prints are produced by scientists who are employed in this sector and get a wage income in return[2]. There is free entry into the R & D sector. The blue-prints are produced as per the following:

$$\frac{\dot{m}}{m} = b^c s^c \varphi(s^c) \qquad and \qquad \frac{\dot{n}}{n} = b^d s^d \varphi(s^d), \qquad \dots\dots\dots (3.9)$$

$b^c, b^d > 0$ are the different productivity parameters of the scientists in the respective sector. $\varphi(s)$ is a continuously differentiable and decreasing function which is also a productivity term of the scientists. The following holds for it

$$\varphi(s) > 0, \qquad \frac{d(s.\varphi(s))}{ds} > 0, \qquad \varphi' < 0, \quad \varphi(0) < \infty. \quad \dots\dots\dots (3.10)$$

Above implies that there are intra-temporal decreasing returns to R&D. This might be due to the fact that scientists crowd out each other in competing for the invention of similar type of intermediate machines. It is assumed that the crowding effect is not internalised by the individual R&D firms so that each of them takes the productivity of allocating one more scientist to each of the two sectors as given. The closure condition is given by $s^c + s^d \leq 1$. One important thing to note here is that the R&D sector

2. Scientists also constitute the consumers of the final good as well as owners of the firms.

reflects the "building on the shoulder of the giants" phenomenon so that there are knowledge spillovers from past research.

3.2 Production Side Equilibrium

Since Y is produced in a competitive market, profit maximization would yield the demand for the intermediate inputs $Y_c, and\ Y_d$.

$$P_c = \left[Y_c^{\frac{\sigma}{\sigma-1}} + Y_d^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma-1}} Y_c^{-\frac{1}{\sigma}} \quad and \quad P_d = \left[Y_c^{\frac{\sigma}{\sigma-1}} + Y_d^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma-1}} Y_d^{-\frac{1}{\sigma}}. \quad \ldots\ldots\ldots(3.11)$$

So, we equate the prices of the respective input with their marginal product. $P_c\ and\ P_d$ are the prices of the two intermediate inputs respectively. The relative demand for the two intermediates is thus given by

$$P \equiv \frac{P_c}{P_d} = \left\{\frac{Y_c}{Y_d}\right\}^{-\frac{1}{\sigma}}. \quad \ldots\ldots\ldots(3.12)$$

This is the usual relative demand function exhibiting an inverse relation between the relative quantity demanded and the relative prices. Normalizing the price of final good to 1, we get

$$[P_c^{1-\sigma} + P_d^{1-\sigma}]^{\frac{1}{1-\sigma}} = 1 \ldots\ldots\ldots(3.13)$$

The above expression indicates the contribution of price of both clean and dirty sector in the pricing of the final good (which is normalized to 1).

Thus,

$$P_c = [1 + P^{\sigma-1}]^{\frac{1}{\sigma-1}} \quad or\ P_d = [1 + P^{1-\sigma}]^{\frac{1}{\sigma-1}} \ldots\ldots\ldots(3.14)$$

The prices of clean and dirty intermediate inputs are expressed in terms of the relative price of clean and dirty inputs.

The intermediate inputs $Y_c\ and\ Y_d$ are also produced in competitive markets; profit maximization yields their demand as follows:

$$\frac{P_c(j)}{P_c} = \left\{\frac{y_c(j)}{Y_c}\right\}^{\beta-1} \quad and \quad \frac{P_d(j)}{P_d} = \left\{\frac{y_d(j)}{Y_d}\right\}^{\beta-1} \quad \dots\dots\dots\dots\dots (3.15)$$

Here too, we equate the prices of respective intermediate inputs to their market determined price. The intermediate machines y_j^c *and* y_j^d are produced by monopolists. Since their demands are iso-elastic, the profit maximisation would give a constant mark-up over the marginal cost which is r for $y_c(j)$ and τ for $y_d(j)$. These profits are very critical to our analysis (as we will see later) as they are the guiding factor for the direction of technical change. The maximisation problem of the monopolist can be expressed as

$$Max\, \pi_c(j) = P_c(j)y_c(j) - r.k_j \quad and \quad Max\, \pi_d(j) = P_d(j)y_d(j) - \tau.d_j \;\dots\dots (3.16)$$

which yields the prices for the intermediate machines supplied as

$$P_c(j) = \frac{r}{\beta} \quad and \quad P_d(j) = \frac{\tau}{\beta} \quad \dots\dots\dots (3.17)$$

Since, $\beta \in (0,1)$ the price of intermediate inputs is a constant mark-up over their marginal costs. The above condition means that the price of the intermediate input proportionately increases with an increase in the marginal cost of its input.

Irrespective of the type of machine, the price of all capital-intensive and resource intensive machines are same. Therefore, the equilibrium demand for these machines would also be the same.

$$y_j^c = k_j = \frac{K}{m} \quad and \quad y_j^d = d_j = \frac{D}{n}. \quad \dots\dots\dots.. (3.18)$$

Substituting (3.18) into (3.4) we arrive at,

$$Y_c = m^{\frac{1-\beta}{\beta}}.K \quad and \quad Y_d = n^{\frac{1-\beta}{\beta}}.D. \quad \dots\dots\dots (3.19)$$

Similarly, substituting (3.15) and (3.18) into (3.17) we get,

$$r = \beta P_c m^{\frac{1-\beta}{\beta}} \quad and \quad \tau = \beta P_d n^{\frac{1-\beta}{\beta}}. \quad \dots\dots\dots (3.20)$$

The above condition clearly states that with an increase in the price of the intermediate input, the profitability of the monopolists increase and this increases the demand for K

and D and, hence, raises the rental rate and the price of natural resource. Similarly, with the increase in the varieties of machine through augmented technical change, the demand for K and D and, hence, the rental rate and the price of natural resource are increased.

Using (3.17) and (3.18) in (3.16) the monopoly profits can be written as

$$\pi_c \equiv \pi_c(j) = \frac{1-\beta}{\beta}.\frac{r.K}{m} \quad and\ \pi_d \equiv \pi_d(j) = \frac{1-\beta}{\beta}.\frac{\tau.D}{n} \text{ ……. } (3.21)$$

Finally, using (3.19) in (3.12) the relative price of the capital intensive good can be written as

$$P \equiv \frac{P_c}{P_d} = \left[\left(\frac{m}{n}\right)^{\frac{1-\beta}{\beta}}.\frac{K}{D}\right]^{-\frac{1}{\sigma}} \text{ ……….. } (3.22)$$

The value of discounted profits of a monopolist who invents a new c or d intermediate machine, is

$$V_f(t) = \int_t^{\infty} e^{-\left[\int_t^s (r(\omega).d\omega\right]}.\pi_f(v).dv \quad where \quad f = \{c, d\} \text{ ……….. } (3.23)$$

$$where \quad \pi_c = \frac{1-\beta}{\beta}.\frac{r.K}{m} \quad and \quad \pi_d = \frac{1-\beta}{\beta}.\frac{\tau.D}{n}$$

The term for profits can be interpreted as an increase in profits with higher rental rate and capital for the clean sector and similarly, higher price of resource and the extraction of natural resource for the dirty sector. But with a higher technology level, since the intermediate inputs are gross substitutes there is some profit stealing effect from the new machine producers to the existing ones and as a result, profits fall.

Scientists earn a wage of $w^c\ or\ w^d$, depending on which sector they are employed in. Since there is free entry into the R&D sector and competition between the two sectors, this wage is the maximum of their contribution to the value of monopolists in the two sectors. As already stated, since individual monopolists are private decision makers, does not internalise the effect of crowding out scientists, the marginal value of allocating an additional scientist to the invention of a new clean or dirty machine would be $b^c s^c \varphi(s^c) m V_c$ and $b^d s^d \varphi(s^d) n V_d$ respectively. Thus, wages of the scientist is given by

$$w = Max\{b^c s^c \varphi(s^c) m V_c \, , b^d s^d \varphi(s^d) n V_d\}. \quad \ldots\ldots\ldots.(3.24)$$

This implies that competition in the two sectors would drive the expected profits for all monopoly firms at all point of time would be zero so that $\Pi = 0$. Π is the sum of monopoly profits from both the sectors.

Finally, in the nest section we find out the consumers' optimum, i.e., his behaviour while trying to maximize his utility accounting for the constraints binding on him.

3.3 Consumer's Optimum

The consumer optimizes his lifetime utility subject to its budget constraint. Hence, the consumer's optimization involves maximising the objective function

$$Max\ U = \int_0^{\infty} \frac{\left[C(Z)^{-\phi} H(t)^{\psi}\right]^{1-\theta}}{1-\theta} e^{-\rho t}.dt$$

subject to the three constraints

$$I \equiv \dot{K} = r.K + \tau.D + w_c.s + w_d.(1-s) + \Pi - C$$

$$\dot{Z} = \alpha D - \eta Z$$

$$\dot{H} = \xi H - D$$

We set up this dynamic optimization exercise using the method of optimal control

The Hamiltonian can be expressed as.

$$\mathcal{H} = \frac{1}{(1-\theta)}\left[C(Z)^{-\phi} H^{\psi}\right]^{1-\theta}.e^{-\rho t} + \mu\{r.K + \tau.D + w^c.s + w^d.(1-s) + \Pi - C\} + \lambda\{\alpha D - \eta Z\} + \varepsilon\{\xi H - D\} \quad \ldots\ldots\ldots\ldots.(3.25)$$

where *C, s, D* are the control variables, *K, Z* are the state variables and $\mu, \lambda\ and\ \varepsilon$ are the co-state variables.

The first order conditions for the above optimization are:

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Rightarrow C^{-\theta} Z^{-\phi(1-\theta)} H^{\psi(1-\theta)} . e^{-\rho t} = \mu \quad \ldots\ldots\ldots (3.25.1)$$

$$\frac{\partial \mathcal{H}}{\partial D} = 0 \Rightarrow \mu\tau = \varepsilon - \lambda\alpha \quad \ldots\ldots\ldots\ldots (3.25.2)$$

$$\frac{\partial \mathcal{H}}{\partial s} = 0 \Rightarrow \mu\{w^c - w^d\} = 0 \Rightarrow \frac{b^c \varphi(s^c) m V_c}{b^d \varphi(s^d) n V_d} = 1 \quad \ldots\ldots\ldots (3.25.3)$$

$$\frac{\partial \mathcal{H}}{\partial K} = -\dot{\mu} \Rightarrow r = \frac{-\dot{\mu}}{\mu} \quad \ldots\ldots\ldots (3.25.4)$$

$$\frac{\partial \mathcal{H}}{\partial Z} = -\dot{\lambda} \Rightarrow -\frac{C}{Z}\phi\mu - \lambda\eta = -\dot{\lambda} \Rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{\phi C\mu}{Z\lambda} + \eta \ldots\ldots\ldots (3.25.5)$$

$$\frac{\partial \mathcal{H}}{\partial H} = -\dot{\varepsilon} \Rightarrow \frac{C}{H}\mu\psi + \xi\varepsilon = -\dot{\varepsilon} \Rightarrow -\frac{\dot{\varepsilon}}{\varepsilon} = \frac{\psi C\mu}{H\varepsilon} + \xi \ldots\ldots\ldots (3.25.6)$$

And the set of transversality conditions are given by:

$$\lim_{t\to\infty} K.\mu = 0. \quad \ldots\ldots\ldots (3.25.7)$$

$$\lim_{t\to\infty} Z.\lambda = 0. \quad \ldots\ldots\ldots (3.25.8)$$

$$\lim_{t\to\infty} H.\varepsilon = 0. \quad \ldots\ldots\ldots (3.25.9)$$

Taking the time derivative of (3.25.1) and using (3.25.4) we get,

$$-\theta\frac{\dot{C}}{C} - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H} = \frac{\dot{\mu}}{\mu} + \rho$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[r - \rho - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H}\right] \ldots\ldots\ldots (3.26)$$

This is the Euler equation which yields the growth rate of consumption under the marginal conditions of consumption being satisfied, i.e., the marginal utility of consumption equals the shadow price of investment.

Observation 1: The growth rate of consumption in this framework is different from than what it is typically observed in standard models as $\frac{\dot{C}}{C} = \frac{1}{\theta}[r - \rho]$ .This is the arbitrage equation for the consumption and savings such that by not consuming the individuals get a return of r which but in future the utility is discounted by the term ρ, so on the net his return is $(r - \rho)$, in terms of the elasticity of marginal utility of consumption within two periods, θ. A higher (lower) elasticity of marginal utiltiy would mean that individuals' utility is more (less) sensitive to changes in consumption and so growth rate of consumption is lower (higher). In this modified framework, there is a departure from the original result, because of the pollution stock and natural resource stock arguments in the utility function. So, with the rate of return on savings adjusted for rate of time preference, there is also accounting for the change of the stock of pollution and natural resources weighted with the elasticity of marginal utility and their respective intensities in the utility function.

Taking time derivative of (3.25.2) we get,

$$\frac{\dot{\tau}}{\tau} = -\frac{\dot{\mu}}{\mu} + \frac{1}{\mu\tau}\left[\dot{\varepsilon} - \alpha\dot{\lambda}\right]$$

$$\Rightarrow \frac{\dot{\tau}}{\tau} = r + \frac{1}{\mu\tau}\left[\dot{\varepsilon} - \alpha\dot{\lambda}\right] \ldots\ldots\ldots (3.27)$$

Observation 2: This equation gives the pricing of the natural resource in the economy. This is the modified Hotelling rule for the pricing of the natural resource in the economy. In the original Hotelling rule without any externality, the growth rate of the price of natural resource equals the interest rate giving the arbitrage between the price of capital and that of the natural resource. $\dot{\varepsilon}$ gives the change in the shadow price of the stock of renewable resource whereas $\alpha\dot{\lambda}$ is the change in damage due to the change in the shadow price of pollution times the stock of pollution. The first is a benefit to the consumer, the second one exerts negativity, thus $\frac{1}{\mu\tau}\left[\dot{\varepsilon} - \alpha\dot{\lambda}\right]$ expressing the net benefit or loss (depending on $\dot{\varepsilon} \gtreqless \alpha\dot{\lambda}$) measured in the shadow price of capital per unit of the resource price. With this modified rule, the growth rate of the price of the natural resource is lower than the rate of interest if the net loss from pollution is higher than the benefit from the resource due to a change in the shadow prices. As a result, the price of the natural resource would grow at a slower rate. But, on the other hand, if the benefit from the change in shadow price outweighs the cost from the change in the value of pollution, the growth rate of the resource price is over and above the rate of interest. With the changes in shadow prices such that they exactly cancel off, we get back the original Hotelling rule. This is observed because of the adverse impact of pollution and the positive impact of the existence value of natural resource in the individual's utility function.

3.4 Direction of Technical Change

As the consumers are the scientists, the remuneration from their R&D forms a part of their income which they would maximise. Hence, they can direct research in the sector which is more profitable. So, from (3.25.3)

$$\frac{b^c \varphi(s^c) m V_c}{b^d \varphi(s^d) n V_d} = 1 \Rightarrow \frac{m V_c}{n V_d} = \frac{b^d \varphi(s^d)}{b^c \varphi(s^c)}$$

This gives us the allocation of scientists in the two sectors. According to the above condition, the marginal contribution of an additional scientist to research in the clean and dirty sectors, i.e., the wages in the clean and dirty sectors must be equalised. While doing this, the consumers do not internalise the crowding out effect of additional scientists on their marginal productivity and instead take that as given.

The ratio $\frac{m V_c}{n V_d}$ is the relative profitability of the clean versus the dirty sector. Since, $s^c + s^d \leq 1$, and one additional scientist always increases the total returns in any sector, clean or dirty, the resource constraint would be binding. It will be $s^c + s^d = 1$. The above condition for the optimum allocation of scientists to R&D sector can be written as

$$\frac{m V_c}{n V_d} = \frac{b^d \varphi(1-s)}{b^c \varphi(s)}, \qquad \text{.........} (3.28)$$

Equation (3.28) gives the allocation of the scientists at every point in time. It is worth mentioning here that the direction of technical change is an out-of-steady-state property as transitional dynamics imply that scientists move to a relatively more profitable sector, which cannot be a phenomenon in the steady state.

$$\Phi(s) \equiv \frac{\varphi(1-s)}{\varphi(s)} = \frac{b^c m V_c}{b^d n V_d}.$$

Using (3.21) and (3.23), the above equation can be written as

$$\Phi(s) \equiv \frac{b^d}{b^c}\int_t^\infty \left\{\frac{r(v).K(v)}{\tau(v).D(v)}\right\}.dv\,.$$

Using, (3.20) and (3.22) in the above equation, we get

$$\Phi(s) \equiv \frac{b^d}{b^c}\int_t^\infty \left\{\frac{M(v).K(v)}{N(v).D(v)}\right\}^{\frac{\sigma-1}{\sigma}}.dv.$$

Let us write $M = m^{\frac{1-\beta}{\beta}}$ *and* $N = n^{\frac{1-\beta}{\beta}}$.

Also let, $x = \frac{MK}{ND}$ where x is the technology augmented capital-resource ratio (also defined in the Appendix).

Then,

$$\Phi(s) \equiv \frac{b^d}{b^c}\int_t^\infty x(v)^{\frac{\sigma-1}{\sigma}}.dv, \quad \dots\dots\dots(3.29)$$

Since,

$$\Phi'(s) \equiv -\left\{\frac{\varphi'(1-s)}{\varphi(1-s)} + \frac{\varphi'(s)}{\varphi(s)}\right\} > 0 \Rightarrow (\Phi^{-1})' > 0,$$

It implies that,

$$s = \Phi^{-1}\left\{\frac{b^d}{b^c}\int_t^\infty x(v)^{\frac{\sigma-1}{\sigma}}.dv\right\} > 0 \dots\dots\dots\dots(3.30)$$

Observation 3: The expression (3.30) guides the direction of technical change. So, if the relative profitability of the clean versus the dirty sector increases it would move scientists towards the cleaner sector and vice-versa. Here, the elasticity of substitution between the intermediate inputs plays a very critical role. If the elasticity of substitution greater than 1, i.e., when the intermediate goods are gross substitutes then, with a higher technology augmented capital resource ratio, the relative profitability of clean versus dirty sector increases and thus scientists move towards cleaner sector. On the other hand, if the elasticity of substitution is less than 1, in case of inputs being gross complements, with a higher technology augmented capital resource ratio, the relative profitability of clean versus dirty sector falls and thus scientists move towards the dirty sector. This is how the dynamics of the direction of technical change moves with the relative profitability and the capital-resource ratio.

3.5 Steady State Analysis

A steady state is defined as the state when all variables grow at a constant rate (not necessarily the same constant rate). So, *Y, C, K, D, H and Z* all grow at a constant rate.

Proposition 1: *At decentralized market equilibrium, in the presence of environmental constraints in the form of resource use and the adverse affect of pollution on consumers' utility, when there are two intermediate production sectors clean and dirty each having a separate R&D sector, the steady state requires that only the dirty sector grows. The growth*

(i) is not sustainable if $r + (1-\theta)(\psi - \phi)\gamma_D < \rho$

(ii) is sustainable if $r + (1-\theta)(\psi - \phi)\gamma_D > \rho$

Proof: The proof follows from the equations (A.1) to (A.14) in the Appendix.

All the variables grow at a constant rate:

$$\frac{\dot{C}}{C} = \gamma_C \ ; \ \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \gamma_D + \gamma_N \ ; \ \frac{\dot{D}}{D} = \frac{\dot{Z}}{Z} = \frac{\dot{H}}{H} = \gamma_D \ ; \ \frac{\dot{N}}{N} = \gamma_N \ ; \ \frac{\dot{M}}{M} = \frac{\dot{s}}{s} = 0$$

From (A.11),

$$\frac{\dot{C}}{C} = \frac{1}{\theta}[r - \rho + (1 - \theta)(\psi - \phi)\gamma_D] \equiv \gamma_C$$

Hence the steady state exists.

The basic difference between the capital and the natural resource is that the capital can be accumulated through the market mechanism depending upon the rate of interest but the natural resource cannot. The accumulation of capital or savings from the consumer's point (or rather the consumption growth) directly depends on the arbitrage through the rate of return on capital. For a constant growth rate of consumption so as for the steady state to exist, this arbitrage must be constant and hence should be the rate of return. The rate of return also depends on the technological progress so that with an increased technology for capital, the return increases. But on the other hand, the other factor which is the natural resource is rarely accumulated (when the extraction is very low, even lower than the regeneration which is almost negligible). So, to keep pace with the increased stocks of capital the effective stock of the natural resource would increase through the increase in the technology level for this sector. Thus all technical change is labour augmenting in the long run.

When, the effective rate of return, i.e., the rate of return adjusting for the net intensity of disutility from pollution and utility from the existence value due to the presence of natural resource stock weighted by elasticity of marginal utility is lower than the rate of time preference, the growth is not sustainable. Intuitively this is explainable as the consumers find that the return on savings is not sufficiently high enough to make up for the net intensity of disutility or utility from the pollution and the existence value of natural resources. This is possible when the rate of return on capital or the interest rate is sufficiently low or the intensity of pollution sufficiently dominates the intensity of positive utility of the existence value of the natural resource. Thus, due to this lack of incentive a positive sustained growth in consumption is not possible. On the other hand, we witness a case of sustainable growth if either the rate of return is very high or the net effect of pollution disutility gets dominated by the net benefit in the utility due to the existence value. The above also very significantly, depends on the nature of risk behaviour of the consumers. With, the same rate of return on capital and the same values of the disutility intensity of pollution and utility intensity of existence value of natural resources, the results might be reversed if the individual is risk-lover or risk-averse (i.e. contingent upon the value of θ). This is because with the different behaviour of consumers towards risk, the effective rate of return differs as a risk lover individual would discount the rate of return due to pollution differently than the risk-averse individual. So, even if the rate of return is sufficiently low and there is net disutility form pollution over existence value, due to his risk-loving nature he would still save and maintain a positive growth rate of consumption.

Chapter 4: Social Planner's equilibrium for under environmental constraints

In the previous chapter we have been looking at the market outcome for the economy, when the stock of pollution is generated by the use of a non-tradable renewable natural resource in the economy, the pollution adversely affects the consumers' utility and there is an existence value attached to the stock of natural resources due to which it positively affects utility. The market outcome is a sub-optimal one with not internalising for the externalities in various forms like the allocation of research scientists, monopoly production of machines, the pollution externality.

In this chapter we find a socially desirable sustainable outcome for the same economy when a social planner is trying to maximize the social welfare.

The structure of the economy is same as before. The difference arises in that the R&D sector is no longer governed by sole profit motives but rather the social planner chooses a socially desirable allocation of scientists to the two R&D sectors. Moreover, the planner internalises the crowding effect of the scientists in the R&D sector which is ignored when an individual maximises net private benefits. Even the extraction of *D*, the resources, and hence the pollution is optimally chosen by the planner so that the growth is sustainable.

We would once again briefly discuss the structure of the economy. The utility function is same as before

$$U = \int_0^\infty \frac{\left[C(t)(Z(t))^{-\emptyset}H(t)^{\psi}\right]^{1-\theta}}{1-\theta} e^{-\rho t}.dt \;\; where\; \theta < 1, \emptyset > \psi > 0, \rho > 0 \;\; .... (4.1)$$

But the budget constraint of the social planner will now be given by macro resource constraint unlike the individual consumer's budget constraint as given in (3.2). Specifically, the budget constraint the planner faces is $I \equiv \dot{K} = Y - C$.

Substituting (3.19) into (3.3), we get,

$$Y = \left[\left(m^{\frac{1-\beta}{\beta}}.K\right)^{\frac{\sigma}{\sigma-1}} + \left(n^{\frac{1-\beta}{\beta}}.D\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}},$$

which is the derived production function of the final good in terms of primary factor inputs. While writing the final output in terms of the primary factor inputs, all the other market clearing conditions both in the final good in the intermediate goods sector are met (i.e., (3.12), (3.15), (3.17) and (3.18) holds). The derived production function is obtained from profit maximising exercise in each of the intermediate input markets.

Using, $M = m^{\frac{1-\beta}{\beta}}$ *and* $N = n^{\frac{1-\beta}{\beta}}$ in the above equation we get,

$$\dot{K} = \left[(M.K)^{\frac{\sigma}{\sigma-1}} + (N.D)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} - C . \quad \ldots\ldots\ldots (4.2)$$

This is the aggregate change in capital stock of the economy, which is the total production of final goods less consumption.

The dynamics of pollution and natural resource stock are given by (3.7) and (3.8) as earlier. The R&D sector is also the same as before (given in (3.9)) in the previous chapter. Expressing *m* and *n* it in terms of *M* and *N*, we get the rate of technical change in the clean and dirty sectors to be:

$$\frac{\dot{M}}{M} = \left(\frac{1-\beta}{\beta}\right) b^c s^c \varphi(s^c) \qquad and \qquad \frac{\dot{N}}{N} = \left(\frac{1-\beta}{\beta}\right) b^d s^d \varphi(s^d). \quad \ldots\ldots\ldots (4.3)$$

4.1 **Social Planner's Optimization**

The objective of the social planner is to maximise the aggregate social welfare given by the felicity function subject to the economic and environmental constraints. Thus,

$$Max\ U = \int_0^{\infty} \frac{\left[C(Z)^{-\phi} H(t)^{\psi}\right]^{1-\theta}}{1-\theta} e^{-\rho t}.dt$$

Subject to the following constraints:

$$\dot{K} = \left[(M.K)^{\frac{\sigma}{\sigma-1}} + (N.D)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} - C$$

$$\dot{Z} = \alpha D - \eta Z$$

$$\dot{H} = \xi H - D$$

$$\frac{\dot{M}}{M} = \left(\frac{1-\beta}{\beta}\right) b^c s^c \varphi(s^c)$$

$$\frac{\dot{N}}{N} = \left(\frac{1-\beta}{\beta}\right) b^d s^d \varphi(s^d)$$

Notably, unlike the decentralized market outcome, the scientists' allocation in the R&D sector is not made through guiding profit motives. Rather, the social planner allocates resources optimally to both the sector which are guided by the consumers' preference sustainability concerns.

We set up this dynamic optimization exercise using the method of optimal control

The Hamiltonian can be written as.

$$\mathcal{H}_s = \frac{1}{(1-\theta)}\left[C(Z)^{-\phi} H^{\psi}\right]^{1-\theta}.e^{-\rho t} + \mu_p\left\{\left[(M.K)^{\frac{\sigma}{\sigma-1}} + (N.D)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} - C\right\} +$$
$$\lambda_p\{\alpha D - \eta Z\} + \varepsilon_p\{\xi H - D\} + q_M\left\{\left(\frac{1-\beta}{\beta}\right) b^c s\varphi(s) M\right\} +$$
$$q_N\left\{\left(\frac{1-\beta}{\beta}\right) b^d (1-s)\varphi(1-s) N\right\} \qquad \ldots\ldots\ldots\ldots(4.4)$$

where *C, s, D* are the control variables, *K, Z, H, M, N* are the state variables and $\mu_p,\ \lambda_p,\ \varepsilon_p,\ q_M \text{ and } q_N$ are the co-state variables.

We first express $y = \frac{Y}{ND} \text{ and } x = \frac{MK}{ND}$

The first-order conditions for the above optimization are:

$$\frac{\partial \mathcal{H}_s}{\partial C} = 0 \Rightarrow C^{-\theta} Z^{-\phi(1-\theta)} H^{\psi(1-\theta)}.e^{-\rho t} = \mu_p \quad \ldots\ldots\ldots(4.4.1)$$

$$\frac{\partial \mathcal{H}_s}{\partial D} = 0 \Rightarrow \mu_p\left[(M.K)^{\frac{\sigma}{\sigma-1}} + (N.D)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma-1}} N^{\frac{\sigma-1}{\sigma}} D^{-\frac{1}{\sigma}} = \varepsilon_p - \lambda_p \alpha$$

$$\Rightarrow \mu_p Y^{\frac{1}{\sigma}} N^{1-\frac{1}{\sigma}} D^{-\frac{1}{\sigma}} = \varepsilon_p - \lambda_p \alpha \Rightarrow \mu_p y^{\frac{1}{\sigma}} N = \varepsilon_p - \lambda_p \alpha \quad \ldots\ldots\ldots\ldots(4.4.2)$$

$$\frac{\partial \mathcal{H}_s}{\partial s} = 0 \Rightarrow \frac{q_N}{q_M} = \frac{b^c\{\varphi(s) + s.\varphi'(s)\}M}{b^d\{\varphi(1-s) + (1-s).\varphi'(1-s)\}N} \quad \dots\dots\dots(4.4.3)$$

$$\frac{\partial \mathcal{H}_s}{\partial K} = -\dot{\mu}_p \Rightarrow \mu_p Y^{\frac{1}{\sigma}} M^{1-\frac{1}{\sigma}} K^{-\frac{1}{\sigma}} = -\frac{\dot{\mu}_p}{\mu_p} \Rightarrow y^{\frac{1}{\sigma}} x^{-\frac{1}{\sigma}} M = -\frac{\dot{\mu}_p}{\mu_p} \quad \dots\dots\dots(4.4.4)$$

$$\frac{\partial \mathcal{H}_s}{\partial Z} = -\dot{\lambda}_p \Rightarrow -\frac{C}{Z}\phi\mu_p - \lambda_p\eta = -\dot{\lambda} \Rightarrow \frac{\dot{\lambda}_p}{\lambda_p} = \frac{\phi C \mu_p}{Z\lambda_p} + \eta \quad \dots\dots\dots(4.4.5)$$

$$\frac{\partial \mathcal{H}_s}{\partial H} = -\dot{\varepsilon}_p \Rightarrow \frac{C}{H}\psi\mu_p + \xi\varepsilon_p = -\dot{\varepsilon}_p \Rightarrow -\frac{\dot{\varepsilon}_p}{\varepsilon_p} = \frac{\psi C \mu_p}{H\varepsilon_p} + \xi \quad \dots\dots\dots(4.4.6)$$

$$\frac{\partial \mathcal{H}_s}{\partial M} = -\dot{q}_M \Rightarrow \mu_p Y^{\frac{1}{\sigma}} M^{-\frac{1}{\sigma}} K^{1-\frac{1}{\sigma}} + q_M\left(\frac{1-\beta}{\beta}\right) b^c s\varphi(s) = -\dot{q}_M$$

$$\Rightarrow \frac{\mu_p}{q_M} y^{\frac{1}{\sigma}} x^{-\frac{1}{\sigma}} K + \left(\frac{1-\beta}{\beta}\right) b^c s\varphi(s) = -\frac{\dot{q}_M}{q_M} \quad \dots\dots\dots\dots(4.4.7)$$

$$\frac{\partial \mathcal{H}_s}{\partial N} = -\dot{q}_N \Rightarrow \mu_p Y^{\frac{1}{\sigma}} N^{-\frac{1}{\sigma}} D^{1-\frac{1}{\sigma}} + q_N\left(\frac{1-\beta}{\beta}\right) b^d (1-s)\varphi(1-s) = -\frac{\dot{q}_N}{q_N}$$

$$\Rightarrow \frac{\mu_p}{q_N} y^{\frac{1}{\sigma}} D + \left(\frac{1-\beta}{\beta}\right) b^d (1-s)\varphi(1-s) = -\frac{\dot{q}_N}{q_N} \quad \dots\dots\dots\dots(4.4.8)$$

And the set of transversality conditions are given by:

$$\lim_{t\to\infty} K.\mu_p = 0. \quad \dots\dots\dots(4.4.9)$$

$$\lim_{t\to\infty} Z.\lambda_p = 0. \quad \dots\dots\dots(4.4.10)$$

$$\lim_{t\to\infty} H.\varepsilon_p = 0. \quad \dots\dots\dots(4.4.11)$$

$$\lim_{t\to\infty} M.q_M = 0. \quad \dots\dots\dots(4.4.12)$$

$$\lim_{t\to\infty} N.q_N = 0. \quad \dots\dots\dots(4.4.13)$$

Taking the time derivative of (4.4.1) and using (4.4.4) we get,

$$-\theta\frac{\dot{C}}{C} - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H} = \frac{\dot{\mu}}{\mu} + \rho$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M - \rho - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H}\right] \ldots\ldots\ldots (4.5)$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[r - \rho - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H}\right] \ldots\ldots\ldots (3.26)$$

Observation 4: Note that (4.5) is similar to the Euler equation in chapter 3 (equation (3.26)) except for the fact that instead of r we have the marginal productivity of capital in the production of final output. In (3.26) the rate of return is the rate of interest which is $\beta y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M < y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M$, since $\beta \in (0,1)$. This divergence arises between the market and the social planner's equilibrium due to the fact that, the social planner corrects for the monopoly distortion in the market for the production of intermediate machines, and thus unlike the market outcome where individual consumers are paid the market interest rate, the social planner pays the marginal product of capital as the return to the savings by the consumers which is higher than the market rate of interest.

The condition in (4.4.7) and (4.4.8) can be rewritten as

$$\frac{\mu_p}{q_M} y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}K = -\frac{\dot{q}_M}{q_M} - \frac{\dot{M}}{M} \qquad and \qquad \frac{\mu_p}{q_N} y^{\frac{1}{\sigma}}D = -\frac{\dot{q}_N}{q_N} - \frac{\dot{N}}{N} \ldots\ldots\ldots\ldots (4.6)$$

Taking the time derivative of (4.4.3) we get,

$$\frac{\dot{q}_M}{q_M} + \frac{\dot{M}}{M} - \frac{\dot{q}_N}{q_N} - \frac{\dot{N}}{N} = \left[\frac{-2\varphi'(1-s) - (1-s).\varphi''(1-s)}{\varphi(1-s) + (1-s).\varphi'(1-s)}\right]\dot{s} - \left[\frac{2\varphi'(s) + s.\varphi''(s)}{\varphi(s) + s.\varphi'(s)}\right]\dot{s}$$

$$\Rightarrow \frac{\dot{q}_M}{q_M} - \frac{\dot{q}_N}{q_N} + \frac{\dot{M}}{M} - \frac{\dot{N}}{N} = -\left[\frac{2\varphi'(1-s) + (1-s).\varphi''(1-s)}{\varphi(1-s) + (1-s).\varphi'(1-s)} + \frac{2\varphi'(s) + s.\varphi''(s)}{\varphi(s) + s.\varphi'(s)}\right]\dot{s}$$

Substituting (4.6) in the above equation, we get,

$$\frac{\mu_p}{q_N} y^{\frac{1}{\sigma}} D - \frac{\mu_p}{q_M} y^{\frac{1}{\sigma}} x^{-\frac{1}{\sigma}} K = \Omega(s) \frac{\dot{s}}{s}$$

where $\Omega(s) = -\left[\frac{2\varphi'(1-s)+(1-s).\varphi''(1-s)}{\varphi(1-s)+(1-s).\varphi'(1-s)} + \frac{2\varphi'(s)+s.\varphi''(s)}{\varphi(s)+s.\varphi'(s)}\right] s$

$$\Rightarrow \frac{\dot{s}}{s} = \Omega(s)^{-1} \mu_p y^{\frac{1}{\sigma}} \left[\frac{D}{q_N} - \frac{K}{q_M}\right] \quad \dots\dots\dots\dots (4.7)$$

4.2 Steady State Analysis

In a steady state, all variables will be growing at a constant rate. So, Y, C, K, D, H and Z all grow at a constant rate. We state and prove the following proposition.

Proposition 2: *In a social planner's equilibrium, in the presence of environmental constraints in the form of resource use and the adverse affect of pollution on consumers' utility, when there are two intermediate production sectors clean and dirty each having a separate R&D sector, the steady state requires that only the dirty sector grows but the growth is sustainable.*

Proof: The proof follows from the equations (A.1) to (A.6) and (A.15) to (A.24) in the Appendix.

All the variables grow at a constant rate:

$$\frac{\dot{C}}{C} = \widetilde{\gamma_C} \ ; \ \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \gamma_N \ ; \ \frac{\dot{D}}{D} = \frac{\dot{Z}}{Z} = \frac{\dot{H}}{H} = 0 \ ; \ \frac{\dot{N}}{N} = \gamma_N \ ; \ \frac{\dot{M}}{M} = \frac{\dot{s}}{s} = 0$$

Hence compared to the decentralised equilibrium, not only the steady state exists but also the growth is sustainable as the planner chooses an optimal extraction of natural resources (and hence a tolerable level of pollution.) The planner chooses a level of resource extraction such that pollution does not grow over time consequently, leading to sustainable growth.

It is the same story here for the growth of the dirty sector technology and the technological stagnation of the clean sector which we have explained for the decentralised equilibrium. The same economic intuition applies to the proposition here too which was established in the previous chapter. The surprising result is that even if the social planner tries to redirect technical change to cleaner sectors, ultimately a long run steady state witnesses a purely resource augmenting technical change.

This is compared to Acemoglu et. al. (2010) where they show technical change to be clean or dirty or both in the long run depending on the elasticity of substitution and the government policy to redirect technical change. But the difference arises due to the fact that they assume non-accumulating factors of production. Instead, we show that when capital when accumulates over time. Thus, our results give a more general picture of the real world. Our analysis also characterises the Balanced Growth Path for the social planner who chooses a level of extraction of the natural resource so that the pollution does not grow, and hence growth is sustainable with rising consumption in steady state.

4.3 Comparison between Decentralised and Planner's equilibrium

In our analysis, the outcome for both the social planner and the market equilibrium seems to be similar in respect of the fact that both rely on the concern for the environment in the form of pollution and the existence value of natural resource stock in the individual's utility function. However, in spite of the similarity a key difference arises as the growth is always sustainable in the social planner's equilibrium which may or may not be the case in the decentralized market equilibrium. As one would expect that market equilibrium may generate unsustainable growth by warranting a higher growth rate of output and capital accumulation along with higher extraction of natural resource every period, thus increasing pollution. Again, another difference is that, in the decentralised equilibrium, the scientists do not internalise the crowding effects of an additional scientist but the social planner does it while allocating the scientists to research sectors. Therefore, in the short-run, in a market equilibrium, the technical change might get directed towards the sector which is apparently more profitable but actually it might not be more profitable in the margin due to the intra-temporal returns in the R&D and lowering of productivity of existing scientist due to crowding-out effect of an additional scientist in the R&D sector. But, the social planner takes due care of this and internalises the crowding out effect of an additional scientist. As we have explained in observation 4, the rate of return on capital lower in the case of the decentralised equilibrium than in the social planner's equilibrium due to the presence of monopoly distortion in the intermediate machines market which the social planner corrects for. Moreover, the growth rates of consumption are different depending on the strength of the adverse effect of pollution relative to the gain in utility from the existence value of natural resource stock and the risk-taking behaviour of the individuals.

Chapter 5: Conclusions

We started with the idea of endogenous growth models with R&D as purposeful investment activities. We discussed the different endogenous growth models with R&D as a rational investment activity which is the driving factor of growth in the economy. On the same lines we introduced the idea of newly developed concept of endogenous technical change in which the profit incentives shape the bias of technical change. We also established the communion of the relationship between growth and environment in various dimensions. Finally we tried to present a view of the relevance of this endogenous technical change in the context of environmental issues considered within the growth framework.

In chapter 3 and chapter 4, we set up the basic model with two forms of environmental constraints – one in the form of emissions into the pollution and the other the use of a non-tradable renewable resource. We characterise the economy in steady state, in both the decentralised and the planner's equilibrium. Our results in this context are different from Acemoglu et. al. (2010) and Grimaud and Rouge (2008) as both of these research points at a successful redirection of research through optimal mix of tax-subsidy schemes so that in the long run the moves to a regime of clean (or green) research shifting away from dirty (or grey research). But we show that in the long run, a steady state is ensured only with a purely resource augmenting technical change. This result differs precisely because of the nature of the input we have taken which accumulates over time. So, all efforts to redirect technical change to a clean sector are effective only in the short run, but it loses out in the long run as only a dirty sector technological change ensures a steady states.

Also, we have shown that the growth is sustainable in the social planner's case and might not be in the market outcome as consumption is not growing at some constant rate. In the planner's equilibrium the output and the capital growth rate is less than that along the decentralised balanced growth path. Further, the extraction of the natural resource, stock of pollution and the stock of natural resources are constant in a social planner's balance growth path as compared to the market outcome where they all grow at a constant rate. An interesting result is that, in spite of all the R&D going into the dirty sector, the social planner's outcome is optimal and generates sustainable growth.

The reason is that, with every new technology used for the dirty sector, the amount of natural resource used by each of intermediate dirty sector reduces and hence the extraction of natural resources is not growing and remains a constant.

We also derive the out-of-steady-state characteristics of the R&D sector in a decentralised equilibrium. These are guided by profit motive and the research flows to that sector which is relatively more profitable. It critically depends upon the value of the elasticity of substitution between the clean and the dirty intermediate input. This can be affected by the social planner by use of policy instruments to redirect research in desired direction. But the long run research neither depends on the elasticity of substitution nor can be redirected by any means of policy. From Barro and Sala-i-Martin, (Pg 53 and Pg78: Proposition 1.5.3) "**The Necessity for Technological Progress to be Labour Augmenting:** Suppose that we consider only constant growth rates of technological progress. Then , in the neo-classical growth model with a constant rate of population growth, only labour augmenting technological change turns out to be consistent with the existence of steady state, that is with constant growth rates of the various quantities in the long run." This is the very well established neo-classical result and our results conform to this. In steady state the technical change has to be purely resource augmenting (synonymous to labour-augmenting technical change) and the factor shares are constant in the long run.

In the decentralised equilibrium we go on to showing a modified Hotelling rule which differs from the original one on account of the presence of the stock of pollution and the existence value of the natural resource stock in the preference structure.

These are the main findings of our research arriving at the decentralised and the social planner's equilibrium. We also compare the two equilibria in terms of the rate of growth of consumption and output. We go on explaining the results which reconfirms the neo-classical proposition and the possible explanation of departure from the existing literature.

This is not an exhaustive set of results but I will provide some future directions for research from here. Endogenizing the scientists such that whether to move into a scientists occupation would be decision variable which would also be affected by relative profitability and other opportunities in the economy. Also, when both the factors are of similar nature i.e. both factors can be accumulated would be an interesting analysis to follow. Another interesting feature can emerge with uncertainties in the environment process which are generally stochastic rather than determinant expository we have taken. A framework allowing for integration or trade between countries in the presence of such environmental constraints would be very relevant for a global policy objective.

Appendix

Let us redefine some variables so as to facilitate the analysis.

$$Y = F(MK, ND) = \left[(MK)^{\frac{\sigma-1}{\sigma}} + (ND)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$\Rightarrow y \equiv \frac{Y}{ND} = \left[1 + \left\{\frac{MK}{ND}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = \left(1 + x^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \equiv f(x) \ldots\ldots\ldots (A.1)$$

$$where\ x = \frac{MK}{ND}$$

Thus, the left hand side is the technology augmented resource output ratio and x (as we have already defined earlier) is the technology augmented capital-resource ratio. Here, we have expressed the production function in resource intensive form.

Thus, from (11) we can write the expressions for the price of intermediate inputs as

$$P_c = f_x(x) = \left(\frac{f(x)}{x}\right)^{\frac{1}{\sigma}}\ and\ P_d = f(x)^{\frac{1}{\sigma}} \ldots\ldots (A.2)$$

Substituting (32) in (20), we get

$$r = \beta\left(\frac{f(x)}{x}\right)^{\frac{1}{\sigma}} M \quad and \quad \tau = \beta P_d n^{\frac{1-\beta}{\beta}}$$

Also, since

Taking time derivative of the above we get,

$$\frac{\dot{\tau}}{\tau} = \frac{\dot{P}_d}{P_d} + \frac{\dot{N}}{N} = \frac{\dot{x} f_x(x)}{\sigma f(x)} + \frac{\dot{N}}{N} \quad and \quad \frac{\dot{r}}{r} = \frac{\dot{P}_c}{P_c} + \frac{\dot{M}}{M} = \frac{1}{\sigma}\left(\frac{\dot{f}(x)}{(f(x)} - \frac{\dot{x}}{x}\right) + \frac{\dot{M}}{M} \ldots\ldots\ldots (A.3)$$

Taking the time derivative of (A.1), the growth rate of aggregate output can be written

$$\frac{\dot{Y}}{Y} = \frac{\dot{N}}{N} + \frac{\dot{D}}{D} + \frac{\dot{x}}{x}\left(\frac{x f_x(x)}{f(x)}\right). \quad \ldots\ldots\ldots\ldots (A.4)$$

In a balanced growth equilibrium or the steady state, by definition all of, Y, N, D, x must grow at a constant rate. Hence, the coefficient term of the $\frac{\dot{x}}{x}$ within the parentheses must also be a constant, which requires that

$$\frac{\dot{x}}{x}+\frac{\dot{f}_x(x)}{f_x(x)}-\frac{\dot{f}(x)}{(f(x)}=0.$$

From (A.2) and then taking the time derivative of the same we get,

$$f_x(x)=\left(\frac{f(x)}{x}\right)^{\frac{1}{\sigma}} \Rightarrow \dot{f}_x(x)=\frac{1}{\sigma}\left(\frac{\dot{f}(x)}{(f(x)}-\frac{\dot{x}}{x}\right).$$

Hence,

$$\left(1-\frac{1}{\sigma}\right)\left(\frac{\dot{f}(x)}{(f(x)}-\frac{\dot{x}}{x}\right)=0 \Rightarrow \frac{\dot{f}(x)}{(f(x)}=\frac{\dot{x}}{x}\ and\ \dot{f}_x(x)=0.$$

This implies that P_c is constant in the steady state.

$$\frac{\dot{f}_x(x)}{f_x(x)}=0 \Rightarrow \frac{1}{\sigma}\frac{\dot{x}}{x}\left[\frac{x^{\frac{\sigma-1}{\sigma}}}{1+x^{\frac{\sigma-1}{\sigma}}}-1\right]=0.$$

The term in the parenthesis can never be zero. In fact it is < 0, for $x > 0$. Thus for the above equality to hold,

$$\frac{\dot{x}}{x}=0. \quad \ldots\ldots\ldots\ldots\ldots(A.5)$$

Putting, the above in (A.3), we get $\frac{\dot{\tau}}{\tau}=\frac{\dot{N}}{N}$

Now, from the definition of x, the growth rate of x, can be written as below and then again using (A.5) we get,

$$\frac{\dot{x}}{x}=\frac{\dot{K}}{K}+\frac{\dot{M}}{M}-\frac{\dot{D}}{D}-\frac{\dot{N}}{N} \quad \Rightarrow \quad \frac{\dot{K}}{K}+\frac{\dot{M}}{M}=\frac{\dot{D}}{D}+\frac{\dot{N}}{N} \quad \ldots\ldots\ldots\ldots(A.6)$$

Decentralised Market Outcome

From Euler equations since,

$$\Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[r - \rho - \phi(1-\theta).\frac{\dot{Z}}{Z} + \psi(1-\theta).\frac{\dot{H}}{H}\right]$$

For consumption to grow at a constant rate, it requires r to be a constant. From (A.3) $\frac{\dot{r}}{r} = \frac{\dot{M}}{M}$ which means that $\frac{\dot{M}}{M} = 0$ for r to be constant. This implies the steady state growth must be resource augmenting technical change or absence of any technical change in clean sector. This only reconfirms the neoclassical result.

Thus,

$$\frac{\dot{M}}{M} = 0 \ldots\ldots.. (A.7)$$

As a result,

$$\frac{\dot{N}}{N} = \left(\frac{1-\beta}{\beta}\right) b^d \varphi(1) \equiv \gamma_N \quad \ldots\ldots\ldots (A.8)$$

Further, besides r to be constant, in balanced growth equilibrium, $\frac{\dot{Z}}{Z}$ *and* $\frac{\dot{H}}{H}$ must also be constant. From (3.7) and (3.8),

$$\frac{\dot{H}}{H} = \xi - \frac{D}{H} \quad and \quad \frac{\dot{Z}}{Z} = \alpha\frac{D}{Z} - \eta$$

Thus in the steady state, $\frac{D}{H}$ *and* $\frac{D}{Z}$ must be constant. As a result, the growth rate of, D, Z and H must be the same implying, $\frac{H}{Z}$ must to be constant too. That is,

$$\frac{\dot{H}}{H} = \frac{\dot{Z}}{Z} = \frac{\dot{D}}{D} = \gamma_D. \quad \ldots\ldots\ldots\ldots (A.9)$$

Using (A.9) in (3.7) and (3.8), we get,

$$\xi - \frac{D}{H} = \alpha\frac{D}{Z} - \eta \ \Rightarrow D = \frac{(\xi + \eta)ZH}{(Z + \alpha H)} \quad \ldots\ldots\ldots.. (A.10)$$

$$\frac{\dot{C}}{C}=\frac{1}{\theta}[r-\rho+(1-\theta)(\psi-\phi)\gamma_D] \equiv \gamma_C \;\ldots\ldots\ldots\ldots (A.11)$$

Substituting for (A.5), (A.8) and (A.9) in (A.4), we get

$$\frac{\dot{Y}}{Y}=\gamma_N+\gamma_D \;\ldots\ldots\ldots. (A.12)$$

Similarly, using (A.7), (A.8) and (A.9) in (A.6) we get,

$$\frac{\dot{K}}{K}=\gamma_N+\gamma_D \;\ldots\ldots\ldots\ldots (A.13)$$

Taking the time derivative of (29) and using the Leibniz's rule, we get

$$\Phi'(s)\dot{s} \equiv= \frac{b^d}{b^c}\left[\frac{\sigma-1}{\sigma}\int_t^{\infty} x(v)^{\frac{\sigma-1}{\sigma}}.\frac{\dot{x}}{x}.dv+\{x(\infty)-x(0)\}\right]$$

But since $\frac{\dot{x}}{x}=0$, $x(v)$ is constant for all $t \in (0,\infty)$ which implies that $x(\infty)=x(0)$

As a result,

$$\frac{\dot{s}}{s}=0 \;\ldots\ldots\ldots (A.14)$$

Social Planner's Outcome

From the Euler equation, we have,

$$\Rightarrow \frac{\dot{C}}{C}=\frac{1}{\theta}\left[y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M-\rho-\phi(1-\theta).\frac{\dot{Z}}{Z}+\psi(1-\theta).\frac{\dot{H}}{H}\right].$$

which implies that for consumption to grow at a constant rate it requires $y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M$, the marginal product of capital, is to be constant. From (A.5) x is constant, implies y will also be constant. Then, for the marginal product of capital to be constant, it must be hold that $\frac{\dot{M}}{M}=0$. This implies the steady state growth must entail resource augmenting technical change alone which again reconfirms the neoclassical result.

Specifically,

$$\frac{\dot{M}}{M} = 0 \dots\dots\dots (A.15)$$

$$\frac{\dot{N}}{N} = \left(\frac{1-\beta}{\beta}\right) b^{d}\varphi(1) \equiv \gamma_{N} \quad \dots\dots\dots (A.16)$$

Furthermore, besides the marginal product capital being constant, in a balanced growth equilibrium, $\frac{\dot{Z}}{Z}$ *and* $\frac{\dot{H}}{H}$ must also be constant. But the planner chooses the amount of usage of the natural resources and hence pollution such that the growth is sustainable.

From (3.7) and (3.8),

$$\frac{\dot{H}}{H} = \xi - \frac{D}{H} \quad and \quad \frac{\dot{Z}}{Z} = \alpha\frac{D}{Z} - \eta$$

Thus in the steady state, $\frac{D}{H}$ *and* $\frac{D}{Z}$ must be constant. As a result, the growth rate of all the three must be the same. Moreover, $\frac{H}{Z}$ must be constant. But for sustainable growth to be realized, $\frac{\dot{Z}}{Z} = 0$ is a necessary condition. Therefore,

$$\frac{\dot{H}}{H} = \frac{\dot{Z}}{Z} = \frac{\dot{D}}{D} = 0 \quad \dots\dots\dots\dots (A.17)$$

Using this in the above equation we get,

$$\xi - \frac{D}{H} = \alpha\frac{D}{Z} - \eta \;\Rightarrow D = \frac{(\xi+\eta)ZH}{(Z+\alpha H)} \quad and\; Z = \frac{\xi H\alpha}{\eta} \dots\dots\dots (A.18)$$

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left[y^{\frac{1}{\sigma}}x^{-\frac{1}{\sigma}}M - \rho\right] \equiv \widetilde{\gamma_{C}} \quad \dots\dots\dots\dots (A.19)$$

Substituting for (A.5), (A.16) and (A.17) in (A.4), we get

$$\frac{\dot{Y}}{Y} = \gamma_{N} \quad \dots\dots\dots (AI.20)$$

Similarly, using (A.15), (A.16) and (A.17) in (A.6) we get,

$$\frac{\dot{K}}{K} = \gamma_N \quad \dots\dots\dots\dots (A.21)$$

In steady state, $\dot{s} = 0$, must hold, so from (4.7), it must be true that in steady state,

$$\left[\frac{D}{q_N} - \frac{K}{q_M}\right] = 0 \;\Rightarrow\; \frac{D}{q_N} = \frac{K}{q_M} \Rightarrow \frac{q_M}{q_N} = \frac{K}{D}$$

Using (4.4.3) in the above,

$$\frac{b^d\{\varphi(1-s) + (1-s).\varphi'(1-s)\}N}{b^c\{\varphi(s) + s.\varphi'(s)\}\,M} = \frac{K}{D}$$

$$\Rightarrow \frac{b^d\{\varphi(1-s) + (1-s).\varphi'(1-s)\}}{b^c\{\varphi(s) + s.\varphi'(s)\}} = \frac{MK}{ND} \equiv x* \quad \dots\dots\dots. (A.22)$$

In steady state, as we have shown that all research is put into the dirty sector and the clean sector is stagnant technologically. Putting $s = 0$ in the above equation, we get

$$x *= \frac{b^d\{\varphi(1) + \varphi'(1)\}}{b^c\{\varphi(0)\}}, \quad \dots\dots. (A.23)$$

which is the steady state value of the technology augmented capital-resource ratio. Similarly,

$$y *= \left(1 + (x*)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \quad \dots\dots\dots (A.24)$$

This is the steady state value of output per unit of technology augmented resource.

Both of these are constant in steady state.

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