Government Expenditure and Economic Growth: A Demand-side Analysis

Dissertation submitted to the Jawaharlal Nehru University in partial fulfillment of the requirements for the award of the degree of

MASTER OF PHILOSOPHY

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DECLARATION

This is to certify that the dissertation entitled "***Government Expenditure and Economic Growth: A Demand-side Analysis***" submitted by me is in partial fulfillment of the requirement for the award of the degree of Master of Philosophy of Jawaharlal Nehru University. This dissertation has not been submitted for the award of any other degree in this University or any other University and is my own work.

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CERTIFICATE

We recommend that this dissertation be placed before the examiners for the evaluation.

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Chapter 1

Introduction

Recent financial crisis which started in August, 2007 and gathered pace after the default of Lehman Brothers' in September, 2008 resulted in a severe economic crisis for the USA as well as for the rest of the world. This has forced economists and policymakers to rethink conventional wisdom regarding economic theories and policies. The old debate as to whether the government expenditure is able to stimulate the economic growth, has once again emerged before us in a new way. The view generally held by Keynesians is that government involvement in economic activity is vital for growth while others say that government operations are inherently bureaucratic and inefficient and therefore rather than promoting growth, stifle it. Recently there has been a debate on the fact that whether government expenditure should increase or decrease to stimulate the economic growth.

There are several institutions in the literature discussing the relationship between government expenditure and the growth rate starting from Keynes. But we can find more formal analysis in the literature beginning with the work of Barro (1990). In his work, government expenditure enters into the private production function as a complementary input. His work explains the role of public expenditure in economic growth from the supply side of the economy. The demand side analysis incorporating the effect of effective demand on economic growth is absent there.

Formal analysis regarding the role of government expenditure on growth from the demand perspective more or less starts with You and Dutt (1996). While aiming to address the question of whether government debt worsens income distribution, their analysis also implied that fiscal expansion has a significant effect on government debt-capital ratio, growth rate and income distribution. In the short run

Differentiating the equation (5.7) with respect to π we get,

$$\frac{dr^*}{d\pi} = u^* + \pi \frac{du^*}{d\pi}$$

$$= \frac{(s_W - g_2)g_0}{[\{(s_P - s_W) - g_1\}\pi + s_W - g_2]^2}$$

The economy is in conflictive-stagnationist if $\frac{dr^*}{d\pi} > 0$ and it is in cooperative-stagnationist if $\frac{dr^*}{d\pi} < 0$.

$$\text{Thus } \frac{dr^*}{d\pi} \gtrless 0 \text{ according to whether } s_W \gtrless g_2.$$

Thus we have seen that even using the 'Kalecki-Steindl' type of investment function, we can achieve the same result as Blecker. Only the conditions regarding the stability, exhilarationist regime, profit-led and wage-led growth, cooperative and conflictive-stagnationist regime changes.

different from the previous literature. In our model we incorporate the fact that certain kinds of investment expenditure can influence labour productivity. Labour productivity on the other hand through its impact on share of profit influences the current profitability of the private capital formation. The novelty of the model of this dissertation lies in taking into account this fact. Unlike Commendatore and Pinto (2011) and Dutt (2013), in our model, the investment function also depends on the rate of profit. The main objective of this dissertation is to know whether both kinds of government expenditure have a positive impact on degree of capacity utilization and economic growth. We also want to know whether allowing the government to run in deficit and incur debt necessarily leads to the public debt to rise without bound.

In the next chapter of our dissertation we discuss the existing literature which analyze the effect of fiscal policies on aggregate demand and economic growth from a demand-side perspective. First section of that chapter contains a discussion of the basic 'post-Keynesian' growth model developed by Dutt (1984). Then we discuss Bhaduri and Marglin's criticism regarding the investment function used in this model. Then we move to a discussion of a paper by You and Dutt (1996) regarding the possibility of profit-led growth regime even in the stagnationist regime. We show that instead of Bhaduri-Marglin kind of investment function, using Kalecki-Steindl type of investment function we can attain the same result. The last part of that section is about the discussion in Blecker (2002) that points out that by introducing positive saving out of wages, fiscal policy, progressivity of taxation and international capital mobility, exhilarationist regime can be achieved even using an investment function that would otherwise imply stagnationism. The next section contains the literature regarding fiscal expansion and its impact on aggregate demand, level of capacity utilization and growth. It contains a brief discussion of the work of You and Dutt (1996), Commendatore and Pinto (2011) and Dutt (2013).

In the third chapter we present our model. Constructing a one sector, simple 'post-Keynesian' closed economy growth model, we try to analyse the impact of various kinds of government expenditures on aggregate demand, equilibrium level of capacity utilization and the equilibrium accumulation rate. Unlike Dutt (2013) a switch in government expenditure from consumption to investment purposes does not always leads to a rise in degree of capacity utilization and growth rate. In this case our findings are similar to Commendatore and Pinto (2011). But the main reason behind it in their model is influence of a change in capital productivity on the equilibrium degree of capacity utilization, while in our analysis it is the change in labour productivity that influences the equilibrium level of capacity utilization. Then we consider a more general case where workers also save and try to see whether the possibility of exhilarationist regime arises. We then analyze the impact of fiscal expansion on the aggregate demand and the growth rate. The next section is about the impact of changes in fiscal policy on the equilibrium employment rate. In the last section we consider the effect of deficit and government debt on the economy in the short as well as in the long-run.

The last chapter is about the conclusion which summarises and discusses the results of the model contained in chapter 3.

Chapter 2

Critical Review of the Literature

Modern theory of economic growth starts with the work of Harrod and Domar as both aimed to extend the work of Keynes beyond the short run. According to Harrod, in a closed economy in which capital output ratio (v), saving propensity (s), population growth rate(n), and labour productivity growth rate ($\hat{a}$) all are constants, it is almost impossible for the economy to grow at a steady state. If the conomy is converging to such a growth path, it is only by accident. Then he was concerned with the stability issue. Even if the economy grows at a steady state, it is unstable. Finally Harrod asks whether the economy can grow steadily along with full employment of labour. As $v, s, n, \hat{a}$ all are exogenously given, there is no reason why the warranted growth rate ($\frac{s}{v}$) which is the growth rate in steady state will be equal to the natural growth rate ($n + \hat{a}$) which is required for full employment..

Neo-classical growth theory model pioneered by Solow and Swan responded to the questions posed by Harrod by assuming that there is substitutability between labour and capital. Flexibility between capital and labour makes the capital-output ratio variable which by adjusting itself ensures equality between warranted and natural growth rates and hence stable steady-state growth path with full employment is achieved. The central problem with this kind of model is that it assumes investment is always equal to saving or discrepancy between investment and saving is taken care of automatically while considering the long run. Aggregate demand has no role to play in this kind of growth model. Further this kind of growth theory does not emphasize distributional issues separately as it assumes distribution to be determined through the equilibrium of competitive factor markets.

On the other hand the neo-Keynesian growth model pioneered by Kaldor (1956) and Passinetti (1962) emphasizes the role of income distribution which influences the average saving propensity out of income. So any imbalance between warranted and natural growth rate leads to change in income distribution which in turn makes the average saving propensity endogenous. Adjustment in average saving propensity ensures the equality between those two growth rates[1].

Later on, the basic post-Keynesian growth model, formulated independently by Robert Rowthorn (1981) and Amitava Krishna Dutt (1984), showed that steady state growth can be attained with involuntary unemployment even in the long run through the expansion in degree of capacity utilization.

Now we will discuss the basic differences between neo-Keynesian and neo-Kaleckian tradition both of which can be said to form part of the post-Keynesian framework. Growth models that follow Keynesian tradition can be divided into two parts, the older one developed by Kaldor (1957), Robinson (1962) and Passinetti (1962) are called the neo-Keynesian models and the newer ones developed independently by Robert Rowthorn (1981) and Amitava Krishna Dutt (1984), which are known to as the neo-Kaleckian models. The neo-Kaleckian growth model is also recognized as the Kalecki-Steindl or structuralist(Taylor; 1983) growth model. There are two major differences between these two groups of model. Firstly, in the neo-Keynesian models, the market is competitive in nature where firms are price takers, while in the neo-Kaleckian models the market is oligopolistic in nature where firms set prices over markup on prime costs. Second, in the neo-Keynesian model, in the long run in the economy there is either full capacity or the rate of capacity utilization is fixed at a given normal level, whereas in the neo-Kaleckian model, the rate of capacity utilization is endogenous and is below full capacity or below a normal level, even in the long run. Thus in the neo-Keynesian model, in the short run output is fixed while price adjusts due to change in aggregate demand. So when the demand falls, firms are forced to reduce the price to clear the goods market. As a result for a given money wage rate, real wage is higher while real profit falls and so firms have no incentive to invest further. Hence the economy stagnates. Thus, in neo-Keynesian theory, because of insufficient demand, it is price which changes and causes stagnation. But in the neo-Kaleckian model, in the short run output varies in response to change in aggregate demand. So when the demand is depressed, in response to that, firms reduce the production of output, while price is fixed. It reduces both the level of capacity utilization and profit rate. As a consequence investment falls which in turn reduces capital accumulation and finally the economy stagnates. Thus in neo-Kaleckian theory, it is output and hence the

[1]In their model, it is again assumed that full employment is attained in the long run.

capacity level which falls and leads to stagnation. We will briefly discuss the basic neo-Kaleckian growth model developed by Dutt in the next section.

2.1 Post-Keynesian growth model

The basic purpose of Dutt's (1984) model is to examine the relationship between growth and income distribution in an underdeveloped economy in order to analyse the cause of stagnation in the Indian industrial economy in the middle nineteen sixties and the nineteen seventies. A brief discussion of the model is given below.

Using homogeneous inputs, capital and labour, the economy produces industrial good, where the production function is of Leontief type i.e.

$$Y = min\{aL, bK\} \tag{2.1}$$

where Y is the amount of industrial good produced, L is total amount of labour employed, K is available capital stock, a and b are fixed output-labour and potential output-capital ratio respectively. There is excess supply of labour. According to Dutt, a large reservoir of labour exists either in the form of a reserve army or as employed in a subsistence sector having no other interaction with the industrial sector. There is excess capacity in the economy. So, $Y \leq bK$, where the equality represents the full capacity level of output or potential level of output (Y^P)[2]. So, from equation (2.1) we get,

$$Y = aL \tag{2.2}$$

The industrial sector is oligopolistic in nature. Price is determined by mark-up (λ) on the unit prime cost $(\frac{W}{a})$, where W is money wage[3]. Thus the price level can be represented by the following equation,

$$p = (1 + \lambda)\frac{W}{a} \tag{2.3}$$

The rate of mark-up (λ), where $\lambda > 0$, is assumed to be given at a point in time and is determined by the 'degree of monopoly' as suggested by Kalecki (1971). The share of profit (π) is then given by,

$$\pi = \frac{pY - WL}{pY} = \frac{\lambda}{1 + \lambda} \tag{2.4}$$

[2] $b = \frac{Y^P}{K}$

[3] Dutt assumes that the money wage is fixed, either through bargains or by the government. He also assumes money wage to be fixed at least at a level that ensures subsistence consumption at a large range of prices.

So, the share of profit is determined by the 'degree of monopoly power'. The rate of profit is given by,

$$r = \frac{P}{K} = \frac{P}{Y}\frac{Y}{Y^P}\frac{Y^P}{K} = \pi u \tag{2.5}$$

where, P is aggregate profit, $\frac{P}{Y}$ is the share of profit-π, and u is degree of capacity utilization. As long as potential output-capital ratio is fixed, actual output-capital ratio can be used as a proxy for degree of capacity utilization. Following in the tradition of the classical economists as well as Kalecki (1971), Kaldor (1956) and Pasinetti (1962), Dutt assumes that there are two groups - workers and capitalists in the economy with two different saving propensities. Workers do not save, while capitalists save a fraction, s, of their income[4]. So the total savings can be represented as

$$S = sP \tag{2.6}$$

Thus the saving-capital ratio is

$$\frac{S}{K} = s\frac{P}{K} = sr \tag{2.7}$$

Investment decisions are made with regard to both, the rate of profit and the rate of capacity utilization and the investment function can be written as,

$$\frac{I}{K} = \alpha + \beta r + \gamma u \tag{2.8}$$

where α, β, γ are positive constants. The first term α represents the state of 'animal spirits' in the economy. Investment depends positively on the rate of profit and the rate of capacity utilization.

Rate of profit enters in the investment function as a proxy for the expected rate of return. It also provides internal funding for accumulation plans. For firms depending on external finance, it is also easier to raise that external finance while rate of profit is higher[5]. For simplicity Dutt assumes that actual rate of profit is equal to the expected profit rate.

The argument for rate of capacity utilization entering in the investment function comes from Steindl (1952). According to Steindl, because of indivisibilities in capital equipment, it is profitable for profit maximizing firms to have a certain desired level of excess capacity due to fluctuations in demand or expected growth in demand. Thus when capacity utilization rises above the desired level, firms would like to invest more; while the capacity utilization falls below the desired level, firm

[4] Where $0 < s < 1$

[5] See Dutt (1984, 1987), Rowthorn (1981), Robinson (1956)

would like to increase utilization by disinvesting and hence by reducing the stock of capital. β and γ are the coefficients measuring the responsiveness of investment-capital ratio due to the change in rate of profit and degree of capacity utilization respectively. In equilibrium, savings must be equal to investment. Thus,

$$\frac{I}{K} = \frac{S}{K}$$

$$\Rightarrow \alpha + \beta r + \gamma u = sr$$

Substituting the value of r from equation (2.5) in the above equation and solving for u, we get the equilibrium level of capacity utilization as,

$$u^* = \frac{\alpha}{(s-\beta)\pi - \gamma} \tag{2.9}$$

The equilibrium is stable if the induced increase in saving, as u rises, is greater than the induced increase in investment i.e.

$$s\pi > \beta\pi + \gamma$$

$$\text{or, } (s-\beta)\pi - \gamma > 0 \tag{2.10}$$

In other word, for the equilibrium to be stable the denominator of u^* must be positive. Since $\alpha > 0$, the condition $(s-\beta)\pi > \alpha + \gamma$, not only satisfies the stability requirement, but also ensures the existence of excess capacity. But this condition also sets a lower bound on the share of profit, $\pi > \frac{\alpha+\gamma}{(s-\beta)}$. Substituting u^* in equation (2.5) we get the equilibrium rate of profit as,

$$r^* = \pi u^* \tag{2.11}$$

Substituting r^* in equation (2.8) we get the equilibrium rate of accumulation as,

$$g^* = \alpha + \beta r^* + \gamma u^* \tag{2.12}$$

Suppose the saving propensity of capitalists decreases. Due to a fall in saving propensity, consumption demand of capitalists and aggregate consumption demand increases which in turn leads to an increase in the aggregate demand. Thus aggregate demand and hence degree of capacity utilization increases. Mathematically,

$$\frac{\partial u^*}{\partial s} = -\frac{\alpha\pi}{[(s-\beta)\pi - \gamma]^2} < 0 \tag{2.13}$$

Equilibrium rate of profit depends on the share of profit and the degree of capacity utilization. So, as s decreases, r^* rises. Differentiating equilibrium value of profit rate with respect to s we get,

$$\frac{\partial r^*}{\partial s} = \pi\frac{\partial u^*}{\partial s} = -\frac{\alpha\pi^2}{[(s-\beta)\pi-\gamma]^2} < 0 \tag{2.14}$$

Thus, due to fall in s, both r^* and u^* increase. Hence the equilibrium level of growth rate would also rise. Differentiating equation (2.12) with respect to s we get,

$$\frac{\partial g^*}{\partial s} = \beta\frac{\partial r^*}{\partial s} + \gamma\frac{\partial u^*}{\partial s} = -\frac{(\beta\pi+\gamma)\alpha\pi}{[(s-\beta)\pi-\gamma]^2} < 0 \tag{2.15}$$

Thus, a fall in s not only increases u^* but g^* also.

An increase in profit-share leads to a decrease in the degree of capacity utilization. As the share of profits increases, a shift of income from wage income to profit income leads to a decrease in consumption demand. On the other hand due to a rise in the share of profit, for a given degree of capacity utilization, investment demand increases. As long as $s > \beta$, the negative effect of a rise in profit share on consumption demand dominates the positive effect on investment demand. Aggregate demand and hence the equilibrium degree of capacity utilization falls. Mathematically,

$$\frac{\partial u^*}{\partial \pi} = -\frac{\alpha(s-\beta)}{[(s-\beta)\pi-\gamma]^2} < 0 \tag{2.16}$$

Due to a rise in profit-share, rate of capacity utilization decreases. But what is the impact of π on the rate of profit ? Differentiating equation (2.11) with respect to π we get

$$\frac{\partial r^*}{\partial \pi} = u^* + \pi\frac{\partial u^*}{\partial \pi}$$
$$= -\frac{\alpha\gamma}{[(s-\beta)\pi-\gamma]^2} < 0 \tag{2.17}$$

Thus due to a rise in π, r^* also decreases.

As due to a rise in π both the equilibrium degree of capacity utilization and the equilibrium profit rate fall, the equilibrium value of the rate of accumulation should also decrease. Differentiating equation (2.12) with respect to π we get,

$$\frac{\partial g^*}{\partial \pi} = \beta\frac{\partial r^*}{\partial \pi} + \gamma\frac{\partial u^*}{\partial \pi} = -\frac{s\alpha\gamma}{[(s-\beta)\pi-\gamma]^2} < 0 \tag{2.18}$$

There is a positive relation between the rate of mark up(λ) and the share of profit (π)[6] . Therefore due to an increase in λ, the equilibrium values of u,r, and g all will decrease.

[6]Because $\frac{d\pi}{d\lambda} = \frac{1}{(1+\lambda)^2} > 0$

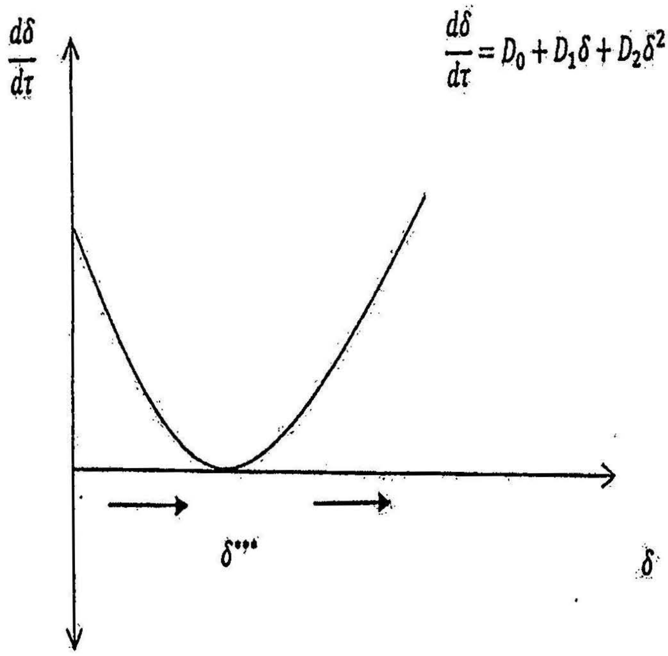

Figure-2

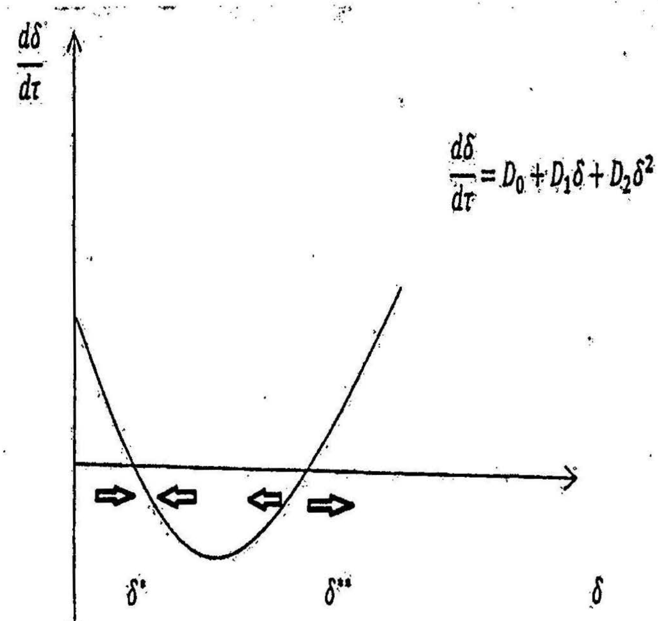

Figure-3

$$\text{Then, } \frac{dg^*}{d\pi} = \beta + \gamma \frac{\partial u^*}{\partial \pi} \tag{2.20}$$

If the economy is in a stagnationist regime then due to a rise in profit-share, the equilibrium rate of capacity utilization falls (i.e. $\frac{\partial u^*}{\partial \pi} < 0$). But growth can be both wage-led (i.e. $\frac{dg^*}{d\pi} < 0$) or profit-led (i.e. $\frac{dg^*}{d\pi} > 0$).

$$\text{So, } \frac{dg^*}{d\pi} \gtrless 0 \text{ according to whether } \left|\frac{\partial u^*}{\partial \pi}\right| \lesseqgtr \frac{\beta}{\gamma} \tag{2.21}$$

A one unit rise in profit share leads to a fall in the equilibrium level of capacity utilization by $|\frac{\partial u^*}{\partial \pi}|$ units. A fall in capacity utilization has a negative effect on the investment demand. Thus due to one unit rise in profit share, through the effect of capacity utilization investment demand decreases by $\gamma\,|\frac{\partial u^*}{\partial \pi}|\,K$ unit. On the other hand a one unit rise in profit share directly raises the investmentment demand by βK unit. If the indirect effect is greater than the direct effect (i.e. if $|\frac{\partial u^*}{\partial \pi}| > \frac{\beta}{\gamma}$), the effect of a rise in profit share on the equilibrium level of accumulation rate is negative and hence we get wage-led growth regime. On the other hand, when the latter dominates the previous one, the equilibrium growth rate rises and we get profit-led growth regime. But when the economy is in exhilarationist regime, we always get profit led growth.

Instead of the Bhaduri-Marglin kind of investment function, we can use the Kalecki-Steindl type of investment function[8] to obtain the same result. In that case equilibrium level of accumulation rate can be represented as,

$$g^* = \alpha + \beta r^* + \gamma u^*$$

$$\text{Then, } \frac{dg^*}{d\pi} = \beta u^* + (\beta\pi + \gamma)\frac{\partial u^*}{\partial \pi} \tag{2.22}$$

$$\text{Thus, } \frac{dg^*}{d\pi} \gtrless 0 \text{ according to whether } |\frac{\partial u^*}{\partial \pi}| \lesseqgtr \frac{\beta u^*}{\beta\pi + \gamma} \tag{2.23}$$

Here again, depending on how strong the effect of change in degree of capacity utilization is due to change in share of profit, growth can be wage-led or profit-led.

Blecker (2002) shows that even with a Bhaduri-Marglin kind of investment function exhilarationism is not possible when the investment function is of linear form or the Cobb-Douglas form. He also points out that in the presence of positive saving out of wages, fiscal policy and progressive taxation or international competition and capital mobility, the exhilarationist regime can be achieved even using an investment function that would otherwise imply stagnationism. Assuming a

[8] In this type of investment function, investment depends on the rate of profit. For our purpose $I = [\alpha + \beta r + \gamma u]\,K$ represents the Kalecki-Steindl type of investment function.

Bhaduri-Marglin kind of investment function and introducing saving out of wages he shows that exhilarationism can be achieved when saving propensity out of wages is large enough. As the workers are also savers, because of redistribution of income to profits, there is relatively small loss in consumption demand compared to increase in investment demand. He also shows that in the stagnationist regime, growth can be profit-led as well.

But in his analysis, he uses Bhaduri-Marglin kind of investment function. However we can get similar results even in the presence of a Kalecki-Steindl type of investment function. In the appendix-A we have discussed how positive saving out of wages creates the possibilitiy of an the exhilarationist regime even if we use the 'Kalecki-Steindl' type of investment function.

2.2 Government expenditure and growth

In this section we will discuss different kinds of government expenditure and its impact on growth in the neo-Kaleckian framework. In Keynesian analysis, although sufficient attention was paid regarding the implication of various kinds of government expenditures by its pioneers, the subject was largely overlooked later on. According to Commendatore and Pinto (2011), though Kaldor presented interesting insights regarding the relationship between the composition of government expenditure and long run growth, there was no formal analysis. The formal analysis of the impact of government expenditure on growth more or less starts with You and Dutt (1996).

While trying to address the question of whether or not government debt worsens income distribution, You and Dutt show that fiscal expansion has a significant effect on the government debt-capital ratio, economic growth and income distribution. Their analysis implies a positive relationship between fiscal expansion[9] and the rate of economic growth rate in the short-run.. As fiscal expansion increases, aggregate demand and the degree of capacity utilization rises which in turn enhances the growth rate. But in the long run, the effect of fiscal expansion on the growth rate is ambiguous. This is because while fiscal expansion, through an increase in aggregate demand and the degree of capacity utilization, raises the growth rate, it can either increase or decrease the government debt-capital ratio (δ). An increase

[9]The increase in fiscal expansion is represented as an increase in the ratio of government expenditure to capital stock (γ)

in the government debt-capital ratio has a positive impact on the growth rate[10]. When a rise in fiscal expansion raises the government debt-capital ratio, fiscal expansion unambiguously enhances the growth rate. However, when due to a rise in the fiscal expansion, the government debt-capital ratio falls then its effect on the growth rate is ambiguous and depends on the strength of change in debt-capital ratio(δ) due to change in the ratio of government expenditure to capital (γ). We can represent it mathematically as well.

Differentiating equilibrium growth rate with respect to γ we get,

$$\frac{dg^*}{d\gamma} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\gamma} \tag{2.24}$$

$$= \frac{\partial g^*}{\partial u^*}\left[\frac{\partial u^*}{\partial \delta}\frac{d\delta}{d\gamma} + \frac{\partial u^*}{\partial \gamma}\right]$$

$$= \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \delta}\frac{d\delta}{d\gamma} + \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \gamma} \tag{2.25}$$

$\frac{\partial g^*}{\partial u^*}, \frac{\partial u^*}{\partial \delta}, \frac{\partial u^*}{\partial \gamma}$ are all positive. Therefore if $\frac{d\delta}{d\gamma}$ is positive, g^* unambiguously rises with to a rise in γ. But if $\frac{d\delta}{d\gamma}$ is negative then ,

$$\frac{dg^*}{d\gamma} \gtrless 0 \text{ according to whether } \left|\frac{d\delta}{d\gamma}\right| \lessgtr \frac{\frac{\partial u^*}{\partial \gamma}}{\frac{\partial u^*}{\partial \delta}} \tag{2.26}$$

Throughout their analysis Dutt and You however fail to take account of the fact that fiscal expansion can also influence labour productivity. On the other hand, labour productivity itself can influence the share of profit which has an impact on the growth rate. Certain kinds of government expenditure enhance labour productivity. An increase in labour productivity increases the profit share[11].Thus if we incorporate this argument, then equation (2.24) can be modified as

$$\frac{dg^*}{d\gamma} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\gamma} + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\gamma} \tag{2.27}$$

$$= \frac{\partial g^*}{\partial u^*}\left[\frac{\partial u^*}{\partial \pi}\frac{d\pi}{d\gamma} + \frac{\partial u^*}{\partial \delta}\frac{d\delta}{d\gamma} + \frac{\partial u^*}{\partial \gamma}\right] + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\gamma}$$

$$= \left[\frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \pi} + \frac{\partial g^*}{\partial \pi}\right]\frac{d\pi}{d\gamma} + \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \delta}\frac{d\delta}{d\gamma} + \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \gamma}$$

[10]This is because, as debt-capital ratio rises, interest income of the capitalists increases which leads to a rise in consumption demand which in turn increases aggregate demand and the degree of capacity utilization. Increase in the degree of capacity utilization through the accelerator effect enhances the rate of investment and capital accumulation.

[11]This will be discussed in detail in the next chapter (page no.- 23).

$$= \frac{dg^*}{d\pi}\frac{d\pi}{d\gamma} + \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \delta}\frac{d\delta}{d\gamma} + \frac{\partial g^*}{\partial u^*}\frac{\partial u^*}{\partial \gamma} \tag{2.28}$$

Now the analysis is more complex. Even when $\frac{d\delta}{d\gamma}$ is positive, the effect of fiscal expansion on growth rate is ambiguous .

In a later contribution in the neo-Kaleckian tradition, Commendatore and Pinto (2011) analyze the impact of different kinds of government expenditure on capacity utilization and growth. There analysis is briefly discussed below.

In a single-good closed economy framework the output is given as

$$Y = aL \leq bK = Y^P \tag{2.29}$$

The price of the good is assumed to be numeraire. They introduce two different types of public expenditure: government consumption expenditure (C_G) and public provision of capital (I_G).

Because of balanced budget assumption the following equation must hold :

$$T = C_G + I_G$$

$$\Rightarrow \frac{T}{K} = \frac{C_G}{K} + \frac{I_G}{K} \tag{2.30}$$

They assume public provision of capital positively affects the capital productivity by enhancing potential output-capital ratio i.e.

$$\frac{Y^P}{K} = b = b(\frac{I_G}{K})$$

where $b'(\frac{I_G}{K}) > 0$ and $b''(\frac{I_G}{K}) \leq 0$

For $b(\frac{I_G}{K})$ they have choosen a linear form:

$$b(\frac{I_G}{K}) = b_0 + b_1(\frac{I_G}{K}) \text{ with } b_0 \text{ and } b_1 \text{ both are positive.}$$

he rate of profit can be written as

$$r = \frac{P}{K} = \frac{P}{Y}\frac{Y}{Y^P}\frac{Y^P}{K} = \pi u b \tag{2.31}$$

Unlike the previous section (section 2.1) since the potential output-capital ratio is no longer fixed, $\frac{Y}{K}$ can't be used as a proxy for the degree of capacity utilization. Rather $\frac{Y}{Y^P}$ is used as the degree of capacity utilization (u). Now equation (2.30) can be written as

$$tbu = \frac{C_G}{K} + \frac{I_G}{K} \tag{2.32}$$

where t is the tax rate.

In the economy workers do not save, while capitalists save a fraction, s, of their income. So the after-tax total savings can be represented as

$$S = sP(1-t) \tag{2.33}$$

Thus the saving-capital ratio can be written as

$$\frac{S}{K} = s(1-t)\frac{P}{K} = sr(1-t) \tag{2.34}$$

Investment decisions can be written as,

$$\frac{I}{K} = \alpha + \beta u + \gamma(\frac{I_G}{K}) \tag{2.35}$$

where α, β, γ are positive constants. The first term α represents the 'animal spirit'. Investment depends positively on the rate of capacity utilization and on public investment. According to them a higher $(\frac{I_G}{K})$ may strengthen firms' incentive to invest through crowding-in effect.

Their conclusions on the effects of government expenditure on equilibrium capacity utilization and growth can be summarized as follows:

1. When government size is allowed to vary, the effect of government consumption expenditure in terms of capital stock on both equilibrium capacity utilization and equilibrium growth rate is unambiguously positive, while the effect of government capital expenditure in terms of capital stock, on the rate of capacity utilization and capital accumulation is ambiguous and depends on the responsiveness of capital productivity to government investment-capital ratio (b_1) and the strength of 'crowding in' effect (γ). When the effect of b_1 is sufficiently small or the effect of γ is sufficiently large, an increase in government investment-capital ratio can increase the degree of capacity utilization and the growth rate[12]. Those two parameters are also crucial to determine the relative effectiveness of these two kinds of government expenditure.

2. When the government size is fixed, the effect of a change in the composition of public expenditure on the growth rate depends on the ratio $(\frac{b_1}{\gamma})$ as follows:

[12]in both cases, government budget is balanced

For a sufficiently large value of $(\frac{b_1}{\gamma})$ a shift in the composition in favour of public investment expenditure enhances growth; whereas for smaller values of $(\frac{b_1}{\gamma})$ various possibilities may occur:

- starting from a zero public investment expenditure, a shift in favour of public investment expenditure can slow down the growth initially and increase it subsequently. Here, the highest growth rate is achieved when the government consumption expenditure is zero(i.e. government spends its entire expenditure on investment purposes)
- starting from a zero public investment expenditure, a shift in favour of public investment expenditure can slow down growth initially and increase it subsequently. Here, the highest growth rate is achieved when the government investment expenditure is zero(i.e. government spends its expenditure fully on consumption purposes)
- an increase in government investment expenditure slows down growth for any possible public expenditure composition

In the analysis by Commendatore and Pinto (2011), government investment expenditure influences capital productivity. On the other hand capital productivity itself has a negative impact on the equilibrium degree of capacity utilization. Thus whether an increase in public investment expenditure increases $u^*$and g^* depends on the strength of negative effect on aggregate demand which comes through the enhancement of capital productivity and the strength of positive effect on aggregate demand which comes from the increase in investment demand due to crowding-in effect. But the ratio of physical capital to output is nearly constant.It is one of the stylized facts given by Kaldor. The long-term data also shows the same result[13]. On the other hand, government investment expenditure like expenditure on streets and highways, electricity, gas and water supply, hospital, education can enhance labour productivity as well. But the analysis of impact of government expenditure on labour productivity is absent here (in Commendatore and Pinto (2011)).

Dutt (2013) analyzes the impact of different kinds of government expenditure on aggregate demand and growth in the short run as well as in the long run in a single-good closed economy framework. Unlike Commendatore and Pinto (2011), in his analysis $\frac{Y^P}{K}$ is fixed and is not influenced by the government investment

[13]Barro and Sala-i-Martin (2004)

expenditure. Thus $\frac{Y}{K}$ can be used as a proxy for the degree of capacity utilization. In his analysis he assumes that the government budget is balanced and the government does not carry any debt. So the balanced budget equation can be written as

$$tY = C_G + I_G$$

He assumes that the government investment expenditure is proportional to the aggregate real income i.e. $I_G = \theta Y$. Thus the above equation can be written as

$$tY = C_G + \theta Y$$

So when government budget is fixed, a rise in θ represents the shift of government expenditure from consumption to investment purposes. He assumes that there is a homogenous class of people who save a fraction, s, of disposable income. So the aggregate consumtion demand can be given as

$$C = (1 - s)(1 - t)Y$$

He also assumes the private investment function depends positively on u and $(\frac{I_G}{K})$ as follows :

$$\frac{I}{K} = \gamma + \gamma_1 u + \gamma_2(\frac{I_G}{K})$$

The good market is in equilibrium when

$$Y = C + I + C_G + I_G$$

$$\Rightarrow u^* = \frac{\gamma}{s(1 - t) - \gamma_1 - \gamma_2\theta}$$

The equilibrium rate of accumulation can be given by,

$$g^* = \gamma + \gamma_1 u^* + \gamma_2 \theta u^*$$

The result of the analysis regarding the impact of government expenditure on the rate of capacity utilization and the growth rate are summarized below.

In the short run, both kinds of government expenditure - government consumption and investment expenditure enhance aggregate demand and hence degree of capacity utilization. Increase in capacity utilization increases the growth rate. So, both kinds of government expenditure have positive effects on aggregate demand, degree of capacity utilization and accumulation rate. But government investment expenditure due to its 'crowding-in' effect on private investment increases investment and hence aggregate demand further. Thus the degree of capacity utilization and the growth rate both are higher in this case compared to the case of an increase in government consumption expenditure. In a balanced budget situation, when total revenue is given in the short run, a switch from government consumption to investment expenditure, does not increase the level of aggregate demand directly, but its indirect effect through 'crowding-in' of private investment increases aggregate demand, the degree of capacity utilization and the growth rate.

Then he introduces the endogenous technological change where the long-run rate of growth of the economy is determined by both demand and supply forces. In the long run, both kinds of government expenditure have positive effect on growth rate. But a switch from government consumption to government investment expenditure increases the growth rate. In other word, government investment expenditure is more effective in the long run too. The reason is two folded. First, it 'crowds-in' private investment. Second, it influences the speed of adjustment for technological change positively.

Then he relaxes the balanced budget assumption by allowing the government to run a deficit and incur debt. Now government tax revenue can be represented by

$$T = t(Y + iD)$$

where t is the tax rate, Y is the real aggregate productive income, i is the interested rate that is paid by the government, and D is the real stock of government debt.

He assumes that government consumption expenditure and government investment expenditure depend on the income level of the economy.

Thus current government consumption expenditure is now given by

$$C_G = \eta Y$$

Investment expenditure is given by

$$I_G = \theta Y$$

Dutt assumes the entire government deficit is financed by issuing government debt. So, the change in debt with respect to time is given by,

$$\frac{dD}{d\tau} = (C_G + I_G) - T + iD$$

The consumption function is now

$$C = (1-s)(1-t)(Y+iD)$$

Investment function is given by,

$$\frac{I}{K} = [\gamma + \gamma_1 u + \gamma_2.\theta.u - \gamma_3\delta]$$

where γ_4 is the coefficient measuring responsiveness of investment due to change in δ. Here the fourth term entering in the investment function, represents the financial crowding-out effect[14]. The short run equilibrium degree of capacity utilization for this model is

$$u^* = \frac{\gamma + [(1-s)(1-t)i - \gamma_3]\delta}{s(1-t) + t - \gamma_1 - \theta(1+\gamma_2) - \eta}$$

From the above equation it is clear that in the short run, the effect of a rise in δ on u^* is ambiguous and it depends on the sign of $(1-s)(1-t)i - \gamma_3$. If the financial crowding out is weak (strong), i.e. if γ_3 is small (large) then the effect of a rise in δ on u^*may be positive (negative). When $\frac{du^*}{d\delta} < 0$, the effect of a rise in δ on $g^*$is negative. But if $\frac{du^*}{d\delta} > 0$, the effect of a rise in δ on $g^*$is then unambiguous.

In Dutt (2013), the long-run the dynamics of the ratio of debt to capital is given as,

$$\frac{d\delta}{d\tau} = \frac{(\eta+\theta-t)\gamma}{\Lambda} + \left[\frac{(\eta+\theta-t)}{\Lambda} + i(1-t) - \gamma - \frac{\{\gamma_1+\gamma_2\theta\}\gamma}{\Lambda}\right]\delta - \left[\frac{\{\gamma_1+\gamma_2\theta\}\Gamma}{\Lambda} - \gamma_3\right]\delta^2$$

where $\Gamma = [(1-s)(1-t)i - \gamma_3]$ and $\Lambda = [s(1-t) + t - \gamma_1 - \theta(1+\gamma_2) - \eta] > 0$. We can write the above equation as

$$\frac{d\delta}{d\tau} = D_0 + D_1\delta - D_2\delta^2$$

Here D_0 is positive. So depending on the signs of D_1 and D_2 various shapes regarding the relation between $\frac{d\delta}{d\tau}$ and δ can take place. If D_1 and D_2 both have positive signs then the equation $\frac{d\delta}{d\tau} = D_0 + D_1\delta - D_2\delta^2$ would be inverted "U" shaped. On that case, the existing unique equilibrium is stable equilibrium. If $D_1 < 0$ and $D_2 > 0$ then the equation $\frac{d\delta}{d\tau} = D_0 + D_1\delta - D_2\delta^2$would be negatively sloped. On that case again, the existing unique equilibrium is stable equilibrium. On the other hand if D_1 and D_2 both have negative signs then the equation $\frac{d\delta}{d\tau} = D_0 + D_1\delta - D_2\delta^2$ would be "U" shaped. Thus although the government runs in deficit and incurs debt, it is not necessarily the case that the ratio of public debt to capital rises without bound.

[14]According to Dutt (2013) alhough the empirical evidence on whether $\gamma_3 > 0$ is not clear, he includes the negative effect to allow crowding out effect.

In the next chapter we make a neo-Kaleckian growth model to see the impact of government expenditure on aggregate demand, employment rate and the economic growth. Our analysis departs from Commendatore and Pinto (2011) on the ground that here instead of capital productivity, labour productivity is influenced by public investment expenditure. Unlike Dutt (2013), allowing different classes in the economy, we come to a conclusion which is different from Dutt (2013).

Chapter 3

The Model

We assume a simple one-sector neo-Kaleckian growth model in which the economy consists of two classes: capitalists and workers. Workers consume whatever they earn while capitalists save a fraction of their income. Capitalists' saving propensity out of the current income is given by "s" where $0 < s < 1$.

Income is distributed between wages and profits:

$$pY = WL + rpK \tag{3.1}$$

where p is price level, Y is real income, W is nominal wage rate, L is total amount of labour employment, K is the existing capital stock, r is the real rate of profit.

There is excess supply of labour and no depreciation of capital in the economy. The production function is of Leontief type i.e.

$$Y = min\{aL, bK\} = aL, b = \frac{Y^P}{K} > \frac{Y}{K} \tag{3.2}$$

Where $Y^P$is the potential output level. So the actual output is below the potential output level.

The market is oligopolistic in nature where price is determined by mark-up on prime cost. For simplicity we assume away cost of raw materials and overhead cost. We assume here that the only cost is the labour cost. So price is given by

$$p = (1 + \lambda)\frac{WL}{Y}$$

$$\Rightarrow p = (1 + \lambda)\frac{W}{a} \tag{3.3}$$

where λ is the rate of mark-up and $a = \frac{Y}{L}$ is labour productivity.

Total wage share $= \frac{WL}{pY} = \frac{w}{a}$, where w is real wage rate.

So, share of profit $\pi = (1 - \frac{w}{a})$

From this equation we can conclude that share of profit depends on labour productivity and real wage rate.

Real wage rate itself depends on labour productivity i.e. $w = w(a)$. But the rate of change in the real wage rate with respect to labour productivity depends on the bargaining power of the workers which in turn depends on the prevailing employment rate and the extent of unionization. We assume $\varepsilon_{w,a} < 1$ i.e.elasticity of real wage rate with respect to labour productivity is less than one[1]. As a consequence, if labour productivity increases, wage share $\frac{w}{a}$ decreases which in turn increases the share of profit. Thus, $\pi'(a) > 0$ i.e. change in share of profit due to change in labour productivity is positive.

We assume that there are two types of government expenditure: government consumption expenditure, denoted by C_G and government investment expenditure denoted by I_G. We also assume that government investment expenditure is proportional to the aggregate real income i.e. $I_G = \theta Y$, where θ represents government investment-output ratio. Government raises revenue through an income tax. Total tax revenue is $T = tY$, where t is the tax rate. For simplicity, we assume that the government budget is balanced. So,

$$t.Y = C_G + I_G \tag{3.4}$$

If t and θ are fixed, this equation can be satisfied through adjustment in C_G. Given the tax rate, if θ increases then government consumption expenditure must fall. Thus for a given aggregate government expenditure, a change in the parameter θ represents a change in fiscal policy i.e. here changes in θ represents changes in fiscal policy related to the government's decision as to how much to spend on consumption and how much to spend on investment.

Total savings as a proportion of capital stock is expressed as

$$\frac{S}{K} = s(1-t)r = s(1-t)\pi(a).u \tag{3.5}$$

[1]In developing countries, a large number of workers are employed in unorganized sectors(eg. India, Pakistan, Bangladesh) where either they don't have any organized labour union or the union is too weak to have strong bargaining power. On the other hand, in developed countries as well, the workers may not be able to fully internalize the increase in productivity through proportionate increases in the real wage rate. (Carter (2007), Sharpe et al. (2008a), (2008b))

where, $r = \frac{P}{K} = \frac{P}{Y}.\frac{Y}{K} = \pi(a).u$

P represents total profit, $\frac{P}{Y}$ = share of profit $= \pi(a)$ and u is the output-capital ratio which is used as a proxy for degree of capacity utilization[2] (Dutt 1984, 1987, 1990).

We assume that there is excess capacity in the economy (i.e $u < 1$)

The investment function in the economy is given by

$$I = [\gamma + \gamma_1 u + \gamma_2(1-t)r + \gamma_3(\frac{I_G}{K})]K \tag{3.6}$$

$$\text{or, } \frac{I}{K} = [\gamma + \gamma_1 u + \gamma_2(1-t).\pi(a).u + \gamma_3.\theta.u] \tag{3.7}$$

where $\gamma, \gamma_1, \gamma_2, \gamma_3$ all are positive parameters.

γ represents the autonomous part of the investment function. Following Rowthorn(1981) and Dutt(1984,1987), we assume that investment depends positively on the degree of capacity utilization (u), the rate of profit (r) and the ratio of government investment to capital stock ($\frac{I_G}{K}$). γ_1 indicates the responsiveness of investment to a change in u. Similarly γ_2 and γ_3 indicate the responsiveness of investment due to a change in the rate of profit and the ratio of government investment to capital respectively. The positive effect of u is the static equivalent of the accelerator effect. The relationship between investment and the degree of capacity utilization and investment and the rate of profit has been discussed in the previous chapter. Here let us focus on the last variable in the investment function- the ratio of government investment expenditure to capital stock. Following Dutt (2013) and Taylor (1991) we can say that government investment expenditure has a positive impact on private investment because of a "crowding in[3]" effect.

Certain kinds of government investment expenditure (like expenditure on part of infrastructure, education and health facilities, water and electricity supply) have a positive impact on labour productivity as well. So we can say $a = a(\theta)$ and $a'(\theta) > 0$ i.e. labour productivity depends positively on the ratio of government investment to output[4].

[2]As long as the potential output-capital ratio is fixed, actual output-capital ratio can be used as a proxy for the degree of capacity utilization

[3]Government investment on infrastructure, education, water supply, health facilities etc boosts private investment through its complementary and other external effects. Again it raises the future profitability of private capital formation.

[4]Government investment expenditures like expenditure on health, roads, electricity and water supply have an impact on labour productivity within a fairly short period. On the other

3.1 Short-run equilibrium

In the short run, the goods market is cleared through changes in the level of output and capacity utilization.

In equilibrium, saving must be equal to investment.

$$\text{i.e. } \frac{I}{K} = \frac{S}{K}$$

$$\Rightarrow \gamma + \gamma_1 u + \gamma_2(1-t).\pi(a).u + \gamma_3.\theta.u = s(1-t).\pi(a).u$$

$$\Rightarrow u^* = \frac{\gamma}{(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta} \tag{3.8}$$

$u^*$is the equilibrium level of capacity utilization.

The equilibrium is stable if and only if the induced increase in saving as u rises is greater than the induced increase in investment i.e.

$$s(1-t)\pi(a) > \gamma_2(1-t)\pi(a) + \gamma_1 + \gamma_3\theta$$

$$\text{or, } (s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta > 0 \tag{3.9}$$

In other word, for the equilibrium to be stable the denominator of u^* must be positive.

Putting the equilibrium value of degree of capacity utilization in equation (3.7) we get the equilibrium value of growth rate as,

$$g^* = \gamma + \gamma_1 u^* + \gamma_2(1-t)\pi(a)u^* + \gamma_3\theta u^* \tag{3.10}$$

3.1.1 Comparative statics

In this subsection we will focus on the effect of a change in parameters on the equilibrium degree of capacity utilization.

Proposition 1 : An increase in either of s or π causes a fall in the equilibrium level of u while an increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3,$ and t leads to an increase in the equilibrium level of u.

hand, a few kind of government investment expenditures like expenditure on education affects labour productivity with a long time lag. For simplicity, we assume that government investment expenditure is mainly of the first type. This type of government expenditure has the potential to influence the current profitability of private investment and thus this is different from the crowding in effect where the future profitability of private capital formation is influenced.

Proof : Differentiating equation (3.8)with respect to s we get,

$$\frac{du^*}{ds} = \frac{-\gamma(1-t)\pi(a)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} < 0 \tag{3.11}$$

Differentiating equation (3.8)with respect to π we get,

$$\frac{du^*}{d\pi} = \frac{-\gamma(s-\gamma_2)(1-t)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} < 0 \tag{3.12}$$

Differentiating equation (3.8) with respect to γ we get,

$$\frac{du^*}{d\gamma} = \frac{1}{(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta} > 0 \tag{3.13}$$

Differentiating equation (3.8)with respect to γ_1 we get,

$$\frac{du^*}{d\gamma_1} = \frac{\gamma}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} > 0 \tag{3.14}$$

Differentiating equation (3.8) with respect to γ_2 we get,

$$\frac{du^*}{d\gamma_2} = \frac{\gamma(1-t)\pi(a)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} > 0 \tag{3.15}$$

Differentiating equation (3.8)with respect to γ_3 we get,

$$\frac{du^*}{d\gamma_3} = \frac{\gamma\theta}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} > 0 \tag{3.16}$$

Differentiating equation (3.8) with respect to t we get,

$$\frac{du^*}{dt} = \frac{\gamma(s-\gamma_2)\pi(a)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} > 0 \tag{3.17}$$

■■■

The equilibrium level of u increases with the level of autonomous investment γ. But it falls with a rise in the saving rate of the capitalists[5].

Here $\frac{du^*}{d\pi} < 0$ i.e the economy is in a stagnationist regime. This is because $s > \gamma_2$, i.e positive effect on investment demand due to rise in π is less than the negative effect on consumption demand due to rise in π.

As γ_1 increases, the accelerator effect of u on investment demand rises, which in turn raises aggregate demand and hence equilibrium level of u.

[5]This is the paradox of thrift(Rowthorn,1981).

As γ_2 increases, equilibrium level of u also increases. This is because, an increase in γ_2, for a given profit rate, leads to an increase in investment demand which in turn increases the aggregate demand and hence the degree of capacity utilization. Similarly, as γ_3 increases, for a given θ, investment demand increases which in turn raises the aggregate demand and hence the degree of capacity utilization.

The tax rate has a positive impact on the equilibrium degree of capacity utilization. This is mainly because of the balanced budget assumption. Per unit increase in tax rate reduces consumption for capitalists by $(1-s)\pi$ unit, while the consumption for workers decreases by $(1-\pi)$ unit. By reducing the after tax profit rate, it also reduces investment demand by $\gamma_2\pi$ unit. But the entire tax revenue is spent by the government and so the aggregate demand increases by one unit. As the increase in the government spending is higher than the reduction of consumption and investment demand[6] , an increase in the tax rate increases the equilibrium level of degree of capacity utilization.

A rise in t leads to an increase in the government consumption expenditure for a given θ because of the balanced budget assumption. Thus we get a positive relationship between government consumption expenditure and the equilibrium degree of capacity utilization. All these results are qualitatively similar to those obtained by Dutt (2013).

Next we focus on fiscal policy and its impact on aggregate demand and growth. Before we proceed further, let us discuss about the sign of $\frac{\partial g^*}{\partial u^*}, \frac{\partial g^*}{\partial \pi}, \frac{d\pi}{d\theta}, \frac{\partial g^*}{\partial \theta}, \frac{\partial u^*}{\partial \theta}, \frac{\partial u^*}{\partial \pi}$ which are essential for the proof of proposition 2, 3 and 4.

$$\frac{\partial g^*}{\partial u^*} = [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta] > 0$$

$$\frac{\partial g^*}{\partial \pi} = \gamma_2(1-t)u^* > 0$$

$$\frac{\partial g^*}{\partial \theta} = \gamma_3 u^* > 0$$

$$\frac{du^*}{d\pi} = \frac{\partial u^*}{\partial \pi} = \frac{-\gamma(s-\gamma_2)(1-t)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} < 0$$

$$\frac{\partial u^*}{\partial \theta} = \frac{\gamma\gamma_3}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} > 0$$

$$\frac{d\pi}{d\theta} = \frac{d\pi}{da}.\frac{da}{d\theta} > 0$$

6 as $(1-\pi) + (1-s)\pi + \gamma_2\pi < 1$

Proposition 2 : The effect of a switch in government expenditure from consumption to investment purposes on the equilibrium degree of capacity utilisation depends on the product of $\varepsilon_{\pi,a}$ *and* $\varepsilon_{a,\theta}$ *as follows:* $\frac{du^*}{d\theta} \gtreqless 0$ *according to whether* $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \psi$*; where* $\psi = \frac{\gamma_3\theta}{(s-\gamma_2)(1-t).\pi(a)}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ *and* $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$

Proof: Differentiating equation (3.8) with respect to θ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}\frac{d\pi}{da}\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

Putting the values of $\frac{\partial u^*}{\partial \pi}$ and $\frac{\partial u^*}{\partial \theta}$ on the above equation we get,

$$\frac{du^*}{d\theta} = \frac{-\gamma[(s-\gamma_2)(1-t)\frac{d\pi}{da}.\frac{da}{d\theta} - \gamma_3]}{[(s-\gamma_2)(1-t).\pi(a) - \gamma_1 - \gamma_3\theta]^2}$$

$$\Rightarrow \frac{du^*}{d\theta} = \frac{-\gamma\frac{\pi(a)}{\theta}[(s-\gamma_2)(1-t).\varepsilon_{\pi,a}.\varepsilon_{a,\theta} - \gamma_3\frac{\theta}{\pi(a)}]}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2}$$

So, $\frac{du^*}{d\theta} \gtrless 0$ according to whether $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \frac{\gamma_3\theta}{(s-\gamma_2)(1-t).\pi(a)} = \psi$

■■■

It follows that if the product of the elasticity of profit share with respect to labour productivity ($\varepsilon_{\pi.a}$) and the elasticity of labour productivity with respect to the ratio of government investment to output ($\varepsilon_{a,\theta}$) is greater (less) than a critical value (let's say ψ) then effect of θ on u^* is negative (positive). But when the product of $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$ is equal to the critical level ψ, then government consumption and government investment expenditure both have same degree of impact on the degree of capacity utilization and so a switch in government expenditure from consumption to investment does not raises the equilibrium degree of capacity utilization.

The economic intuition behind the result is that a rise in θ raises labour productivity which in turn raises profit share. Due to a rise in profit share investment demand rises. But on the other hand due to redistribution of income from wages to profits, consumption demand decreases. Given our assumptions, the latter effect dominates the previous one and so the rise in share of profit reduces the degree of capacity utilization[7]. On the other hand, a rise in θ directly raises the investment demand through the crowding-in effect leading to a rise in aggregate demand and the degree of capacity utilization.

[7]Bhaduri and Marglin(1990) named it as the stagnationist regime.

So the final impact of a change in θ on the degree of capacity utilization depends on the relative strength of the direct effect of θ on u and its indirect effect on u through the change in the share of profit. When the elasticities have lower values then a change in θ has lower impact on labour productivity and a change in labour productivity has lower impact on profit share. So the indirect effect of θ on u through the change in the profit share is comparatively lower and as a result the direct effect of θ on u dominates the indirect effect. On the other hand when the above elasticities have sufficiently high values then the indirect effect of θ on u dominates the direct effect and thus the impact of θ on u is negative.

Proposition 1 implies that the economy is always in the stagnationist regime. Although there is wage-led expansion in the economy, whether the growth is of wage-led or profit-led will be cleared from the following proposition.

Proposition 3 : In this stagnationist regime, only wage-led growth can be attained.

Proof : Differentiating equation (3.10) with respect to π we get,

$$\frac{dg^*}{d\pi} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\pi} + \frac{\partial g^*}{\partial \pi}$$

$$= \{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\frac{du^*}{d\pi} + \gamma_2(1-t)u^*$$

Putting the values of $\frac{du^*}{d\pi}$ and u^* in the above equation we get,

$$\frac{dg^*}{d\pi} = \{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\left\{\frac{-\gamma(s-\gamma_2)(1-t)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2}\right\}$$

$$+\gamma_2(1-t)\left\{\frac{\gamma}{(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta}\right\}$$

$$= \frac{-\gamma s(1-t)(\gamma_1 + \theta\gamma_3)}{[(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta]^2} < 0$$

$$\text{Thus } \frac{dg^*}{d\pi} < 0$$

Thus in our discussion, in the stagnationist regime we always get wage-led growth.

■□■

From proposition 2 we come to know the relationship between θ and the equilibrium degree of capacity utilization. Now in proposition 4 we will discuss about the impact of a change in θ on the equilibrium rate of capital accumulation.

Proposition 4 : The effect of a switch in government expenditure from consumption to investment purposes on the equilibrium rate of accumulation depends on the product of $\varepsilon_{\pi,a}$ ***and*** $\varepsilon_{a,\theta}$ ***as follows:*** $\frac{dg^*}{d\theta} \gtreqless 0$ ***accoding to whether*** $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \rho$ ***; where*** $\rho = \frac{\gamma_3\theta}{\gamma_1+\gamma_3\theta}$

Proof : Differentiating the equilibrium rate of capital accumulation with respect to θ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}.\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$= [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta]\frac{du^*}{d\theta} + \gamma_2(1-t)u^*\frac{d\pi}{d\theta} + \gamma_3 u^*$$

$$= [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta]\left[\frac{-\gamma[(s-\gamma_2)(1-t)\frac{d\pi}{d\theta} - \gamma_3]}{[(s-\gamma_2)(1-t).\pi(a) - \gamma_1 - \gamma_3\theta]^2}\right]$$

$$+ \left[\gamma_2(1-t)\frac{d\pi}{d\theta} + \gamma_3\right]\left[\frac{\gamma}{(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta}\right]$$

$$= \frac{s\gamma(1-t)\pi\left[\gamma_3\theta - (\gamma_1+\gamma_3\theta)\varepsilon_{\pi.a}.\varepsilon_{a,\theta}\right]}{\theta[(s-\gamma_2)(1-t).\pi(a) - \gamma_1 - \gamma_3\theta]^2}$$

$$\text{Thus } \frac{dg^*}{d\theta} \gtreqless 0 \text{ according to whether } \varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \frac{\gamma_3\theta}{\gamma_1+\gamma_3\theta} = \rho$$

■□■

θ can affect g in three ways: first its direct effect on g which we call "crowding in" effect; second, its effect through the share of profit and finally its effect through u^*. So even if θ has a negative effect on u^*, if the positive effect of θ on the share of profit and its direct effect on g^* ("crowding in" effect) is very high, then these two can more than compensate the negative effect. That is when $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} > \psi$, $\frac{du^*}{d\theta} < 0$. But, as $\rho > \psi$, if $\psi < \varepsilon_{\pi.a}.\varepsilon_{a,\theta} < \rho$ then $\frac{dg^*}{d\theta} > 0$ and if $\rho < \varepsilon_{\pi.a}.\varepsilon_{a,\theta}$ then $\frac{dg^*}{d\theta} < 0$. Of course when the effect of θ on u^* is positive then due to rise in θ, g^* rises unambiguously. That is when $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} < \psi$, $\frac{du^*}{d\theta} > 0$. And as $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} < \psi$, so $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} < \rho$ and then $\frac{dg^*}{d\theta} > 0$.

3.1.2 A more general case: when workers also save

So far our analysis has been based on the assumption that workers do not save. Now we drop the assumption to observe whether there is any qualitative change in term of the results in the previous section. Let's assume workers save a fraction s_W of their wage income. We also assume capitalists saving propensity (s_P) is higher than that of workers.

So total private saving in the economy is given by,

$$S = s_P(1-t)P + s_W(1-t)W$$

$$\Rightarrow \frac{S}{K} = (s_P - s_W)(1-t)r + s_W(1-t)u \tag{3.18}$$

The economy is in equilibrium when,

$$\frac{I}{K} = \frac{S}{K}$$

$$\Rightarrow \gamma + \gamma_1 u + \gamma_2(1-t)\pi(a)u + \gamma_3\theta u = (1-t)(s_P - s_W)\pi(a)u + s_W(1-t)u$$

$$\Rightarrow u^* = \frac{\gamma}{(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta} \quad . \tag{3.19}$$

For stability, induced increase in saving as u rises must be greater than the induced increase in investment. That is, we require,

$$(s_P - s_W)(1-t)\pi(a) + s_W(1-t) > \gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta \tag{3.20}$$

$$\text{or, } (s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta > 0 \tag{3.21}$$

In other words, we need the denominator of u^* to be positive.

So the stability condition can be satisfied if

$$s_W > \frac{\gamma_1 + \gamma_3\theta - (s_P - \gamma_2)(1-t)\pi}{(1-t)(1-\pi)} \tag{3.22}$$

The following proposition due to Blecker (2002) provides the sufficient condition for the economy to be in a stagnationist regime or in an exhilarationist regime.

Proposition 5 : ***Whether the economy is in a stagnationist regime or in an exhilarationist regime depends on the value of*** s_W ***as follows : (i) if*** $s_W < (s_P - \gamma_2)$ ***then the economy is in a stagnationist regime and (ii) if*** $s_W > (s_P - \gamma_2)$ ***then the economy is in an exhilarationist regime.***

Proof : Differentiating the equilibrium level of u^* with respect to π we get,

$$\frac{du^*}{d\pi} = \frac{-\gamma(s_P - s_W - \gamma_2)(1-t)}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

$$\text{Thus if } s_W < (s_P - \gamma_2) \text{ then } \frac{du^*}{d\pi} < 0 \tag{3.23}$$

$$\text{And if } s_W > (s_P - \gamma_2) \text{ then } \frac{du^*}{d\pi} > 0 \tag{3.24}$$

■□■

If the saving propensity out of wages is high enough then due to redistribution of income from workers to capitalists, consumption demand falls by $(s_P - s_W)$ per unit income transferred from wages to profits. On the other hand, due to increase in profitability, investment demand rises by γ_2 per unit increase in profits. As (3.24) shows, if the latter effect dominates the former, the equilibrium degree of capacity utilization rises due to a rise in the share of profit. Following a similar argument, we can say that when $(s_P - s_W - \gamma_2) > 0$ then the economy is in a stagnationist regime (i.e. $\frac{du^*}{d\pi} < 0$). So depending on the sign of $(s_P - s_W - \gamma_2)$ the economy may be either in a stagnationist or in an exhilarationist regime.

Now we will analyse the impact of a rise in θ on u^* and g^*. Before we proceed further, let us discuss about the sign of $\frac{\partial g^*}{\partial u^*}, \frac{\partial g^*}{\partial \pi}, \frac{d\pi}{d\theta}, \frac{\partial g^*}{\partial \theta}, \frac{\partial u^*}{\partial \theta}, \frac{\partial u^*}{\partial \pi}$.

$$\frac{\partial g^*}{\partial u^*} = [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta] > 0$$

$$\frac{\partial g^*}{\partial \pi} = \gamma_2(1-t)u^* > 0$$

$$\frac{\partial g^*}{\partial \theta} = \gamma_3 u^* > 0$$

$$\frac{du^*}{d\pi} = \frac{\partial u^*}{\partial \pi} = \frac{-\gamma(s_P - s_W - \gamma_2)(1-t)}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

$$\text{So, } \frac{du^*}{d\pi} \gtreqless 0 \text{ according to whether } (s_P - \gamma_2) \lesseqgtr s_W$$

$$\frac{\partial u^*}{\partial \theta} = \frac{\gamma\gamma_3}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2} > 0$$

$$\frac{d\pi}{d\theta} = \frac{d\pi}{da}.\frac{da}{d\theta} > 0$$

Let's discuss the effect of a rise in θ on u^* first.

Proposition 6 : When workers also save, an increase in θ leads to an unambiguous increase in the equilibrium degree of capacity utilisation in the exhilarationist regime, while in the stagnationist regime, the effect of a rise in θ on the equilibrium degree of capacity utilisation depends on the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows: $\frac{du^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \lesseqgtr \psi'$; where $\psi' = \frac{\gamma_3\theta}{(s_P - s_W - \gamma_2)(1-t)\pi(a)}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.

Proof: Let's discuss the exhilarationist regime first.

Differentiating the quilibrium degree of capacity utilisation with respect to θ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

If the economy is in exhilarationist regime then $\frac{\partial u^*}{\partial \pi} > 0$. $\frac{d\pi}{d\theta}$ and $\frac{\partial u^*}{\partial \theta}$ are also positive. So, $\frac{du^*}{d\theta}$ is unambiguously positive.

Now suppose the economy is in stagnationist regime.Then,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

$$= \left\{ \frac{-\gamma(s_P - s_W - \gamma_2)(1-t)}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2} \right\} \frac{d\pi}{d\theta} + \left\{ \frac{\gamma\gamma_3}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2} \right\}$$

$$= \frac{-\gamma\left[(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{da}.\frac{da}{d\theta} - \gamma_3\right]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

$$= \frac{-\gamma\frac{\pi(a)}{\theta}\left[(s_P - s_W - \gamma_2)(1-t)\varepsilon_{\pi,a}.\varepsilon_{a,\theta} - \gamma_3\frac{\theta}{\pi(a)}\right]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

So, $\frac{du^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \lesseqgtr \frac{\gamma_3\theta}{(s_P - s_W - \gamma_2)(1-t)\pi(a)} = \psi'$

■□■

Above proposition implies that while in the exhilarationist regime an increase in θ always has a positive impact on the equilibrium degree of capacity utilization, in the stagnationist regime the effect of a rise in θ on u^* depends on some critical value of the product of $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$. If the product of $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$ exceeds (less than) the critical value (let's say ψ') then the effect of a rise in θ on $u^*$is negative (positive). In that sense there is no qualitatively difference from the situation where workers do not save. Only the critical value changes.

In propositions 7 and 8 we discuss the impact of a change in θ on the equilibrium rate of capital accumulation.

Proposition 7 : When workers also save, an increase in θ leads to an unambiguous increase in the equilibrium rate of accumulation in the exhilarationist regime.

Proof : Differentiating the equilibrium rate of accumulation with respect to θ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}.\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$\frac{\partial g^*}{\partial u^*}, \frac{\partial g^*}{\partial \pi}, \frac{d\pi}{d\theta}, \frac{\partial g^*}{\partial \theta} \text{ are all positive.}$$

From Proposition 6 we know that when the economy is in exhilarationist regime $\frac{du^*}{d\theta}$ is positive.

$$\text{So, } \frac{dg^*}{d\theta} > 0$$

Proposition 8: *When workers also save, in the stagnationist regime the effect of a switch in government expenditure from consumption to investment purposes on the equilibrium rate of accumulation depends on the product of* $\varepsilon_{\pi,a}$ *and* $\varepsilon_{a,\theta}$ *as follows:* $\frac{dg^*}{d\theta} \gtreqless 0$ *accoding to whether* $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \rho'$ *; where* $\rho' = \frac{\{(s_P - s_W)\pi + s_W\}\gamma_3\theta}{\pi\{(s_P - s_W)(\gamma_1+\gamma_3\theta) - s_W\gamma_2(1-t)\}}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ *and* $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.

Proof : Differentiating the equilibrium rate of capital accumulation with respect to θ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}.\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$= [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta]\frac{du^*}{d\theta} + \gamma_2(1-t)u^*\frac{d\pi}{d\theta} + \gamma_3 u^*$$

$$= [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta]\left[\frac{-\gamma[(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{d\theta} - \gamma_3]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}\right]$$

$$+ \left[\gamma_2(1-t)\frac{d\pi}{d\theta} + \gamma_3\right]\left[\frac{\gamma}{(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta}\right]$$

$$= \frac{\gamma(1-t)\pi\left[\{(s_P - s_W)\pi + s_W\}\gamma_3\frac{\theta}{\pi} - \{(\gamma_1+\gamma_3\theta)(s_P - s_W) - s_W\gamma_2(1-t)\}\varepsilon_{\pi.a}.\varepsilon_{a,\theta}\right]}{\theta[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

Thus $\frac{dg^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \frac{\{(s_P - s_W)\pi + s_W\}\gamma_3\theta}{\pi\{(s_P - s_W)(\gamma_1+\gamma_3\theta) - s_W\gamma_2(1-t)\}} = \rho'$

■■■

So when the economy is in an exhilarationist regime, due to a rise in θ, the equilibrium rate of capital accumulation unambiguously rises. But when the economy is in a stagnationist regime, the effect of a rise in θ on g^* depends on the value of the product of $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$. If the product of $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$ exceeds (less than) the critical value (let's say ρ') then the effect of a rise in θ on g^* is negative (positive). In that sense there is no qualitatively difference from the situation where workers do not save. Only the critical value changes.

3.2 Issues regarding the employment rate

In this section we focus on the employment rate in the economy.

Equilibrium level of employment rate e^* can be written as:

$$e^* = \frac{L}{N} = \frac{Y}{K}\frac{L}{Y}\frac{K}{N} = u^*\frac{1}{a}k_0 = u^*k \tag{3.25}$$

where, N is the total supply of labour which is fixed in the short-run, k_0 is the ratio of capital stock to total supply of labour and $k(=\frac{K}{aN})$ is the ratio of capital stock to the productive labour supply.

Proposition 9 : An increase in s causes a fall in the equilibrium level of employment rate while an increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3,$ ***and t leads to an increase in the*** e^*.

Proof : We know,

$$e^* = u^*k \tag{3.26}$$

Differentiating equation (3.25) with respect to s we get,

$$\frac{de^*}{ds} = k\frac{du^*}{ds} < 0$$

Similarly differentiating equation (3.25) with respect to $\gamma, \gamma_1, \gamma_2, \gamma_3, t$ we get,

$$\frac{de^*}{d\gamma} = k\frac{du^*}{d\gamma} > 0$$

$$\frac{de^*}{d\gamma_1} = k\frac{du^*}{d\gamma_1} > 0$$

$$\frac{de^*}{d\gamma_2} = k\frac{du^*}{d\gamma_2} > 0$$

$$\frac{de^*}{d\gamma_3} = k\frac{du^*}{d\gamma_3} > 0$$

$$\frac{de^*}{dt} = k\frac{du^*}{dt} > 0$$

■■■

As in the short-run K and N both are fixed, k_0, which is the ratio of the capital stock to the labour supply, is also fixed in the short-run. Then as long as the labour productivity is not influenced by any change in parameters, k is also fixed.

Thus a change in any parameter which does not have an impact on a, can change the equilibrium rate of employment only through change in u^*.

Proposition 10 : *The effect of a rise in θ on the equilibrium level of employment rate depends on $\varepsilon_{\pi.a}$ and $\varepsilon_{a,\theta}$ as follows : $\frac{de^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi.a} \lesseqgtr \frac{\gamma_3\theta+(\gamma_1+\gamma_3\theta)\varepsilon_{a,\theta}}{(s-\gamma_2)(1-t).\pi(a)\varepsilon_{a,\theta}} - 1$, where $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.*

Proof : We know,

$$e^* = u^*.\frac{1}{a}.k_0$$

Differentiating it with respect to θ we get,

$$\frac{de^*}{d\theta} = \frac{\partial e^*}{\partial u^*}.\frac{du^*}{d\theta} + \frac{\partial e^*}{\partial a}.\frac{da}{d\theta}$$

$$\Rightarrow \frac{de^*}{d\theta} = \frac{k_0}{a}.\frac{du^*}{d\theta} - \frac{u^* k_0}{a^2}\frac{da}{d\theta}$$

Now putting the value of $\frac{du^*}{d\theta}$ and u^* in this equation we get,

$$\Rightarrow \frac{de^*}{d\theta} = \frac{k_0}{a}\left\{\frac{-\gamma[(s-\gamma_2)(1-t)\frac{d\pi}{da}.\frac{da}{d\theta} - \gamma_3]}{[(s-\gamma_2)(1-t).\pi(a) - \gamma_1 - \gamma_3\theta]^2}\right\} - \frac{k_0}{a^2}\left\{\frac{\gamma}{(s-\gamma_2)(1-t)\pi(a) - \gamma_1 - \gamma_3\theta}\right\}\frac{da}{d\theta}$$

$$\Rightarrow \frac{de^*}{d\theta} = \nabla\left[\left\{(s-\gamma_2)(1-t)\left(a\frac{d\pi}{d\theta} + \pi\frac{da}{d\theta}\right)\right\} - \left\{(\gamma_1+\gamma_3\theta)\frac{da}{d\theta} + \gamma_3 a\right\}\right]$$

$$\Rightarrow \frac{de^*}{d\theta} = \nabla\left[\frac{\pi a}{\theta}\left\{(s-\gamma_2)(1-t)\left(\frac{d\pi}{da}\frac{a}{\pi}\frac{da}{d\theta}\frac{\theta}{a} + \frac{\theta}{a}\frac{da}{d\theta}\right)\right\} - \frac{a}{\theta}\left\{(\gamma_1+\gamma_3\theta)\frac{da}{d\theta}\frac{\theta}{a} + \gamma_3\theta\right\}\right]$$

$$\Rightarrow \frac{de^*}{d\theta} = \nabla\left[\frac{\pi a}{\theta}\{(s-\gamma_2)(1-t)\left(\varepsilon_{\pi.a}.\varepsilon_{a,\theta} + \varepsilon_{a,\theta}\right)\} - \frac{a}{\theta}\{(\gamma_1+\gamma_3\theta)\varepsilon_{a,\theta} + \gamma_3\theta\}\right]$$

$$\text{where, } \nabla = -\frac{k_0\gamma}{[a\{(s-\gamma_2)(1-t).\pi(a) - \gamma_1 - \gamma_3\theta\}]^2} < 0$$

$$\text{Thus } \frac{de^*}{d\theta} \gtreqless 0 \text{ according to whether } \varepsilon_{\pi.a} \lesseqgtr \frac{\gamma_3\theta + (\gamma_1+\gamma_3\theta)\varepsilon_{a,\theta}}{(s-\gamma_2)(1-t).\pi(a)\varepsilon_{a,\theta}} - 1$$

■■■

From the above proposition, it follows that when the impact of a change in θ on the equilibrium degree of capacity utilisation is negative, the equilibrium employment rate unambiguously falls due to a rise in θ. On the other hand, when $\frac{du^*}{d\theta} < 0$, the effect of a change in θ on the equilibrium employment rate is ambiguous. A change in θ affects the equilibrium employment rate in two ways. First, it has a positive impact on labour productivity which itself in turn leads to reduction of employment rate. Second, its impact on employment rate through the change in the equilibrium degree of capacity utilisation. The effect of θ on u^*has already been discussed in proposition 2. Thus the final effect of a change in θ on the equilibrium employment rate depends on its effect on u^* and labour productivity.

3.3 Effect of government deficits and debt

So far we have assumed that the government budget is balanced and there is no government debt. Now we will relax the assumption. Let's assume there is a budget deficit and the government incurs debt.

Let's assume that the aggregate government tax revenue is given by

$$T = t(Y + iD) \tag{3.27}$$

where t is the tax rate, Y is the real aggregate productive income, i is the interested rate that is paid by the government, and D is the real stock of government debt.

In this section we assume that government consumption expenditure and government investment expenditure depend on the income level of the economy.

Let us assume that current government consumption expenditure is now given by

$$C_G = \eta Y \tag{3.28}$$

Investment expenditure is given by

$$I_G = \theta Y \tag{3.29}$$

For the sake of simplicity we ignore monetary and other assets. The entire government deficit is financed by issuing government debt. So, the change in debt with respect to time is given by,

$$\frac{dD}{d\tau} = (C_G + I_G) - T + iD \tag{3.30}$$

Aggregate private saving in the economy is given by,

$$S = s(1 - t)(P + iD)$$

So, the ratio of saving to capital stock is,

$$\frac{S}{K} = s(1 - t)(r + i\delta) \tag{3.31}$$

where $\delta = \frac{D}{K} =$ debt-capital ratio.

Investment function is given by,

$$I = [\gamma + \gamma_1 u + \gamma_2(1-t)r + \gamma_3(\frac{I_G}{K}) - \gamma_4\delta]K$$

$$\text{So, } \frac{I}{K} = [\gamma + \gamma_1 u + \gamma_2(1-t).\pi(a).u + \gamma_3.\theta.u - \gamma_4\delta] \tag{3.32}$$

where γ_4 is the coefficient measuring responsiveness of investment due to change in δ.

Here the fifth term entering in the investment function, represents the financial crowding out effect[8].

Total government expenditure is represented by

$$G = C_G + I_G$$

So the ratio of government expenditure to capital is

$$\frac{G}{K} = (\eta + \theta)u \tag{3.33}$$

The ratio of tax revenue to capital is

$$\frac{T}{K} = t(u + i\delta) \tag{3.34}$$

3.3.1 The short-run analysis

In the short run equilibrium, the following equation must be satisfied,

$$\frac{S}{K} + \frac{T}{K} = \frac{I}{K} + \frac{G}{K} \tag{3.35}$$

Substituting from equations (3.43), (3.44), (3.45) and (3.46) in (3.47) we get the equilibrium degree of capacity utilization as

$$u^* = \frac{\gamma - [\{s(1-t)+t\}i + \gamma_4]\delta}{(s-\gamma_2)(1-t)\pi + t - \gamma_1 - \theta(1+\gamma_3) - \eta} \tag{3.36}$$

[8]We introduce it to show that, even allowing the neo-classical argument of financial crowding-out of private investment due to rise in public debt, when we introduce government deficits and the dynamics of the government debt into our analysis, the model does not necessarily become unstable and δ does not rises without bound. We also will show that our long-run result differs from Dutt (2013) as here are two equilibrium values of δ and the smaller one represents the stable equilibrium value while in Dutt (2013) this is not necessarily the case.

The equilibrium is stable when the induced increase in private savings and revenue income as u rises must be greater than the induced increase in private investment and government expenditure. That is when the following equation is satisfied :

$$s(1-t)\pi + t > \gamma_1 + \gamma_2(1-t)\pi + \gamma_3\theta + (\eta + \theta) \tag{3.37}$$

$$\text{That is, } (s-\gamma_2)(1-t)\pi + t - \gamma_1 - \theta(1+\gamma_3) - \eta > 0 \tag{3.38}$$

In other word, for the equilibrium to be stable the denominator of u^* must be positive.

But for a meaningful positive equilibrium degree of capacity utilization the numerator also should be positive. So we need,

$$\gamma - [\{s(1-t)+t\}i + \gamma_4]\delta > 0$$

$$\text{That is, } \gamma > [\{s(1-t)+t\}i + \gamma_4]\delta \tag{3.39}$$

Putting the equilibrium value of degree of capacity utilization in equation (3.44) we get the equilibrium value of growth rate as,

$$g^* = \gamma + \gamma_1 u^* + \gamma_2(1-t)\pi(a)u^* + \gamma_3\theta u^* - \gamma_4\delta \tag{3.40}$$

3.3.1.1 Comparative statics

Now we will focus on the effect of a change in model parameters on the equilibrium degree of capacity utilization.

Proposition 11 : An increase in either of $s, \delta, i, \gamma_4, t$ ***causes a fall in the equilibrium level of*** u ***while an increase in either of*** $\gamma, \gamma_1, \gamma_2, \gamma_3,$ ***and*** η ***leads to an increase in the equilibrium level of*** u***.***

Proof : Differentiating equation (3.36) with respect to s we get,

$$\frac{du^*}{ds} = \frac{-[(1-t)i\delta\Lambda + (1-t)\pi(\gamma - \Gamma\delta)]}{\Lambda^2} < 0$$

where, $\Gamma = \{s(1-t)+t\}i+\gamma_4 > 0$ and $\Lambda = (s-\gamma_2)(1-t)\pi+t-\gamma_1-\theta(1+\gamma_3)-\eta > 0$

Differentiating equation (3.36) with respect to δ we get,

$$\frac{du^*}{d\delta} = \frac{-\Gamma}{\Lambda} < 0$$

Differentiating equation (3.36) with respect to i we get,

$$\frac{du^*}{di} = \frac{-\{s(1-t)+t\}\delta}{\Lambda} < 0$$

Differentiating equation (3.36) with respect to γ_4 we get,

$$\frac{du^*}{d\gamma_4} = \frac{-\delta}{\Lambda} < 0$$

Differentiating equation (3.36) with respect to t we get,

$$\frac{du^*}{dt} = \frac{-\left[\Lambda(1-s)i\delta + (\gamma - \Gamma\delta)\{1-(s-\gamma_2)\pi\}\right]}{\Lambda^2} < 0$$

Differentiating equation (3.36) with respect to γ we get,

$$\frac{du^*}{d\gamma} = \frac{1}{\Lambda} > 0$$

Differentiating equation (3.36) with respect to γ_1 we get,

$$\frac{du^*}{d\gamma_1} = \frac{\gamma - \Gamma\delta}{\Lambda^2} > 0$$

Differentiating equation (3.36) with respect to γ_2 we get,

$$\frac{du^*}{d\gamma_2} = \frac{(\gamma - \Gamma\delta)(1-t)\pi}{\Lambda^2} > 0$$

Differentiating equation (3.36) with respect to γ_3 we get,

$$\frac{du^*}{d\gamma_3} = \frac{(\gamma - \Gamma\delta)\theta}{\Lambda^2} > 0$$

Differentiating equation (3.36) with respect to η we get,

$$\frac{du^*}{d\eta} = \frac{(\gamma - \Gamma\delta)}{\Lambda^2} > 0$$

In the short run an increase in δ decreases the equilibrium degree of capacity utilization. Due to one unit increase in δ, the ratio of private saving to capital stock increases by $s(1-t)i$ units while the ratio of government revenue income to capital stock increases by it units. Thus due to one unit increase in δ, consumption demand decreases by $\{s(1-t)+t\}iK$ unit. On the other hand due to one unit rise in δ , investment demand decreases by $\gamma_4 K$ unit.[9] Thus aggregate demand and hence the equilibrium degree of capacity utilization decreases.

An increase in γ_4 , for a given δ, decreases investment through the crowding-out effect which in turn decreases the aggregate demand and hence the equilibrium degree of capacity utilization. An increase in i , for a given δ, increases private saving by $s(1-t)\delta$ unit while government revenue income increases by $t\delta$ unit. Thus due to one unit increase in i, consumption demand decreases by $\{s(1-t)+t\}\delta K$ unit. Hence agrregate demand and so the degree of capacity utilization falls.

A rise in η , the ratio of government consumption expenditure to output, increases the aggregate demand and the equilibeium degree of capacity utilization.

Here we will focus on the effect of θ on u^*. Before we proceed further, let us discuss about the sign of $\frac{\partial g^*}{\partial u^*}, \frac{\partial g^*}{\partial \pi}, \frac{d\pi}{d\theta}, \frac{\partial g^*}{\partial \theta}, \frac{\partial u^*}{\partial \theta}, \frac{\partial u^*}{\partial \pi}$ which are essential for the proof of proposition 12, 13 and 14.

$$\frac{\partial g^*}{\partial u^*} = [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta] > 0$$

$$\frac{\partial g^*}{\partial \pi} = \gamma_2(1-t)u^* > 0$$

$$\frac{\partial g^*}{\partial \theta} = \gamma_3 u^* > 0$$

$$\frac{du^*}{d\pi} = \frac{\partial u^*}{\partial \pi} = \frac{-(\gamma - \Gamma\delta)(s-\gamma_2)(1-t)}{\Lambda^2} < 0$$

$$\frac{\partial u^*}{\partial \theta} = \frac{(\gamma - \Gamma\delta)(1+\gamma_3)}{\Lambda^2} > 0$$

$$\frac{d\pi}{d\theta} = \frac{d\pi}{da}.\frac{da}{d\theta} > 0$$

Prposition 12 : *The effect of a rise in θ on the equilibrium degree of capacity utilisation depends on the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows: $\frac{du^*}{d\theta} \gtrless 0$ according to whether $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \psi''$; where $\psi'' = \frac{(1+\gamma_3)\theta}{(s-\gamma_2)(1-t).\pi}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.*

Proof: Differentiating equation (3.36) with respect to θ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}\frac{d\pi}{da}\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

[9] This is because of the financial crowding out effect.

Putting the values of $\frac{\partial u^*}{\partial \pi}$ and $\frac{\partial u^*}{\partial \theta}$ on the above equiation we get,

$$= \frac{-[\gamma - \Gamma\delta]\left[(s-\gamma_2)(1-t)\frac{d\pi}{da}.\frac{da}{d\theta} - (1+\gamma_3)\right]}{\Lambda^2}$$

$$= \frac{-[\gamma - \Gamma\delta]\frac{\pi}{\theta}\left[(s-\gamma_2)(1-t).\varepsilon_{\pi,a}.\varepsilon_{a,\theta} - \frac{(1+\gamma_3)\theta}{\pi}\right]}{\Lambda^2}$$

$$\text{So, } \frac{du^*}{d\theta} \gtreqless 0 \text{ according to whether } \varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \frac{(1+\gamma_3)\theta}{(s-\gamma_2)(1-t).\pi} = \psi''$$

■■■

It follows that if the product of the elasticity of profit share with respect to labour productivity ($\varepsilon_{\pi.a}$) and the elasticity of labour productivity with respect to the ratio of government investment to output ($\varepsilon_{a,\theta}$) is greater than (less than) a critical value (let's say ψ'') then the effect of θ on u^* is negative (positive).

A rise in θ raises labour productivity which in turn raises profit share. In the stagnationist regime a change in proft share has a negative impact on the equilibrium degree of capacity utilization. On the other hand, a rise in θ directly raises the investment demand through the crowding-in effect leading to a rise in aggregate demand and the degree of capacity utilization. So the final impact of a change in θ on the degree of capacity utilization depends on the relative strength of the direct effect of θ on u and its indirect effect on u through the change in the share of profit. When the elasticities have lower values the indirect effect of θ on u through the change in the profit share is comparatively lower and as a result the direct effect of θ on u dominates the indirect effect. On the other hand when the above elasticities have sufficiently high values then the indirect effect of θ on u dominates the direct effect and thus the impact of θ on u is negative.

Let's focus on the effect of θ on g^* now.

Prposition 13 : The effect of a rise in θ on the equilibrium rate of accumulation depends on the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows: $\frac{dg^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \rho''$; where $\rho'' = \frac{\theta\{\gamma_1+\gamma_3(t-\eta)+(1-t)\pi(\gamma_2+s\gamma_3)\}}{(1-t)\pi\{s(\gamma_1+\theta\gamma_3)-(t-\eta-\theta)\gamma_2\}}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.

Proof : Differentiating the equilibrium rate of capital accumulation with respect to θ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}.\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}.\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$= \{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\frac{du^*}{d\theta} + \gamma_2(1-t)u^*\frac{d\pi}{d\theta} + \gamma_3 u^*$$

$$= \{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\left[\frac{-(\gamma - \Gamma\delta)\left\{(s-\gamma_2)(1-t)\frac{d\pi}{d\theta} - (1+\gamma_3)\right\}}{\Lambda^2}\right]$$

$$+\left\{\gamma_2(1-t)\frac{d\pi}{d\theta} + \gamma_3\right\}\left\{\frac{(\gamma - \Gamma\delta)}{\Lambda}\right\}$$

$$= \frac{(\gamma - \Gamma\delta)}{\Lambda^2}\left[-\{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\left\{(s-\gamma_2)(1-t)\frac{d\pi}{d\theta} - (1+\gamma_3)\right\}\right]$$

$$+\frac{(\gamma - \Gamma\delta)}{\Lambda^2}\left[\left\{\gamma_2(1-t)\frac{d\pi}{d\theta} + \gamma_3\right\}\{(s-\gamma_2)(1-t)\pi + t - \gamma_1 - \theta(1+\gamma_3) - \eta\}\right]$$

$$= \frac{(\gamma - \Gamma\delta)}{\Lambda^2}\frac{\pi}{\theta}\left[\{\gamma_1 + \gamma_3(t-\eta) + (1-t)\pi(\gamma_2 + s\gamma_3)\}\frac{\theta}{\pi}\right]$$

$$-\frac{(\gamma - \Gamma\delta)}{\Lambda^2}\frac{\pi}{\theta}\left[(1-t)\varepsilon_{\pi.a}.\varepsilon_{a,\theta}\{s(\gamma_1 + \theta\gamma_3) - (t - \eta - \theta)\gamma_2\}\right]$$

So, $\frac{dg^*}{d\theta} \gtreqless 0$ according to whether $\varepsilon_{\pi.a}.\varepsilon_{a,\theta} \lesseqgtr \frac{\theta\{\gamma_1 + \gamma_3(t-\eta) + (1-t)\pi(\gamma_2 + s\gamma_3)\}}{(1-t)\pi\{s(\gamma_1 + \theta\gamma_3) - (t-\eta-\theta)\gamma_2\}} = \rho''$

■□■

Now we will focus on the effect of δ on g^*.

Proposition 14: An increase in δ decreases the equilibrium value of the rate of accumulation.

Proof: Differentiating equation (3.40) with respect to δ we get,

$$\frac{dg^*}{d\delta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\delta} + \frac{\partial g^*}{\partial \delta}$$

$$= -[\{\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta\}\frac{\Gamma}{\Lambda} + \gamma_4] < 0$$

■□■

An increase in δ decreases the equilibrium level of g in two ways : (1) directly through financial crowding-out effect and (2) indirectly through decrease in the equilibrium degree of capacity utilization. Its (δ) effect on u^* has been discussed in proposition 11. A rise in debt-capital ratio due to the financial crowding out effect, directly leads to a fall in the equilibrium growth rate. So, the effect of a change in δ on the equilibrium growth rate is negative.

3.3.2 The long-run analysis

Now in this section, we will analyse the long-run dynamics of the government debt and the capital stock. We will say that long-run equilibrium is attained when the government debt-capital ratio (δ) remains constant over time.

$$\text{We know, } \delta = \frac{D}{K}$$

$$\text{So, } \hat{\delta} = \hat{D} - \hat{K}$$

From equation (3.30) we get,

$$\frac{\frac{dD}{d\tau}}{D} = \frac{1}{D}[(C_G + I_G) - T + iD]$$

$$\Rightarrow \hat{D} = (\eta + \theta - t)\frac{u^*}{\delta} + i(1-t)$$

$$\text{Now, } \hat{\delta} = \hat{D} - \hat{K}$$

$$\Rightarrow \hat{\delta} = (\eta + \theta - t)\frac{u^*}{\delta} + i(1-t) - g^*$$

$$\Rightarrow \frac{d\delta}{d\tau} = (\eta + \theta - t)u^* + i(1-t)\delta - g^*\delta$$

$$\Rightarrow \frac{d\delta}{d\tau} = \frac{(\eta+\theta-t)\gamma}{\Lambda} + \left[-\frac{(\eta+\theta-t)\Gamma}{\Lambda} + i(1-t) - \gamma - \frac{\{\gamma_1 + \gamma_2(1-t)\pi + \gamma_3\theta\}\gamma}{\Lambda}\right]\delta$$
$$+ \left[\frac{\{\gamma_1 + \gamma_2(1-t)\pi + \gamma_3\theta\}\Gamma}{\Lambda} + \gamma_4\right]\delta^2$$

$$\Rightarrow \frac{d\delta}{d\tau} = D_0 + D_1\delta + D_2\delta^2$$

Here $D_2 > 0$. Let's assume $D_0 > 0$. This assumption ensures that even when $\delta = 0$, the government runs a deficit and so δ increases over time.

In Dutt (2013) both of D_1 and D_2 don't have any definite sign. So various possibilities regarding $\frac{d\delta}{d\tau}$ may occur depending on the sign of D_1 and D_2. But in our analysis D_2 is unambiguously positive. Then depending on the sign of D_1, δ can either have a stable equilibrium value or it can rise without bound.

Now let us discuss the conditions for existence and stability of equilibrium.

If the interest rate (i) is not too high (if $i < \frac{\frac{(\eta+\theta-t)\Gamma}{\Lambda} + \gamma + \frac{\{\gamma_1+\gamma_2(1-t)\pi+\gamma_3\theta\}\gamma}{\Lambda}}{(1-t)}$) then D_1

can have a negative value. Then the change in debt-capital ratio with respect to δ would be "U" shaped and there is a possibility of existence of equilibrium.

Then the necessary and sufficient condition for existence of equilibrium is : minimum value of $\frac{d\delta}{d\tau}$ must be ≤ 0. Minimum value of $\frac{d\delta}{d\tau}$ can be attained at $\delta = -\frac{D_1}{2D_2}$. Then the minimum value of $\frac{d\delta}{d\tau} = D_0 - \frac{D_1^2}{4D_2}$. Thus the necessary and sufficient condition for existence of equilibrium is $\left(D_0 - \frac{D_1^2}{4D_2}\right) \leq 0$.

The necessary and sufficient condition for existence of a stable equilibrium is : minimum value of $\frac{d\delta}{d\tau}$ must be < 0. Thus $\left(D_0 - \frac{D_1^2}{4D_2}\right) < 0$ ensures the necessary and sufficient condition for existence of a stable equilibrium.

Three different diagrams are given below. In figure 1 there is no equilibrium. Figure 2 represents existence of unique but unstable equilibrium. Figure 3 represents existence of multiple equilibria where one of them [10]is stable.

But if $i \geq \frac{\frac{(\eta+\theta-t)\Gamma}{\Delta}+\gamma+\frac{\{\gamma_1+\gamma_2(1-t)\pi+\gamma_3\theta\}\gamma}{\Delta}}{(1-t)}$ then δ increases without bound. In diagram 2 there is only one equilibrium value of δ which is unstable.

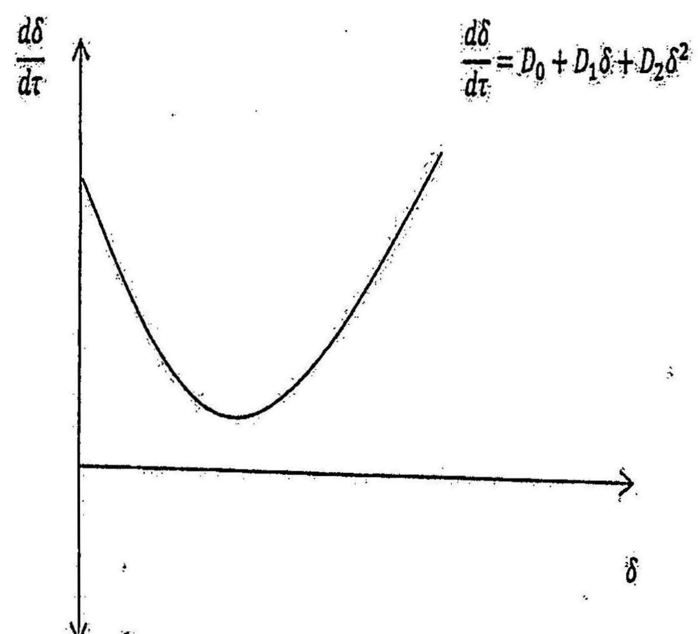

Figure-1

[10]Note that between those two equilibrium values of δ , the low equilibrium value of δ (i.e. δ^*) gives the stable equilibrium.

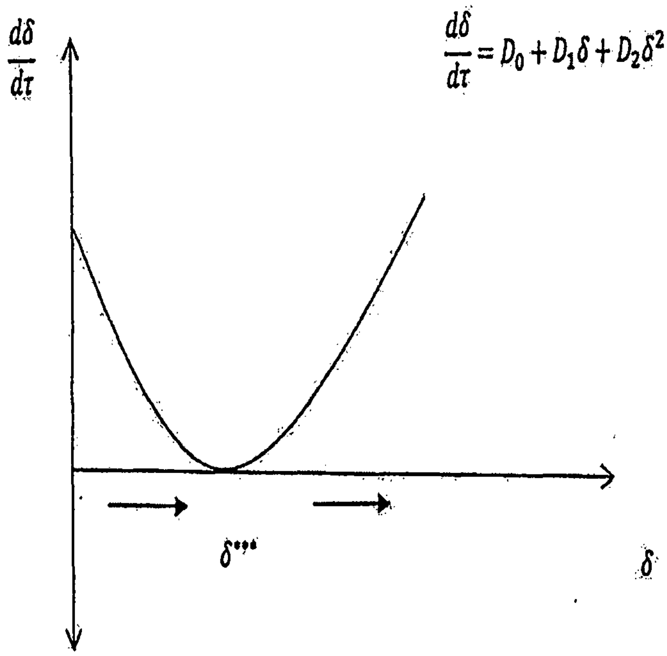

Figure-2

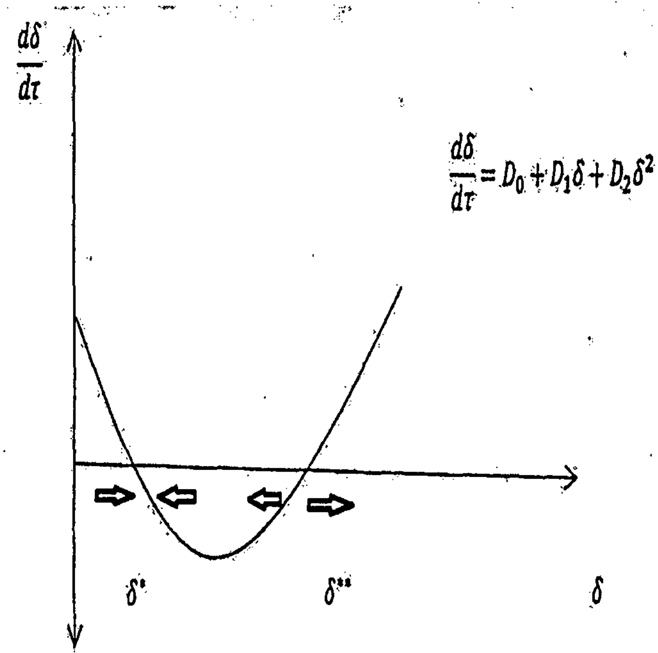

Figure-3

Chapter 4

Conclusion

Following a neo-Kaleckian framework we have tried to analyze the impact of expansionary fiscal policy on the growth rate. In the short-run, we have found that an increase in the government consumption expenditure increases the aggregate demand, equilibrium level of capacity utilization and the equilibrium growth rate.

Unlike Dutt (2013) a switch in government expenditure from consumption to investment purposes does not always lead to a rise in the equilibrium degree of capacity utilization and the equilibrium growth rate. As a rise in θ represents simply a switch in government expenditure from consumption to investment purposes, it does not increase aggregate demand and capacity utilization directly. It may raise the aggregate demand through its indirect 'crowding in' effect . On the other hand public investment expenditure through its effect on labour productivity can lead to a rise in share of profit in the economy which in turn decreases[1] aggregate demand and the degree of capacity utilization. Thus the final outcome of a rise in θ depends on the relative magnitudes of these opposing effects. In this case, although our findings are similar to Commendatore and Pinto (2011), the reason behind it is that unlike a change in capital productivity here it is a change in labour productivity that influences the equilibrium level of capacity utilization which in turn has an impact on the equillibrium growth rate.

When the balanced budget assumption is dropped, an increase in government debt-capital ratio leads to a decrease in the equilibrium level of capacity utilization and the equilibrium growth rate. This is in contrast to the analysis by Dutt (2013), where a rise in the government debt-capital ratio has an ambiguous effect on the equilibrium levels of capacity utilization and the accumulation rate. We also find

[1] when the economy is in stagnationist regime

that a rise in the current government consumption expenditure to output ratio raises the aggregate demand, capacity utilization and the accumulation rate. But a rise in θ has an ambiguous effect on both the equilibrium level of capacity utilization and the accumulation rate. This result differs from Dutt (2013) as there is a positive relation between θ and u^* and θ and g^* in his analysis. This is because a rise in public investment expenditure through its effect on labour productivity leads to a rise in share of profit which in turn mitigates the positive effect of a rise in θ on u^* and g^*.

We also have seen that a change in any model parameter, which increases the equilibrium degree of capacity utilization without affecting labour productivity, necessarily increases the employment rate. But the effect of a rise in θ on the employment rate is ambiguous and it depends on the elasticity of the degree of capacity utilization and the elasticity of labour productivity with respect to the ratio of government investment to output.

Following Blecker (2002) we have shown that when workers also save, the possibility of an exhilarationist regime arises. When the economy is in exhilarationist regime, an increase in θ unambiguously raises both the equilibrium level of capacity utilization and the equilibrium growth rate. But if the economy is in the stagnationist regime, the result is similar to that when the workers' saving propensity is zero.

We have also seen that in the long run, a stable government debt-capital ratio is possible, provided that the interest rate is smaller than a critical value.

It should be noted that the results of our analysis are based on a very simple model. We have taken a homogenous tax rate for different classes in the economy. Introduction of different tax rates may change the results. Further, our model is based on the closed economy assumption. Introduction of an open economy framework may significantly change our findings.

In the long-run, we only have considered the dynamics of the government debt-capital ratio and the capital stock. If instead of assuming constant level of labour supply, the profit share and the technological growth, we allow these to vary in the long-run, then the analysis will be more interesting and the results may vary. Hope later on we might incorporate those issues and try to make the analysis more robust.

Chapter 5

Appendix

A

When workers also save a fraction 's_W' of their income, then the saving function of the economy can be written as

$$\frac{S}{K} = [s_P\pi + s_W(1-\pi)]u \tag{5.1}$$

where s_P and s_W are saving propensity of capitalists and workers respectively.

Blecker's investment function which is meant to capture the Bhaduri and Marglin's critique of the Kalecki-Steindl type of investment function is as follows :

$$\frac{I}{K} = g_0 + g_1\pi\nu + g_2 u \tag{5.2}$$

where g_0 represents the autonomous part of the investment function, $g_1$and g_2 are coefficients indicating the responsiveness of the investment-capital ratio to changes in the share of profit and capacity utilization respectively and ν is the 'full-capacity' output capital ratio which is fixed.

If we take 'Kalecki-Steindl' type of investment function then the investment-capital ratio can be represented by,

$$\frac{I}{K} = g_0 + g_1\pi u + g_2 u \tag{5.3}$$

Then in equilibrium,

$$\frac{I}{K} = \frac{S}{K}$$

$$\Rightarrow g_0 + g_1\pi u + g_2 u = (s_P - s_W)\pi u + s_W u$$

$$\Rightarrow u^* = \frac{g_0}{\{(s_P - s_W) - g_1\}\pi + s_W - g_2} \tag{5.4}$$

The equilibrium is stable if the induced increase in saving as u rises is greater than the induced increase in investment. So for the 'Kalecki-Steindl' type of investment function the equilibrium is stable when

$$(s_P - s_W)\pi + s_W > g_1\pi + g_2 \tag{5.5}$$

The equilibrium value of the accumulation rate can be obtained by substituting the equilibrium level of u^* in equation (5.3) as

$$g^* = g_0 + g_1\pi u^* + g_2 u^* \tag{5.6}$$

Now let's focus on the conditions for existence of the stagnationist and the exhilarationist regimes. The economy is in a stagnationist regime if $\frac{du^*}{d\pi} < 0$ and vice-versa.

Differentiating equation (5.4) with respect toπ we get,

$$\frac{du^*}{d\pi} = \frac{-g_0(s_P - s_W - g_1)}{[\{(s_P - s_W) - g_1\}\pi + s_W - g_2]^2}$$

$$\text{Thus } \frac{du^*}{d\pi} \gtreqless 0 \text{ according to whether } s_W \gtreqless (s_P - g_1)$$

Now let's check the condition for the profit-led growth in the stagnationist regime. Differentiating equation (5.6) with respect to π we get,

$$\frac{dg^*}{d\pi} = g_1 u^* + (g_1\pi + g_2)\frac{du^*}{d\pi}$$

As in the stagnationist regime $\frac{du^*}{d\pi} < 0$,

$$\frac{dg^*}{d\pi} \gtreqless 0 \text{ according to whether } \left|\frac{du^*}{d\pi}\right| \lesseqgtr \frac{g_1 u^*}{g_1\pi + g_2}$$

So, even if the economy is in stagnationist regime, growth can be profit-led depending on whether $\left|\frac{du^*}{d\pi}\right| < \frac{g_1 u^*}{g_1\pi + g_2}$.

We know, the equilibrium rate of profit is

$$r^* = \pi u^* \tag{5.7}$$

Differentiating the equation (5.7) with respect to π we get,

$$\frac{dr^*}{d\pi} = u^* + \pi\frac{du^*}{d\pi}$$

$$= \frac{(s_W - g_2)g_0}{[\{(s_P - s_W) - g_1\}\pi + s_W - g_2]^2}$$

The economy is in conflictive-stagnationist if $\frac{dr^*}{d\pi} > 0$ and it is in cooperative-stagnationist if $\frac{dr^*}{d\pi} < 0$.

$$\text{Thus } \frac{dr^*}{d\pi} \gtrless 0 \text{ according to whether } s_W \gtrless g_2.$$

Thus we have seen that even using the 'Kalecki-Steindl' type of investment function, we can achieve the same result as Blecker. Only the conditions regarding the stability, exhilarationist regime, profit-led and wage-led growth, cooperative and conflictive-stagnationist regime changes.

Bibliography

[1] Barro, R. and Sala-i-Martin, X. (2004), *Economic Growth*, Prentice-Hall of India, New Delhi

[2] Bhaduri, A. and Marglin, S.A.(1990), "Unemployment and the Real Wage: the Economic Basis for Contesting Political Ideologies", *Cambridge Journal of Economics*, Vol. 14, No. 4, pp. 375-393

[3] Blecker, R.A. (2002), "Distribution, Demand and Growth in neo-Kaleckian Macro Models", in Settereld, M. (ed.) (2002), *The Economics of Demand- led Growth: Challenging the Supply Side Vision of the Long run,* Edward Elgar, Cheltenham, UK

[4] Carter, S. (2007), " Real wage productivity elasticity across advanced economies, 1963-1996", *Journal of Post Keynesian Economics*, Vol. 29, No. 4 , pp. 573-600

[5] Commendatore, P. and Pinto A.(2011), " Public expenditure composition and growth: a neo-Kaleckian analysis", *Papers in Political Economy*, L'Harmattan, Vol. 61, pp. 187-222.

[6] Dutt, A.K. (1984), "Stagnation, Income Distribution and Monopoly Power", *Cambridge Journal of Economics*, Vol. 8, No. 1, pp. 25-40

[7] Dutt, A.K. (1987), "Alternative Closures Again: A Comment on Growth, Distribution and In ation", *Cambridge Journal of Economics*, Vol. 11, No. 1, pp. 75-82

[8] Dutt, A.K. (1990), *Growth, Distribution and Uneven Development*, Cambridge University Press, Cambridge.

[9] Dutt, A. K. (2006), "Maturity, stagnation and consumer debt: a steindlian approach", *Metroeconomica*, Vol. 57, No. 3, pp. 339–364.

[10] Dutt, A. K. (2009), "Path Dependence, Equilibrium and Economic Growth" , in Arestis, P. and Sawyer, M. (ed.) (2009), *Path Dependency and Macroeconomics,*Palgrave Macmillan, UK

[11] Dutt, A. K. (2010), "Keynesian Growth Theory in the 21st Century" , in Arestis, P. and Sawyer, M. (ed.) (2010), *21st Century Keynesian Economics,*Palgrave Macmillan, UK

[12] Dutt, A. K. (2011), "Economic Growth and Income Distribution: Kalecki, the Kaleckians and Their Critics", in Arestis, P. (ed.) (2011), *Microeconomics, Macroeconomics and Economic Policy: Essays in Honour of Malcolm Sawyer,* Palgrave Macmillan, UK

[13] Dutt, A.K. (2013), "Government spending, aggregate demand, and economic growth", *Review of Keynesian Economics*, Vol. 1, No. 1, pp. 105-119

[14] Kaldor, N. (1956), "Alternative Theories of Distribution", *Review of Economic Studies*, Vol. 23, No. 2, pp. 83-100

[15] Kaldor, N. (1957) 'A model of economic growth', *Economic Journal*, 67 (December), 591–624.

[16] Kalecki, M. (1971), *Selected Essays on the Dynamics of the Capitalist Economy*, Cambridge: Cambridge University Press

[17] Lavoie, M. (1995), "The Kaleckian Model of Growth and Distribution and its neo-Ricardian and neo-Marxian Critiques", *Cambridge Journal of Economics*, Vol. 19, No. 6, pp. 789-818

[18] Pasinetti, L. (1962), "Rate of Prot and Income Distribution in Relation to the Rate of Growth", *Review of Economic Studies*, Vol. 29, No. 4, pp. 267-279

[19] Robinson, J. (1956), *The Accumulation of Capital*, Macmillan, London

[20] Robinson, J. (1962), *Essays in the Theory of Growth*, Macmillan, London

[21] Rowthorn, R.E. (1981), "Demand, Real Wages and Economic Growth", *Thames Papers in Political Economy*, 1981, pp. 1-39

[22] Sharpe, A., J. Arsenault and P. Harrison (2008a), "The Relationship between Productivity and Real Wage Growth in Canada and OECD Countries: 1961-2006," *CSLS Research Report 2008-8*, December

[23] Sharpe, A., J. Arsenault and P. Harrison (2008b), "Why Have Real Wages Lagged Labour Productivity Growth in Canada? ", *International Productivity Monitor*, No. 17, Fall 2008

[24] Steindl, J. (1952): *Maturity and Stagnation in American Capitalism*, New York: Monthly Review Press

[25] Taylor, L. (1983), *Structural Macroeconomics*, New York: Basic Books.

[26] Taylor, L. (1985), "A Stagnationist Model of Economic Growth", *Cambridge Journal of Economics*, Vol. 9, No. 4, pp. 383-403

[27] Taylor, L. (1991), *Income Distribution, Inflation, and Growth: Lectures on Structuralist Macroeconomic Theory* ,Cambridge, MA: MIT Press, USA.

[28] You, J.-II, Dutt, A. (1996), "Government debt, income distribution and growth", *Cambridge Journal of Economics*, Vol. 20, No. 3, pp. 335–351.